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Cyclical Part-Time Employment in an Estimated New Keynesian Model with Search Frictions*

Toshihiko Mukoyama†, Mototsugu Shintani‡ and Kazuhiro Teramoto§

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Abstract

This paper analyzes the dynamics of full-time employment and part-time employment over the business cycle. We first document basic macroeconomic facts on these employment stocks using the U.S. data and decompose their cyclical dynamics into the contributions of different flows into and out of these stocks. Second, we develop and estimate a New Keynesian search-and-matching model with two labor markets to uncover the fundamental driving forces of the cyclical dynamics of employment stocks. We find that the procyclicality of the net flow from part-time to full-time employment is essential in accounting for countercyclical patterns of part-time employment.

Key Words: Part-time employment; Bayesian estimation; DSGE model; Search, matching and bargaining.

JEL Classification: E24; E32.

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1 Introduction

Asymmetric roles of full-time and part-time workers in business cycles have attracted growing attention in recent years. In the United States, the number of part-time workers increased dramatically in the process of recovery from the 2007–2009 Great Recession, while there were relatively small increase in the full-time employment.\footnote{A similar asymmetry has also been observed in other countries. For example, Borowczyk-Martins (2017) documents that many of major European countries saw the hikes in involuntary part-time workers in contrast to the sharp declines in the full-time employment in the aftermath of the Great Recession.} In fact, the share of part-time workers in the total work force has become nearly 20 percent after 2010. It is, therefore, natural to infer that this heterogeneous behavior can play an important role in analyzing the business cycle dynamics in the context of the aggregate labor market.\footnote{In a seminal study, Blanchard and Diamond (1990) also emphasize the importance of considering the ‘primary’ and ‘secondary’ workers in understanding the business cycle property of the aggregate employment in the United States. Finegan, Penalova, and Shintani (2008) reconfirm their findings using an updated time-series data.}

In this study, we first use the Current Population Survey (CPS) dataset to document the cyclical patterns of full-time and part-time employment in the United States. We separate employed workers into full-time and part-time employed workers following the CPS distinction: full-time employed workers are the ones who report that they usually work 35 hours or more per week, and part-time employed workers work for less than 35 hours per week. Under this distinction, we observe that the full-time employment rate has a clear procyclical pattern, while the part-time employment rate exhibits a less pronounced business cycle dynamics during tranquil times and a sharply countercyclical pattern in deep recessions, such as the ones in early 1980s and the Great Recession.

To uncover which labor market flows are responsible for the cyclical dynamics of employment stocks, we decompose the dynamics of stocks into the contributions of different flows. More specifically, using the rotated survey sample of the CPS, we calculate the monthly transition across five labor market states (full-time employment, part-time employment, full-time unemployment, part-time unemployment, and nonparticipation) and decompose the changes in each labor market population into different net flows.\footnote{The terms “full-time unemployment” and “part-time unemployment” represent the unemployed workers who look for full-time employment and the unemployed workers who look for part-time employment. We describe the details of the data in Section 2.} Since the consistent transition calculation is available only after 1996, we focus on one event: a sharp decline in the full-time employment rate and a hike in the part-time employment rate during the Great Recession. We highlight two features. First, the flows between employment and unemployment strongly contributed to the decrease in the full-time employment during the Great Recession, while they did not contribute much to the increase...
in the part-time employment. Second, the flows within employment, that is, the direct transitions between full-time employment and part-time employment, were among the most important contributors to the change of the part-time employment stock.\footnote{This result is consistent with the results of the empirical studies on the rise in part-time employment in the aftermath of the Great Recession, e.g., Canon et al. (2014), Cajner et al. (2014), and Borowczyk-Martins and Lalé (2018). These studies find that the changes in the composition of employment stocks were more important contributor to the increase in involuntary part-time workers (or those working part-time for economic reasons) than the changes in flows between employment and non-employment.}

Another notable fact from the CPS is that the majority of the unemployed who are looking for full-time jobs (conditional on finding a job) transition into full-time jobs, while the majority of the unemployed who are looking for part-time jobs (conditional on finding a job) transition into part-time jobs. This suggests that their labor markets are segmented to a considerable extent, and the job characteristics of full-time workers and part-time workers are different and therefore reallocating an employee part-time to full-time is not frictionless. In other words, even when such reallocation is within a same firm, it can require substantial cost when assigning a right worker to a right job.

Motivated by these empirical regularities, we next construct a model which introduces the Diamond-Mortensen-Pissarides (DMP) search and matching frictions in a Dynamic Stochastic General Equilibrium (DSGE) framework. We mainly follow the modeling strategy of Gertler, Sala, and Trigari (2008) (GST henceforth). The estimated model is used to evaluate the importance of the part-time labor market in the aggregate fluctuations and the economy’s responses to various shocks. The model analysis enables us to uncover the fundamental driving forces of the cyclical dynamics of employment stocks.

An important feature of our model, which differs from GST, is that it incorporates two segmented labor markets (full-time and part-time) and an endogenous transition across different employment stocks. In particular, the following characteristics of the dual labor market structure are included in our model. First, two labor markets (the full-time market and the part-time market) are separated and a part-time job requires less work hours than a full-time job, reflecting the fact that in the data part-time workers are defined as people who usually work fewer than 35 hours a week. Second, the time-varying productivity, matching efficiency, separation rate, and the wage stickiness differ across two labor markets. Third, a direct transition from part-time employment to full-time employment is possible through the on-the-job search by the part-time workers. We embed this dual labor market structure into an otherwise standard medium-scale DSGE model that has been popularly used in the business cycle analysis of the U.S. economy, such as Christiano, Eichenbaum, and Evans (2005) (CEE henceforth) and Smets and Wouters (2007) (SW henceforth).
Using the U.S. quarterly data, we estimate our model with Bayesian methods. Because of the rich features, our estimated model can generate realistic cyclical behavior of full-time employment and part-time employment. In particular, in the estimated model, conventional aggregate shocks can generate the asymmetric dynamics of two labor market flows. As a result, our model is successful in matching the lead-lag pattern of the part-time employment stock with business cycle fluctuations.

We further evaluate the importance of the endogenous transition within employment stocks from two viewpoints. First, we decompose the impulse responses of the employment stocks into the contribution of the flows between employment and unemployment and the contribution of the flows within employment (from part time to full time). This decomposition shows that the direct transitions from part-time to full-time employment behave procyclically, and this is essential in generating the countercyclical response of the part-time employment stock. Second, we conduct a simple counterfactual analysis by setting the job-finding probability of a part-time worker (i.e., the probability that a part-time worker finds a new full-time job) constant across time. We find that in the absence of the procyclical direct transition from part-time to full-time, part-time employment would no longer be countercyclical and this results in an about 1 percentage point higher unemployment rate in the counterfactual in the aftermath of the Great Recession.

In the recent DSGE literature, there have been a number of studies that consider labor market search and matching frictions along with wage bargaining between workers and firms. To the best of our knowledge, however, our study is the first attempt to estimate a DSGE model that explicitly considers dual labor markets of full-time and part-time workers. In our dual labor market framework, an unemployed worker either searches in full-time or part-time labor markets and a part-time worker decides whether to conduct an on-the-job search for a full-time job. This framework is distinct from the model structure that analyzes the part-time utilization margin by firms, which is often adopted in recent papers that examine the business cycle fluctuations of involuntary part-time employment. For example, Warren (2017) considers such margin within a competitive search model with heterogeneous firms and Lariau (2017) integrates this margin into a random search model with heterogeneous workers.

An earlier work by Trigari (2009) considers both intensive and extensive margins of labor adjustment within a monetary DSGE model to study how the incorporation of the two margins changes the inflation dynamics. In our model, full-time employment, part-time employment, and unemployment are considered distinct labor market states, and the intensive margin choice is abstracted away. Instead, the average hours per worker fluctuate through the change in the composition of full-time and part-time employment. Our modeling decision is motivated by the empirical evidence of Borowczyk-Martins and Lalé (2018), which show that changes in composition of full-time and part-time employment, rather than changes in hours within full-time and part-time work, are more important drivers of the cyclical movements in hours per worker.

The remainder of this paper is organized as follows. Stylized facts regarding full-time and part-time labor markets are first presented in Section 2. Our model of two labor markets with an endogenous transition within employment stocks is introduced in Section 3. Quantitative performance of the model is explained and then a counterfactual experiment is conducted in Section 4. Concluding remarks are made in Section 5.

2 Part-time employment and the U.S. labor market

This section empirically analyzes the behavior of full-time and part-time employment over the recent years. Our main data source is the CPS, which is a primary source of labor force statistics in the United States. The statistics in this section cover all civilians 16 years old and over.

2.1 Cyclical patterns of employment stocks

As is well known, the unemployment rate is strongly countercyclical. The employment rate, defined here as the employment stock divided by the labor force, is therefore strongly procyclical. Our approach is to divide the total employment into full-time employment and part-time employment in analyzing the dynamics of employment. We follow the CPS distinction of full time and part time: a full-time employed worker works 35 hours or more per week, and a part-time employed worker works less than 35 hours per week. As Figure 1 shows, the full-time employment rate (i.e., full-time employment as a fraction of labor force) is clearly procyclical. In contrast, the cyclicality of part-time employment is less pronounced, and in deep recessions, such as in early 1980s and during the Great Re-

\footnote{See Appendix A.1 for the detailed descriptions on the necessary adjustments to the time series because of the redesign of the CPS implemented in January 1994. See also Valletta and Bengali (2013).}
Figure 1: Full-time and part-time employment divided by labor force

Note: This figure shows the time series of the full-time employment as a fraction of the labor force with a solid line (left scale) and the part-time employment as a fraction of the labor force with a dashed line (right scale) from 1979 to 2016. The shaded areas correspond NBER recessions.

Table 1: Correlation with per capita real GDP

<table>
<thead>
<tr>
<th>leads-lags (quarter)</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr($EF_h$, $Y$)</td>
<td>0.161</td>
<td>0.265</td>
<td>0.372</td>
<td>0.473</td>
<td>0.549</td>
<td>0.570</td>
<td>0.538</td>
<td>0.470</td>
<td>0.384</td>
</tr>
<tr>
<td>Corr($EP_h$, $Y$)</td>
<td>0.042</td>
<td>-0.061</td>
<td>-0.169</td>
<td>-0.270</td>
<td>-0.366</td>
<td>-0.417</td>
<td>-0.426</td>
<td>-0.397</td>
<td>-0.347</td>
</tr>
</tbody>
</table>

Note: $EF_h$ and $EP_h$ denote the full-time employment rate and the part-time employment rate with $h$ quarters lag, respectively. $Y$ denotes per capita real GDP in the log-deviations from the HP-filter trend with the smoothing parameter 1600. The sample period is from 1979:Q1 to 2016:Q4.

cession, the part-time employment rate exhibits a clear countercyclical pattern. Another notable fact is that in the aftermath of the Great Recession, the part-time employment rate has stayed high even when the unemployment rate kept falling.

These patterns are confirmed by the correlations with the cyclical components of real GDP shown in Table 1. This table also shows that the movement of employment stocks tends to lag behind the business cycle; in particular, we observe a two quarter lag for the part-time employment rate.\(^7\)

\(^7\)This pattern is different from the cyclical pattern of involuntary part-time employment documented by Lariau (2017); in her analysis, the contemporaneous correlation is the strongest.
2.2 Cyclical patterns of unemployment stocks

On the unemployment side, the CPS contains a question about what type of jobs the unemployed workers are looking for. Although the answer to that question does not restrict the worker’s actual behavior (for example, a worker who is looking for a full-time work can transition into a part-time job), it does provide some information for analyzing why part-time employment exhibits a countercyclical pattern. Note that this distinction is indeed informative in light of eventual behavior: in Appendix B.1, we show that a majority (around 2/3) of the unemployed who are looking for full-time jobs (conditional on finding a job) transition into full-time jobs while a majority (around 4/5) of the unemployed who are looking for part-time jobs (conditional on finding a job) transition into part-time jobs. This implies that for the majority of workers flowing from the unemployment state, the full-time and the part-time jobs are different type of jobs.\footnote{This is also consistent with the existing evidences; for example, Canon et al. (2014) document that the occupational characteristics of part-time jobs and full-time jobs are substantially different.}

Motivated by this observation, we consider the part-time labor market and the full-time labor market as separate markets in Section 3. This feature distinguishes our study from the existing models of (single) labor market search with a flexible intensive margin, such as Trigari (2009), or those with part-time utilization by firms, such as Warren (2017) and Lariau (2017).

Turning to the unemployment stocks (plotted in Figure 2), we find that both series are countercyclical, and the cyclicality is stronger for the unemployed looking for full-time work. This indicates that the main reason that the part-time employment increases in recessions (especially compared to full-time employment) is not because there are so many unemployed workers looking for a part-time job during recessions. For this reason, below, it is important to investigate all possible flows that can change the level of the part-time employment stock.

2.3 Which labor market flows are responsible for the cyclical dynamics of employment stocks?

To further investigate the cyclical dynamics of full-time and part-time employment, below, we conduct a (net) flow decomposition. In particular, we consider the particular case of the Great Recession era, and decompose the changes in employment stocks into different net flows.\footnote{Recent literature argues that, despite its severeness, the labor market dynamics during the Great Recession is qualitatively very similar to the previous recessions, except for the behavior of long-term unemployment. See Elsby et al. (2011).} The formal steps are as follows.

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\textsuperscript{8}This is also consistent with the existing evidences; for example, Canon et al. (2014) document that the occupational characteristics of part-time jobs and full-time jobs are substantially different.

\textsuperscript{9}Recent literature argues that, despite its severeness, the labor market dynamics during the Great Recession is qualitatively very similar to the previous recessions, except for the behavior of long-term unemployment. See Elsby et al. (2011).
Consider the stock of labor market state $j \in \{1, \ldots, k\}$ in a month $t$, and denote it $S^j_t$. The value of $S^j_t$ changes during the time period $t = 0$ to $t = T$. Let $F^ij_{t,t+1}$ denote the net flow from state $i$ to state $j$ between a month $t$ and the next month $t + 1$. Then, the change in stock (changes are expressed with $\Delta$) can be decomposed into net flows by

$$\Delta S^j_{t,t+1} = S^j_{t+1} - S^j_t = \sum_{i=1}^{k} F^ij_{t,t+1} \text{ for } t = 0, \ldots, T - 1. \quad (1)$$

We define $r^j_{t,t+1}$ as the (monthly) rate of change in stock $j$ between period $t$ and $t + 1$: $r^j_{t,t+1} = \Delta S^j_{t,t+1}/S^j_t$. Furthermore, we normalize the net flow as $f^ij_{t,t+1} = F^ij_{t,t+1}/S^j_t$ and call it the net flow rate. Then, dividing both sides of (1) by $S^j_t$, we have

$$r^j_{t,t+1} = \sum_{i=1}^{k} f^ij_{t,t+1}. \quad (2)$$

Let us denote the (long-run) time-series average values of the rate of change in stocks by

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**Note:** This figure shows the time series of unemployed workers looking for part-time work as a fraction of labor force (solid line) and unemployed workers looking for full-time work as a fraction of labor force (dashed line). The shaded areas correspond to NBER recessions.

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10Here, we consider five labor market states ($k = 5$): full-time employment, part-time employment, full-time unemployment, part-time unemployment, and nonparticipation.

11We set $t = 0$ at January 1996 and $t = T$ at December 2016 since there are missing observations in 1995 due to the failure of individual identifiers in the CPS.
Table 2: Net flow decomposition of full-time employment and part-time employment during the Great Recession period

<table>
<thead>
<tr>
<th>The rate of change in stock of state $j$</th>
<th>$j = EF$</th>
<th>$j = EP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net flow rate from state $i$ to state $j$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i = EF$</td>
<td>-0.49</td>
<td>0.38</td>
</tr>
<tr>
<td>$i = EP$</td>
<td>-0.13</td>
<td>-0.01</td>
</tr>
<tr>
<td>$i = UF$</td>
<td>-0.25</td>
<td>0.02</td>
</tr>
<tr>
<td>$i = UP$</td>
<td>-0.03</td>
<td>-0.11</td>
</tr>
<tr>
<td>$i = O$</td>
<td>-0.07</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Note: Average monthly flow (%) during December 2007 ($t = T_1$) to December 2009 ($t = T_2$), compared to the long-run average of January 1996 ($t = 0$) to December 2016 ($t = T$). $EF$ is full-time employment, $EP$ is part-time employment, $UF$ is the unemployed looking for a full-time job, $UP$ is the unemployed looking for a part-time job, and $O$ is out of labor force.

$$\bar{r}^j = \frac{1}{T} \sum_{t=0}^{T-1} r^j_{i,t+1} = \frac{1}{T} \sum_{t=0}^{T-1} \left( \sum_{i=1}^{k} f^j_{i,t+1} \right) = \sum_{i=1}^{k} \left( \frac{1}{T} \sum_{t=0}^{T-1} f^j_{i,t+1} \right) = \sum_{i=1}^{k} f^j_{i}$$

(3)

holds. Now, let $T_1$ and $T_2$ be the month the Great Recession started and the month it ended, respectively ($0 < T_1 < T_2 < T$). We denote the average value of the monthly rate of change in stocks during the Great Recession period by $\bar{r}^j_{GR}$ and the average value of the net flow rates by $\bar{f}^j_{GR}$. Similarly to above,

$$\bar{r}^j_{GR} = \frac{1}{T_2 - T_1} \sum_{t=T_1}^{T_2-1} r^j_{t} = \frac{1}{T_2 - T_1} \sum_{t=T_1}^{T_2-1} \left( \sum_{i=1}^{k} f^j_{i,t+1} \right) = \sum_{i=1}^{k} \left( \frac{1}{T_2 - T_1} \sum_{t=T_1}^{T_2-1} f^j_{i,t+1} \right) = \sum_{i=1}^{k} f^j_{i,GR}.$$  

(4)

Subtracting (3) from (4),

$$\bar{r}^j_{GR} - \bar{r}^j = \sum_{i=1}^{k} \left( f^j_{i,GR} - f^j_{i} \right)$$

(5)

holds. This is our decomposition formula. This formula decomposes the deviation of the rate of change in stock during the Great Recession into the deviations of the net flow rates.

Table 2 applies the decomposition (5) to the Great Recession period (December 2007

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12The NBER’s Business Cycle Dating Committee decided that the Great Recession lasted from December 2007 to June 2009. In this decomposition analysis, so as to avoid seasonality problems, we regard the Great Recession started in December 2007 and ended in December 2009.
to November 2009) for full-time employment (EF) and part-time employment (EP).\textsuperscript{13} The first row is the change in stock: compared to the average change in stock during the entire sample, the full-time employment stock as a share of the entire population experienced 0.49% decline per month during the Great Recession. In total, it declined about 12\% during this two-year span (0.49\% × 24(months) = 11.76\%: this magnitude can also be seen from Figure 1—there, the full-time employment rate fell from 79\% to 72\%). The part-time employment stock, in contrast, increased at the rate of 0.38\% per month (once again, this magnitude is consistent with Figure 1). The second to sixth rows are net flow components. For the full-time employment, the largest contributors are the net flows from the unemployed looking for a full-time job and the employed part-time workers. Note that the flow from the unemployed looking for a part-time job has almost no contribution. For the part-time employment, the main contributor is the net flow from the employed full-time workers. These results suggest that the cyclical behavior of transition probabilities between EF and EP (that are shown in Appendix B.1 in detail) is a crucial driving force behind the cyclical pattern of the two employment stocks.\textsuperscript{14}

In sum, we find that, to analyze the cyclical behavior of the full-time employment and the part-time employment, it is essential to (i) explicitly incorporate the flow between full-time employment and part-time employment; and to (ii) separately model the job-finding behavior of the unemployed workers who are looking for full-time jobs and the unemployed workers who are looking for part-time jobs. These two facts motivate our model formulation in the next section.

3 Model

In this section, we set up a DSGE model with frictional labor markets. By constructing and estimating the model, we will be able to evaluate the role of the part-time labor market in business cycle dynamics. While the basic structure of the model follows that of GST, there are important departures as we incorporate part-time employment into the model. An important modeling decision here is that we model separate labor markets

\textsuperscript{13}Since the CPS rotates the survey sample, there is discrepancy between the sample used in calculating net flow decomposition and changes in stock data. This induces gaps between the sum of flows and the change in stock, which is called a margin error. In order to correct this margin error, we use the method employed by Elsby, Hobijn, and Sahin (2015). See Appendix A.2 for the detail.

\textsuperscript{14}This is consistent with the findings of recent empirical studies that focus on the involuntary part-time employment. For example, Warren (2017) finds that the transition probability of involuntary part-time employment from and to full-time employment is more strongly correlated with output than the transition probability from and to unemployment and Lariau (2017) finds that the countercyclical movement in the transition probability from full-time to involuntary part-time employment is the key driving force of the fluctuation in the involuntary part-time employment.
for full-time jobs and part-time jobs. This is motivated by the fact that the workers who look for a full-time job ($UF$) tend to move to full-time employment ($EF$) and the workers who look for a part-time job ($UP$) tend to move to part-time employment ($EP$). It is also motivated by the finding in Table 2 that the net flow from and to $UF$ plays an prominent role in accounting for the cyclical movement of $EF$, while $UP$ has almost no influence on the cyclicality of $EP$.

Another important ingredient of our model is that we allow for direct transitions between $EF$ and $EP$ and explicitly model this (net) flow. In Table 2, we find that this flow is an important contributor to the cyclicality of both $EF$ and $EP$.

The model consists of households, wholesale firms, retail firms, and the government. Households consume, invest, rent the capital stock, and supply both full-time and part-time labor. Each wholesale firm has two internal divisions (full-time and part-time division), and each division produces a homogeneous intermediate good using capital and either full-time or part-time labor. Retail firms use the intermediate goods and produce differentiated retailed goods, which are combined to the final good and used for consumption and investment. The government conducts monetary and fiscal policy based on pre-specified rules. In what follows, we use the superscript $F$ to indicate the full-time labor market and $P$ to represent the part-time labor market.

### 3.1 Unemployment, matching, and labor market dynamics

We consider the dynamics of workers’ flows among the four employment states: full-time employment, part-time employment, unemployment looking for a full-time job, and unemployment looking for a part-time job. The total labor force is assumed to be one. In our model, the labor market is segmented into the full-time labor market and the part-time labor market. Also, different from usual labor search models, not only unemployed job searchers but also employed part-time workers are allowed to do on-the-job search and look for a full-time job.

The timing assumptions of the model are summarized in Figure 3. For convenience, we divide one period into two sub-periods, denoted by the reallocation stage and the production stage.

At the beginning of the period, the values of all shocks are revealed. Then the reallocation stage starts, at which point the labor markets open and the job searchers and the vacancy meet through matching functions. At the same time, the job separation takes place and a constant fraction of employed workers lose their jobs. Following GST, we assume that a job searcher who matches a firm in period $t$ starts working immediately.
at the production stage in period $t$, while a worker who is separated in period $t$ is allowed to start job hunting at the reallocation stage in period $t + 1$.\footnote{As will be described in detail below, at the reallocation stage, there are additional (exogenous) reallocations for the unemployed job searchers and the workers who have just separated.}

Next, at the production stage, the employed workers negotiate the contracted wages with the employers (the wholesale firms) and work to produce intermediate goods. It is assumed that the employed full-time workers spend all their time working at the wholesale firm, while the employed part-time workers spend a constant fraction of their time working and the remaining time staying at home.

We turn to describe the evolution of the number of workers in each employment state. Let $n^F_t$ and $n^P_t$ be the number of the employed full-time workers and the employed part-time workers, respectively, \textit{at the production stage} in period $t$. In other words, these are the numbers after all reallocations in period $t$ took place. Let $u^F_t$ be the number of unemployed workers looking for a full-time job and $u^P_t$ be the number of unemployed workers looking for a part-time job \textit{at the beginning of the reallocation stage} in period $t$. These are the numbers before the labor markets open in period $t$. Thus, the number of total unemployed job searchers at the reallocation stage, $u^F_t + u^P_t$, is equal to the number of workers who are not employed at the production stage in the previous period:

$$u^F_t + u^P_t = 1 - n^F_{t-1} - n^P_{t-1}. \quad (6)$$

Let $h^F_t$ and $v^F_t$ be the number of job searchers (including workers conducting on-the-job search) and the aggregate vacancy, respectively, at the market $\ell (= F, P)$. Employed part-time workers are also allowed to search for a full-time job and, in equilibrium, a certain fraction $\varphi_t$ of them joins the full-time labor market. Note that, as will be explained in Section 3.5, the participation decision of the part-time employed workers in the full-time labor market (on-the-job search) is endogenous. Therefore, $h^F_t$ is respectively
given by

$$h^F_t = u^F_t + \varphi_t n^P_{t-1}$$

and

$$h^P_t = u^P_t.$$  

The number of new matches at each labor market $m^\ell_t$ is given by the matching function

$$m^\ell_t = \sigma^\ell_m(h^\ell_t)\sigma(\nu^\ell_t)^{1-\sigma}.$$  

The parameter $\sigma^\ell_m$ represents the matching efficiency. Note that we allow for $\sigma^F_m$ and $\sigma^P_m$ to be different; it is reasonable to think, for example, that the market for part-time workers involves less frictions compared to the market for full-time workers ($\sigma^P_m > \sigma^F_m$).

With this matching function, the probability of filling a vacancy at the market $\ell$, $q^\ell_t = m^\ell_t/\nu^\ell_t$ is decreasing in the market tightness $\theta^\ell_t = \nu^\ell_t/h^\ell_t$. The probability that an unemployed worker finds a job at the market $\ell$ is

$$s^\ell_t = \frac{m^\ell_t}{h^\ell_t} = \sigma^\ell_m(\theta^\ell_t)^{1-\sigma},$$

while the probability that a worker who was a part-time worker in the previous period finds a full-time job is\(^{16}\)

$$s^J_t = \varphi_t \frac{m^F_t}{h^F_t} = \varphi_t s^F_t.$$  

The unemployed pool goes through a job-finding process during the reallocation stage. An unemployed worker who meets with a vacancy starts working immediately. Unemployed workers who did not meet with vacancies stay unemployed. When they stay unemployed, with probability $\xi^\ell$ (for a worker in the market $\ell$) they change the labor market for searching jobs. In sum, unemployed workers in the labor market $\ell$ can have three potential outcomes of this process: (i) find a job (probability $s^\ell_t$), (ii) stay unemployed in the same market (probability $1 - s^\ell_t - \xi^\ell$), and (iii) move to the unemployment in another market (probability $\xi^\ell$). Here, $\xi^F$ and $\xi^P$ are exogenous probabilities of unemployment reallocation: a constant fraction of unemployed workers change the market in which they search for a job. We can interpret this switch as a change in preferences regarding part-time jobs versus full-time jobs.\(^{17}\) An alternative interpretation is that some workers’ skills deteriorate so that they have to apply for a lower-skill job and some

\(^{16}\)Therefore, by construction, $q^F_t \nu^F_t = s^F_t u^F_t + s^J_t u^P_{t-1}$ and $q^P_t \nu^P_t = s^P_t u^P_t$ hold.

\(^{17}\)An alternative formulation would be to allow the workers to choose in which market to search, without resorting to random movements. It turns out that the current formulation allows for a substantially simpler representation of the log-linearized model.
workers successfully re-tool their skills to be able to apply for different types of jobs.

The job separation takes place exogenously. In the reallocation stage, an employed worker with a type $\ell$ job in the previous period loses her job and becomes unemployed with probability $(1 - \rho^\ell)$. As in the case with unemployed job searchers, we consider the unemployment reallocation upon separation of employment: the employed worker who lost her job immediately decides whether she will look for the same type of job she lost or another type of job.$^{18}$ Accordingly, for the full-time employed workers in the previous period, they can (i) stay employed at full-time (probability $\rho^F$), (ii) lose a job and move into the full-time unemployment pool (probability $(1 - \rho^F - \xi^F)$), or (iii) lose a job and move into the part-time unemployment pool (probability $\xi^F$). For the part-time employed workers, they can (i) stay employed at part-time (probability $\rho^P - s^P_\ell$), (ii) move to full-time employment (probability $s^P_\ell$), (iii) lose a job and move into the part-time unemployment pool (probability $(1 - \rho^P - \xi^P)$), or (iv) lose a job and move into the full-time unemployment pool (probability $\xi^P$).

In sum, the law of motion of the employment stocks between the production stage of period $t - 1$ and period $t$ can be expressed as

$$n_{t}^F = \rho^F n_{t-1}^F + s^F_{t} u_{t}^F + s^P_{t} n_{t-1}^P, \quad (7)$$

and

$$n_{t}^P = (\rho^P - s^P_\ell)n_{t-1}^P + s^P_{t} u_{t}^P. \quad (8)$$

The first terms in both equations are the workers who stay at the same employment states, and the second (and the third) terms are new inflows.

On the unemployment side, when the production stage of period $t$ begins (when the reallocation stage ends), there are the number $u_{t+1}^\ell$ of unemployed workers who have decided to look for a job at the market $\ell$ in the next period (i.e., the reallocation stage of period $t + 1$). Their laws of motion between the production stage of period $t - 1$ and period $t$ are given by

$$u_{t+1}^F = (1 - s^F_{t} - \xi^F)u_{t}^F + \xi^P u_{t}^P + (1 - \rho^F - \xi^F)n_{t-1}^F + \xi^P n_{t-1}^P, \quad (9)$$

and

$$u_{t+1}^P = (1 - s^P_{t} - \xi^P)u_{t}^P + \xi^F u_{t}^F + (1 - \rho^P - \xi^P)n_{t-1}^P + \xi^F n_{t-1}^F. \quad (10)$$

The first terms are the ones who did not find a job and stayed in the same labor market.

$^{18}$As we see in Section 3.4, this assumption allows us to treat the newly unemployed workers in the same manner as the existing unemployed workers.
The other terms are inflows. The second term is the inflow from the other unemployment pool. The third and fourth terms are the workers who lost a job in period $t$. For example, in (9), the third term is the workers who lost their full-time jobs and decided to look for a full-time job, while the fourth term is the workers who lost their part-time jobs and decided to look for a full-time job.

### 3.2 Households

We consider a representative household that consists of a continuum of infinitely-lived consumers with the total mass of one. There is a perfect insurance in consumption within a household: the household head collects all resources from the members and optimally allocate them, as in Merz (1995) and Andolfatto (1996). In our setting, this implies equal consumption across consumers within a household. She chooses the sequences of real consumption spending $c_t$, nominal bond holdings $B_t$, real investment expenditure $i_t$, capital utilization $\nu_t$, and physical capital stock $k_t^p$ so as to maximize

$$
\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \varepsilon_{t+s} \log(c_{t+s} - h_c c_{t+s-1}),
$$

where $\mathbb{E}_t$ is the expectation conditional on the information available at period $t$, the parameter $\beta$ is the discount factor and $h_c \in [0, 1)$ is the degree of habit persistence. The intertemporal preference shock $\varepsilon_t^b$ follows the stochastic process

$$
\log(\varepsilon_t^b) = \rho_b \log(\varepsilon_{t-1}^b) + \xi_t^b
$$

where $|\rho_b| < 1$ and $\xi_t^b$ is an innovation independently drawn from the normal distribution with mean zero and variance $\sigma_b^2$.\(^{19}\)

The budget constraint is given by

$$
c_t + i_t + \frac{B_t}{r_t^p p_t} = d_t + r_t^k k_t - \mathcal{A}(\nu_t) k_{t-1}^p + \frac{B_{t-1}}{p_{t-1}} + T_t.
$$

The variable $r_t^p$ is one-period nominal risk-free interest rate, $p_t$ is the price of consumption goods, $d_t$ includes all labor incomes and the flow value from nonworking measured in units of consumption goods, which are described below in detail, $r_t^k$ is the rental rate of capital stock and $k_t$ is the effective capital used for production at period $t$, which is given by $k_t = \nu_t k_{t-1}^p$. The function $\mathcal{A}(\nu_t)$ is the cost of utilization per capital. The steady-state

---

\(^{19}\)Throughout this paper, all other shocks are also assumed to follow the autoregressive models of order 1 with the same stationarity condition and distributional assumption on innovation.
value of $\nu_t$ is 1, and this function satisfies $A(1) = 0$ and $A'(1)/A''(1) = \eta > 0$. The physical capital accumulates according to

$$k_t^p = (1 - \delta)k_{t-1}^p + \left[ 1 - S\left( \frac{\bar{i}_t}{i_{t-1}} \right) \right] \bar{i}_t,$$

where $S(\cdot)$ shows investment adjustment costs which satisfy $S(\gamma_z) = S'(\gamma_z) = 0$ and $S''(\gamma_z) = \eta_k > 0$ under the balanced-growth rate $\gamma_z$. The shock to investment adjustment cost $\varepsilon_t^i$ follows the stochastic process

$$\log(\varepsilon_t^i) = \rho_i \log(\varepsilon_{t-1}^i) + \zeta_t^i.$$

The variable $T_t$ includes the dividends from the firm sector and the lump-sum transfers.

Below, let us describe the components of $d_t$. Let $\mu_b^x \in (0, 1]$ be the working hours of workers with the type $\ell$ $(= F, P)$ job, with an assumption that $\mu_b^p < \mu_b^F = 1$. The total labor income of the household is $w_t^F n_t^F + w_t^p \mu_b^p n_t^P$ where $w_t^F$ and $w_t^p$ represent the average wage of the employed full-time workers and that of the employed part-time workers, respectively. While staying at home, the employed part-time workers receive the amount $(1 - \mu_b^p)b_t^P$ of the flow value from nonworking. Meanwhile, the unemployed workers stay at home all the time and receive the flow value $b_t$. The total amount of the flow value from nonworking becomes $(1 - \mu_b^P)n_t^P b_t^P + (1 - n_t^F - n_t^P)b_t$. Here, the (per hour) flow value from nonworking is allowed to differ between part-time workers and unemployed workers. Using a variable $\bar{\mu}_{b,t}^P$, which is defined as $\bar{\mu}_{b,t}^P \equiv 1 - (1 - \mu_b^P)b_t^P/b_t$, the total flow values from nonworking can be simplified to $(1 - n_t^F - \bar{\mu}_{b,t}^P n_t^P)b_t$. Accordingly, we have

$$d_t = w_t^F n_t^F + w_t^P \mu_b^p n_t^P + (1 - n_t^F - \bar{\mu}_{b,t}^P n_t^P)b_t.$$

### 3.3 Wholesale firms

There is a continuum of wholesale firms indexed by $i \in [0, 1]$. Each firm has two divisions: full-time and part-time division. The full-time (part-time) division owns the employment stock of full-time (part-time) workers $n_{i,t}^F$ $(n_{i,t}^P)$ and, given the capital stock $k_{i,t}^F$ $(k_{i,t}^P)$, produces intermediate goods $y_{i,t}^F$ $(y_{i,t}^P)$. The production functions of full-time

\[\text{Note that } \bar{\mu}_{b,t}^P \text{ is constant when } b_t^P/b_t \text{ is constant over time, which is the case under our assumption below.}\]

\[\text{An alternative interpretation of the model is that there are firms that use only full-time workers and firms that use only part-time workers. We employ the current interpretation because the existing literature suggests that the cyclicality of transitions between } EF \text{ and } EP \text{ is largely accounted for by the movements within the same firm (Borowczyk-Martins and Lalé (2018) and Warren (2017)). In Appendix B.3, we find that about 80% of the transitions between } EF \text{ and } EP \text{ occur within the same firm.}\]
and part-time divisions are respectively given by

\[ y^F_{i,t} = (k^F_{i,t})^\alpha (z_t \mu^F_{i,t} n^F_{i,t})^{1-\alpha}, \]

and

\[ y^P_{i,t} = (k^P_{i,t})^\alpha (z_t \phi_t \mu^P_{i,t} n^P_{i,t})^{1-\alpha}, \]

where \( \alpha \) is the capital share of income and \( z_t \) is the labor productivity with its growth rate \( \varepsilon_t^z = z_t/z_{t-1} \) follows the exogenous stochastic process

\[ \log(\varepsilon_t^z) = (1 - \rho_z) \log \gamma_z + \rho_z \log(\varepsilon_{t-1}^z) + \xi_t^z. \]

The per hour productivity of part-time workers relative to full-time workers \( \phi_t \) exogenously fluctuates according to \( \phi_t = \phi \varepsilon_t^\phi \), where the exogenous shock \( \varepsilon_t^\phi \), which we denote as the part-time labor productivity shock, follows the stochastic process

\[ \log(\varepsilon_t^\phi) = \rho_\phi \log(\varepsilon_{t-1}^\phi) + \zeta_t^\phi. \]

Note that it is allowed that \( \phi \neq 1 \). It is plausible, for example, that \( \phi < 1 \); in this case, part-time workers are less efficient in their capacity, per hour, compared to full-time workers. In this study, we estimate the value for \( \phi \).

In the full-time division, a fraction \( \rho^F \) of employed workers stays employed at the next period, while in the part-time division, a fraction \( \rho^P - s^P_t \) of employed workers in period \( t-1 \) stays employed part-time and a fraction \( s^P_t \) of them starts working full-time in period \( t \). Therefore, the stock of employment at firm \( i \) (at the production stage in period \( t \)) evolves according to the following equations

\[ n^F_{i,t} = (\rho^F + x^F_{i,t}) n^F_{i,t-1} \tag{11} \]

and

\[ n^P_{i,t} = (\rho^P - s^P_t + x^P_{i,t}) n^P_{i,t-1}, \tag{12} \]

where \( x^\ell_{i,t} \) is the hiring rate of type \( \ell \) labor at firm \( i \), defined as \( x^\ell_{i,t} = (q^\ell_t v^\ell_{i,t})/n^\ell_{i,t-1} \). Here, \( v^\ell_{i,t} \) is the vacancy at firm \( i \), which is the firm’s control variable. Alternatively, we can view that the firm decides on the hiring rate, \( x^\ell_{i,t} \).

Now let us describe the optimization problem of the wholesale firms. In this model, intermediate goods are assumed to be homogeneous and thus all their prices are set to be a single value \( p^w_t \). Denote \( w^\ell_{i,t} \) be the nominal wage of type \( \ell \) labor in the firm \( i \). Given
the capital stock, the present value of each division is given by

\[
F^\ell_t(w^n_{i,t}, n_{i,t-1}; k^\ell_t) = \max_{n_{i,t}^\ell} \left[ \frac{w_{i,t}^\ell}{p_t} y_{i,t} - \frac{w_{i,t}^\ell}{p_t} (\mu_0 n_{i,t}^\ell) - \frac{\kappa_t^\ell}{2} (x_{i,t}^\ell)^2 n_{i,t}^\ell + \mathbb{E}_t \left[ \beta_{t,t+1} F^\ell_{t+1}(w^n_{i,t+1}, n_{i,t+1}^\ell; k^\ell_{i,t+1}) \right] \right],
\]

where \( \beta_{t,t+s} \) is the stochastic discount factor between period \( t \) and \( t+s \). The third term in the square bracket represents the hiring cost of new workers. The value of \( \kappa_t^\ell \) controls the hiring cost, and it is formulated as \( \kappa_t^\ell = \kappa^\ell z_t \).

Given the rental rate of capital \( r^k_t \), the firm allocates the capital stock to each division so as to maximize

\[
F^F_t(w^{Fn}_{i,t}, n^{F}_{i,t-1}; k^{F}_t) + F^P_t(w^{Pn}_{i,t}, n^{P}_{i,t-1}; k^{P}_t) - r^k_t(k^F_t + k^P_t).
\]

The optimal level of capital stock satisfies

\[
\rho_t = \frac{p_t w^\ell}{k^\ell_t} = \frac{p_t w^\ell}{k^\ell_t}
\]

and thus all firms choose the same capital-output ratio as well as the same labor-output ratio. Using the marginal product of labor \( a^\ell_{i,t} \), defined as

\[
a^\ell_{i,t} \equiv (1 - \alpha) \frac{y^\ell_{i,t}}{n_{i,t}^\ell} = (1 - \alpha) \frac{y^\ell_{i,t}}{n_{i,t}^\ell} = a^\ell_t,
\]

the optimality condition regarding the hiring rate (or equivalently the vacancy) for the full-time division can be expressed as

\[
k^F_t x^{F}_{i,t} = p_t a^F_t = \frac{w^{Fn}_{i,t} H^F_t}{p_t} + \mathbb{E}_t \left[ \beta_{t,t+1} \frac{\kappa_t^{F}_{i+1}}{2} (x^{F}_{i,t+1})^2 \right] + \rho^F \mathbb{E}_t [\beta_{t,t+1} \kappa_t^{F} x^{F}_{i,t+1}]. \tag{13}
\]

This equation states that the marginal hiring cost of a worker equals the marginal product of labor net out of the wage payment plus the (discounted) marginal benefit from saving the future hiring costs. Similarly, for the part-time division, we have

\[
k^P_t x^{P}_{i,t} = p_t a^P_t = \frac{w^{Pn}_{i,t} H^P_t}{p_t} + \mathbb{E}_t \left[ \beta_{t,t+1} \frac{\kappa_t^{P}_{i+1}}{2} (x^{P}_{i,t+1})^2 \right] + \mathbb{E}_t [\beta_{t,t+1} (\rho^P - s^J_{i+1}) \kappa_t^{P} x^{P}_{i,t+1}]. \tag{14}
\]

For the purpose of formulating the wage bargaining later, we define \( J^\ell_t(w^n_{i,t}) \) as the
value of an additional worker, after the adjustment cost is sunk. That is,
\[
J^\ell_t(w_{i,t}^\ell) = p_t^{\ell} a_t^{\ell} - \frac{w_{i,t}^{\ell} \mu_b^{\ell}}{p_t} + \mathbb{E}_t \left[ \beta_{t,t+1} \left( \frac{\partial F_{t+1}^\ell(w_{i,t+1}^\ell, n_{i,t}^\ell; k_{i,t+1}^\ell)}{\partial n_{i,t}^\ell} \right) \right].
\]

From (13) and (14), this is equivalent to:
\[
J^F_t(w_{i,t}^F) = p_t^{F} a_t^{F} - \frac{w_{i,t}^{F} \mu_b^{F}}{p_t} - \mathbb{E}_t \left[ \beta_{t,t+1} \frac{\kappa_{t,t+1}^F}{2} (x_{i,t+1}^F)^2 \right] + \mathbb{E}_t \left[ (\rho^F + x_{i,t+1}^F) \beta_{t,t+1} J^F_{t+1}(w_{i,t+1}^F) \right]\]

and
\[
J^P_t(w_{i,t}^P) = p_t^{P} a_t^{P} - \frac{w_{i,t}^{P} \mu_b^{P}}{p_t} - \mathbb{E}_t \left[ \beta_{t,t+1} \frac{\kappa_{t,t+1}^P}{2} (x_{i,t+1}^P)^2 \right] + \mathbb{E}_t \left[ ((\rho^P - x_{i,t+1}^P) \beta_{t,t+1} J^P_{t+1}(w_{i,t+1}^P) \right].
\]

Thus \(\kappa_t^\ell x_{i,t} = J^\ell_t(w_{i,t}^\ell)\) holds and the hiring rate is explicitly written as \(x_{i,t} = x_{i,t}^\ell(w_{i,t}^\ell)\).

### 3.4 Workers

Here, we compute the value of a worker in each employment state. With assumptions made above, the value of working full-time at firm \(i\) is given by
\[
V^F_t(w_{i,t}^F) = \frac{w_{i,t}^{F} \mu_b^{F}}{p_t} + \mathbb{E}_t \left[ \beta_{t,t+1} \left( \rho^F V^F_{t+1}(w_{i,t+1}^F) + \xi^F U^P_{t+1} + (1 - \rho^F - \xi^F) U^F_{t+1} \right) \right]
\]
and the value of working part-time at firm \(i\) is given by
\[
V^P_t(w_{i,t}^P) = \frac{w_{i,t}^{P} \mu_b^{P}}{p_t} + (1 - \mu_b^{P}) b_t^{P} + \mathbb{E}_t \left[ \beta_{t,t+1} \left( \rho^P V^P_{t+1}(w_{i,t+1}^P) + \xi^P U^F_{t+1} + (1 - \rho^P - \xi^P) U^P_{t+1} + \Delta_{t+1} \right) \right],
\]

where \(\Delta_t\) is the option value of being able to participate in the on-the-job search (the determination of \(\Delta_t\) will be detailed in Section 3.5). Here, \(U^F_t\) and \(U^P_t\) are the values of being unemployed at the full-time and part-time labor markets, respectively. They are given by
\[
U^F_t = b_t + \mathbb{E}_t \left[ \beta_{t,t+1} \left( s_{t+1}^F V^F_{x,t+1} + \xi^F U^P_{t+1} + (1 - s_{t+1}^F - \xi^F) U^F_{t+1} \right) \right]
\]
and
\[
U^P_t = b_t + \mathbb{E}_t \left[ \beta_{t,t+1} \left( s_{t+1}^P V^P_{x,t+1} + \xi^P U^F_{t+1} + (1 - s_{t+1}^P - \xi^P) U^P_{t+1} \right) \right],
\]
where

\[ V_{x,t} = \int_0^1 V_t^\ell(w_{i,t}^F) \frac{x_{i,t}^F n_{i,t}^F}{x_{i,t-1}^F n_{i,t-1}^F} \, di \]

is the average value of working, conditional on being a newly employed worker at the market \( \ell \) (= \( F, P \)). Note that the worker who moves into a particular labor market due to the job separation is in the same position as the previously unemployed worker, because they also will face the possibility of unemployment reallocation during the period.

As in GST, the present value from nonworking \( b_t \) and \( b_P^t \) evolve proportionally to physical capital: \( b_t = b_k^t \) and \( b_P^t = b_k^t k_P^t \) for \( b > 0 \) and \( b_P > 0 \). In the subsequent quantitative exercise, we assume that for the unemployed workers searching a job in either market is indifferent along the balanced-growth path.

For the purpose of wage bargaining below, we obtain the surpluses from being employed at firm \( i \), \( H_t^F(w_i^F) = V_t^F(w_i^F) - U_t^F \) and their averages conditional on being a newly employed worker, \( H_{x,t}^\ell = V_{x,t}^\ell - U_t^\ell \). From (15), (16), (17), and (18), they are given by

\[
H_t^F(w_i^F) = \frac{w_{i,t}^F \mu_b^F}{p_t} - b_t + \mathbb{E}_t \left[ \beta_{t+1} \left( \rho^F H_{t+1}^F(w_{i,t}^F) - s_{t+1}^F H_{x,t+1}^F \right) \right]
\]

and

\[
H_t^P(w_i^P) = \frac{w_{i,t}^P \mu_b^P}{p_t} - b_t + \mathbb{E}_t \left[ \beta_{t+1} \left( \rho^P H_{t+1}^P(w_{i,t}^P) - s_{t+1}^P H_{x,t+1}^P + \Delta_{t+1} \right) \right].
\]

### 3.5 On-the-job search

The workers who were employed part-time at period \( t - 1 \) (population \( n_{P,t-1}^P \)) decides whether to conduct on-the-job searches for a new full-time job (unlike the usual on-the-job search, this search is conducted within the firm) at the outset of period \( t \), before the labor market at period \( t \) opens. They make that decision after observing aggregate shocks at period \( t \). They also observe their idiosyncratic disutility, which is assumed to be an i.i.d. shock for participating in the market at period \( t \), \( \gamma_{c,t} \). We assume that \( \gamma_{c,t} = \gamma_c w_{t-1}^P \), where \( \gamma_c \) is the idiosyncratic component independently drawn from a logistic distribution with mean \( \mu_y \) and scaling parameter \( \omega^{-1} > 0 \). The time component \( w_{t-1}^P \) implies that the opportunity cost of an on-the-job search at the beginning of period is proportional to the average value of the wage the part-time workers received in the previous period.\(^{22}\)

\(^{22}\)As we will mention below, in equilibrium, all part-time workers receive the same wage (i.e., \( w_{i,t}^F = w_i^P \)) and thus this assumption implies that the opportunity cost is proportional to the wage the worker agreed on in the previous period.
Conditional on the participation, the job searcher can find a new full-time job with probability \( s^F_t \) and therefore the benefit of conducting an on-the-job search is \( s^F_t (V^F_{x,t} - V^P_{x,t}) \), which we denote \( R_t \). Accordingly, the probability that a part-time employee participates in the full-time labor market is given by

\[
\varphi_t = \text{Prob}(s^F_t (V^F_{x,t} - V^P_{x,t}) - \gamma_{c,t} \geq 0) = G_{\gamma_c} \left( \frac{R_t}{w^P_{t-1}}; \mu_g, \omega^{-1} \right),
\]

(19)

where \( G_{\gamma_c}(y; \mu_g, \omega^{-1}) \) is the cumulative distribution function of the logistic random variable \( \gamma_c \). In (19), \( R_t/w^P_{t-1} \) is thus the participation threshold. As shown in Appendix C, the option value of being able to participate in the on-the-job search is expressed by

\[
\tilde{\Delta}_t = \mathbb{E} \left[ \max \{s^F_t (V^F_{x,t} - V^P_{x,t}) - \gamma_{c,t}, 0 \} \right] = R_t - w^P_{t-1} \left[ \mu_g + \omega^{-1} \log(\varphi_t) \right].
\]

Given this specification, the parameter \( \omega \) determines the sensitivity of the participation probability to the change in the aggregate state. As shown in Appendix C, log-linearization of (19) is

\[
\tilde{\varphi}_t = \omega (1 - \varphi) \frac{\bar{R}_{\gamma_z}}{\bar{w}^P} \left( \bar{R}_t + \tilde{\varepsilon}_t^z - \tilde{w}^P_{t-1} \right),
\]

(20)

where the variables with tilde (\( \tilde{\cdot} \)) denote the percentage deviation from their deterministic balanced-growth steady-state value. The parameters \( \bar{R} \) and \( \bar{w}^P \), which are the balanced-growth steady-state values for \( R_t \) and \( w^P_t \) respectively, satisfy the steady-state condition for (19):

\[
\varphi = G_{\gamma_c}(\bar{R}_{\gamma_z}/\bar{w}^P; \mu_g, \omega^{-1}).
\]

(21)

Below, we treat \( \varphi \) and \( \omega \) as parameters and set \( \mu_g(\omega, \varphi) = \bar{R}_{\gamma_z}/\bar{w}^P + \omega^{-1} \log(1/\varphi - 1) \), so that (21) holds.\(^{23}\) Therefore, holding \( \varphi \) constant, a high value for \( \omega \) implies a high elasticity of the participation of an on-the-job search with respect to a change in the participation threshold. In other words, \( \omega \) governs susceptibility of intensity of an on-the-job search by part-time workers to the variation of the macroeconomic conditions.

Note that \( \mathbb{E} [\gamma_c] = \bar{R}_{\gamma_z}/\bar{w}^P + \omega^{-1} \log(1/\varphi - 1) \) and \( \text{Var}(\gamma_c) = \pi^2/(3\omega^2) \). Hence, \( \mathbb{E} [\gamma_c] \to \bar{R}_{\gamma_z}/\bar{w}^P \) and \( \text{Var}(\gamma_c) \to 0 \) as \( \omega \to \infty \). This means that the larger \( \omega \) is, the more part-time workers in the vicinity of the steady-state participation threshold \( (\bar{R}_{\gamma_z}/\bar{w}^P) \) and thus more part-time workers change their participation decision as the

23This is possible because the steady-state values for \( \bar{R} \) and \( \bar{w}^P \) are affected by \( \mu_g \) and \( \omega \) only indirectly through \( \varphi \). Hence, holding \( \varphi \) constant, the values for \( \bar{R} \) and \( \bar{w}^P \) are also constant. See Appendix F for the expressions for \( \bar{R} \) and \( \bar{w}^P \).
macroeconomic conditions change. (See also Figure C.1 for this intuition.)

3.6 Nash bargaining and wage dynamics

3.6.1 Full-time workers

The wages of full-time workers are determined by the staggered multi-period Nash bargaining à la Gertler and Trigari (2009), in which the wages can be negotiated in a firm with probability $1 - \theta_w^F$. The non-negotiation firms adjust their wages following a partial indexation rule:

$$w_{i,t}^{Fn} = \gamma_z(\pi)^{1-\tau_w}(\pi_{t-1})^{\tau_w}w_{i,t-1}^{Fn},$$

where $\pi_t \equiv p_t/p_{t-1}$ is the gross inflation, $\pi$ is its steady-state value, and $\tau_w \in [0,1]$ is the degree of indexation.

The newly contracted wage at period $t$ is the outcome of the Nash bargaining:

$$\text{arg max}_{w_{i,t}^{Fn}} \ H^F_t(w_{i,t}^{Fn}) \ J^F_t(w_{i,t}^{Fn})^{1-\eta^F_t}$$

subject to

$$w_{i,t+k}^{Fn} = \begin{cases} 
\gamma_z(\pi)^{1-\tau_w}(\pi_{t-1})^{\tau_w}w_{i,t+k-1}^{Fn} & \text{with probability } \theta_w^F \\
 w_{i,t+k}^{*Fn} & \text{with probability } 1 - \theta_w^F, 
\end{cases}$$

for $k \geq 1$. Here, $\eta^F_t = \eta^F \varepsilon^F_t$ is the full-time worker’s relative bargaining power, where $\eta^F \in (0,1)$ and $\varepsilon^F_t$ follows the stochastic process

$$\log(\varepsilon^F_t) = \rho_{\eta^F} \log(\varepsilon^F_{t-1}) + \zeta^F_t.$$

As in GST, we obtain the log-linearized equation on the dynamics of the average real wage for the full-time workers, which is given by $w^F_t = \int_0^1 w^F_{i,t} \left( \frac{n_{i,t}^F}{n_t^F} \right) \, di$, as follows:

$$\tilde{w}_t^F = \gamma_b^F \left( \tilde{w}_{t-1}^F - \tilde{\pi}_t + \tau_w \tilde{\pi}_{t-1} - \varepsilon^F_t \right) + \gamma_o^F \tilde{w}_t^{o,F} + \gamma_f^F \tilde{w}_t^{F} \tilde{w}_{t+1}^F \tilde{\pi}_{t+1} - \tau_w \tilde{\pi}_t + \tilde{\varepsilon}^F_{t+1},$$

where $\tilde{w}_t^{o,F}$ is the (log-deviation of) wage that would be chosen if all employed full-time workers and firms were allowed to negotiate wages period-by-period, which is called the economy-wide target wage by GST. Also, the parameters in the coefficients are all positive and satisfy $\gamma_b^F + \gamma_o^F + \gamma_f^F = 1$, which imply that the average wage is the weighted average of the backward looking component, the forward looking component, and the
target wage. Note that $\gamma^F_o$ decreases in the Calvo parameter $\theta^F_w$ and $\gamma^F_o \rightarrow 1$ as $\theta^F_w \rightarrow 0$. See Appendix G for the explicit expressions of $\bar{\nu}^F_t$ and the three parameters.

### 3.6.2 Part-time workers

The wages of part-time workers are perfectly flexible and determined by period-by-period Nash bargaining. They are set to maximize the Nash product $H^P_t(w^F_t^n)\eta^P_t J^P_t(w^F_t^n)^{1-\eta^F_t}$. Here, the part-time worker’s relative bargaining power is given by $\eta^P_t = \eta^P \varepsilon^P_t$ where $\eta^P \in (0,1)$ and $\varepsilon^P_t$ follows the stochastic process

$$\log(\varepsilon^P_t) = \rho^P \log(\varepsilon^P_{t-1}) + \zeta^P_t.$$

Note that the wages of part-time workers are homogeneous across wholesale firms and hence $H^P_t(w^F_t^n) = H^P_{x,t}$ and $V^P_t(w^F_t^n) = V^P_{x,t}$ for all $i \in [0,1]$. See Appendix D for the explicit derivation of the wage function for the part-time worker.

### 3.7 Retailers

There is a continuum of monopolistically competitive retailers indexed by $j \in [0,1]$. Retailers purchase the intermediate good from the wholesale firms and transform it into retail goods and sell them to the households. One unit of the intermediate good is used to produce one unit of a retail good. Retailers set their own prices. We assume that retail prices are sticky.

As in GST, we adopt the Kimball (1995) formulation of product differentiation, which is a generalization of the Dixit-Stiglitz formulation. Let $y_{j,t}$ be the quantity sold by the retailer $j$. The composite final goods, whose quantity is $y_t$, is implicitly defined by

$$\int_0^1 G\left(\frac{y_{j,t}}{y_t} ; \varepsilon^P_t\right) dj = 1,$$

where the demand aggregator $G(\cdot)$ function satisfies $G(1) = 1$, $G'(\cdot) > 0$, and $G''(\cdot) < 0$. The markup shock $\varepsilon^P_t$, which appears in the aggregator, follows the stochastic process

$$\log(\varepsilon^P_t) = (1 - \rho^P) \log(\varepsilon^P) + \rho^P \log(\varepsilon^P_{t-1}) + \zeta^P_t.$$

Under this specification, given the relative price $p_{j,t}/p_t$, the demand curve for the retail goods $j$ is given by

$$y_{j,t} = G'^{-1}\left(\frac{p_{j,t}}{p_t} \tau_t\right) y_t,$$

where $\tau_t \equiv \int_0^1 G'\left(\frac{y_{j,t}}{y_t}\right) \frac{y_{j,t}}{y_t} dj$. 

23
As in CEE, we consider the Calvo price adjustment; a fraction $1 - \theta_p$ of retailers is allowed to optimally choose its price and the other retailers reset its price following the partial indexation rule $p_{jt} = (\pi)^{1-\phi_p}(\pi_{t-1})^{\phi_p}p_{jt-1}$. With these assumptions, the law of large numbers implies the evolution of the price index:

$$ p_t = (1 - \theta_p)p^*_t G^{t-1} \left( \frac{p^*_t}{p_t} \right) + \theta_p \left[ (\pi)^{1-\phi_p}(\pi_{t-1})^{\phi_p}p_{t-1} G^{t-1} \left( \frac{(\pi)^{1-\phi_p}(\pi_{t-1})^{\phi_p}p_{t-1}}{p_t} \right) \right], $$

where $p^*_t$ is the target price that maximizes the present discounted value of future profits

$$ E_t \sum_{s=0}^{\infty} (\theta_p)^s \beta_{t,t+s} \left[ \frac{P_{jt}}{P_{t+s}} \left( \prod_{k=1}^{s} (\pi)^{1-\phi_p}(\pi_{t+k-1})^{\phi_p} \right) - P_{t+s}^w \right] G^{t-1} \left( \frac{P_{jt}}{P_{t+s}} \right) y_{t+s}. $$

### 3.8 Government

The government conducts the monetary policy based on the Taylor rule:

$$ r^v_t = \left( r^v_{t-1} - r^n \right) \phi_r \left[ \left( \frac{\pi_t}{\pi} \right) \phi_\pi \left( \frac{y_t}{y_{nt}} \right) \phi_y \right]^{1-\phi_r} \varepsilon^r_t, $$

where $y_{nt}$ is the natural level of output, $r^n$ is the steady-state nominal interest rate and $\phi_r$, $\phi_\pi$, and $\phi_y$ are positive policy parameters. The monetary policy shock $\varepsilon^r_t$ follows

$$ \log(\varepsilon^r_t) = \rho_r \log(\varepsilon^r_{t-1}) + \zeta^r_t. $$

The fiscal authority sets the government spending $g_t$ as a fraction $\zeta_t$ of aggregate output: $g_t = \zeta_t y_t$. The government spending shock defined by $\varepsilon^g_t = 1/(1 - \zeta_t)$ follows

$$ \log(\varepsilon^g_t) = (1 - \rho_g) \log(\varepsilon^g) + \rho_g \log(\varepsilon^g_{t-1}) + \zeta^g_t. $$

### 4 Quantitative experiments

In this section, we estimate the model using Bayesian methods and investigate the key factor that accounts for labor market fluctuations. To estimate the model in the linear state space form, we approximate the model equations using the first order perturbation in logs, which are presented in Appendix G.
Table 3: Calibrated parameters

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<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\xi$</th>
<th>$\zeta$</th>
<th>$\mu^F_b$</th>
<th>$\sigma$</th>
<th>$s^F$</th>
<th>$s^P$</th>
<th>$\rho^F$</th>
<th>$\rho^P$</th>
<th>$s^J$</th>
<th>$\xi^F$</th>
<th>$\xi^P$</th>
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<tbody>
<tr>
<td>0.33</td>
<td>0.996</td>
<td>0.025</td>
<td>10</td>
<td>0.24</td>
<td>0.501</td>
<td>0.5</td>
<td>0.698</td>
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<td>0.953</td>
<td>0.928</td>
<td>0.056</td>
<td>0.027</td>
<td>0.059</td>
</tr>
</tbody>
</table>

4.1 Dataset

We use nine quarterly series over the sample period from 1979:Q1 to 2016:Q4. In addition to commonly-used set of variables in the literature of medium-scale DSGE model estimation, including (i) per capita real GDP, (ii) per capita real personal consumption expenditures, (iii) per capita real investment, (iv) the real hourly compensation, (v) inflation, (vi) the employment rate, and (vii) the federal funds rate; we utilize the information on (viii) the ratio of the full-time employment rate to the part-time employment rate and (ix) the ratio of the real hourly wage for the full-time workers to that for the part-time workers. See Appendix E for the detailed description of the data and the corresponding observation equations.

4.2 Estimation strategy

We first calibrate some of the parameters. Then we use the Markov chain Monte Carlo methods to compute the posterior distributions of the remaining parameters and the parameters that characterize the stochastic processes.

4.2.1 Calibration

Table 3 summarizes the calibrated parameters. One period corresponds to one quarter. Following GST, we assign values of 0.33 to the capital share $\alpha$, 0.996 to the subjective discount factor $\beta$, 0.025 to the capital depreciation rate $\delta$, and 10 to the curvature of the Kimball aggregator $\xi$. The output share of external demands $\zeta$ is set to 0.24, which is the sample average of the GDP share of external demands. The relative working hours of a part-time employee $\mu^F_b$ is set to 0.501 to be consistent with the empirical evidence from the CPS that the median of usual hours worked by part-time workers is 50.1 percent of those by full-time workers. The elasticity of new matches with respect to labor market tightness, $\sigma$, is set to 0.5. Based on the transition probability matrix, calculated in Section 2, we assign values to the steady-state values of the job finding probabilities for the part-time workers. See Appendix E for the detailed description of the data and the corresponding observation equations.

24This choice is within the range of empirical values reported by Petrongolo and Pissarides (2001). This parameter value is also similar to the values used in existing studies of frictional labor market. For example, Blanchard and Diamond (1989), Andolfatto (1996), and Merz (1995) use 0.4; and Shimer (2005) uses 0.72.
the unemployed workers, \( s^F \) and \( s^P \). The steps are as follows: We first compute the average of the quarterly transition probability matrix \( \bar{Q} \) between 1996 and 2016

\[
\bar{Q} = \begin{bmatrix}
Q_{EF,EF} & Q_{EF,EP} & Q_{EF,UF} & Q_{EF,UP} & Q_{EF,O} \\
Q_{EP,EF} & Q_{EP,EP} & Q_{EP,UF} & Q_{EP,UP} & Q_{EP,O} \\
Q_{UF,EF} & Q_{UF,EP} & Q_{UF,UF} & Q_{UF,UP} & Q_{UF,O} \\
Q_{UP,EF} & Q_{UP,EP} & Q_{UP,UF} & Q_{UP,UP} & Q_{UP,O} \\
Q_{O,EF} & Q_{O,EP} & Q_{O,UF} & Q_{O,UP} & Q_{O,O}
\end{bmatrix},
\]

where \( \bar{Q}_{i,j} \) is the average of quarterly transition probability from a employment state \( i \) to state \( j \)\(^{25} \). Let \( Q_{i,j} \) be the transition probability conditional on staying in the labor force:

\[
Q_{i,j} = \frac{Q_{i,j}}{\sum_{j' \in \{EF,EP,UF,UP\}} Q_{i,j'}} \quad \text{for } i, j \in \{EF,EP,UF,UP\}.
\]

Then, the job finding probabilities are set to be

\[
s^F = Q_{UF,EF} + Q_{UF,EP} = 0.495 + 0.202,
\]

and

\[
s^P = Q_{UP,EF} + Q_{UP,EP} = 0.341 + 0.485.
\]

The remaining five parameters are: the job survival probability for the full-time workers \( \rho^F \); that for the part-time workers \( \rho^P \); the steady-state value for the job finding probability of an employed part-time worker \( s^J \); and the unemployment reallocation probabilities \( \xi^F \) and \( \xi^P \). We target the five statistics on the long-run labor market dynamics: (i) the steady-state value of the labor force share of full-time employment is 76.29\%,\(^{26} \) (ii) the steady-state value of the labor force share of part-time employment is 17.32\%, (iii) the ratio of the two job survival probabilities matches the empirical counterpart, i.e.,

\[
\frac{\rho^F}{\rho^P} = \frac{Q_{EF,EF} + Q_{EF,EP}}{Q_{EP,EF} + Q_{EP,EP}} = 1.027,
\]

(iv) the transition probability from full-time employment to full-time unemployment matches the empirical counterpart, i.e.,

\[
1 - \rho^F - \xi^F = Q_{EF,UF} = 0.020,
\]

\(^{25}\)We obtain the quarterly transition probability matrix by multiplying the monthly transition probabilities of three consecutive months.

\(^{26}\)See Appendix F.1 for the derivation of the steady-state value of the labor force share.
and (v) the transition probability from part-time employment to part-time unemployment matches the empirical counterpart, i.e.,

$$1 - \rho^P - \xi^P = Q_{EP,UP} = 0.013.$$ 

As a result, we set $\rho^F = 0.953$, $\rho^P = 0.928$, $s^F = 0.056$, $\xi^F = 0.027$, and $\xi^P = 0.059$.\textsuperscript{27}

4.2.2 Priors

The specification of the prior distributions for the estimated parameters is summarized in the first block of columns in Table 4. Since the main feature of our model is the dual labor market structure, we describe our choice of priors for the labor market parameters in detail. For the remaining structural parameters and the exogenous processes parameters, our priors are broadly in line with those used in the literature e.g., An and Schorfheide (2007), SW, and Justiniano, Primiceri, and Tambalotti (2010).\textsuperscript{28} For convenience, let $\tilde{b}^F$ denote to be the relative flow value from unemployment to the marginal contribution of a full-time worker to firms, defined as

$$\tilde{b}^F = \frac{\bar{b}}{p^w a^F + \beta(\kappa^F/2)(x^F)^2},$$

where $\bar{x}$ denotes the steady-state value of the detrended variable $x_t/z_t$.

There are seven estimated labor market parameters: the workers’ surplus share in wage bargaining $\eta^F$ and $\eta^P$; the relative flow value $\tilde{b}^F$; the relative per hour productivity of part-time workers $\phi$; the scaling parameter of the idiosyncratic shock to the part-time worker $\omega$; the Calvo parameter regarding the nominal wage for full-time workers $\theta^F_{w}$; and the degree of indexation of non-negotiated wages $\tau_{w}$.

The bargaining power parameters and the relative flow value of unemployment play an important role in determining the cyclical behavior of labor market flows. We choose a Beta prior with mean 0.5 (which is the value often used in a calibration studies) for $\eta^F$ and $\eta^P$. Various values for $\tilde{b}^F$ are used in calibration studies: Shimer (2005) uses 0.4 based on the income replacement rate of unemployment benefits in the United States, while Hall and Milgrom (2008) and Hall (2009) argue that this parameter should include non-monetary benefits from nonworking as well.\textsuperscript{29} Recent empirical studies which

\textsuperscript{27}This implies that 8 percent of employed part-time workers conduct on-the-job search ($\varphi = 0.079$).

\textsuperscript{28}For the elasticity of the capital adjustment cost function $\eta_k$, we follow SW and GST and estimate a parameter $\psi = 1/(1 + \eta_k) \in (0, 1)$. In addition, for the steady-state growth rate (100 log $\gamma_z$), the steady-state nominal interest rate (100 log $r^n$), and the steady-state inflation (100 log $\pi$), the prior means are set based on the respective sample average.

\textsuperscript{29}Chodorow-Reich and Karabarbounis (2016) decompose the relative flow value from unemployment
estimate DSGE models with frictional labor markets tend to produce a value larger than the value used in Shimer (2005). For example, GST’s estimated value is 0.726 and Christiano, Eichenbaum, and Trabandt (2016) estimate 0.88 under the Nash bargaining framework. Considering these recent estimates, we set a Beta prior with mean 0.7.

We set a uniform prior over the unit interval for $\phi$ since this study is the first attempt to estimate a DSGE model with part-time labor market and there is little empirical evidence on the difference in productivity between full-time workers and part-time workers. The parameter $\omega$ has a support on the semi-infinite interval $[0, \infty)$ and thus we select a Gamma prior that covers a wide range of values. For $\theta_w^F$ and $\epsilon_w$, we set a Beta prior with mean 0.75, considering the argument for a substantial degree of wage rigidities in the quantitative New Keynesian literature e.g., CEE, SW and others.\footnote{This choice is also in line with the micro data evidence in Barattieri, Basu, and Gottschalk (2014). They report that the probability of a nominal wage change is around 25 percent per quarter.}

### 4.3 Estimation Results

#### 4.3.1 Parameter Estimates

The posterior means and the 5th and 95th percentiles are reported in the second block of columns in Table 4.\footnote{The posteriors are obtained by the random walk Metropolis-Hastings algorithm based on 500,000 replications with the first 250,000 draws discarded. The parameter scaling the proposal distribution in the algorithm is set such that the average acceptance rates become around 25 percent.} Our estimates of non labor market parameters are in line with the estimates of the previous studies, except that the estimates of $h_c$ and $\epsilon_p$ are somewhat larger. Regarding the wage bargaining parameters, the estimated surplus share of the full-time workers is larger than that of the part-time workers ($\eta^F = 0.90$ and $\eta^P = 0.72$). These values are in line with GST’s result using a DSGE model with a single labor market in which the estimate of the workers’ surplus share is 0.9. Our posterior mean estimate of $\tilde{b}^F$ is 0.93, which is close to the estimate in Christiano, Eichenbaum, and Trabandt (2016). The estimate of $\phi$ is 0.56, which implies that hourly productivity of a part-time worker is about half relative to a full-time worker. This result is consistent with the facts documented by Canon et al. (2014) that part-time workers tend to be less educated than full-time workers and the share of part-time employment tends to be higher in low-skill occupations. The quarterly probability of a wage renegotiation for the full-time workers is around 55 percent ($\theta_w^F = 0.44$). To some extent, this frequency of a wage revision is high compared to ones adopted in the quantitative New Keynesian literature.

\[\text{into public benefits and the value of nonworking time and estimate cyclical patterns of each component.}\]
Table 4: Prior and posterior distributions

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
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<td>Elasticity in utilization rate</td>
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<td>$\sigma_6$</td>
<td>Part-time labor productivity</td>
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</table>

Note: This table reports the prior and posterior distributions of the estimated parameters. The Inverse Gamma distribution is denoted by ‘IG’. The parameters $z_*, r^n_*$, and $\pi_*$ denote $z_* = 100 \log z$, $r^n_* = 100 \log r^n$, and $\pi_* = 100 \log \pi$, respectively.
Figure 4: Bayesian impulse response functions of full-time and part-time employment

Note: This figure shows the posterior medians of impulse response functions of full-time employment with solid thick lines and those of part-time employment with broken thick lines. The thin lines represent the 5th and 95th percentiles. ‘Investment’ represents the investment adjustment cost shock. The magnitude of each shock is set to be the one standard deviation.

4.3.2 Estimate of impulse response functions

Let us examine the impulse responses of the employment stocks in the estimated model. In Figure 4, our model is capable of producing asymmetric responses between the full-time and part-time employment stocks to a large variety of the shocks. It is worth noting that such asymmetric responses are the results of the standard aggregate shocks (the six panels in the top and middle) as well as the shocks that are specific to each type of labor (the three panels in the bottom). For example, a positive technology shock (an unanticipated rise in the technology growth rate) increases full-time employment and decreases part-time employment; an adverse investment adjustment cost shock (an unanticipated deterioration in the investment efficiency) increases part-time employment and decreases full-time employment; and a contractionary monetary policy shock (an unanticipated rise in the nominal interest rate) increases part-time employment and decreases full-time employment.

Explaining the asymmetric responses to the bargaining power shocks and the part-
time labor productivity shock is almost trivial since these shocks are specific to each type of labor. In contrast, explaining the asymmetric responses to the other aggregate shocks is not necessarily straightforward. To cast light on the mechanism behind the asymmetric responses, we then decompose the impulse responses of the employment stocks into the contribution of the net UE flow and the net EE flow. Here, the net UE flow represents the net movement of workers between the unemployment pool and each stock of employment, and the net EE flow represents the net movement of workers across different types of employment. Note that the expression of “EE” should not be confused with the conventional job-to-job transitions; as we have emphasized, the large majority of the movements between full-time jobs and part-time jobs occur within the same firm, and they do not show up in the usual measurement of job-to-job transitions.

4.3.3 Decomposing the impulse response functions

We formally describe the procedure to decompose the impulse response functions of the employment stocks into the response of the net UE flow and the net EE flow.

From the evolution of the employment stocks (7) and (8), the rate of change of each employment stock is respectively given by

$$\frac{n_t^F - n_{t-1}^F}{n_{t-1}^F} = \left[ s_t^F u_t^F \right] + \left[ s_t^F n_{t-1}^F \right] \equiv x_{UE,t}^F + x_{EE,t}^F$$  \hspace{0.5cm} (22)

and

$$\frac{n_t^P - n_{t-1}^P}{n_{t-1}^P} = \left[ s_t^P u_t^P \right] + \left[ -s_t^P \right] \equiv x_{UE,t}^P + x_{EE,t}^P.$$  \hspace{0.5cm} (23)

Here, $x_{UE,t}^F$ ($x_{UE,t}^P$) is the net inflow of workers from the unemployment pool to the full-time (part-time) employment pool between period $t - 1$ and $t$, divided by the number of workers in the full-time (part-time) employment pool at period $t - 1$. Similarly, $x_{EE,t}^F$ ($x_{EE,t}^P$) is the net inflow of workers from part-time (full-time) employment pool to the full-time (part-time) employment pool between period $t - 1$ and $t$, divided by the number of workers in the full-time (part-time) employment pool at period $t - 1$.

The log-linearization of (22) and (23) yields

$$\tilde{n}_t^\ell - \tilde{n}_{t-1}^\ell = x_{UE,t}^\ell \tilde{x}_{UE,t}^\ell + x_{EE,t}^\ell \tilde{x}_{EE,t}^\ell \quad \text{for } \ell = F, P.$$
where $\tilde{x}_t = \log(x_t/x)$. Hence, the response of the employment stock at $t$ periods after the realization of a shock is given by

$$n_t^\ell = \sum_{s=0}^{t} x_{UE}^{\ell} \tilde{x}_{UE,t-s} + \sum_{s=0}^{t} x_{EE}^{\ell} \tilde{x}_{EE,t-s}.$$

This means that the response of the employment stocks can be decomposed into the cumulative sum of the change in flows between employment and unemployment and the change in flows between full-time and part-time employment.

We apply this decomposition to the impulse responses for a positive technology shock in Figure 5.\textsuperscript{32} Both the net $UE$ flow and the net $EE$ flow contribute to elevating the full-time employment (left panel) immediately after the shock. In comparison, for the part-time employment (right panel), the net $UE$ flow and the net $EE$ flow contribute

\textsuperscript{32}The parameter values for the estimated parameters are fixed at the posterior mean.
in opposite directions. While the net UE flow puts an upward pressure on the part-time employment, the net EE flow puts a downward pressure on it. Our estimation suggests that the latter dominates, and consequently the part-time employment decreases in response to the shock.

As shown in Figure 6, a similar mechanism also works for the case of an investment adjustment cost shock. An increase in the adjustment cost decreases the net inflow from unemployment pool to employment pool for both types of jobs. Meanwhile, the net inflow from part-time employment to full-time employment decreases because such a shock discourages part-time workers from moving to full-time jobs.

4.4 Counterfactual experiments

To further evaluate the importance of incorporating the procyclical direct transition from part-time employment to full-time employment, we conduct a counterfactual analysis using an alternative model specification in which the job finding probability of an employed part-time worker $s_t^J$ is constant over time. That is, we assume that $s_t^J = s^J$, where $s^J$ is the probability in the balanced-growth steady state of the estimated model.\(^{33}\)

Throughout the counterfactual experiments in this section, we call the estimated model ‘benchmark’ and fix the values for the estimated parameters at the posterior mean.

In the alternative model, it is assumed that part-time workers are randomly assigned to the on-the-job search every period and all part-time employed workers are obliged to pay a fixed cost $C_w^P t = 1$ instead of the idiosyncratic cost (disutility). This fixed payment is introduced to equalize the ex-ante expected value of participating in the on-the-job search between the benchmark and the alternative on the balanced-growth path. With the assumptions above, the value in the alternative is given by $\Delta^A = s^J (H_x^F - H_x^P) - C(\bar{w}^P / \gamma_z)$ and thus $C$ is chosen to be $s^J (\bar{H}_x^P - \bar{H}_x^P) - C(\bar{w}^P / \gamma_z) = (\bar{w}^P / \gamma_z) \log(1 - \varphi) / \omega$.

4.4.1 The role of time-varying EE transitions within employment stocks

In Figure 7, we compare the impulse responses to a positive technology shock in the alternative model (dashed lines) with those in the benchmark (solid lines). This figure sheds light on the role of the procyclical direct transition channel from part-time to full-time in the labor market cyclical dynamics. First, in the counterfactual case based on the alternative model, due to the lack of the EE fluctuations, the movement of full-

---

\(^{33}\)Recall that $s_t^J = \varphi_t s_t^F$ holds. Thus the probability that a part-time employee conducts on the job search can be “backed out” from $\varphi_t = s^J / s_t^F$ in this case.
Figure 7: Impulse response function to a positive technology shock

Note: This figure shows the impulse responses of the full-time employment rate, the part-time employment rate, the unemployment rate, and the growth rate of the average wage to a positive technology shock of one standard deviation.

time employment is dampened and the movement of part-time employment is almost nonexistent. This is because the time-varying $EE$ transition strengthens the movement of full-time employment, and it is the main driving force of the part-time responses. Second, in the counterfactual case, the response of the unemployment rate is strengthened by about 4 percentage points (the largest decline is 7 percent in the benchmark while it is 11 percent in the counterfactual case). The difference corresponds to 0.25 percentage points in terms of the unemployment rate.\footnote{Recall that the “percentage points” in the figure is the deviation from the steady-state values.} Third, because of the stronger composition effect, the benchmark involves a larger response of the aggregate wage (the average of hourly wages for full-time and part-time workers, weighted by their employment stocks) than the counterfactual case. In the benchmark, in response to a positive technology shock, an employed part-time worker is more likely to become a full-time worker. Since the hourly wage for a full-time worker is higher than that for a part-time worker, this movement increases the aggregate wage.

In Appendix H.1, we show that similar results can be observed for the case of other
Figure 8: Cross correlogram with the real GDP

Note: This figure shows the correlations between the cyclical components of real GDP and the full-time (part-time) employment rate $h$ quarters lagged in the left (right) panel. The horizontal axis is the number of lags, denoted by $h$. The cyclical components of real GDP is measured in terms of log-deviations of per capita real GDP from its HP filter trend. The solid lines without marks represent those simulated from the estimated model, while the solid lines with marks represent the correlations simulated from the alternative model. The dashed lines represent the empirical counterparts.

For example, in response to an adverse investment adjustment cost shock, instead of the opposing responses of the employment stocks in the benchmark, both full-time and part-time employment decrease in the counterfactual case.

We also examine the importance of the endogenous direct transition from part-time to full-time jobs in accounting for the overall cyclical patterns of the employment stocks.

Figure 8 presents the correlations of each employment stock with the cyclical components of real GDP, simulated from the estimated and alternative models. In this figure, we first find that the estimated model is remarkably successful in accounting for the procyclical pattern of the full-time employment rate and the countercyclical pattern of the part-time employment rate. In comparison, if the endogenous direct transition across the two employment stocks were absent, the countercyclical pattern of the part-time employment rate would disappear.

This figure also indicates that our model is able to account for the lead-lag correlations of part-time employment shown in Section 2.1. That is, the cyclical pattern of part-time employment lags half a year behind the business cycles. Indeed, our estimate suggests that the moderate sensitivity of $\varphi_t$ (the fraction that part-time workers conduct on-the-job search, defined in (19)) to the variation of the macroeconomic conditions is crucial in explaining the delayed response. Recall that, in (20), the key parameter for the sensitivity is $\omega$. In Appendix H.2, we show that if the value for $\omega$ were considerably high, the contemporaneous correlation would become strongest.\footnote{The simulated sample is drawn over 12000 periods with the first 20 percent discarded.}\footnote{There, we consider an extreme value of $\omega = 70$ instead of the estimated value of 2.10. The correlation}
4.4.2 A counterfactual analysis on the recent U.S. labor dynamics

To place the last section’s counterfactual experiment into a concrete context, here we analyze how the labor market dynamics would have behaved around the Great Recession period in the absence of the procyclical direct transition channel. To this end, we first utilize the Kalman smoother on the benchmark model and estimate the realization of the shocks \( \{ \varepsilon_t \}_{t=0}^T \) and the path of the (latent) endogenous variables \( \{ \hat{s}_t \}_{t=0}^T \) over the estimation period. Then, using the realization of the shocks, we simulate the path of the endogenous variables in the alternative model after 2006:Q1, \( \{ \hat{s}_t^A \}_{t=T} \). In this experiment, we take the estimate of the endogenous variables in the benchmark at 2006:Q1 as the initial value of the simulation i.e., \( \hat{s}_t^A = \hat{s}_T \).

Figure 9 shows that, starting from the onset of the Great Recession, full-time employment sharply decreases for both scenarios. At the same time, part-time employment rises in the benchmark, while it decreases in the counterfactual case. This difference in the reaction of part-time employment results in an about 1 percentage point higher unemployment rate in the counterfactual economy during the aftermath of the Great Recession.

Note: This figure shows the counterfactual simulation after 2006:Q1 for the full-time employment rate (%), the part-time employment rate (%), and the unemployment rate (%). The solid lines represent the estimated path and the dashed lines represent the counterfactual path. The shaded areas correspond to NBER recessions.
4.5 Additional results

Here, we briefly comment on how total hours worked behave in our model. By construction, the movement of total hours worked is decomposed into that of hours worked per worker and that of total employment stocks according to

$$\text{total hours worked} = \left( \frac{n_t^F \mu_b^F}{n_t^F + n_t^P \mu_b^P} \right) \mu_b^P \times \left( n_t^F + n_t^P \right).$$  \hspace{1cm} (24)

Note that our model does not consider the intensive margin choice explicitly; nevertheless, hours per worker fluctuate through the change in composition of the employment stocks. In Appendix H.3, we report the response of each variable to a contractionary monetary policy shock. We find that, in our estimated model, both hours per worker and total employment decrease in response to the shock, and the response of total employment is larger in magnitude, which are in line with evidence shown in Trigari (2009).

5 Conclusion

This paper studied the asymmetric roles of full-time employment and part-time employment in the business cycle. In the first part of the paper, we used the rotating panel household-level data of the Current Population Survey to document the macroeconomic facts on the cyclical behavior of full-time and part-time employment in the United States.

The key empirical facts we found are as follows. First, the full-time employment rate is procyclical, while the part-time employment rate exhibits a countercyclical pattern, particularly in deep recessions. Second, on the unemployment side, the majority of unemployed workers who search full-time jobs end up in full-time employment, and the majority of unemployed workers who search part-time jobs end up in part-time employment. Thus the labor market is segmented into full-time and part-time markets. The unemployment stocks for full-time jobs have a cyclical pattern substantially distinct from those for part-time jobs. Third, the flows between employment and unemployment strongly contributed to the decrease in full-time employment during the Great Recession, while they did not contribute much to the increase in part-time employment. Fourth, the flows between full-time employment and part-time employment were among the most important contributors to the change of the part-time employment stock.

We then built and estimated a medium-scale DSGE model with search and matching labor market frictions to evaluate the importance of the behavior of the part-time market.
The model incorporates two segmented labor markets and endogenous direct transitions from part-time employment to full-time employment. The model can account for the cyclical patterns in the data, and the major aggregate shocks can give rise to different dynamic properties of two employment stocks. Through the decomposition of the impulse responses for the employment stocks and the counterfactual analysis, we found that incorporating the endogenous direct transitions within employment stocks is essential in generating the countercyclical response of the part-time employment stock.

References


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Appendix

A CPS data

A.1 The redesign of the CPS in January 1994

In computing the (un)employment rate, we use the multiplicative factor constructed in Polivka and Miller (1998) to correct the break attributable to the redesign of the CPS in January 1994. The adjusted (un)employment rate equals the ratio of the adjusted number of (un)employment divided by that of labor force. The adjusted number of labor force is calculated by multiplying the adjusted labor participation rate by the civilian noninstitutional population. Multiplying the adjusted employment-to-population rate by the civilian noninstitutional population computes the adjusted number of employment. The unemployment is calculated by subtracting the number of employment from that of labor force.

Similarly, to compute the adjusted full-time and part-time employment rate (i.e., the full-time and part-time employment as a fraction of labor force) in Figure 1, we use the multiplicative factor for the number of employed workers working part-time as a fraction of total employment.

Figure A.1: Unemployment rate

Note: The shaded areas are NBER recessions.
A.2 Margin adjustment

In order to correct margin errors, we employ the method proposed by Elsby, Hobijn, and Şahin (2015) as following. Let $\Delta s_t$ be the vector of the change of the stocks, defined as

$$\Delta s_t = s_t - s_{t-1} = [E_t^F - E_{t-1}^F, E_t^P - E_{t-1}^P, U_t^F - U_{t-1}^F, U_t^P - U_{t-1}^P]'$$

where $E_t^F$ is the number of full-time employment, $E_t^P$ is the number of part-time employment, $U_t^F$ is the number of full-time unemployment, and $U_t^P$ is the number of part-time unemployment, all at the beginning of period $t$. From the identity that the change in the stock is the sum of the inflows minus the outflows, we have

$$\Delta s_t = X_{t-1}p,$$

where

$$X_{t-1} = \begin{bmatrix}
-E_{t-1}^F & -E_{t-1}^P & -E_{t-1}^P & E_{t-1}^P & 0 & 0 & 0 & U_{t-1}^P & 0 & 0 & 0 & O_{t-1} & 0 & 0 & 0 \\
E_{t-1}^F & 0 & 0 & 0 & -E_{t-1}^F & E_{t-1}^P & -E_{t-1}^P & E_{t-1}^P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & O_{t-1} \\
0 & E_{t-1}^P & 0 & 0 & 0 & E_{t-1}^P & 0 & 0 & -U_{t-1}^F - U_{t-1}^P & -U_{t-1}^P & -U_{t-1}^P & 0 & 0 & U_{t-1}^F & 0 & 0 & O_{t-1} \\
0 & 0 & E_{t-1}^P & 0 & 0 & 0 & E_{t-1}^P & 0 & 0 & 0 & 0 & U_{t-1}^F & -U_{t-1}^F - U_{t-1}^P & -U_{t-1}^P & 0 & 0 & O_{t-1} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & O_{t-1} & 0 & 0 & 0
\end{bmatrix}.$$ 

$O_t$ is the number of out-of-labor-force workers at the beginning of period $t$ and

$$p = \begin{bmatrix}
\text{p}_{E-E} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P} & \text{p}_{E-P}
\end{bmatrix}'.$$

The element $p_{ij}$ denotes the transition probability from state $i$ to state $j$.

Given the vector of the transition probabilities in data $\hat{p}$, the vector of the change of the stocks in data $\Delta s_t$ and the matrix $X_{t-1}$, the vector of corrected transition probabilities is chosen so as to minimize

$$\frac{1}{2}(p - \hat{p})'W(p - \hat{p})$$

subject to

$$\Delta s_t = X_{t-1}p,$$
where the weight matrix satisfies

\[
W = \begin{bmatrix}
W_{EF} & 0 & 0 & 0 & 0 \\
0 & W_{EP} & 0 & 0 & 0 \\
0 & 0 & W_{UF} & 0 & 0 \\
0 & 0 & 0 & W_{UP} & 0 \\
0 & 0 & 0 & 0 & W_N
\end{bmatrix}^{-1},
\]

and

\[
W_{EF} = \begin{bmatrix}
\hat{p}_{EF} \hat{p}_{EF} (1-\hat{p}_{EF}) & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} \\
\hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} \\
\hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} \\
\hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} \\
\hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EF}
\end{bmatrix}
\]

\[
W_{EP} = \begin{bmatrix}
\hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} (1-\hat{p}_{EF}) & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} \\
\hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} \\
\hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} \\
\hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} \\
\hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP} & \hat{p}_{EF} \hat{p}_{EF} \hat{p}_{EP}
\end{bmatrix}
\]

\[
W_{UF} = \begin{bmatrix}
\hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} (1-\hat{p}_{UF}) & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} \\
\hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} \\
\hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} \\
\hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} \\
\hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UF}
\end{bmatrix}
\]

\[
W_{UP} = \begin{bmatrix}
\hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} (1-\hat{p}_{UF}) & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} \\
\hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} \\
\hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} \\
\hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} \\
\hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP} & \hat{p}_{UF} \hat{p}_{EF} \hat{p}_{UP}
\end{bmatrix}
\]

\[
W_N = \begin{bmatrix}
\hat{p}_{NEF} (1-\hat{p}_{NEF}) & \hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} \hat{p}_{NEF} \\
\hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} (1-\hat{p}_{NEF}) & \hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} \hat{p}_{NEF} \\
\hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} (1-\hat{p}_{NEF}) & \hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} \hat{p}_{NEF} \\
\hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} (1-\hat{p}_{NEF}) & \hat{p}_{NEF} \hat{p}_{NEF} \\
\hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} \hat{p}_{NEF} & \hat{p}_{NEF} (1-\hat{p}_{NEF})
\end{bmatrix}
\]
B Additional statistics

B.1 The monthly transition probabilities

Figure B.1 presents the transition probabilities from EF (panel (a)), from EP (panel (b)), from UF (panel (c)), and from UP (panel (d)) from 1994 to 2016.

Figure B.1: Transition probability

(a) Transition probability from EF

(b) Transition probability from EP

(c) Transition probability from UF

(d) Transition probability from UP

Note: All series are seasonally adjusted. Since there are missing observations in 1995 due to the failure of individual identifiers in CPS, we use Tramo (“Time Series Regression with ARIMA Noise, Missing Observations, and Outliers”)/Seats (“Signal Extraction in ARIMA Time Series”) interface to interpolate them along with seasonal adjustment. EF is full-time employment, EP is part-time employment, UF is the unemployed looking for a full-time job, UP is the unemployed looking for a part-time job, and O is out of labor force.

Table B.1 shows the averages of the monthly transition probabilities. Due to the missing observations in 1995, we report the averages of them over 1996:M1–2016:M12. In this table, by construction, the sum of each row is 1. We find that about 24 percent of unemployed workers in full-time labor market find a job and 2/3 of them find a job in full-time position. Also, about 26 percent of unemployed workers in part-time labor
Table B.1: Average transition probability

<table>
<thead>
<tr>
<th></th>
<th>$S_{t+1}^{i}$</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$EF$</td>
<td>$EP$</td>
<td>$UF$</td>
<td>$UP$</td>
<td>$O$</td>
</tr>
<tr>
<td>$EF$</td>
<td>0.936</td>
<td>0.038</td>
<td>0.009</td>
<td>0.001</td>
<td>0.017</td>
</tr>
<tr>
<td>$EP$</td>
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<td>0.730</td>
<td>0.016</td>
<td>0.010</td>
<td>0.078</td>
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<tr>
<td>$S_{t}^{i}$</td>
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<td>0.081</td>
<td>0.548</td>
<td>0.002</td>
<td>0.206</td>
</tr>
<tr>
<td>$UP$</td>
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<td>0.215</td>
<td>0.016</td>
<td>0.341</td>
<td>0.381</td>
</tr>
<tr>
<td>$O$</td>
<td>0.020</td>
<td>0.025</td>
<td>0.018</td>
<td>0.007</td>
<td>0.930</td>
</tr>
</tbody>
</table>

Note: $EF$ is full-time employment, $EP$ is part-time employment, $UF$ is the unemployed looking for a full-time job, $UP$ is the unemployed looking for a part-time job, and $O$ is out of labor force.

market find a job and 4/5 of them find a job in part-time position. These suggest that full-time jobs and part-time jobs are likely to be different type of jobs and workers tend to know whether they want to work full-time or part-time while hunting jobs.

B.2 The time series of direct $EF/EP$ transitions

Figure B.2 plots the number of transitions from $EF$ to $EP$ (red), the number of transitions from $EP$ to $EF$ (blue), and the amount of net flow from $EF$ to $EP$ (black) in the units of labor force share. We observe that the net flow sharply declined during the Great Recession; while the flow from $EP$ to $EF$ is greater than that from $EF$ to $EP$ during tranquil time, the difference was closer and the magnitude was reversed during the Great Recession.

B.3 Do direct $EF/EP$ transitions occur within or across firms?

Here, we investigate whether the direct flow between $EF$ and $EP$ is explained by the intensive margin, that is, changing regular weekly hours over 35 hours within the same firm, or the extensive margin, that is, changing jobs (i.e., changing employer). CPS asks employed workers the question “Last month, it was reported that you worked for (employer’s name). Do you still work for (employer’s name)?” since the redesign in 1994. Here, we classify workers who change their labor force statues between $EF$ and $EP$ and respond “No” to this question into “job change transition” and respond “Yes” into “within firm transition.” Moreover, regardless of the answer to this question, the workers who change the number of jobs they hold are classified into “transition by multiple job holders.” Figure B.3 exhibits the distribution and shows that within transition accounts for around 80 percent of patterns of $EF/EP$ transition.
Figure B.2: Transition from and to full-time and part-time employment, divided by the labor force

![Graph showing transition rates over time](image)

Figure B.3: The intensive versus extensive margin along with the direct EF/EP transition

![Graph showing intensive and extensive margins](image)

*Note*: “Within” corresponds the direct EF/EP transition due to changing regular weekly hours within the firm, “N.A.” corresponds the workers we cannot classify due to the missing responses, “Multiple holders” corresponds the direct EF/EP transition due to the gain or loss of jobs by multiple job holders, and “Job change” corresponds that due to changing jobs (i.e., changing employer). The missing observations in 1995 are regarded as “N.A.”
C The option value of on-the-job search for the part-time workers

C.1 The explicit derivation of the option value \( \hat{\Delta}_t \)

In this section, we derive the expected value for a part-time employee to participate in full-time job market, \( \hat{\Delta}_t \). By construction,

\[
\hat{\Delta}_t = \mathbb{E} \left[ \max \{ R_t - \gamma_c w_{t-1}, 0 \} \right] = \int_{-\infty}^{\mathcal{R}_t} (R_t - y w_{t-1}) g_{\gamma_c}(y; \mu_g, \omega^{-1}) dy,
\]

(C.1)

where \( R_t = s_t^F (V_{x,t}^F - V_{x,t}^P) = s_t^F (H_{x,t}^F - H_{x,t}^P + U_t^F - U_t^P) \) and \( \mathcal{R}_t = R_t / w_{t-1} \). The functions \( g_{\gamma_c}(\cdot; \mu_g, \omega^{-1}) \) and \( G_{\gamma_c}(\cdot; \mu_g, \omega^{-1}) \) are the probability density function and the cumulative distribution function of a logistic random variable \( \gamma_c \) with mean \( \mu_g \) and the scaling parameter \( \omega^{-1} > 0 \):

\[
g_{\gamma_c}(y; \mu_g, \omega^{-1}) = \frac{\omega \exp(-(y - \mu_g)\omega)}{[1 + \exp(-(y - \mu_g)\omega)]^2}
\]

and

\[
G_{\gamma_c}(y; \mu_g, \omega^{-1}) = \frac{1}{1 + \exp(-(y - \mu_g)\omega)} = \frac{\exp((y - \mu_g)\omega)}{1 + \exp((y - \mu_g)\omega)}.
\]

Because \( \varphi = G_{\gamma_c}(\mathcal{R}_t; \mu_g, \omega^{-1}) \), we can rewrite (C.1) as

\[
\hat{\Delta}_t = \varphi_t R_t - w_{t-1} \int_{-\infty}^{\mathcal{R}_t} y g_{\gamma_c}(y; \mu_g, \omega^{-1}) dy
\]

\[
= s_t^l (H_{x,t}^F - H_{x,t}^P + U_t^F - U_t^P) - w_{t-1} \int_{-\infty}^{\mathcal{R}_t} y g_{\gamma_c}(y; \mu_g, \omega^{-1}) dy.
\]

Let us define \( \Delta_t \equiv -w_{t-1} \int_{-\infty}^{\mathcal{R}_t} y g_{\gamma_c}(y; \mu_g, \omega^{-1}) dy \). Applying integration by parts to this expression, we have

\[
\Delta_t = -R_t \varphi_t + w_{t-1} \int_{-\infty}^{\mathcal{R}_t} G_{\gamma_c}(y; \mu_g, \omega^{-1}) dy
\]

\[
= -R_t \varphi_t + \int_{-\infty}^{\mathcal{R}_t} \exp((y - \mu_g)\omega) \frac{1}{1 + \exp((y - \mu_g)\omega)} dy
\]

\[
= -R_t \varphi_t + \int_{-\infty}^{\mathcal{R}_t} \left[ \frac{1}{\omega} \log [1 + \exp((y - \mu_g)\omega)] \right] dy
\]

\[
= -R_t \varphi_t + \frac{w_{t-1}}{\omega} \log [1 + \exp((\mathcal{R}_t - \mu_g)\omega)].
\]

(C.2)
To simplify the expression further, we prove the following lemma.

**Lemma 1** For any \( x \in \mathbb{R} \),

\[
\log(1 + \exp(x)) = x - \log \left[ \frac{1 + \exp(-x)}{1 + \exp(-x)} \right].
\]

**Proof.**

\[
\log(1 + \exp(x)) = \log \left[ \left( 1 + \frac{1}{\exp(x)} \right) \exp(x) \right] = \log(1 + \exp(-x)) + x = x - \log \left[ \frac{1}{1 + \exp(-x)} \right].
\]

Applying Lemma 1 to (C.2) and using \( \varphi_t = G_{\gamma_t}(\mathfrak{R}_t; \mu_g, \omega^{-1}) \), we have

\[
\log \left[ \frac{1 + \exp((\mathfrak{R}_t - \mu_g)\omega)}{\omega} \right] = \mathfrak{R}_t - \mu_g - \frac{1}{\omega} \log(\varphi_t).
\]

Hence, we have

\[
\Delta_t = (1 - \varphi_t)R_t - w_{t-1}^P \left[ \mu_g + \frac{1}{\omega} \log(\varphi_t) \right].
\]

In sum, the option value \( \hat{\Delta}_t \) is given by\(^{37}\)

\[
\hat{\Delta}_t = s_t^J(H_{x,t}^F - H_{x,t}^P + U_{t}^F - U_{t}^P) + \Delta_t \tag{C.3}
\]

where

\[
\Delta_t = (1 - \varphi_t)R_t - w_{t-1}^P \left[ \mu_g + \frac{1}{\omega} \log(\varphi_t) \right]. \tag{C.4}
\]

\(^{37}\)Note that although substituting (C.4) into (C.3) yields a more simplified expression

\[
\hat{\Delta}_t = R_t - w_{t-1}^P \left[ \mu_g + \frac{1}{\omega} \log(\varphi_t) \right],
\]

this simplification makes it inconvenient to derive the log-linearized worker’s surplus and the wage functions. Thus, in the subsequent section, we log-linearize (C.3) and (C.4).
C.2 The steady-state conditions and log-linearization

We first derive the values for $\Delta_t$ and $\Delta_t$ on the balanced-growth path. Let the variables with bar represent the detrended ones (e.g., $\bar{\Delta}_t = \Delta_t / \bar{z}_t$), and then we have

$$R_t = R_t \left( \frac{\bar{z}_t}{\bar{w}^p_{t-1}} \right),$$

$$\tilde{\Delta}_t = \bar{R}_t - \bar{w}^p_{t-1} \left[ \mu_g + \frac{1}{\omega} \log(\varphi_t) \right],$$

and

$$\Delta_t = \tilde{\Delta}_t - \varphi_t \bar{R}_t,$$

where $\bar{R}_t = s_t^F (\bar{H}_{x,t} - \bar{H}_{x,t}^P + \bar{U}_t^F - \bar{U}_t^P)$.

On the balanced-growth path, given the calibrated and estimated parameters for $\omega$, $\varphi$, and $\bar{R}_t = \bar{R} \gamma^2 / \bar{w}^P$, the mean of the logistic distribution $\mu_g$ satisfies

$$\varphi = G_{\gamma_c}(\bar{R}; \mu_g, \omega^{-1}) = \frac{1}{1 + \exp(-(\bar{R} - \mu_g)\omega)}, \quad \text{(C.5)}$$

which implies

$$\mu_g = \bar{R} + \frac{1}{\omega} \log \left( \frac{1}{\varphi} - 1 \right). \quad \text{(C.6)}$$

Plugging this condition into the steady-state condition for $\tilde{\Delta}_t$, we have

$$\tilde{\Delta} = \bar{R} - \frac{\bar{w}^P}{\gamma^2} \left[ \mu_g + \frac{1}{\omega} \log(\varphi) \right] = -\frac{\bar{w}^P}{\gamma^2 \omega} \log(1 - \varphi).$$

Thus, the steady-state condition for $\Delta_t$ is given by

$$\Delta = -\varphi \bar{R} - \frac{\bar{w}^P}{\gamma^2 \omega} \log(1 - \varphi).$$

We then log-linearize $\tilde{\Delta}_t$ and $\varphi_t$ around the balanced-growth steady state. Recall that $\tilde{\Delta}_t = -(\bar{w}^p_{t-1}/\bar{z}_t) \int_{-\infty}^{\bar{R}_t} y g_{\gamma_c}(y; \mu_g, \omega^{-1}) dy$. Using Leibnitz’s rule, we obtain

$$\tilde{\Delta}_t = \psi_\Delta \bar{R}_t + (1 - \psi_\Delta) (\bar{w}^p_{t-1} - \bar{z}_t),$$

where

$$\psi_\Delta = -\frac{\bar{R} g_{\gamma_c}(\bar{R}; \mu_g, \omega^{-1}) \bar{R}}{\Delta}.\quad \text{(49)}$$
and
\[ \tilde{R}_t = \tilde{s}_t^F + \frac{s^F H^F}{R} \tilde{H}_{x,t} + \frac{s^F H^P}{R} \tilde{H}_{x,t} + \frac{s^F \bar{U}}{R} (\bar{U}_t^F - \bar{U}_t^P). \]

For \( \varphi_t = G_{\gamma_c}(R_t; \mu_g, \omega^{-1}) \), we have
\[ \tilde{\varphi}_t = \frac{g_{\gamma_c}(R_t; \mu_g, \omega^{-1}) \Phi}{\varphi} \left( \tilde{R}_t + \tilde{e}_t^z - \tilde{w}_{t-1}^P \right). \]

Here, given the values for \( \varphi \) and \( \Phi \), the density function in the steady state is given by
\[ g_{\gamma_c}(R_t; \mu_g, \omega^{-1}) = \frac{\omega \exp(-(\Phi - \mu_g)\omega)}{(1 + \exp(-(\Phi - \mu_g)\omega))^2} \]
\[ = \varphi^2 \left[ \omega \left( \frac{1}{\varphi} - 1 \right) \right] \quad \text{(from (C.5) and (C.6))} \]
\[ = \omega \varphi (1 - \varphi). \]

Thus, \( \tilde{\varphi}_t \) is given by
\[ \tilde{\varphi}_t = \omega (1 - \varphi) \Phi (\tilde{R}_t + \tilde{e}_t^z - \tilde{w}_{t-1}^P). \]

Note that, as shown in Appendix F, the value for \( \Phi = \tilde{R}_{t+1}/\tilde{w}^P \) is affected by \( \varphi \) but not by \( \omega \), and therefore we have a higher elasticity of the participation of on-the-job search with respect to a variation of the net benefit of conducting on-the-job search with a higher value for \( \omega \).

C.3 The elasticity of the participation decision with different values for \( \omega \)

Figure C.1 illustrates how much the participation rate \( \varphi_t \) changes to a variation in the \( \Phi_t \) around the steady state.

D The wage function for the part-time workers

In this section, we derive the log-linearized wage function for the part-time workers. First, log-linearizing the firm’s surplus, which is given by
\[ J_t^P(\tilde{w}_t^s P_n) = \rho_t^P \tilde{a}_t^P - \frac{\tilde{w}_t^s P_n \mu_n^P}{p_t} + \mathbb{E}_t \left[ \beta_{t,t+1} \frac{\kappa^P}{2} (x_{t+1}^P)^2 \right] + \mathbb{E}_t [(\rho^P - s_{t+1}^P) \beta_{t,t+1} \tilde{J}_{t+1}^P(\tilde{w}_{t+1}^s P_n)], \]
we obtain

\[
J^P_t(\tilde{w}^P_t) = \frac{p^w a_t^P}{J^P_t(\tilde{w}^P_t + \tilde{a}^P_t)} \omega P_t^w \tilde{w}^P_t + \beta x^F \mathbb{E}_t [\tilde{x}^P_{t+1}(\tilde{w}^P_{t+1}) + (1/2)\tilde{\beta}_{t,t+1}]
\]

\[+ \beta (1 - x^P) \mathbb{E}_t [\tilde{J}^P_{t+1}(\tilde{w}^P_{t+1}) + \tilde{\beta}_{t,t+1}] - \beta s^F \mathbb{E}_t \tilde{s}^J_{t+1}. \tag{D.1}\]

Then, log-linearizing the worker’s surplus, which is given by

\[
\bar{H}^P_t(\tilde{w}^P_{t^n}) = \frac{\tilde{w}^P_{t^n} \mu_{b_t}^P}{p_t} - \bar{b}_{t,s} \bar{b}_t + \mathbb{E}_t [\beta_{t,t+1} (\rho^P - s^J_{t+1}) \tilde{H}^P_{t+1}(\tilde{w}^P_{t+1}) - s^P_{t+1} \tilde{H}^P_{x,t+1}]
\]

\[+ \mathbb{E}_t [\beta_{t,t+1} (s^J_{t+1} (\bar{H}^F_{x,t+1} + \bar{U}^F_{t+1} + \bar{U}^P_{t+1}) + \bar{\Delta}_{t+1})], \tag{D.2}\]

we have

\[
\bar{H}^P_t(\tilde{w}^P_{t^n}) = \frac{\bar{w}^P_{t^n} \mu_{b_t}^P}{H^P} - \bar{b}_{t,s} \bar{b}_t + \mathbb{E}_t [\beta_{t,t+1} (\rho^P - s^J_{t+1}) \tilde{H}^P_{t+1}(\tilde{w}^P_{t+1}) + \tilde{\beta}_{t,t+1}]
\]

\[- \beta s^J \mathbb{E}_t \tilde{s}^J_{t+1} - \beta s^P \mathbb{E}_t [\tilde{s}^P_{t+1} + \tilde{\beta}_{t,t+1} + \tilde{H}^P_{x,t+1}]
\]

\[+ \beta s^J \bar{H}^F \mathbb{E}_t [\tilde{s}^J_{t+1} + \tilde{\beta}_{t,t+1} + \bar{H}^F_{x,t+1}] + \beta s^J \bar{U} \mathbb{E}_t [\tilde{U}^F_{t+1} - \tilde{U}^P_{t+1}] + \beta \bar{\Delta} \mathbb{E}_t [\tilde{\Delta}_{t+1} + \tilde{\beta}_{t,t+1}]. \tag{D.2}\]
Log-linearizing the Nash bargaining solution delivers
\[
\tilde{J}_t^P(\tilde{w}_t^{*P}) + (1 - \eta^P)^{-1}\tilde{\varepsilon}_t^P = \tilde{H}_t^P(\tilde{w}_t^{*P}). \tag{D.3}
\]
Substituting (D.1) and (D.2) into (D.3) delivers
\[
\begin{align*}
\frac{p^w\tilde{a}_t}{J^P} (\tilde{p}_t^w + \tilde{a}_t^P) - \frac{\tilde{w}_t^P\mu_b^P}{J^P} \tilde{w}_t^{*P} + \beta x^P \mathbb{E}_t \left[ \tilde{\beta}_{t+1}(\tilde{w}_t^{*P}) + (1/2)\tilde{\beta}_{t,t+1} \right] \\
+ \beta (1 - x^P) \mathbb{E}_t \left[ \tilde{J}_{t+1}(\tilde{w}_t^{*P}) + \tilde{\beta}_{t,t+1} \right] - \beta s^P \mathbb{E}_t \tilde{s}_{t+1}^P + (1 - \eta^P)^{-1}\tilde{\varepsilon}_t^P \\
= \frac{\tilde{w}_t^P\mu_b^P}{H^P} \tilde{w}_t^{*P} - \frac{\tilde{\mu}_b^P}{H^P} \tilde{b}_t + \tilde{\mu}_b^P + \beta (1 - x^P) \mathbb{E}_t \left[ \tilde{H}_{t+1}(\tilde{w}_t^{*P}) + \tilde{\beta}_{t,t+1} \right] - \beta s^P \mathbb{E}_t \tilde{s}_{t+1}^P \\
- \beta s^P \mathbb{E}_t \left[ \tilde{s}_{t+1}^P + \tilde{\beta}_{t,t+1} + \tilde{H}_{t+1}^P \right] + \beta s^P \tilde{H}_{t+1}^P \mathbb{E}_t \left[ \tilde{s}_{t+1}^P + \tilde{\beta}_{t,t+1} + \tilde{H}_{t+1}^P \right] \\
+ \beta s^P \tilde{H}_{t+1}^P \mathbb{E}_t \left[ \tilde{U}_{t+1}^P - \tilde{U}_{t+1} \right] + \beta \Delta \mathbb{E}_t \left[ \Delta_{t+1} + \tilde{\beta}_{t,t+1} \right].
\end{align*}
\]
Multiplying both sides by \(\eta^P J^P/(\tilde{w}_t^P\mu_b^P)\) or \((1 - \eta^P)\tilde{H}_t^P/(\tilde{w}_t^P\mu_b^P)\) and collecting terms deliver
\[
\begin{align*}
\tilde{w}_t^{*P} &= \varphi_a^P(\tilde{p}_t^w + \tilde{a}_t^P) + \varphi_x^P \mathbb{E}_t \left[ \tilde{x}_{t+1}^P(\tilde{w}_t^{*P}) + (1/2)\tilde{\beta}_{t,t+1} \right] + \varphi_b^P(\tilde{b}_t + \tilde{\mu}_b^P) \\
&+ \varphi_x^P \mathbb{E}_t \left[ \tilde{s}_{t+1}^P + \tilde{\beta}_{t,t+1} + \tilde{H}_{t+1}^P \right] + \varphi_b^P \mathbb{E}_t \left[ \tilde{\varepsilon}^P_t - \beta (1 - x^P) \tilde{\varepsilon}^P_t \right] \\
&- \varphi_x^P \mathbb{E}_t \left[ \tilde{\varepsilon}^P_t \right] - \varphi^P \mathbb{E}_t \left[ \Delta_{t+1} + \tilde{\beta}_{t,t+1} \right], \tag{D.4}
\end{align*}
\]
where
\[
\begin{align*}
\varphi_a^P &= \eta^P \frac{p^w\tilde{a}_t}{\tilde{w}_t^P\mu_b^P}, & \varphi_x^P &= \eta^P \frac{x^P \beta J^P}{\tilde{w}_t^P\mu_b^P}, & \varphi_b^P &= (1 - \eta^P) \frac{\tilde{\mu}_b^P}{\tilde{w}_t^P\mu_b^P}, & \varphi_s^P &= (1 - \eta^P) \frac{\beta s^P \tilde{H}_{t+1}^P}{\tilde{w}_t^P\mu_b^P}, \\
\varphi_x^P &= \frac{\eta^P}{1 - \eta^P} \frac{J^P}{\tilde{w}_t^P\mu_b^P}, & \varphi_s^P &= (1 - \eta^P) \frac{\beta s^P J^P}{\tilde{w}_t^P\mu_b^P}, & \varphi_b^P &= (1 - \eta^P) \frac{\beta \Delta}{\tilde{w}_t^P\mu_b^P}, & \varphi^P &= \kappa^P x^P.
\end{align*}
\]
As in GST, we obtain
\[
\mathbb{E}_t \tilde{H}_{t+1}^P = \mathbb{E}_t \tilde{x}_{t+1}^P + (1 - \eta^P)^{-1}\mathbb{E}_t \tilde{\varepsilon}_{t+1}^P \tag{D.5}
\]
and
\[
\mathbb{E}_t \tilde{H}_{t+1}^F = \mathbb{E}_t \tilde{x}_{t+1}^F(\tilde{w}_{t+1}^F) + \theta_w^F(1 - \theta_w^F)^{-1} \Gamma F \mathbb{E}_t [\tilde{x}_{t+1}^F - (\tilde{w}_{t+1}^F - \tilde{\pi}_{t+1}^F + \lambda_w \tilde{\varepsilon}_{t+1}^F)] \\
+ (1 - \chi^F)^{-1} \mathbb{E}_t [\tilde{x}_{t+1}^F(\tilde{w}_{t+1}^F)] + (1 - \eta^F)^{-1} \mathbb{E}_t \tilde{\varepsilon}_{t+1}^F. \tag{D.6}
\]
Plugging (D.5) and (D.6) into (D.4) delivers

\[
\tilde{w}_t^P = \varphi_a^P (\tilde{p}_t^w + \tilde{a}_t^P) + (\varphi_s^P + \varphi_x^P)\tilde{E}_t \tilde{x}_t^P + \varphi_s^P \tilde{E}_t \tilde{z}_t^P - \varphi_s^J \tilde{E}_t \tilde{s}_t^J + \varphi_h^P (\tilde{b}_t + \tilde{\mu}_{h,t}) - \varphi_\Delta \tilde{E}_t \tilde{\Delta}_t + 1
\]

\[
+ (\varphi_s^P - \varphi_s^J - \varphi_\Delta + \varphi_x^P / 2)\tilde{E}_t \tilde{\beta}_{t+1} + \varphi_s^J \tilde{E}_t \tilde{H}_t + \varphi_s^J (\tilde{U} / \tilde{H}_F) \tilde{E}_t \left[ \tilde{U}_{t+1} - \tilde{U}_{t+1}^P \right]
\]

\[
+ \varphi_x^J [1 - \beta (1 - x^P - s^P) \rho_y^P] \tilde{E}_t^P.
\]

Also, note that \( \tilde{E}_t \tilde{\beta}_{t+1} = \tilde{E}_t \tilde{\lambda}_{t+1} - \tilde{\lambda}_t \) where \( \tilde{\lambda}_t \) satisfies (G.1).

### E The dataset and the observation equations

The descriptions of the nine time series we use for the Bayesian estimation are as follows:

1. The growth rate of per capita real GDP (\( \Delta \log Y_t \)).

2. The growth rate of per capita real personal consumption expenditures (PCE) (\( \Delta \log C_t \)): Personal consumption expenditures of durables are classified into investment, instead of consumption.

3. The growth rate of per capita real investment (\( \Delta \log I_t \)): Investment equals to private residential and nonresidential fixed investment plus personal consumption expenditures of durables.

4. The growth rate of the real hourly compensation in nonfarm business sector (\( \Delta \log W_t \)).

5. The inflation rate using GDP deflator (\( \Pi_t \)).

6. The employment rate i.e., the ratio of total employment to labor force (\( N_t \)).

7. The federal funds rate (\( R^u_t \)).

8. The ratio of the full-time employment rate to the part-time employment rate (\( N^F_t / N^P_t \)): The full-time employment rate is defined as the number of the employed workers who usually work full-time as a fraction of labor force, while the part-time employment rate is defined as the number of the employed workers who usually work part-time as a fraction of labor force.

9. The ratio of the real hourly wage for the full-time workers to that of the part-time workers (\( W^F_t / W^P_t \)): We compute the median of usual real hourly earnings of wage and salary workers from the extracts of the CPS established by the Merged Outgoing Rotation Group for full-time and part-time workers respectively. Usual hourly earnings equal to earnings per week divided by hours a wage and salary worker usually works per week. The Consumer Price Index for All Urban Consumers is used.
for computing usual real hourly earnings. The wage and salary workers exclude self-employed and persons who work without payment.

The observation equations for our model are given by

\[
\begin{bmatrix}
\Delta \log Y_t \\
\Delta \log C_t \\
\Delta \log I_t \\
\Delta \log W_t \\
\log \Pi_t \\
\log N_t \\
\log R^n_t \\
\log(N^n_F/N^n_P) \\
\log(W^n_F/W^n_P)
\end{bmatrix}
= \begin{bmatrix}
\log \gamma_z \\
\log \gamma_z \\
\log \gamma_z \\
\log \gamma_z \\
\log \pi \\
\log(n^F + n^P) \\
\log r^n \\
\log(n^F/n^P) \\
\log(\bar{w}^F/\bar{w}^P)
\end{bmatrix}
+ \begin{bmatrix}
\tilde{y}_t - \tilde{y}_{t-1} + z^z_t \\
\tilde{c}_t - \tilde{c}_{t-1} + z^z_t \\
\tilde{i}_t - \tilde{i}_{t-1} + z^z_t \\
\tilde{w}_t - \tilde{w}_{t-1} + z^z_t \\
\tilde{n}_t \\
\tilde{n}_t \\
\tilde{r}^n_t \\
\tilde{n}_t^F - \tilde{n}_t^P \\
\tilde{w}_t^F - \tilde{w}_t^P
\end{bmatrix}.
\]

\section{F Steady-state conditions}

\subsection{F.1 Employment state}

The steady-state conditions for (6) and (7)-(10) imply that the stationary distribution of employment states \((n^F, n^P, u^F, u^P)'\) solves

\[
\begin{pmatrix}
1 - \rho^F & 0 & 0 & 0 \\
0 & 1 - (\rho^P - \xi^P) & 0 & 0 \\
1 - \rho^F - \xi^F & \xi^P & -s^F & \xi^P \\
1 & 1 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
n^F \\
n^P \\
u^F \\
u^P
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
0 \\
1
\end{pmatrix}.
\]

\subsection{F.2 Worker’s value}

The values of each employment state in the balanced-growth steady state \((\bar{V}^F, \bar{V}^P, \bar{U})'\) solves

\[
\begin{pmatrix}
1 - \beta \rho^F & 0 & -\beta(1 - \rho^F) \\
0 & 1 - \beta \rho^P & -\beta(1 - \rho^P) \\
-\beta s^F & 0 & 1 - \beta(1 - s^F)
\end{pmatrix}
\begin{pmatrix}
\bar{V}^F \\
\bar{V}^P \\
\bar{U}
\end{pmatrix}
= \begin{pmatrix}
\bar{w}^P \mu^F_b + (1 - \bar{u}^P_b) \bar{b} + \beta \bar{\Delta} \\
\bar{w}^F \mu^F_b \\
\bar{b}
\end{pmatrix}.
\]

\subsection{F.3 Other steady-state conditions}

- Given the calibrated and estimated parameters, the parameter values and steady-state values \((p^w, x^F, x^P, r^k, \bar{k}^F, \bar{k}^P, \bar{a}^F, \bar{a}^P, \chi^F, \epsilon^F, \mu^F, \bar{H}^F, \bar{H}^P)\) are determined by

\[
p^w = \frac{1}{\epsilon_p},
\]

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\[ x^F = 1 - \rho^F, \]
\[ x^P = 1 - (\rho^P - s^F), \]
\[ r^k = \frac{\gamma^k}{\beta} - 1 + \delta, \]
\[ \frac{\bar{k}^F}{\mu^F n^F} = \frac{\bar{k}^P}{\mu^P \phi n^P} = \left( \frac{r^k}{p^w \alpha} \right)^{-1/(1-\alpha)}, \]
\[ \bar{a}^F = (1 - \alpha) \left( \frac{\bar{k}^F}{\mu^F n^F} \right)^\alpha, \]
\[ \bar{a}^P = (1 - \alpha) \mu^P \phi \left( \frac{\bar{k}^P}{\mu^P \phi n^P} \right)^\alpha, \]
\[ \chi^F = \frac{\eta^F}{\eta^F + (1 - \eta^F) \mu^F / \epsilon^F}, \]
\[ \epsilon^F = \frac{\mu^F}{1 - \beta \theta_w (1 - x^F)}, \]
\[ \mu^F = \frac{\mu^F}{1 - \beta \theta_w}, \]
\[ \bar{H}^F = \kappa^F x^F \left( \frac{\chi^F}{1 - \chi^F} \right), \]
\[ \bar{H}^P = \kappa^P x^P \left( \frac{\eta^P}{1 - \eta^P} \right). \]

- The values for \((\kappa^F, \bar{w}^F, \bar{b})\) are solved out from the following steady-state conditions:

\[ \bar{b}^F = \frac{\bar{b}}{p^w \bar{a}^F + \beta (\kappa^F / 2) (x^F)^2}, \]
\[ \bar{w}^F = p^w \bar{a}^F - (1 - \beta \rho^F) \kappa^F x^F + \beta \left( \frac{\kappa^F}{2} \right) (x^F)^2, \]
and
\[ (1 - \chi^F) \bar{b} = \bar{w}^F - \chi^F \left[ p^w \bar{a}^F + \beta \left( \frac{\kappa^F}{2} \right) (x^F)^2 + \beta s^F \kappa^F x^F \right]. \]

The system of equations solves

\[ \kappa^F = \frac{2p^w \bar{a}^F (1 - \bar{b}^F)(1 - \chi^F)}{x^F [2 - \beta x^F (1 - \bar{b}^F)(1 - \chi^F) - 2 \beta (\rho^F - s^F \chi^F)]}, \]
\[ \bar{w}^F = \frac{2\rho w^A \bar{b}^F (1 - \chi^F)(1 - \beta \rho^F) - \chi^F (\beta \rho^F - \beta s^F - 1)}{2 - \beta x^F (1 - \bar{b}^F)(1 - \chi^F) - 2\beta (\rho^F - s^F \chi^F)}, \]

and

\[ \bar{b} = \frac{2\rho w^A \bar{b}^F [1 - \beta (\rho^F - s^F \chi^F)]}{2 - \beta x^F (1 - \bar{b}^F)(1 - \chi^F) - 2\beta (\rho^F - s^F \chi^F)}. \]

- The values for \((\kappa^P, \bar{w}^P, \tilde{\Delta}, \tilde{\mu}_b^P)\) are solved out by the following steady-state conditions

\[ \mu_b^P \bar{w}^P = p^w \bar{a}^P - [1 - \beta (\rho^P - s^P)] \kappa^P x^P + \beta \left( \frac{\kappa^P}{2} \right) (x^P)^2, \quad \text{(F.1)} \]

\[ (1 - \eta^P)(\mu_b^P \bar{b} - \beta \tilde{\Delta}) = \mu_b^P \bar{w}^P - \eta^P \left[ p^w \bar{a}^P + \beta \left( \frac{\kappa^P}{2} \right) (x^P)^2 + \beta (s^P - s^F) \kappa^P x^P \right], \quad \text{(F.2)} \]

\[ \tilde{\Delta} = -\frac{\bar{w}^P}{\gamma^2 \omega} \log(1 - \varphi), \quad \text{(F.3)} \]

and the steady-state condition that \(\bar{U}^F = \bar{U}^P:\)

\[ \mu_b^P \bar{w}^P = \mu_b^P \bar{b} - \beta \tilde{\Delta} + \frac{(1 + \beta s^P - \beta \rho^P) s^F (\bar{w}^F - \bar{b})}{s^P (1 + \beta s^F - \beta \rho^P)}. \quad \text{(F.4)} \]

Plugging (F.4) into the term \((\tilde{\mu}_b^P \bar{b} - \beta \tilde{\Delta})\) in (F.2) and combining this with (F.1) enable us to solve out \((\kappa^P, \bar{w}^P)\) as

\[ \kappa^P = \frac{\zeta_0}{x^P s^P (1 + \beta s^F - \beta \rho^F) \eta^P}, \]

\[ \bar{w}^P = \frac{1}{\mu_b^P} \left[ p^w \bar{a}^P - \frac{[x^P (1 + \beta s^P - \beta \rho^P) - \beta (x^P)^2 / 2] \zeta_0}{x^P s^P (1 + \beta s^F - \beta \rho^F) \eta^P} \right]. \]

where \(\zeta_0 = (\bar{w}^F - \bar{b})(1 - \eta^P) s^F\). Then equations (F.3) and (F.4) solve \(\tilde{\Delta}\) and \(\tilde{\mu}_b^P\).

### G  Log-linearized model equations

#### Consumption, Investment, and Production

\[ (1 - \beta h_z) \check{\lambda}_t = h_{1,c} (\check{c}_{t-1} - \check{\varepsilon}_t^c + \beta \bar{E}_t \check{c}_{t+1} + \beta \bar{E}_t \check{\varepsilon}_t^{c+1}) - h_{2,c} \check{c}_t + \check{\varepsilon}_t^h - \beta h_z \bar{E}_t \check{\varepsilon}_{t+1}^h, \quad \text{(G.1)} \]

\[ \check{\lambda}_t = \check{\tau}_t^n + \bar{E}_t \check{\nu}_{t+1} + \bar{E}_t \check{\varepsilon}_{t+1}^z - \bar{E}_t \check{\varepsilon}_t^{z+1}, \quad \text{(G.2)} \]

\[ \bar{E}_t \check{\beta}_{t+1} = \bar{E}_t \check{\lambda}_{t+1} - \check{\lambda}_t, \quad \text{(G.3)} \]

\[ \check{\kappa}_t^P = \delta_z (\check{\kappa}_{t-1}^P - \check{\varepsilon}_t^z) + (1 - \delta_z) \check{\iota}_t, \quad \text{(G.4)} \]

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\[ \tilde{k}_t = \tilde{\nu}_t + \tilde{k}_{t-1} - \tilde{\varepsilon}_t^i, \]  
\[ \tilde{\nu}_t = \eta_{\nu} \tilde{r}_t, \]  
\[ \tilde{q}_t^k = \beta \delta_s E_t \tilde{q}_{t+1}^k + (1 - \beta \delta_s) E_t \tilde{r}_t^k - \tilde{r}_t^n + E_t \tilde{n}_{t+1}, \]  
\[ (1 + \beta) \tilde{\nu}_t = \tilde{\nu}_{t-1} - \tilde{\varepsilon}_t^i - \tilde{\varepsilon}_t^i + [1/(\eta_{\nu}(\gamma_s)^2)] \tilde{q}_t^k + \beta E_t [\tilde{v}_{t+1} + \tilde{\varepsilon}_t^i + \tilde{\varepsilon}_t^i], \]  
\[ \tilde{y}_t^F = \alpha \tilde{k}_t^F + (1 - \alpha) \tilde{n}_t^F, \]  
\[ \tilde{y}_t^P = \alpha \tilde{k}_t^P + (1 - \alpha) (\tilde{n}_t^P + \tilde{\varepsilon}_t^i), \]  
\[ \tilde{y}_t = (\tilde{y}_t^F/\tilde{y}_t^P) \tilde{y}_t^F + (\tilde{y}_t^P/\tilde{y}_t^P) \tilde{y}_t^P, \]  
\[ \tilde{r}_t^k = \tilde{r}_t^w + \tilde{y}_t^F - \tilde{k}_t^F, \]  
\[ \tilde{r}_t^k = \tilde{r}_t^w + \tilde{y}_t^P - \tilde{k}_t^P, \]  
\[ \tilde{a}_t^F = \tilde{y}_t^F - \tilde{n}_t^F, \]  
\[ \tilde{a}_t^P = \tilde{y}_t^P - \tilde{n}_t^P, \]  
\[ \tilde{b}_t = \tilde{k}_t^P, \]  
\[ \tilde{\mu}_t^P = 0, \]  
\[ \tilde{k}_t = (\tilde{k}_t^F/\tilde{k}_t^F + (\tilde{k}_t^P/\tilde{k}_t^P)), \]  
\[ \tilde{y}_t = y_c \tilde{c}_t + y_i \tilde{i}_t + y_g \tilde{g}_t + y_o \tilde{o}_t + y_x^F(2 \tilde{x}_t^F + \tilde{n}_{t-1}^F) + y_x^P(2 \tilde{x}_t^P + \tilde{n}_{t-1}^P) - y_\Delta (\tilde{\Delta} + \tilde{n}_{t-1}^P), \]  
\[ \tilde{n}_t = t_o \tilde{n}_{t-1} + t_o \tilde{\mu}_{t-1} + t_f E_t \tilde{n}_{t+1} + \tilde{\varepsilon}_t^P, \]

where

\[ h_{1,c} = h_z/(1 - h_z), \quad h_{2,c} = (1 + \beta(h_z)^2)/(1 - h_z), \quad h_z = h_c/\gamma_z, \]
\[ \delta_z = (1 - \delta)/\gamma_z, \quad \eta_{\nu} = (1 - \psi_{\nu})/\psi_{\nu}, \quad y_c = 1 - (y_i + \zeta + y_o + y_x^F + y_x^P - y_\Delta), \]
\[ y_i = (1 - \delta z)(\tilde{k}/\tilde{y}), \quad y_o = r^k(\tilde{k}/\tilde{y}), \quad y_x^F = (\kappa^F/2)[n^F(x^F)^2/\tilde{y}], \quad y_x^P = n^P \tilde{\Delta}/\tilde{y}, \]
\[ t_o = t_p \phi_p, \quad t_o = [(1 - \theta_p)(1 - \beta \theta_p)/\theta_p][1 + (\epsilon_p - 1)\tilde{x}]^{-1} \phi_p, \]
\[ t_f = \beta \phi_p, \quad \text{and} \quad \phi_p = 1/(1 + \beta t_p). \]

Labor markets

\[ \tilde{h}_t^F = (u^F/(u^F + \varphi n^P)) \tilde{u}_t^F + (\varphi n^P/(u^F + \varphi n^P))(\tilde{n}_{t-1}^P + \varphi t), \]  
\[ \tilde{m}_t^F = \sigma \tilde{h}_t^F + (1 - \sigma) \tilde{v}_t^F, \]  
\[ \tilde{m}_t^P = \sigma \tilde{u}_t^P + (1 - \sigma) \tilde{v}_t^P, \]
\[
\tilde{q}_t^F = \tilde{m}_t^F - \tilde{v}_t^F, \quad \text{(G.24)}
\]
\[
\tilde{q}_t^P = \tilde{m}_t^P - \tilde{v}_t^P, \quad \text{(G.25)}
\]
\[
\tilde{s}_t^F = \tilde{m}_t^F - \tilde{h}_t^F, \quad \text{(G.26)}
\]
\[
\tilde{s}_t^I = \tilde{q}_t + \tilde{m}_t^F - \tilde{h}_t^F, \quad \text{(G.27)}
\]
\[
\tilde{s}_t^P = \tilde{m}_t^P - \tilde{u}_t^P, \quad \text{(G.28)}
\]

**Evolutions of employment stocks**

\[
\tilde{n}_t^F = \tilde{n}_{t-1}^F + x^F \tilde{x}_t^F, \quad \text{(G.29)}
\]
\[
\tilde{n}_t^P = \tilde{n}_{t-1}^P + x^P \tilde{x}_t^P - s^J \tilde{s}_t^J, \quad \text{(G.30)}
\]
\[
\tilde{u}_t^F = (1-s^F-\xi^F)\tilde{u}_{t-1}^F - s^F \tilde{s}_{t-1}^F + (\xi^P u^F/u^F)\tilde{u}_{t-1}^P + (n^F(1-\rho^F-\xi^F)/u^F)\tilde{n}_{t-2}^F + (\xi^P n^P/u^F)\tilde{n}_{t-2}^P, \quad \text{(G.31)}
\]
\[
n^F \tilde{n}_{t-1}^F + n^P \tilde{n}_{t-1}^P + u^F \tilde{u}_t^F + u^P \tilde{u}_t^P = 0, \quad \text{(G.32)}
\]
\[
\tilde{x}_t^F = \tilde{q}_t^F + \tilde{v}_t^F - \tilde{n}_{t-1}^F, \quad \text{(G.33)}
\]
\[
\tilde{x}_t^P = \tilde{q}_t^P + \tilde{v}_t^P - \tilde{n}_{t-1}^P, \quad \text{(G.34)}
\]
\[
\tilde{z}_t^F = x_a^F (\tilde{p}_t^w + \tilde{a}_t^w) - x_w^F \tilde{w}_t^P + x_\lambda^F \mathbb{E}_t \tilde{\beta}_{t+1} + \beta \mathbb{E}_t \tilde{x}_t^F, \quad \text{(G.35)}
\]
\[
\tilde{z}_t^P = x_a^P (\tilde{p}_t^w + \tilde{a}_t^P) - x_w^P \tilde{w}_t^P + x_\lambda^F \mathbb{E}_t \tilde{\beta}_{t+1} + \beta \mathbb{E}_t \tilde{x}_t^P - \beta s^J \mathbb{E}_t \tilde{s}_t^J, \quad \text{(G.36)}
\]

where

\[
\begin{align*}
x_a^F &= p^w \bar{a}_t^f / (\kappa^F x^f), \\
x_w^F &= (\bar{w}_t^P \mu_b) / (\kappa^F x^f), \\
x_\lambda^F &= \beta(1 + \rho^F)/2, \quad \text{and} \quad x_\lambda^P = \beta(1 + (\rho^P - s^J))/2.
\end{align*}
\]

**Within employment flows**

\[
\tilde{R}_t = \tilde{s}_t^F + (s^F H^F/R) \tilde{H}_x^F - (s^F H^P/R) \tilde{H}_x^P + (s^F \mathbb{U}/R)(\tilde{U}_t^F - \tilde{U}_t^P), \quad \text{(G.37)}
\]
\[
\tilde{\varphi}_t = \omega(1 - \varphi) \mathbb{R}(\tilde{R}_t + \tilde{\varepsilon}_t^z - \tilde{w}_t^P), \quad \text{(G.38)}
\]
\[
\tilde{\Delta}_t = \psi_\Delta \tilde{R}_t + (1 - \psi_\Delta)(\tilde{\varepsilon}_t^P + \tilde{\varepsilon}_t^z), \quad \text{(G.39)}
\]

where

\[
\tilde{R} = s^F (\bar{H}^F - \bar{H}^P), \quad \mathbb{R} = \tilde{R}_x^z / \tilde{w}^P, \quad \text{and} \quad \psi_\Delta = -\mathbb{R} g_\gamma(\mathbb{R}; \mu_g, \omega^{-1}) \tilde{R}/\tilde{\Delta}.
\]

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Worker’s surplus

\[
\mathbb{E}_t \tilde{H}_{x,t+1} = \mathbb{E}_t \bar{x}_{t+1}^F + \theta_w^F (1 - \theta_w^F)^{-1} \Gamma^F \mathbb{E}_t \left[ \bar{\omega}_{t+1}^F - (\bar{w}_{t+1}^F - \bar{\pi}_{t+1} + \ell_w \bar{\pi}_t - \bar{\varepsilon}_{t+1}^F) \right] \\
+ (1 - \chi^F)^{-1} \mathbb{E}_t \chi_{t+1}^F + (1 - \eta^F)^{-1} \rho_{\eta^F} \bar{\varepsilon}_{t+1}^F, \tag{G.40}
\]

\[
\tilde{U}_{t}^F = (\bar{b}/\bar{U})_t + \varrho_s \mathbb{E}_t \tilde{\pi}_{t+1} + \varphi^F (\mathbb{E}_t \bar{s}_{t+1}^F + \mathbb{E}_t \tilde{H}_{x,t+1}^F) + \beta [\xi^F \mathbb{E}_t \tilde{U}_{t+1}^P + (1 - \xi^F) \mathbb{E}_t \tilde{U}_{t+1}^P], \tag{G.41}
\]

\[
\tilde{U}_{t}^P = (\bar{b}/\bar{U})_t + \varrho_s \mathbb{E}_t \tilde{\pi}_{t+1} + \varphi^P (\mathbb{E}_t \bar{s}_{t+1}^P + \mathbb{E}_t \tilde{H}_{x,t+1}^P) + \beta [\xi^F \mathbb{E}_t \tilde{U}_{t+1}^F + (1 - \xi^F) \mathbb{E}_t \tilde{U}_{t+1}^F], \tag{G.42}
\]

where

\[
\Gamma^F = (1 - \eta^F x^F \beta^F \mu^F) \mu^F \bar{z}_w^F/\eta^F, \quad \varrho_b = (\bar{U} - \bar{b})/\bar{U}, \quad \text{and} \quad \varphi^F = \beta (s^F \tilde{H}^F/\bar{U}).
\]

Wage dynamics

\[
\tilde{\chi}_{t}^F = - (1 - \chi^F) (\bar{\mu}_{t}^F - \bar{\varepsilon}_{t}^F), \tag{G.43}
\]

\[
\tilde{\varepsilon}_{t}^F = \rho^F \theta_w^F \beta^F \mathbb{E}_t \left[ \tilde{\pi}_{t+1} + \rho_s \tilde{\pi}_{t+1} - \bar{\varepsilon}_{t+1}^F \right], \tag{G.44}
\]

\[
\tilde{\mu}_{t}^F = (x^F \theta_w^F \beta^F) \mathbb{E}_t \tilde{\pi}_{t+1}^F - (x^F \theta_w^F \beta^F) (x_w^F \mu^F) \mu^F \mathbb{E}_t \left[ \bar{w}_{t+1}^F - \bar{\pi}_{t+1} + \rho_s \tilde{\pi}_{t+1} - \bar{\varepsilon}_{t+1}^F \right] \\
+ (\theta_w^F \beta^F) \mathbb{E}_t \left[ \bar{\pi}_{t+1} + \rho_s \tilde{\pi}_{t+1} + \bar{w}_{t+1}^F - \bar{\varepsilon}_{t+1}^F \right], \tag{G.45}
\]

\[
\bar{w}_{t}^F = \varphi^F (\bar{p}_{t}^w + \bar{a}_{t}^F) + (\varphi_x^F + \varphi^F) \mathbb{E}_t \tilde{\pi}_{t+1}^F + \varphi^F \mathbb{E}_t \tilde{H}_{x,t+1}^F + \varphi^F \tilde{b}_t^F + \varphi^F \tilde{b}_t^F \\
+ \varphi_x^F \left[ \tilde{\chi}_{t}^F - (\rho^F - s^F) \beta \mathbb{E}_t \tilde{\chi}_{t+1}^F \right] + \varphi_x^F (1 - \chi^F) (1 - \eta^F)^{-1} [1 - (\rho^F - s^F) \beta \rho_{\eta^F}] \bar{\varepsilon}_{t+1}^F, \tag{G.46}
\]

\[
\bar{w}_{t}^P = \varphi^P (\bar{p}_{t}^w + \bar{a}_{t}^P) + (\varphi_x^P + \varphi^P) \mathbb{E}_t \tilde{\pi}_{t+1}^P + \varphi^P \mathbb{E}_t \tilde{H}_{x,t+1}^P + \varphi^P \tilde{b}_t^P + \varphi^P \tilde{b}_t^P \\
- \varphi_x^P \tilde{b}_t^P - \bar{H}_{x,t+1}^F - \bar{H}_{x,t+1}^P + (\varphi_x^P [1 - (1 - x^F - s^F) \beta \rho_{\eta^F}] \bar{\varepsilon}_{t+1}^P, \tag{G.47}
\]

\[
\bar{w}_{t}^F = \gamma_{\bar{w}}^F (\bar{w}_{t-1}^F - \bar{\pi}_t + \rho_s \tilde{\pi}_{t-1} - \bar{\varepsilon}_{t}^F) + \gamma_{\bar{w}^F}^F \bar{w}_{t}^F + \gamma_{\bar{w}^F}^F \mathbb{E}_t \left[ \bar{w}_{t+1}^F + \bar{\pi}_{t+1} - \rho_s \tilde{\pi}_{t+1} + \bar{\varepsilon}_{t+1}^F \right], \tag{G.48}
\]
where

\[ \varphi_s^\ell = \chi_h^\ell p^w a^\ell (w^\ell \mu^\ell_h)^{-1}, \quad \varphi_s^o = \chi_h^o \beta \kappa_h^\ell (x^\ell)^2 (w^\ell \mu^\ell_h)^{-1}, \quad \varphi_s^p = (1 - \chi^\ell) s^\ell \beta \bar{H} (w^\ell \mu^\ell_h)^{-1}, \]

\[ \varphi_b^F = (1 - \chi^F) \mu^F_b b (w^F \mu^F_b)^{-1}, \quad \varphi_b^P = (1 - \eta^P) \bar{\mu}_b^P b (w^P \mu^P_b)^{-1}, \quad \varphi_x^\ell = \chi^\ell \kappa_h^F [(1 - \chi^\ell) w^\ell \mu^\ell_h]^{-1}, \]

\[ \varphi_s^J = \beta s^J (1 - \eta^P) \bar{H} (w^P \mu^P_b)^{-1}, \quad \varphi_s^{JU} = \varphi_s^J (U/\bar{H}^F), \quad \varphi_s^\Delta = \beta (1 - \eta^P) \bar{\Delta} (w^P \mu^P_b)^{-1}, \]

\[ \varphi_s^F = \varphi_s^o + \varphi_s^p / 2, \quad \varphi_s^p = (\varphi_s^o - \varphi_s^J - \varphi_s^\Delta + \varphi_s^J / 2), \]

\[ \gamma_b^F = (1 + \tau_2^F) / \Phi_F, \quad \gamma_o^F = \varsigma^F / \Phi_F, \quad \gamma_f^F = (\tau^F / \theta^F_w - \tau_1^F) / \Phi_F, \]

\[ \Phi_F = (1 + \tau_2^F) + \varsigma^F + (\tau^F / \theta^F_w - \tau_1^F), \quad \varsigma^F = (1 - \theta^F_w)(1 - \tau^F) / \theta^F_w, \]

\[ \tau^F = \varphi^F (1 + \varphi^F)^{-1}, \quad \varphi^F = \chi^F \beta \theta^F_w \mu^F + (1 - \chi^F)(1 - x^F) \beta \theta^F_w \mu^F, \]

\[ \tau_1^F = [z^F_w \mu^F \varphi_x^F + \varphi_x^F (1 - \chi^F)(x^F \beta \theta^F_w)(z^F_w \mu^F) \mu^F ((1 - x^F) \beta) + \varphi_s^F \Gamma^F] (1 - \tau^F), \]

\[ \tau_2^F = -(z^F_w \mu^F \varphi_x^F (1 - \chi^F)(x^F \beta \theta^F_w) \mu^F (1 - \tau^F), \quad \text{and} \quad \chi^F = \eta^P. \]

Monetary policy and Government spending

\[ \tilde{\pi}_t^n = \varphi_r \tilde{\pi}_{t-1}^n + (1 - \phi_r) [\phi_x \tilde{\pi}_t + \phi_y (\bar{y}_t - \bar{y}_{nt})] + \tilde{\varepsilon}_t^n, \quad \text{(G.49)} \]

\[ \tilde{y}_t = \bar{y}_t + ((1 - \zeta) / \zeta) \tilde{\varepsilon}_t^\theta, \quad \text{(G.50)} \]

**H Additional results**

In this section, we report some additional results.

**H.1 Additional results for the counterfactual experiment**

Here we present some additional results from the counterfactual experiments. In Figure H.1, we compare the impulse response functions of the full-time employment rate, the part-time employment rate, the unemployment rate, and the growth rate of the average wage to an adverse investment adjustment cost in the alternative model (dashed lines) with those in the benchmark (solid lines). We observe that this shock has almost no impacts or slightly negative impacts on the part-time employment rate in the alternative model, in contrast to a clear positive response of it in the benchmark. We also find that, in the alternative model, the lack of asymmetric responses between full-time employment and part-time employment results in a bigger reaction of the unemployment rate. As shown in Figure H.2, similar results are observed for the impulse response functions to a contractionary monetary policy.
Figure H.1: Impulse response function to an investment adjustment cost shock

Note: This figure shows the impulse responses of the full-time employment rate, the part-time employment rate, the unemployment rate, and the growth rate of the average wage to an adverse investment adjustment cost shock of one standard deviation.

H.2 What explains the lagging pattern of part-time employment?

Figure H.3 presents the correlations of the employment stocks with the cyclical component of real GDP simulated from the model with $\omega = 70$ (solid lines with marks), instead of the estimate 2.10. For the sake of comparison, we also plot those simulated from the estimated model (solid lines without marks) and the empirical counterparts (dashed lines).

We observe that a considerably high value for $\omega$ changes the lead-lag pattern of cyclical behavior of part-time employment. In the estimated model, the countercyclical movement of part-time employment is behind around two quarters the business cycles; in comparison, when the value for $\omega$ is large, part-time employment is even countercyclical but tends to react contemporaneously to the business cycles.
Figure H.2: Impulse response function to a monetary policy shock

![Impulse response function to a monetary policy shock](image)

**Note:** This figure shows the impulse responses of the full-time employment rate, the part-time employment rate, the unemployment rate, and the growth rate of wage to a contractionary monetary policy shock of one standard deviation.

Figure H.3: Cross correlogram with real GDP under a high value for $\omega$

![Cross correlogram with real GDP under a high value for $\omega$](image)

**Note:** This figure shows the correlations between the cyclical components of real GDP and the employment stocks $h$ quarters lagged. The left panel is for the full-time employment rate and the right panel is for the part-time employment rate. The horizontal axis is the number of lags, denoted by $h$. The cyclical components of real GDP is measured in terms of log-deviations of per capita real GDP from its HP filter trend. The solid lines without marks represent those simulated from the estimated model, while the solid lines with marks represent those simulated from the model in which $\omega = 70$. The dashed lines represent the empirical counterparts over 1979:Q1–2016:Q4.
H.3 The response of total hours worked

Figure H.4 displays the responses of total hours worked, hours worked per worker, and total employment to a contractionary monetary policy shock. As is expressed in (24), total hours worked is product of hours worked per worker and total employment and thus, by construction, the sum of the response of hours worked per worker and that of total employment equals the response of total hours worked.

Figure H.4: Impulse response functions of total hours worked, hours worked per worker, and total employment to a contractionary monetary policy shock

Note: This figure shows the impulse responses of total hours worked, hours worked per worker, and total employment to a contractionary monetary policy shock of one standard deviation. The parameter values are fixed at the posterior means.