Does E-Commerce Ease or Intensify Tax Competition?
Destination Principle vs. Origin Principle

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Abstract
By constructing a commodity tax competition model with product differentiation, this paper studies the relationship between the development of e-commerce and the intensity of tax competition under two different tax principles to be applied to e-commerce: the destination principle and the origin principle. Our main findings are as follows: (i) tax competition between two symmetric countries under the destination principle is more intense than tax competition under the origin principle, and (ii) the development of e-commerce raises the tax rate under the origin principle, but lowers it under the destination principle. An analysis of tax competition among asymmetric countries was also conducted. The finding was that in some cases, the development of e-commerce has changed the tax rate set by large and small countries in opposite directions.

Keywords: Tax competition; product differentiation; e-commerce; origin principle; destination principle

JEL Classification Codes: H21; H71; H87
1 Introduction

Analyzing the tax on e-commerce is challenging from at least three perspectives. First, the design of taxation on e-commerce will play an important role in securing stable tax revenue for countries around the world. The e-commerce market continues to expand, and in the presence of various indicators, many predict that the size of the global retail e-commerce market will grow more than five-fold in less than a decade, approaching around $10 trillion by the mid-2020s. Effective taxation of this massive ever-expanding tax base will greatly contribute to securing tax revenue for individual countries and will expand the consumption tax’s role as a core tax. Conversely, if countries build tax systems that allow tax evasion or the avoidance of tax payments for e-commerce, or induce tax reductions among countries, they will suffer significant tax revenue losses. Second, compared to taxation on goods purchased in brick-and-mortar stores, the negative effects of each country applying its own taxation rules to e-commerce will be more serious. This is because the purchase of goods online allows transactions to take place over a much larger spatial area and involves far more cross-border transactions than buying goods in brick-and-mortar stores. E-commerce enables consumption on a global scale; therefore, the number of countries involved in a strategic tax policy relationship is also increasing. This is why there is growing interest in digital taxation, including the taxation of goods purchased online (Organisation for Economic Cooperation and Development [OECD], 2015). Third, the issue of taxing e-commerce poses a new challenge to the standard view of taxation because it adds the option to consume through a new means, i.e., online, to the traditional tax theory, which assumes that goods are purchased only in brick-and-mortar stores. The classic taxation principles may also be forced to change as firms and countries are forced to compete in electronic space, in addition to competing in real space.

The development of e-commerce and the associated change in the role of consumption tax have led researchers to study taxation on e-commerce, for example, Bacache-Beauvallet (2018), Birg (2019), and Agrawal and Wildasin (2020). This study also intends to present new theoretical hypotheses in this field. Specifically, the purpose of our study is to provide one possible answer to the question of whether the imposition of taxes on e-commerce should be based on the origin principle or the destination principle. Our theoretical hypothesis is that taxing e-commerce under the origin principle rather than the destination principle will be superior. That is, commodity taxation should be based on the origin principle, regardless of whether the purchase is made in a brick-and-mortar store or online. This result is derived under a model that allows for the purchase of multiple goods, product differentiation, and price setting by firms under imperfect competition, which is, in some respects, a more generalized version of some of the previous studies mentioned above.

Many classic studies that do not address e-commerce have analyzed the choice between the origin and destination principles. There is a general consensus that the destination-based principle is superior to the origin-based principle when assuming a competitive market (Keen and Lahiri, 1998, p.325). In the case of taxation under the destination principle, the post-tax price is the same in all countries, regardless of where consumption and production activities take place; therefore, production efficiency is not impaired. In the case of taxation under the origin principle, however, economic agents’ post-tax price differs depending on the country in which they operate, which distorts the allocation of resources. Subsequent studies have broadened the scope of analysis to include imperfectly competitive markets and markets with factors such as trade costs, spillovers, and unemployment. Some of those studies have confirmed that the superiority of the destination principle still holds, while others have indicated the possibility of a counterview (Keen and Lahiri, 1998; Lockwood et al., 1995; Lockwood, 2001; Haufler and Pfüger, 2004; Haufler et al., 2005; Hashimzade et al., 2011; Antoniou et al., 2016, 2019; Agrawal and Mardan, 2019). Our paper’s contribution to these studies is to extend the model to include online purchases of goods, but such an analysis will be inevitably complicated by the addition of new purchase options to these models in which consumers only purchase goods in brick-and-mortar stores. In order to avoid complexity and present our main findings with analytical solutions, we focus on symmetric equilibrium for the majority of the paper. The findings when asymmetry is included are presented in the Discussion section.

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1 For example, in Mexico and Canada, which are adjacent to the United States, about 65% of e-commerce consumers have purchased goods online from abroad, and only 35% use domestic e-commerce (Paypal, 2019). This number is much higher for some Western European countries. In Ireland and Austria, for example, more than 80% of e-commerce consumers buy goods online from abroad. Even in large countries such as the United Kingdom, Germany, France, and the United States, more than 30% of consumers who buy goods online have purchased them from outside their home country.

2 See Lockwood et al. (1994a, 1994b) and Genser (1996) for studies that identify the conditions under which the two tax principles could be equivalent.
There are three studies that are closely related to our analysis of the taxation principles as applied to e-commerce. The first is Bacache-Beauvallet’s (2018), which develops a model of taxation on e-commerce in which firms are competitive, and a certain percentage of consumers use the Internet to buy one unit of goods, while the remainder buy goods in brick-and-mortar stores. She compares tax revenue under the origin and destination principles to show that large countries always prefer the destination principle for e-commerce, whereas small countries prefer the origin principle. The second study, Birg’s (2019), finds that when the cost of buying online is low (high), the welfare of the country in which the online retailer is located is higher under the origin (destination) principle, while the welfare of the other country is higher under the destination (origin) principle. Additionally, total welfare is higher when taxes on online shopping are levied according to the destination principle. The third study is Agrawal and Wildasin’s (2020), which extends the commodity tax competition models to include remote commerce, assuming that e-commerce is taxed at the destination, while purchases of goods in brick-and-mortar stores are subject to tax based on the origin principle. They show that as the cost of e-commerce declines, tax rates decrease in the core region where essential goods that can be purchased online are produced, but increase in peripheral ones, thus reducing tax differentials.3 On the one hand, all of these studies share the same objective as our study, which is to elucidate equilibrium tax rates for this new means of conducting transactions called e-commerce. However, there are at least two important differences between these papers and ours, allowing us to present different theoretical hypotheses. First, their studies assume that consumers purchase a unit of goods inelastically. That is, there is no change in the quantity of goods consumers purchase, even if prices or tax rates change. While this might be an acceptable assumption for certain goods, it would be difficult to apply to goods and services in online retail markets. This assumption also blocks the possibility that consumers will change the quantity of goods they purchase in response to changes in taxation principles; hence, the impact of this path on equilibrium may be overlooked. We generalize the demand structure using a model in which demand is elastic in response to changes in prices and tax rates. Second, in previous studies, analysis has been conducted assuming competitive firms to discard firms’ behavior. Considering the tax theory development process so far, explicit depiction of firms’ response to changes in tax rates and tax principles is expected to affect the equilibrium solution. We build a model that assumes an international oligopoly of firms producing differentiated goods and analyze firms’ pricing decisions endogenously.

Assuming that goods purchased in brick-and-mortar stores are taxed under the origin principle, the main conclusions from a more general analysis, with the addition of the new element of shopping online and taxing online goods according to two different principles, are as follows. First, tax competition will be less intense if an origin-based tax is applied to goods purchased online than if a destination-based tax is applied. As Agrawal and Fox (2017) have noted, there is a trend toward the destination-based taxation of goods purchased online. Our results highlight the inherent challenges in this trend. Second, the expansion of the online market, expressed in lower online purchase costs, increases the equilibrium tax rate under origin-based tax competition and decreases the equilibrium tax rate under destination-based tax competition. This suggests that the expansion of the online market intensifies destination-based tax competition, while easing origin-based tax competition. The main reason for this is that while taxation under the destination principle reduces excessive competition for cross-border customers, it creates new incentives for the government to increase the domestic tax base by shifting the way domestic consumers make purchases from in brick-and-mortar stores to online, which generates negative fiscal externalities for other countries.

This paper is organized as follows. Section 2 introduces the model setting. In Sections 3 and 4, we derive the symmetric equilibrium of tax competition when taxes are imposed under the origin and destination principles, respectively. By comparing the results in these two sections, we show the superiority of origin-based taxation on e-commerce. In Section 5, we present additional conclusions obtained under the assumption of two countries with different population sizes. Section 6 offers the conclusions from this study.

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3 Agrawal (2021) empirically clarifies the relationship between the development of online consumption and tax rates and provides evidence that higher Internet penetration generally results in lower local tax rates, but raises tax rates in some jurisdictions.
2 Model

2.1 Basic setup

Consumers are uniformly distributed in the line space of \( x \in [-1, 1] \), and there are independent countries on the left and right sides of the space, bordered by point \( b \in [0, 1] \). Country 0 is located to the left of the border, and country 1 is located on the right. Based on the assumption of the range of \( b \), country 0 is a large country with a large population, and country 1 is a small country. If \( b = 0 \), then the two countries are symmetric.

Firms in countries 0 and 1 produce goods 0 and 1, respectively, but the two goods are differentiated. When consumers purchase two differentiated goods, they can choose to purchase each good either by visiting a store or by purchasing it online. Each country imposes a unit tax on goods. Tax per unit of goods imposed by country \( i \) is denoted by \( t_i (i = 0, 1) \). Let \( s_i \) denote cost per unit of the good, excluding the cost that a consumer incurs when they purchase a good from country \( i \), which is explained below: it depends on whether the consumer purchases the good in a store or online.

**Purchasing goods in stores.** When residents of country \( i \) purchase goods produced in country \( i \) (their own country) from stores, the per unit purchase cost is \( s_i = t_i \). When residents of country \( i \) purchase goods produced in country \( j (\neq i) \) (another country) from stores, the per unit purchase cost is \( s_j = t_j + \delta d \). Since the in-store purchase of goods is taxed based on the origin principle, the consumer pays the tax imposed by the country where the goods are purchased. Here, \( d \) represents the distance from the consumer’s point of residence to the border. We assume that a consumer in country \( i \) can purchase goods in country \( j \) by traveling to the border (Bacache-Beauvallet, 2018). \( \delta > 0 \) denotes the travel cost per distance. There is no travel cost when a consumer purchases a good from a store in their home country.

**Purchasing goods online.** When goods are purchased online, governments impose a tax on the goods based on either the origin principle or the destination principle. If a consumer in country \( i \) purchases a good in country \( j (\neq i) \) online, the per unit cost of purchasing the good is \( s_j = t_j + e \) if it is taxed under the origin principle, where \( e (> 0) \) is the cost associated with purchasing the good online. When purchasing goods online, the cost of traveling to a store is not a factor. Instead, consumers bear certain costs for shipping and Internet equipment, or costs associated with the risk of purchasing goods that are different from those they had imagined because they do not see the actual goods in stores before purchasing them online. These are represented by cost \( e \), which is independent of the distance. When goods are taxed under the destination principle, regardless of where the goods are purchased, the tax rate in the place of residence applies, so consumers in country \( i \) bear the cost of \( s_i = s_j = t_i + e \), in addition to the price, regardless of the country from which they purchase the goods.

The consumer purchases the good either in a brick-and-mortar store or online, whichever has the lower purchase cost. When taxed under the principle of destination, the cost of purchasing a good in the home country is higher when purchasing it online than when purchasing it in a store.\(^4\) Therefore, consumers will always buy from brick-and-mortar stores when purchasing home country goods and are only likely to buy goods online if the goods are from another country. For a consumer residing in location \( x \in [-1, 1] \), the classification of purchase methods and the cost of purchasing goods in each case are summarized in Figures 1a and 1b.

Let us consider Figure 1a. Consumers living in country 0, which is located to the left of the border \( b \), buy good 0 in brick-and-mortar stores in country 0 and pay taxes to country 0 under the origin principle. Among them, consumers residing closer to the border purchase good 1 at a brick-and-mortar store in country 1, while consumers residing farther from the border (closer to \( x = -1 \)) shop online. Under the origin principle of taxation, they pay tax to country 1 when they purchase good 1. The purchase patterns of consumers living to the right of the border can be interpreted in the same way. Figure 1b shows a case in which governments tax e-commerce according to the destination principle. There is no significant difference in the purchase pattern compared to Figure 1a, but it should be noted that consumers residing far from the border pay taxes to their country of residence when they purchase goods online. There are two key differences that arise under the two taxation principles. The first is that consumers who purchase goods online pay taxes in other countries under the origin principle, but they pay taxes in their home country under the destination principle. The second is that the critical point that divides

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\(^4\)Recall that when consumers make purchases in their home country, there is no burden other than tax on in-store purchases, but for online purchases, there is the cost \( e \), in addition to the tax.
online and in-store purchases is affected by tax rates when taxed under the destination principle (Figure 1b), but not when taxed under the origin principle (Figure 1a). On the one hand, with taxation under the origin principle, consumers pay taxes to the country where the goods are produced, regardless of the purchase method, so the location of the critical point for determining the purchase method is not affected by the tax rate. However, in the case of taxation under the destination principle, the tax rate will affect consumers’ choice of purchase method, since the tax rate will be different for the same good when purchased online versus in a brick-and-mortar store.

Based on the classification according to the two taxation principles shown in Figures 1a and 1b, it can be intuitively gleaned that under the destination principle, there is an incentive for the government to manipulate tax rates, so that consumers in the home country purchase goods from other countries online as much as possible. There is no such incentive when taxing under the origin principle, since changing the tax rate will not influence consumers’ choice of purchase method.

### 2.2 Consumers and firms

The consumer’s utility function is assumed to be as follows:

\[
U(q_0, q_1, X) = \frac{\alpha}{1 - \gamma} (q_0 + q_1) - \frac{1}{2(1 + \gamma)} (q_0 + q_1)^2 + X, \tag{1}
\]

where \(q_0\) and \(q_1\) are differentiated goods produced in countries 0 and 1, and \(X\) is a numeraire good. \(\alpha\) and \(\gamma\) are the parameters, and the former is assumed to be sufficiently large to ensure \(q_i > 0\), as described below. \(\gamma\) represents the substitutability between the two goods and is assumed to take values between 0 and 1 to ensure that the utility function is concave. If the price of each good is \(p_i\), and income is \(y\), the budget constraint faced by a consumer residing at point \(x\) is given by:

\[
y = [p_0 + s_0(x)]q_0 + [p_1 + s_1(x)]q_1 + X. \tag{2}
\]

The consumer chooses the amount of consumption of the good that maximizes (1), with (2) as the constraint. The demand function for good \(i\) is as follows:
\[ q_i(x) = \alpha - [p_i + s_i(x)] + \gamma[p_j + s_j(x)]. \tag{3} \]

The profit of a firm in country \( i \) can be expressed as follows:

\[ \pi_i = (p_i - c) \int_{-1}^{1} q_i(x) dx, \tag{4} \]

where \( c \) is the marginal cost of production.

### 2.3 Governments

We assume that each government aims to maximize its tax revenue using either the origin or destination principle of taxation. The tax revenue of country \( i \) under the origin principle is as follows:

\[ R_i^O = t_i \int_{-1}^{1} q_i(x) dx, \tag{5} \]

where \( O \) in the superscript represents the value when taxed according to the origin principle.

When goods are taxed under the destination principle, tax revenue in country 0 and country 1, respectively, is expressed as follows:

\[ R_0^D = t_0 \left( \int_{-1}^{\text{Mdn} \left( b, b+\frac{s_0(t_0-t_1)}{2}\right)} q_0(x) dx + \int_{-1}^{\text{Mdn} \left( -1, b-\frac{s_0(t_0-t_1)}{2}\right)} q_1(x) dx \right), \tag{6} \]

\[ R_1^D = t_1 \left( \int_{\text{Mdn} \left( b, b+\frac{s_0(t_0-t_1)}{2}\right)}^{1} q_0(x) dx + \int_{\text{Mdn} \left( -1, 1-\frac{s_0(t_0-t_1)}{2}\right)} q_1(x) dx \right), \tag{7} \]

where where \( D \) in the superscript represents the value when taxed according to the destination principle and Mdn is the abbreviation for median.

### 3 Symmetric equilibrium under the origin principle of taxation

We begin by describing the symmetric model, that is, \( b = 0 \), in its simplest form, deferring discussion of equilibrium between asymmetric countries until later. This enables us to focus on the efficiency of tax competition in the context of online consumption under two different tax principles by eliminating the effects of disparities between countries.

The decision making takes place according to the following sequence. At the first stage, each government chooses the tax rate that maximizes its own tax revenue, given the tax principles and other countries’ tax rates. At the second stage, firms choose the price that maximizes their profit. At the third stage, consumers choose the quantity and method of purchasing the two goods in order to maximize their own utility. Using the concept of backward induction, we derive the subgame perfect Nash equilibrium (SPNE) by solving from the decision-making of the third stage.

#### 3.1 Third stage

The consumer’s choice at the third stage is shown in Figure 1a. The consumer demand function is divided into four categories according to consumers’ place of residence. In Figure 1a, the space \([-1,1]\) is divided into four sections, and the following Cases 1 through 4 correspond to each section in the figure, with \( b = 0 \).

**Case 1.** \(-1 < x < \text{Mdn}[-1, -e/\delta, 0]\). A consumer residing in \( x \) buys good 0 from a store in his home country and purchases good 1 online. Cost of purchase, other than the price, is \( t_0^O \) for the former and \( e + t_1^O \) for the latter. The demand function for each good can be obtained from (3) as follows:

\[ q_0^O(x) = \alpha - (p_0^O + t_0^O) + \gamma(p_1^O + t_1^O + e) \quad \text{and} \quad q_1^O(x) = \alpha - (p_1^O + t_1^O + e) + \gamma(p_0^O + t_0^O). \]
Case 2. Mdn[−1,−c/δ,0] ≤ x < 0. The consumer buys two goods in a store and does not make any online purchases. The purchase costs other than price are \( t_0^O \) for good 0 and \(-\delta x + t_1^O\) for good 1. The first term in the latter term represents the cost of moving from the consumer’s place of residence \( x \) to the border, \( b = 0 \). The demand function for each good is:

\[
q_i^0(x) = \alpha - (p_i^0 + t_0^O) + \gamma(p_i^0 + t_1^O - \delta x) \quad \text{and} \quad q_i^1(x) = \alpha - (p_i^1 + t_1^O - \delta x) + \gamma(p_i^0 + t_0^O + \delta x).
\]

Case 3. 0 ≤ x < Mdn[0, c/δ, 1]. A consumer living at a location corresponding to this case purchases both goods at a store. The purchase cost, excluding the price, is \( \delta x + t_0^O \) for good 0 and \( t_1^O \) for good 1. Since this consumer travels from country 1 to the border to purchase good 0, he bears the cost of travel, in addition to the tax in country 0 when purchasing good 0. The demand function for each good can be obtained as follows:

\[
q_i^0(x) = \alpha - (p_i^0 + t_0^O + \delta x) + \gamma(p_i^0 + t_1^O) \quad \text{and} \quad q_i^1(x) = \alpha - (p_i^1 + t_1^O) + \gamma(p_i^0 + t_0^O + \delta x).
\]

Case 4. Mdn[0, c/δ, 1] ≤ x < 1. The consumers buy good 0 online and good 1 at a store in their home country. Excluding price, the cost of purchasing good 0 is \( \epsilon + t_0^O \); for good 1, it is \( t_1^O \). The demand function for each good can be obtained as:

\[
q_i^0(x) = \alpha - (p_i^0 + t_0^O + \epsilon) + \gamma(p_i^0 + t_1^O) \quad \text{and} \quad q_i^1(x) = \alpha - (p_i^1 + t_1^O) + \gamma(p_i^0 + t_0^O + \epsilon).
\]

To validate Cases 1 through 4, we make the following assumption:

**Assumption 1.** \( \epsilon < \delta \).

If this assumption does not hold, the cost of purchasing goods online is too high, and consumers who purchase goods online will disappear from the economy.

### 3.2 Second stage

Inserting the demand function in Cases 1 through 4 into (4), the profit that firms in country \( i \) try to maximize at the second stage under Assumption 1 is given by:

\[
\pi_i^O = 2(p_i^O - c) \left \{ \alpha - \left [ p_i^0 + t_i^O + \frac{2\delta - \epsilon}{4\delta} e \right ] + \gamma \left [ p_i^0 + t_i^O + \frac{2\delta - \epsilon}{4\delta} e \right ] \right \}.
\]

Profit-maximization gives the price levels:

\[
p_i^{O*} = \frac{\alpha + c}{2 - \gamma} - \frac{2 - \gamma^2}{4 - \gamma^2} \left [ t_i^0 + \frac{2\delta - \epsilon}{4\delta} e \right ] + \frac{\gamma}{4 - \gamma^2} \left [ t_i^0 + \frac{2\delta - \epsilon}{4\delta} e \right ].
\]  

(8)

where superscript * denotes the equilibrium at each stage. From (8), we have

\[
\frac{\partial p_i^{O*}}{\partial t_i^O} = -\frac{2 - \gamma^2}{4 - \gamma^2} < 0 \quad \text{and} \quad \frac{\partial p_i^{O*}}{\partial t_j^O} = \frac{\gamma}{4 - \gamma^2} > 0.
\]

(9)

Behind the sign in (9), there are two effects of a change in the tax rate: a direct change in demand for goods and substitution of demand. Lower demand for good \( i \) from consumers in both countries due to higher tax in country \( i \) will result in a lower price \( p_i^{O*} \) set in country \( i \). The first equation in (9) shows this direct effect. In addition to this effect, a high tax rate in country \( i \) will cause a shift in demand from good \( i \) to good \( j \), which increases the price of the goods produced in country \( j \). The second equation in (9) illustrates this substitution effect. If there is no substitution relationship between the two goods (\( \gamma = 0 \)), then when \( t_i \) rises, there is no substitution from good \( i \) to good \( j \); thus, price \( p_i \) does not change. If the two goods are perfectly substitutable (\( \gamma = 1 \)), then when \( t_j \) rises, there will be an increase in demand for good \( i \) that is the same size as the decrease in demand for good \( j \), resulting in an increase in price \( p_i \) that is equal to the decrease in price \( p_j \) in absolute value, that is, \( |\partial q_i^{O*}/\partial t_i^O| = |\partial q_j^{O*}/\partial t_j^O| \) if \( \gamma = 1 \).
3.3 First stage

Substituting (8) into (5), the government’s objective function in country \( i \) is obtained as follows:

\[
R_O^i = 2^{4^{2^{2^{2^{e^{4^{4^{e}}}}}}}} \left\{ (2 + \gamma)A - (2 - \gamma^2) \left[ t_O^i + \frac{2\delta - e}{4\delta} \right] + \gamma \left[ t_O^j + \frac{2\delta - e}{4\delta} \right] \right\} t_O^i,
\]

where \( A \equiv \alpha - e(1 - \gamma) \). The first-order condition for maximizing \( R_O^i \) with respect to \( t_O^i \) is:

\[
\frac{\partial R_O^i}{\partial t_O^i} = 2^{4^{2^{2^{2^{e^{4^{4^{e}}}}}}}} \left\{ (2 + \gamma)A - (2 - \gamma^2)t_O^i + \gamma t_O^j - (2 - \gamma - \gamma^2) \frac{2\delta - e}{4\delta} \right\} - \frac{2(2 - \gamma^2)}{4 - \gamma^2} t_O^i = 0, \tag{10}
\]

which represents the two conflicting effects of increasing \( t_i \) on tax revenue. The first term in (10) represents an increase in tax revenue per tax base, and the second term represents a decrease in tax revenue due to a contraction of the tax base, caused by a decrease in demand. Here, we derive the effect of changes in online purchase cost \( e \) on marginal revenue to connect to the discussion in the next section.

Now, let us express the progress of e-commerce as a decline in \( e \) (Agrawal and Wildasin, 2020). From (12), we can show the impact of the development of online purchasing on the equilibrium tax rate in the context of tax competition under the origin principle of taxation analytically as follows:

\[
R_O^i = \frac{1 - \gamma}{2 - \gamma} \left( 1 - \frac{e}{\delta} \right) < 0. \tag{11}
\]

The sign is from Assumption 1. (11) implies that the government will raise the tax rate when the online cost decreases, since a decrease in online cost will increase marginal tax revenue. This property can also be confirmed by explicitly solving for the tax rate and looking at its relationship with the cost of online purchases. Using (10) for \( i \) and \( j \), we can solve the tax rate of SPNE under the origin principle of taxation analytically as follows:

\[
t_{O^*}^i = \frac{(2 + \gamma)A}{4 - \gamma - 2\gamma^2} - \frac{(1 - \gamma)(2 + \gamma)e}{2(4 - \gamma - 2\gamma^2)} + \frac{(1 - \gamma)(2 + \gamma)}{4(4 - \gamma - 2\gamma^2)} \frac{e^2}{\delta}. \tag{12}
\]

Now, let us express the progress of e-commerce as a decline in \( e \) (Agrawal and Wildasin, 2020). From (12), we can show the impact of the development of online purchasing on the equilibrium tax rate in the context of tax competition under the origin principle as follows:

**Proposition 1.** If online purchases of goods are taxed under the origin principle, the tax rate in symmetric equilibrium rises as the cost of online consumption falls, \( \partial t_{O^*}^i / \partial e < 0 \).

**Proof.** Using (12), we have:

\[
\frac{\partial t_{O^*}^i}{\partial e} = -\frac{(1 - \gamma)(2 + \gamma)(\delta - e)}{2\delta(4 - \gamma - 2\gamma^2)} < 0. \tag{13}
\]

The sign is from Assumption 1. □

A decline in \( e \) can affect the tax rate through two channels. The first is through changes in the purchase method. In the following, this will be referred to as the purchase method effect. The decrease in \( e \) changes the critical point (indicated by the dotted line in Figure 1a) in the direction of more online buyers in both countries. For example, some consumers in country \( 1 \) who were buying goods \( 0 \) in brick-and-mortar stores in country \( 0 \) before the fall of \( e \) will now buy online. However, the change in purchase method does not bring about a change in the quantity purchased. Therefore, for the government of country \( 0 \), there is no change in the tax base, and the tax rate does not change through the path of change in the purchase method when \( e \) decreases.

The sign in equation (13) is due to the second path, which is given by the price effect. A decline in the cost of purchasing goods online increases demand from consumers who used to purchase goods online. In other words, the consumer in country \( 0(1) \) who purchased good \( 0(1) \) online will increase the quantity of such purchases. At the same time, they reduce their purchases of good \( 0(1) \) by \( \gamma \) units because demand substitution occurs. Thus, for net value, as \( e \) decreases, the demand for goods \( 0(1) \) increases by \( 1 - \gamma \), respectively. Since the tax base is larger, the additional tax revenue that each country gets when it raises \( t_i \) is larger, suggesting that \( t_i \) will be raised by lowering \( e \). When the two goods are
perfectly substitutable ($\gamma = 1$), however, the demand for the good (i.e., the tax base) does not change even if $e$ decreases. Therefore, when $\gamma = 1$, the tax rate does not change even if $e$ decreases.

The policy implications of Proposition 1 are thus clear. As the cost of e-commerce falls, and more goods are purchased online, the competition for lower taxes will be eased under the origin principle of taxation, except in the special case of the perfect substitution of two goods, $\gamma = 1$.

In our analysis as detailed above, we made the following assumption, which ensures the normal situation in which the consumption of goods in equilibrium is positive, $q_i > 0$, for any location and any $e$ under Assumption 1.

**Assumption 2.**

$$\alpha > \alpha \equiv (1 - \gamma)c + \frac{13 - 6\gamma - 8\gamma^2 + 2\gamma^3 + \frac{4}{2(2 - \gamma^2)}}{\epsilon} \iff A > \frac{13 - 6\gamma - 8\gamma^2 + 2\gamma^3 + \frac{4}{2(2 - \gamma^2)}}{\epsilon}. \quad (14)$$

(14) is also a condition for the demand for goods to be positive at all locations under the classic brick-and-mortar purchase model, with no online purchase option.

4 Symmetric equilibrium under the destination principle of taxation

4.1 Third stage

As in the previous section, consumer choices are organized into four categories based on the consumers’ place of residence. Each of the four cases below corresponds to the four segments of the [-1,1] space in Figure 1b.

**Case 1.** $-1 < x < \text{Mdn}[\bar{x}, -|e + (t_0^0 - t_1^0)|/\delta, 0]$. The consumers living in a location satisfying this case buys good 0 at their home store and purchases good 1 online. The purchase cost per unit is $t_0^0$ for the former and $e + t_0^0$ for the latter. The demand function for each good in this case is:

$$q_0^D(x) = \alpha - (p_0^0 + t_0^0) + \gamma(p_1^0 + t_1^0 + e) \quad \text{and} \quad q_1^D(x) = \alpha - (p_1^0 + t_1^0 + e) + \gamma(p_0^0 + t_0^0).$$

**Case 2.** Mdn$[-1, -|e + (t_0^0 - t_1^0)|/\delta, 0] < x < 0$. A consumer who satisfies this case purchases both goods at the store. Excluding price, the purchase costs of goods 0 and 1 are $t_0^0$ and $-\delta x + t_1^0$, respectively. Since this consumer travels to the border to purchase good 1, the cost of the latter includes the cost of travel, in addition to the tax paid to the government in country 1. The demand function can be obtained as follows:

$$q_0^D(x) = \alpha - (p_0^0 + t_0^0) + \gamma(p_1^0 + t_1^0 - \delta x) \quad \text{and} \quad q_1^D(x) = \alpha - (p_1^0 + t_1^0 - \delta x) + \gamma(p_0^0 + t_0^0).$$

**Case 3.** $0 < x < \text{Mdn}[0, |e - (t_0^0 - t_1^0)|/\delta, 1]$. A consumer residing at point $x$ purchases two goods at a store. The purchase cost of the goods, excluding the price, is $\delta x + t_0^0$ for good 0 and $t_1^0$ for good 1. The demand function for each good is given by the following equation:

$$q_0^D(x) = \alpha - (p_0^0 + t_0^0 + \delta x) + \gamma(p_1^0 + t_1^0) \quad \text{and} \quad q_1^D(x) = \alpha - (p_1^0 + t_1^0) + \gamma(p_0^0 + t_0^0 + \delta x).$$

**Case 4.** Mdn$[0, |e - (t_0^0 - t_1^0)|/\delta, 1] < x < 1$. In this case, the consumer purchases good 0 online and good 1 in a store. The purchase costs are $e + t_0^0$ and $t_1^0$, respectively. The demand function in this case can be given as:

$$q_0^D(x) = \alpha - (p_0^0 + t_0^0 + e) + \gamma(p_1^0 + t_1^0) \quad \text{and} \quad q_1^D(x) = \alpha - (p_1^0 + t_1^0) + \gamma(p_0^0 + t_0^0 + e).$$
4.2 Second stage

Substituting the demand function in Cases 1 to 4 into (4), under \(0 < |e - (t_{i}^{D} - t_{j}^{D})|/\delta < 1\), we can rewrite firms’ profits as:

\[
\pi^{D} = 2(p_{i}^{D} - c) \left\{ \alpha - \left[ p_{i}^{D} + t_{i}^{D} + \frac{2\delta - E_{i}}{4\delta} E_{i} \right] + \gamma \left[ p_{j}^{D} + t_{j}^{D} + \frac{2\delta - E_{j}}{4\delta} E_{j} \right] \right\},
\]

where \(E_{i} = e - (t_{i}^{D} - t_{j}^{D})\). Profit maximization gives the price levels as follows:

\[
p_{i}^{D*} = \frac{\alpha + c}{2 - \gamma} - \frac{2 - \gamma^2}{4 - \gamma^2} \left( t_{i}^{D} + \frac{2\delta - E_{i}}{4\delta} E_{i} \right) + \frac{\gamma}{4 - \gamma^2} \left( t_{j}^{D} + \frac{2\delta - E_{j}}{4\delta} E_{j} \right).
\]

(15)

Evaluating with \(t_{i}^{D} = t_{j}^{D}\), the effect of a change in the tax rate on price is given by:

\[
\frac{\partial p_{i}^{D*}}{\partial t_{i}^{D}} = -\frac{2 - \gamma^2}{4 - \gamma^2} + \frac{(2 - \gamma^2)(\delta - e)}{2\delta(4 - \gamma^2)} + \frac{\gamma(\delta - e)}{2\delta(4 - \gamma^2)} \quad \text{and} \quad \frac{\partial p_{j}^{D*}}{\partial t_{j}^{D}} = \frac{\gamma}{4 - \gamma^2} - \frac{(2 - \gamma^2)(\delta - e)}{2\delta(4 - \gamma^2)} - \frac{\gamma(\delta - e)}{2\delta(4 - \gamma^2)}.
\]

(16)

Comparing (16) with (9), we can see that the last two terms in the two equations in (16) reduce the impact of tax rate changes on prices. The details of these terms, which appear due to differences in taxation principles, will be explained below.

In the case of the destination principle, a change in the tax rate changes the quantity of goods purchased, which, in turn, changes the price. The impact through this pathway also occurs in the case of taxation under the origin principle. In addition, for the destination principle, a change in the tax rate causes a change in the purchase method; a change in the tax rate changes the critical point (indicated by the dotted line in Figure 1b). However, the purchase method effect does not change the price through this path because it only changes the purchase method, not the quantity purchased. Hence, a distinctive feature of the impact of tax rate changes under the destination principle is the difference in the content of the effect through a change in demand. When \(t_{i}\) increases, as in the case of taxation under the origin principle, there is a decrease in demand for good \(i\); thus, \(p_{i}\) falls. This is shown in the first term of the first equation in (16). However, under destination-based taxation, consumers in country \(j\) who purchase goods in country \(i\) online will not reduce the quantity of goods \(i\) purchased. This is because they are subject to the tax imposed by country \(j\) and are not affected by the tax rate of country \(i\). Therefore, the decrease in price \(p_{i}\) when \(t_{i}\) increases will be sufficiently small that they do not reduce their purchases. This is shown in the second term of the first equation in (16). An increase in \(t_{i}\) also reduces the amount of goods that consumers in country \(i\) purchase online in country \(j\), which causes the substitution of goods \(j\) with goods \(i\). This triggers a decline in price \(p_{i}\) when \(t_{i}\) decreases, which is shown by the third term of the first equation in (16). If there is no substitution relationship between the two goods (\(\gamma = 0\)), there will be no impact on prices due to the substitution of goods, and the third term will therefore be zero.

The effect of a change in the tax rate on the prices of goods in other countries can be interpreted by the same decomposition as above. When \(t_{j}\) changes, there is a change in the purchase method. However, there is no change in price through the purchase method effect, since there is no change in the quantity the consumer purchases. The effects on price through a change in demand can be organized as follows: Suppose that \(t_{j}\) rises. As in origin-based taxation, the demand for good \(j\) falls, and \(p_{i}\) rises, owing to demand substitution from good \(j\) to good \(i\). This is shown in the first term of the second equation in (16). If there is no substitution relationship between the two goods (\(\gamma = 0\)), then the first term will be zero. However, in the case of destination-based taxation, consumers in country \(i\) who purchase goods from other countries online do not change the quantity of goods \(j\) they purchase. This is because consumers in country \(i\) are now subjected to taxes in country \(i\). Therefore, since there is no substitution of demand, there is no increase in demand for good \(i\), and the increase in price \(p_{i}\) when \(t_{j}\) rises is small. This is shown in the second term of the second equation in (16). In addition, an increase in \(t_{j}\) reduces the quantity of good \(i\) that consumers in country \(j\) purchase. Since demand substitution occurs, the online purchase of good \(i\) by consumers in country \(j\) will increase. This is shown in the third term of the second equation in (16). If the two goods are independent (\(\gamma = 0\)), no price impact emerges through this route, and then the third term is zero.
4.3 First stage

By inserting the demand function derived at the third stage into (6), we obtain the tax revenue of country $i$:

$$R_i^d = t_i^D \left\{ \left(1 - \frac{E_i}{\delta}\right) [2\alpha - (1 - \gamma)(p_i^{D^*} + p_j^{D^*} + 2t_i^D + e)] + \frac{2e}{\delta} [\alpha - (p_i^{D^*} + t_i^D) + \gamma(p_j^{D^*} + t_j^D)] - \frac{E_i^2}{2\delta} + \delta \frac{E_j^2}{2\delta} \right\},$$

where $p_i^{D^*}$ is given by (15). We obtain the first-order condition to maximize $R_i^D$, and evaluating at the symmetric equilibrium, $t_i^D = t_j^D$, it is given as:

$$\frac{\partial R_i^D}{\partial t_i^D} = \frac{2}{4 - \gamma^2} \left\{ (2 + \gamma) A - (2 - \gamma) t_i^D + \gamma t_i^D - (2 - \gamma - \gamma^2) \frac{2\delta - e}{4\delta} \right\} - \frac{2(2 - \gamma^2)}{4 - \gamma^2} t_i^D$$

$$+ 2\gamma \left( \frac{1 - e}{\delta} \right) t_i^D - \left( \frac{1 - e}{\delta} \right) \left( \frac{1 + \gamma}{2 + \gamma} \right) t_i^D - \frac{2}{\delta} \left[ \frac{A}{2 - \gamma} - \frac{1 - \gamma}{2 - \gamma} t_i^D - \frac{2(3 - \gamma^2)\delta + (1 - \gamma)^2 e}{4(2 - \gamma)\delta} \right] t_i^D.$$

(17)

The first two terms on the first line in (17) correspond to (10). The third and subsequent terms of (17) show the additional effects that would occur if the tax were imposed under the destination principle. The third term $2\gamma(1 - e/\delta)t_i^D$ shows the direct effect of the change in $t_i$. When taxation is based on the origin principle, country $i$ does not receive any tax revenue from the consumption of good $j$. Therefore, when $t_i$ is raised, and demand shifts from good $i$ to good $j$, there can be no increase in tax revenue caused by the increase in demand for good $j$. However, this is not the case when taxation is based on the destination principle. Country $i$ can obtain tax revenue from its citizens’ consumption of goods $j$. Therefore, when demand shifts from good $i$ to good $j$ by raising $t_i$, there will be an increase in tax revenue caused by the increased demand for good $j$. This is presented in the third term. Of course, if demand substitution does not occur ($\gamma = 0$), then this effect does not exist, and the third term does not appear. The fourth term captures the indirect effect (price effect) of changes in $p_i$ and $p_j$ due to changes in $t_i$. When $t_i$ increases, firm $i$ does not pass on the entire tax increase to consumers; the firm also bears some of the tax increase, so the pre-tax price $p_i$ decreases, increasing the demand for goods $i$. However, from (16), because the decrease in the pre-tax price $p_i$ due to $t_i$ is smaller under destination-based taxation than under origin-based taxation, the increase in demand for good $i$ is also necessarily smaller. This makes the marginal increase in tax revenue when raising $t_i$ smaller under the destination principle of taxation. The third and fourth terms disappear when $e$ is so high that no online purchases exist, that is, when $e = \delta$. The final term represents the effect of increasing tax revenue by manipulating the critical point (indicated by the dotted line in Figure 1b) by changing the tax rate. This captures the fact that when $t_i$ rises, the number of consumers in country $j$ who make purchases in country $i$’s stores decreases, so the tax revenue in country $i$ decreases. It also reduces the number of consumers in country $i$ who buy goods $j$ online when $t_i$ rises. It is also shown that under destination-based taxation, tax revenue for country $i$ is reduced because its consumers pay taxes to country $i$.

Next, we evaluate (17) in terms of $t_i^D$ to determine how the tax rates differ between the two tax principles. Using (12) and (17), we have:

$$\frac{\partial R_i^D}{\partial t_i^D} \bigg|_{t_i^D = t_j^D = e_i} = t_i^D H(e)$$

where

$$H(e) \equiv - \frac{2(2 - \gamma^2)}{(2 - \gamma)(4 - \gamma - 2\gamma^2)\delta} \left[ A - 5 - 5\gamma^2 + \gamma^4 \frac{e - \frac{(3 - \gamma^2)(1 - \gamma)^2 e^2}{2(2 - \gamma^2)\delta}}{\gamma^0} \right]$$

$$- \frac{(\gamma + 1)^2}{\gamma + 2} \left[ 1 - \frac{e}{\delta} \right] \left( \frac{e}{\delta} - \frac{\gamma^2 + 2\gamma - 1}{(\gamma + 1)^2} \right).$$

(18)

See Appendix B for the sufficient condition for the existence of an equilibrium.
The sign of the bracket in the first term is positive from Assumption 2 (see Appendix A), but the sign of (18), consisting of the two terms, is nontrivial. However, it is immediately clear that if \( \gamma \) is small, for example, \( \gamma \rightarrow 0 \), or if \( e/\delta \) is large, for example, \( e/\delta \rightarrow 1 \), then the second term will be non-positive, so \( \partial R^D_i/\partial t^D_i |_{t^D_i=\delta t^D_i} < 0 \rightarrow t^D_i > t^P_i \). Moreover, we can formally determine the sign of (18) as in Proposition 2 below:

**Proposition 2.** The tax rate is lower under the destination principle than under the origin principle.

\[ t^D_i < t^P_i. \]

**Proof.** \( \partial R^D_i / \partial t^D_i |_{t^D_i=\delta t^D_i} = t^D_i H(e) < 0 \) for \( 0 < e < \delta \), because \( H(\delta) < 0 \) and

\[
\frac{\partial H(e/\delta)}{\partial (e/\delta)} = \frac{2}{\delta} \left[ \frac{(1 - \gamma)(5 - 3\gamma - 2\gamma^2 + \gamma^3)}{(2 - \gamma)(4 - \gamma - 2\gamma^2)} + \frac{14 + 9\gamma^2 - 4\gamma^3 + \gamma^4 + \gamma^5}{(4 - \gamma^2)(4 - \gamma - 2\gamma^2)} \right] \overline{e} > 0. \]

The intuitive mechanism of the result is as follows. In the case of taxing online purchases of goods under the origin principle, as can be seen in Figure 1b, by lowering the tax rate, the government in country \( i \) can induce consumers in country \( j \) who purchase goods online from country \( i \) to make purchases in stores. At the same time, a reduction in the tax rate in country \( i \) can induce home country consumers who used to buy goods in brick-and-mortar stores in country \( j \) to buy them online. These will lead to an increase in the tax base and provide an incentive for country \( i \) to lower its tax rate. This effect does not occur under the origin principle of taxation, and therefore, the equilibrium tax rate under the destination principle will be lower, compared to the tax rate under the origin principle. This explains why \( t^D_i > t^P_i \).

Next, we consider how the tax rate would change if the cost of online purchases declines. The equilibrium tax rate under the destination principle is determined at the level where (17) is set to zero.\(^6\)

To see how \( t^D_i \) changes as \( e \) decreases, we differentiate (17) by \( e \):

\[
\left. \frac{\partial^2 R^D_i}{\partial t^D_i \partial e} \right|_{t^D_i=\delta t^D_i} = -\frac{1 - \gamma}{2 - \gamma} \left( \frac{1 - e}{\delta} \right) + \left[ \frac{(3 - \gamma)(1 - \gamma)}{2 - \gamma} \right] \left( \frac{6 + 3\gamma - \gamma^2}{(2 - \gamma)(2 + \gamma)} \right) \overline{e} \left( t^D_i - \delta \right) \left( \frac{3 - \gamma}{3 - \gamma} \right). \quad (19)
\]

The first term in (19) is the same as the one derived in (11), which captures the effect on marginal revenue of lower online costs when taxed under the origin principle; because a decline in \( e \) leads to an increase in taxable online consumption, countries that tax the purchase of goods have an incentive to raise the tax rate \( t_i \). In the case of taxation under the destination principle, the second term is added. Lowering the cost of online purchases affects the choice between online and in-store shopping and increases each country’s tax base under the destination principle. This creates an incentive for countries to decrease their tax rates, which is captured by the second term in (19).

If the negative effect of the first term in (19) exceeds the positive effect of the second term, a decrease in \( e \) will increase \( t^D_i \). Conversely, if the negative effect is less than the positive effect, a decrease in \( e \) will decrease \( t^D_i \). The former is the same as that shown in Proposition 1: that when taxing under the origin principle, a decrease in \( e \) causes an increase in the tax rate. What is interesting here is that contrary to the origin principle of taxation, a decline in \( e \) may cause a decline in tax rates, exacerbating the race to lower taxes. Now, noting that the right-hand side of (19) is an increasing function of \( e/\delta \), by substituting \( e/\delta = 0 \) into it, we obtain:

\[
\left. \frac{\partial^2 R^D_i}{\partial t^D_i \partial e} \right|_{t^D_i=\delta t^D_i} > -\frac{1 - \gamma}{2 - \gamma} \left[ \frac{(3 - \gamma)(1 - \gamma)}{2 - \gamma} \right] \left. t^D_i \right|_{t^D_i=\delta t^D_i} + \left( \frac{1 - \gamma}{(2 - \gamma)(2 + \gamma)} \right) \left( t^D_i - \delta \right). \quad (20)
\]

From (20), if \( t^D_i > \delta/(3 - \gamma) \), and it is proved to hold in the following proposition under Assumptions 1 and 2, the positive effect of the second term in (19) outweighs the negative effect of the first term, and a decline in the cost of online purchases accelerates the race to lower tax rates under the destination principle of taxation.

The formal result is shown in the following proposition.

\(^6\)The explicit solution is given in Appendix C.
Proposition 3. If online purchases of goods are taxed under the destination principle, the tax rate in the symmetric equilibrium will definitely decrease as the cost of online consumption decreases, $\frac{\partial t_i^{O*}}{\partial e} > 0$.

Proof. See Appendix D. □

The conclusion of Proposition 3 is in contrast to Proposition 1. When goods purchased online are taxed under the origin principle, the increase in demand associated with the lower cost of online consumption reduces the incentive for countries to lower their tax rates, resulting in higher equilibrium tax rates. However, when goods purchased online were taxed under the destination principle, the government in country $i$ had a incentive to lower the tax rate in order to reduce the cost of online shopping and induce customers to go to stores. This is the reason the sign of $\frac{\partial t_i^{D*}}{\partial e}$ is positive; the smaller the value of $e$, the more likely consumers are to buy goods online, so in order to induce consumers in country $j$ to buy in brick-and-mortar stores in country $i$, the tax rate in country $i$ needs to be reduced. After all, contrary to the result under origin-based taxation, a decline in the cost of online purchases lowers tax rates and worsens tax competition under destination-based taxation.

We can derive useful policy implications from Propositions 1 to 3. First, taxing online purchases of goods under the origin principle is superior to taxing them under the destination principle because it eases competition for the low online taxes. Second, as the cost of online purchases declines and online consumption expands, the advantage of taxing goods purchased online under the origin principle is strengthened.

5 Extension: Tax competition among asymmetric countries

Among several assumptions that have been made in the analysis so far, this section discusses the properties of equilibrium when countries are asymmetric. We have assumed that the two countries are symmetric, which is a necessary assumption in order to get analytical solutions. In this section, with the help of numerical analysis, we examine the properties of equilibrium in the case of two countries of different sizes. Specifically, we extend the analysis to the case where the populations of the two countries are different ($b > 0$) and focus on how the tax rate will change as the online market expands due to lower online costs. The process of finding the SPNE is the same as that in the previous sections; however, we focus on new findings that can be drawn owing to the population difference between the countries.

5.1 Equilibrium under the origin principle of taxation

The SPNE tax rates under the origin principle of taxation are obtained as follows (see Appendix E):

$$t_0^{O*} = \frac{(2 + \gamma)A}{4 - \gamma - 2\gamma^2} \left( \frac{(1 - \gamma)(2 + \gamma)}{2(2 - \gamma)(1 - \gamma)} - \frac{(1 + \gamma)(2 - \gamma)}{2(4 + \gamma - 2\gamma^2)} \right) e + \frac{(1 - \gamma)(2 - \gamma)}{2(4 + \gamma - 2\gamma^2)} \frac{e^2}{\delta}, \quad (21)$$

$$t_1^{O*} = \frac{(2 + \gamma)A}{4 - \gamma - 2\gamma^2} \left( \frac{(1 - \gamma)(2 + \gamma)}{2(2 - \gamma)(1 - \gamma)} + \frac{(1 + \gamma)(2 - \gamma)}{2(4 + \gamma - 2\gamma^2)} \right) e + \frac{(1 - \gamma)(2 + \gamma)}{2(4 + \gamma - 2\gamma^2)} \frac{e^2}{\delta}. \quad (22)$$

Note that (21) and (22) reduce to (12) if $b = 0$. Comparing these equations, we have the following proposition:

Proposition 4. When country 0 has a larger population than country 1 ($b \geq 0$), country 0 has a higher tax rate and higher prices than country 1, $t_0^{O*} \geq t_1^{O*}$ and $p_0^{O*} \geq p_1^{O*}$.

Proof. Comparing (21) and (22) gives $t_0^{O*} \geq t_1^{O*}$, since $b \geq 0$. Using (21) and (22), we obtain equilibrium prices as follows:

$$p_0^{O*} = \frac{(2 - \gamma^2)\alpha + (6 - 4\gamma - 2\gamma^2 + \gamma^3)c + e(2 - 2\delta)(2 - \gamma^2)(2 - \gamma)}{(2 - \gamma)(4 - \gamma - 2\gamma^2)} + \frac{(1 + \gamma)(2 - \gamma^2)(1 - \gamma)}{2(2 + \gamma)(4 + \gamma - 2\gamma^2)}$$

$$p_1^{O*} = \frac{(2 - \gamma^2)\alpha + (6 - 4\gamma - 2\gamma^2 + \gamma^3)c + e(2 - 2\delta)(2 - \gamma^2)(1 - \gamma)}{(2 - \gamma)(4 - \gamma - 2\gamma^2)} + \frac{(1 + \gamma)(2 - \gamma^2)(1 - \gamma)}{2(2 + \gamma)(4 + \gamma - 2\gamma^2)}$$
These equations give \( p_0^{O*} \geq p_1^{O*} \), since \( b \geq 0 \). \( \square \)

The conclusion that countries with larger populations set higher tax rates has been a well-known view since Kanbur and Keen (1993). Proposition 4 confirms that this feature of the equilibrium tax rate holds robustly when the model is extended to a situation where the consumption of goods is elastic and online purchases are possible. Furthermore, many studies, including Kanbur and Keen’s (1993), have assumed that the price of a good is constant (as equal to its marginal cost), but when that assumption is removed, it is also shown that the price of goods in large countries is higher.

The comparative statistics of (21) and (22) also give the following results:

**Proposition 5.** In countries with small populations, origin-based tax increases monotonically as the cost of online consumption \( e \) declines, \( \partial t_1^{O*} / \partial e < 0 \). In countries with large populations, change in the cost of online consumption \( e \) and the tax rate are non-monotonic. When \( e \) is large, origin-based tax falls as \( e \) falls, \( \partial t_0^{O*} / \partial e > 0 \), but when \( e \) is small, taxes rise as \( e \) falls, \( \partial t_0^{O*} / \partial e > 0 \).

**Proof.** From (21) and (22), we have:

\[
\frac{\partial t_0^{O*}}{\partial e} = - \frac{1}{2} \left[ \frac{(1 - \gamma)(2 + \gamma)}{4 - \gamma - 2\gamma^2} \left( 1 - \frac{e}{\delta} \right) - \frac{(1 + \gamma)(2 - \gamma)}{4 + \gamma - 2\gamma^2} b \right],
\]

(23)

\[
\frac{\partial t_1^{O*}}{\partial e} = - \frac{1}{2} \left[ \frac{(1 - \gamma)(2 + \gamma)}{4 - \gamma - 2\gamma^2} \left( 1 - \frac{e}{\delta} \right) + \frac{(1 + \gamma)(2 - \gamma)}{4 + \gamma - 2\gamma^2} b \right].
\]

(24)

\( \partial t_1^{O*} / \partial e < 0 \) is straightforward from (24). (23) shows that \( \partial t_0^{O*} / \partial e < 0 \) if \( e \) is sufficiently small to satisfy

\[
0 < \frac{e}{\delta} < 1 - \frac{(1 + \gamma)(2 - \gamma)(4 - \gamma - 2\gamma^2)}{(1 - \gamma)(2 + \gamma)(4 + \gamma - 2\gamma^2)} b,
\]

and \( \partial t_0^{O*} / \partial e > 0 \) if \( e \) satisfies

\[
1 - \frac{(1 + \gamma)(2 - \gamma)(4 - \gamma - 2\gamma^2)}{(1 - \gamma)(2 + \gamma)(4 + \gamma - 2\gamma^2)} b < \frac{e}{\delta} < 1 - b.
\]

This result can be intuitively explained as follows. The decline in \( e \) has two effects on demand. First, it increases demand from consumers who buy goods online. Second, it reduces the purchase of goods at brick-and-mortar stores, which are substituted with goods purchased online. In a market with this feature, the size of the online market is relatively large in a small country. Therefore, the first effect outweighs the second effect, and the tax base increases, owing to the decrease in \( e \). This will lead to higher tax rates in small countries. In large countries, the size of the online market is relatively small. Therefore, the second effect tends to be larger than the first effect. In this case, the tax rate in the large country will decrease because the decrease in \( e \) will lead to a smaller tax base. However, as the size of the online market expands, owing to the decline in \( e \), the magnitudes of the two effects eventually reverse. Thus, if \( e \) declines further from a sufficiently low level, the large country’s tax rate will start to rise.

### 5.2 Equilibrium under the destination principle of taxation

Even in our simple setting, it is difficult to analytically solve for equilibrium when asymmetric countries are taxed under the destination principle. Therefore, we rely on numerical calculations to show the relationship between tax rates and the decline in the cost of online purchases in each country.

Figures 2(a) and 2(b) show the tax rates in large and small countries for cases where the substitutability of the two goods is large (\( \gamma = 0.75 \)) or small (\( \gamma = 0.25 \)) under a set of plausible parameters within the range that satisfies the conditions corresponding to Assumptions 1 and 2: \( \delta = 1, c = 1, \alpha = 3.2, \) and \( b = 0.2 \). The blue lines are the large country’s tax rates, and the red lines are the small country’s tax rates. In Figures (a) and (b), the lines at the high level represent the tax rate under the origin principle, and the lines at the low level represent the tax rates under the destination principle. From these figures, we can identify several features of the equilibrium tax rate under the two taxation principles. The first is that under both tax principles, tax rates in larger countries are higher than those in smaller countries. This means that the result shown in Proposition 4, derived under the origin principle of taxation,
Figure 2. Tax rates in two asymmetric countries ($b > 0$)

Note. The blue line represents the tax rate for large country 0, and the red line represents that of small country 1. In (a), the lines where the tax rates are greater than 3.5 are the equilibrium tax rates under the origin principle, and the lines where the tax rates are less than 1.5 are the equilibrium tax rates under the destination principle. The same is true for (b): the lines at the high level are the tax rates under the origin principle, and the lines at the low level are the tax rates under the destination principle. The dotted line represents the tax rate for the case of two symmetric countries. The parameters are $\delta = 1, c = 1, \alpha = 3.2$, and $b = 0.2$.

is also valid for taxation under the destination principle. The second is that a decrease in $e$ increases the equilibrium tax rate under origin-based taxation, but decreases it under destination-based taxation. This means that the results shown in Propositions 1 and 3 are valid, even if we assume two asymmetric countries, clearly showing the difference in the change in the tax rate as the cost of online purchases declines under the origin and destination principles, respectively. Third, a decrease in $e$ reduces regional differences in tax rates when taxed under the destination principle, but increases them when taxed under the origin principle. These tendencies are maintained in almost all situations, even when the value of the parameter is changed.

These results are only insights based on numerical calculations; they are not the results of analytical solutions in general. Thus, what can be said of the nature of equilibrium in an environment where there is tax competition between asymmetric countries under the destination principle is that in “many cases”, the expansion of e-commerce as online shopping costs fall intensifies competition to lower taxes and increases regional differences in tax rates.

6 Conclusion

In this study, we examined the efficiency of two different tax principles in a spatial tax competition model in the context of e-commerce. The notable feature of our approach, compared with preceding tax competition studies, is that consumers are assumed to decide not only where to buy goods, but also how to buy them, that is, whether to buy in brick-and-mortar stores or online. Goods purchased in brick-and-mortar stores will be taxed under the origin principle, but the government will be able to tax goods purchased online under the destination principle. This raises the question of whether the origin or destination principle should be applied to online purchases.

We compare tax competition equilibrium under two different taxation principles and show that applying origin-based taxation not only to brick-and-mortar purchases, but also to online purchases mitigates the excessive tax reductions associated with tax competition. Furthermore, we show that the expansion of the online market through the offer of lower online purchase costs raises the equilibrium tax rate in an origin-based tax competition environment, but lowers it in a destination-based tax competition environment. Thus, the main argument emerging from our study is that a move toward applying the destination principle when taxing online purchases could accelerate the race to lower the tax. Applying the same taxation principle to both brick-and-mortar stores and online purchases would not only reduce tax ad-
administration costs, but also curb tax competition. Destination principle taxation creates inefficiencies because it incentivizes governments to induce their own consumers to shop online. If a consumer in one country buys a good in a brick-and-mortar store in a neighboring country, the tax payment is made to the neighboring country. However, if the consumer purchases goods online from the neighboring country, the tax can be paid to the home country. In addition, by encouraging consumers in neighboring countries to shift from online purchases to brick-and-mortar purchases of goods produced in their own countries, governments can also increase their tax revenues. As a result, governments are in a race to steal other countries’ tax bases by inducing their own consumers to buy online.

The basic results are derived from a symmetric tax competition model. In the second half of the paper, the model was extended to two countries with different population sizes. This extension has shown that under either taxation principle, tax rates are lower in countries with small populations than in countries with large populations. Furthermore, we have confirmed that the results under symmetric tax competition are generally valid, regardless of whether the origin or destination principle is applied, as long as online costs are sufficiently low.

While our study achieves some generalizations, such as assuming situations where demand is elastic with respect to price or where there is differentiation of goods, it retains some specific assumptions. The first assumption is that the government’s objective function is to maximize tax revenue. This is the setting used in many tax competition models, but for more generalization, it may be necessary to change the government’s objective. In such a case, we would need to adopt a different approach than analytically solving the model. Second, while our model allows for the inclusion of consumer purchases of goods online and the behavior of oligopolies in the analysis, it does not explicitly model the platform companies that are dominant in the online market. One of the characteristics of the services platform companies supply is the existence of network externalities. In addition to fiscal externalities in tax competition, it would be worthwhile to analyze the nature of taxation principles when other externalities coexist. However, these are beyond the scope of our analysis of basic tax competition studies on taxation principles that include e-commerce; however, such issues should be addressed in the next step.

Appendices
Appendix A
Substituting (12) into (8), we get:

\[ p_i^{O*} = \frac{4(2 - \gamma^2)\alpha + 4(6 - 4\gamma - 2\gamma^2 + \gamma^3)c - (1 - \gamma)(2 - \gamma^2)(2e - e^2/\delta)}{4(2 - \gamma)(2 - \gamma^2)} \].

Given \( x, q_i \) in Case 4 is the smallest among the four cases. Therefore, if \( q_i \) in Case 4 is positive, \( q_i > 0 \) is guaranteed in all other cases. We substitute \( p_i^{O*} \) into \( q_i^O \) in Case 4 to find the smallest demand in the four cases as follows:

\[ q_i^{O*} = \frac{2 - \gamma^2}{(2 - \gamma)(4 - \gamma - 2\gamma^2)} \left( A - \frac{(5 - 5\gamma^2 + \gamma^4)e}{2 - \gamma^2} - \frac{(1 - \gamma)(3 - \gamma^2)}{2(2 - \gamma^2)} \cdot \frac{e^2}{\delta} \right) > 0 \]

where the first sign in the second line is from Assumption 1, and the second sign in the third line is from Assumption 2. This suggests that the equilibrium quantity under origin-based taxation is positive for any \( e \) under two assumptions.
Appendix B

Expressing the first-order condition as $\partial R^D_i / \partial t^D_i |_{t^D_i = t^D} \equiv f(t^D_i) = 0$, from (17), we have:

$$f(t^D_i) = \left\{ \frac{6 + 3\gamma - \gamma^3}{(2 - \gamma)(2 + \gamma)\delta^2} e^2 + \frac{(1 - \gamma)(3 - \gamma)}{(2 - \gamma)\delta} e - \frac{2A + (1 - \gamma)(5 - \gamma)\delta}{(2 - \gamma)\delta} \right\} t^D_i + \frac{2(1 - \gamma)}{(2 - \gamma)\delta} (t^D_i)^2 + \frac{1 - \gamma}{2(2 - \gamma)\delta} e^2 - \frac{1 - \gamma}{2 - \gamma} e + \frac{2A}{2 - \gamma} = 0. \tag{25}$$

$f(t^D_i)$ is a quadratic function that is convex downward with respect to $t^D_i$. We also express the second-order condition as $\partial^2 R^D_i / \partial (t^D_i)^2 |_{t^D_i = t^D} \equiv s(t^D_i) < 0$, where

$$s(t^D_i) = \frac{7(1 - \gamma)}{2 - \gamma} \cdot \frac{t^D_i}{\delta} + \frac{6 + 3\gamma - \gamma^3}{2(2 - \gamma)(2 + \gamma)} \frac{e}{\delta}^2 + \frac{2(1 - \gamma)(3 - \gamma)}{(2 - \gamma)} \frac{e}{\delta} - \frac{2(2A + (1 - \gamma)(3 - \gamma)\delta}{(2 - \gamma)\delta}.$$  

$s(t^D_i)$ is a monotonically increasing linear function.

Now, the solution of $s(t^D_i) = 0$ is denoted by $t^S_i$:

$$t^S_i = \frac{4A}{7(1 - \gamma)} + \frac{2(3 - \gamma)(\delta - e)}{7} - \frac{(6 + 3\gamma - \gamma^3)e^2}{7(1 - \gamma)(2 + \gamma)\delta^2}.$$  

If $f(t^S_i) > 0$ and $f'(t^S_i) > 0$, the two solutions satisfy $f(t^S_i) = 0$, which satisfies the second-order condition. If $f(t^S_i) < 0$, then only the smaller of the two solutions satisfying $f(t^S_i) = 0$ meets the second-order condition. Otherwise, the two solutions satisfying $f(t^D_i) = 0$ do not satisfy the second-order condition.

Next, we have:

$$f(t^S_i) = -\frac{24A^2}{49(2 - \gamma)(1 - \gamma)\delta} - \frac{6A(5 - 4\gamma)}{49(2 - \gamma)} - \frac{2(3 - \gamma)(1 - \gamma)(23 - 3\gamma)\delta}{49(2 - \gamma)} + \left( \frac{24A(3 - \gamma)}{49(2 - \gamma)\delta} + \frac{(1 - \gamma)(13 - 2\gamma)(11 - 6\gamma)}{49(2 - \gamma)} \right) e^2$$

$$+ \left( \frac{12A(6 + 3\gamma - \gamma^3)}{49(2 - \gamma)(1 - \gamma)(2 + \gamma)\delta^2} + \frac{266 + 323\gamma + 73\gamma^2 - 12\gamma^3 + 49\gamma^4}{98(2 - \gamma)(2 + \gamma)\delta} \right) e^4 - \frac{6(3 - \gamma)(6 + 3\gamma - \gamma^3)^2}{49(2 - \gamma)(2 + \gamma)\delta^2} e^3.$$  

$$f'(t^S_i) = \frac{1}{7(2 - \gamma)} \left[ \frac{2A}{\delta} - (1 - \gamma)(11 - \gamma) - (1 - \gamma)(3 - \gamma) (e/\delta) - \frac{6 + 3\gamma - \gamma^3}{2(2 + \gamma)} (e/\delta)^2 \right].$$

Here, $f(t^S_i) > 0$ and $f'(t^S_i) > 0$ do not hold for $0 < \gamma < 1$, $0 < e/\delta < 1$, and $\alpha > \bar{\alpha}$, where $\bar{\alpha}$ is defined in Assumption 2. Therefore, the sufficient condition for the existence of an equilibrium is $f(t^S_i) < 0$, which is satisfied when $A \rightarrow \infty$ is sufficiently large.

Appendix C

From (17), revenue maximization with respect to $t^D_i$ gives:

$$t^D_i = \frac{2A + (1 - \gamma)(5 - \gamma)\delta}{4(1 - \gamma)} = \frac{3 - \gamma}{4} e - \frac{6 + 3\gamma - \gamma^3}{8(1 - \gamma)(2 + \gamma)\delta} e^2 \pm \frac{\sqrt{\Phi}}{8(1 - \gamma)(2 + \gamma)\delta}. \tag{26}$$

where $\Phi \equiv \frac{4(2 + \gamma)^2}{(1 - \gamma)^2(5 - \gamma)^2\delta^2} + 4A[4 + (1 - \gamma)\delta^2] \frac{\delta^2}{(2 - \gamma)(2 + \gamma)^2} - 8(1 - \gamma)(2 + \gamma)^2(2(3 - \gamma)A + (1 - \gamma)(11 - 8\gamma + \gamma^2)\delta) \frac{\delta^2}{(2 + \gamma)(2 - \gamma)(2 + \gamma)^2 - 8(2 + \gamma)(A(6 + 3\gamma - \gamma^3) + (1 - \gamma)(5 - \gamma)(1 + \gamma^2)\delta) \frac{\delta^2}{(2 + \gamma)(2 - \gamma)(2 + \gamma)(6 + 3\gamma - \gamma^3)\delta e^3 + (6 + 3\gamma - \gamma^3)^2 e^4}$. Appendix B shows that among the two solutions in (26), the larger value does not satisfy the second-order condition, but the smaller value does under Assumptions 1 and 2 if $f(t^S_i) < 0$. Therefore, the smaller value in (26) is the equilibrium tax rate.
Figure 3. Value of \( f(t_{i}^D)_{\alpha=\bar{\alpha}} \).

Appendix D

Since \( \partial t_i^{D*}/\partial e = -[\partial f(t_i^{D*})/\partial e]/[\partial f(t_i^{D*})/\partial t_{i}^{D*}] \), we derive \( \partial f(t_i^{D*})/\partial e \) and \( \partial f(t_i^{D*})/\partial t_{i}^{D*} \). From (25), we first obtain:

\[
\frac{\partial f(t_i^{D})}{\partial t_{i}^{D}} = \frac{4(1-\gamma) t_i^{D}}{(2-\gamma) \delta} + \frac{6 + 3 \gamma - \gamma^2}{2(2-\gamma)(2+\gamma)} \left( \frac{e}{\delta} \right)^2 + \frac{(1-\gamma)(3-\gamma)}{2(2-\gamma)} \left( \frac{e}{\delta} \right) - \frac{2A + (1-\gamma)(5-\gamma) \delta}{(2-\gamma) \delta},
\]

which shows that \( \partial f(t_i^{D})/\partial t_{i}^{D} \) is a linear function that is monotonically increasing with respect to \( t_{i}^{D} \) and monotonically decreasing with respect to \( \alpha \). Therefore, if the solution of \( \partial f(t_i^{D})/\partial t_{i}^{D} = 0 \) is \( t_{i}^{T} \), the sufficient condition for \( \partial f(t_i^{D})/\partial t_{i}^{D} < 0 \) in \( \alpha > \bar{\alpha} \) is \( f(t_{i}^{T})_{\alpha=\bar{\alpha}} < 0 \), where \( \bar{\alpha} \) is defined in Assumption 2.

From (25), we have:

\[
f(t_{i}^{T})_{\alpha=\bar{\alpha}} = \left[ \frac{8 \gamma^7 - 56 \gamma^6 + 80 \gamma^5 + 167 \gamma^4 - 452 \gamma^3 + 46 \gamma^2 + 524 \gamma - 321}{8(2-\gamma)(1-\gamma)(2-\gamma)^2} - \frac{8 \gamma^4 - 31 \gamma^3 + 11 \gamma^2 + 69 \gamma - 61}{4(2-\gamma)(2-\gamma^2)} \left( \frac{e}{\delta} \right)^2 + \frac{\gamma^7 - 14 \gamma^6 + 11 \gamma^5 + 74 \gamma^4 - 39 \gamma^3 - 150 \gamma^2 + 15 \gamma + 118}{8(2-\gamma)(1-\gamma)(2-\gamma)^2} \left( \frac{e}{\delta} \right)^2 - \frac{(3-\gamma)(6 + 3 \gamma - \gamma^2)}{8(2-\gamma)(2+\gamma)} \left( \frac{e}{\delta} \right)^2 - \frac{(6 + 3 \gamma - \gamma^2)(e^2)}{32(2-\gamma)(2+\gamma)(2+2 \gamma)(1-\gamma)(2+\gamma)} \left( \frac{e}{\delta} \right) \right] \delta.
\]

Figure 3 shows \( f(t_{i}^{T})_{\alpha=\bar{\alpha}} < 0 \) for \( e/\delta \in (0, 1) \) and \( \gamma \in (0, 1) \). This suggests that \( \partial f(t_i^{D})/\partial t_{i}^{D} < 0 \) is satisfied in \( \alpha > \bar{\alpha} \).

Next, from (25), if \( t_i^{D*} > \delta/(3-\gamma) \),

\[
\frac{\partial f(t_i^{D})}{\partial e} = \frac{\partial^2 R_i^{D}}{\partial t_i^{D} \partial e} \bigg|_{t_i^{D} = t_i^{D}} > 0.
\]

Substitution of \( t_i^{D} = \delta/(3-\gamma) \) into (25) gives:

\[
f \left( \frac{\delta}{3-\gamma} \right) = \left[ \frac{3 + \gamma}{(3-\gamma)(2+\gamma)} \left( \frac{e}{\delta} \right)^2 + \frac{2A}{(3-\gamma) \delta} - \frac{(1-\gamma)( \gamma^2 - 8 \gamma + 13)}{(3-\gamma)^2(2-\gamma)} \right] \delta.
\]

Given \( e/\delta \in (0, 1) \) and \( \alpha > \bar{\alpha} \), substituting \( e/\delta = 0 \) and \( \alpha = \bar{\alpha} \) into (27), we obtain:

\[
f \left( \frac{\delta}{3-\gamma} \right) > \frac{(52 + 45 \gamma)(1-\gamma)^2 + (28 - 18 \gamma - 3 \gamma^2 - 4 \gamma^3 + \gamma^4) \gamma^2}{(3-\gamma)^2(2-\gamma)(2-\gamma^2)} \delta > 0.
\]
Furthermore, when (25) is differentiated by \( t_i^D \) and evaluated at \( t_i^D = \delta/(3 - \gamma) \), we have:

\[
\frac{\partial f \left( \frac{\delta}{t_i} \right)}{\partial t_i} = \frac{6 + 3\gamma - \gamma^3}{2(2 - \gamma)(2 + \gamma)} \left( \frac{e}{\delta} \right)^2 + \frac{(3 - \gamma)(1 - \gamma)}{2 - \gamma} \left( \frac{e}{\delta} \right) - \frac{2A}{(2 - \gamma)^2} - \frac{(1 - \gamma)(\gamma^2 - 8\gamma + 11)}{(3 - \gamma)(2 - \gamma)}. \tag{28}
\]

Given \( e/\delta \in (0, 1) \) and \( \alpha > \bar{\alpha} \), substituting \( e/\delta = 1 \) and \( \alpha = \bar{\alpha} \) into (28), we obtain:

\[
\frac{\partial f \left( \frac{\delta}{t_i} \right)}{\partial t_i} < - \frac{(136 + 196\gamma + 138\gamma^2)(1 - \gamma)^2 + (129 - 111\gamma - 9\gamma^2 - \gamma^3)\gamma^3}{2(3 - \gamma)(2 - \gamma)(2 + \gamma)(2 - \gamma^2)} < 0.
\]

Therefore, if \( \alpha > \bar{\alpha} \), then \( t_i^{D*} > \delta/(3 - \gamma) \) is always valid, and thus \( \partial f(t_i^{D*})/\partial e > 0 \). Therefore, we have \( \partial t_i^{D*}/\partial e > 0 \) for \( e > 0 \).

**Appendix E**

The first-order conditions for profit maximization are

\[
\frac{\partial \pi^O_i}{\partial p_i^O} = 2 \left\{ \alpha - \left[ 2p_i^O + t_i^O + \frac{2(1 - b)\delta - e}{4\delta} - e \right] + \gamma \left[ p_i^O + t_i^O + \frac{2(1 + b)\delta - e}{4\delta} \right] \right\} = 0,
\]

\[
\frac{\partial \pi^D_i}{\partial p_i^D} = 2 \left\{ \alpha - \left[ 2p_i^D + t_i^D + \frac{2(1 - b)\delta - e}{4\delta} - e \right] + \gamma \left[ p_i^D + t_i^D + \frac{2(1 + b)\delta - e}{4\delta} \right] \right\} = 0.
\]

Solving these equations, we get:

\[
p_i^{O*} = \frac{(\alpha + c)(2 + \gamma)}{4 - \gamma^2} \left[ \frac{1}{t_i^O} + \frac{2(1 - b)\delta - e}{4\delta} - e \right] + \frac{\gamma}{4 - \gamma^2} \left[ \frac{1}{t_i^O} + \frac{2(1 + b)\delta - e}{4\delta} - e \right],
\]

\[
p_i^{D*} = \frac{(\alpha + c)(2 + \gamma)}{4 - \gamma^2} \left[ \frac{1}{t_i^D} + \frac{2(1 - b)\delta - e}{4\delta} - e \right] + \frac{\gamma}{4 - \gamma^2} \left[ \frac{1}{t_i^D} + \frac{2(1 + b)\delta - e}{4\delta} - e \right].
\]

Substituting these equations into (5), we have:

\[
R_i^O = \frac{2}{4 - \gamma^2} \left\{ (2 + \gamma)A - (2 - \gamma^2) \left[ \frac{1}{t_i^O} + \frac{2(1 - b)\delta - e}{4\delta} - e \right] + \gamma \left[ \frac{1}{t_i^O} + \frac{2(1 + b)\delta - e}{4\delta} - e \right] \right\} t_i^O,
\]

\[
R_i^D = \frac{2}{4 - \gamma^2} \left\{ (2 + \gamma)A - (2 - \gamma^2) \left[ \frac{1}{t_i^D} + \frac{2(1 - b)\delta - e}{4\delta} - e \right] + \gamma \left[ \frac{1}{t_i^D} + \frac{2(1 + b)\delta - e}{4\delta} - e \right] \right\} t_i^D.
\]

The first-order conditions for tax revenue maximization are obtained as:

\[
\frac{\partial R_i^O}{\partial t_i^O} = \frac{2}{4 - \gamma^2} \left\{ (2 + \gamma)A - (2 - \gamma^2) \left[ 2t_i^O + \frac{2(1 - b)\delta - e}{4\delta} - e \right] + \gamma \left[ t_i^O + \frac{2(1 + b)\delta - e}{4\delta} - e \right] \right\} = 0,
\]

\[
\frac{\partial R_i^D}{\partial t_i^D} = \frac{2}{4 - \gamma^2} \left\{ (2 + \gamma)A - (2 - \gamma^2) \left[ 2t_i^D + \frac{2(1 - b)\delta - e}{4\delta} - e \right] + \gamma \left[ t_i^D + \frac{2(1 + b)\delta - e}{4\delta} - e \right] \right\} = 0.
\]

Solving these equations, we obtain (21) and (22).

**References**


