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Noritaka Kudoh
Nagoya University
Hiroaki Miyamoto
International Monetary Fund
Masaru Sasaki
Osaka University

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Noritaka Kudoh[2]
Nagoya University

Hiroaki Miyamoto[3]
International Monetary Fund

Masaru Sasaki[4]
Osaka University

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[2]E-mail: kudoh@soec.nagoya-u.ac.jp

[3]E-mail: HMiyamoto@imf.org

[4]E-mail: sasaki@econ.osaka-u.ac.jp

Abstract

This paper studies a large-firm search-matching model with variable hours of work to investigate how firms utilize the intensive and extensive margins of labor adjustment over the business cycle. Introduction of variable hours of work introduces the Frisch elasticity parameter into the analysis, and this is a key determinant of the magnitude of fluctuations in hours of work. The model replicates the observed cyclical behavior of the Japanese labor market, in which fluctuations in hours of work account for 79 percent of the variations in total labor input, well. Total labor input in the model is as volatile as that in the data, and is 25 times as volatile as that in the model without the intensive margin.

JEL classification: E32, J20, J64.

Keywords: search, hours of work, employment, business cycles, multi-worker firms.

1 Introduction

Firms adjust their labor inputs over the business cycle through the intensive margin (hours of work per employee) and the extensive margin (the number of employees). In their survey, Hall *et al.* (2000) found that 62 percent of firms consider overtime as the primary reaction to a demand boom. In a perfectly competitive labor market, firms do not need to utilize overtime because they can employ extra workers instantly at the going wage rate. This suggests the importance of *frictions* in understanding how firms utilize the intensive and extensive margins over the business cycle. However, little attention has been paid to the *composition* of labor demand in frictional labor market models.

This paper studies the composition of labor demand over the business cycle in a search-matching model. To this end, we develop a *large-firm* search-matching model with the intensive margin. The large-firm search-matching model with intra-firm bargaining is now a standard framework for studying the labor market.[1] However, its business cycle properties have not been fully explored in the literature.[2] Our contribution is to explore this relatively uncharted area with special attention given to the intensive margin. Our model captures the fact that firms need to engage in time-consuming search-matching process when hiring new employees while changes in hours of work per employee are instantaneous.

A novel property of our model is that firms facing productivity shocks choose both vacancies and hours of work per employee. A caveat is that with variable hours of work, the hourly wage rate may depend on the level of hours of work. To allow for the possibility of nonlinear compensation scheme, we assume that each firm and its employees bargain over a state-contingent contract in the form of an *earnings schedule* which maps hours of work per employee into earnings (Cooper *et al.*, 2007; Kudoh and Sasaki, 2011). The equilibrium earnings schedule turns out to be

[1]A partial list of related contributions is Bertola and Caballero (1994), Smith (1999), Cahuc and Wasmer (2001), Koeniger and Prat (2007), Cahuc *et al.* (2008), Felbermayr and Prat (2011), Kudoh and Sasaki (2011), and Acemoglu and Hawkins (2014).

[2]Leading examples of business cycle studies using the large-firm paradigm are Cooper *et al.* (2007), Elsby and Michaels (2013), Krause and Lubik (2013), and Fujita and Nakajima (2016).

a convex function of hours of work, and this specifies the *marginal hourly wage rate* for the firm of choosing hours of work.

Our model replicates the observed cyclical behavior of the Japanese labor market well. In Japan, the intensive margin accounts for a large proportion of cyclical fluctuations in total labor input. Our empirical analysis reveals that the intensive margin accounts for 79 percent of the variations in total labor input, while the extensive margin accounts for 21 percent of the variations.[3] Clearly, in understanding labor market fluctuations in Japan, it is a serious omission if one uses a model without the intensive margin of labor adjustment. Our model replicates much of the observed fluctuations in total labor input, hours of work per employee, employment, unemployment, and vacancies. In particular, variations in hours of work per employee account for 84 percent of the variations in the aggregate labor input, while variations in the number of employees account for 19 percent of the variations.[4]

We find that the intensive margin magnifies fluctuations in total labor input and improves the model's ability to replicate the data. Specifically, in our basic model, total labor input fluctuates 25 times as much as that in the model *without* hours of work. Introduction of variable hours of work introduces the Frisch elasticity parameter into the analysis, and this is a key determinant of the magnitude of fluctuations in hours of work. In sharp contrast with the standard real business cycle model, our model can generate a realistic magnitude of hours fluctuations with Frisch elasticity much less than one.

Introduction of the intensive margin also magnifies labor market fluctuations along the *extensive margin* and helps resolve the unemployment volatility puzzle (Shimer, 2005). Our model's ability to generate realistic magnitudes of fluctuations in unemployment and vacancies comes partly from disutility from hours of work because it plays the same role as having a high unemployment benefit, generating a small surplus for the firm. This is essentially the mechanism often pointed out in the literature (Hagedorn and Manovskii, 2008). With variable hours of work, our

[3]This is in sharp contrast with the labor market fluctuations in the U.S, in which 79 percent of the variations are accounted for by the *extensive* margin.

[4]Our decomposition of variations admits some discrepancy so that the sum of variations may deviate from unity.

model generates a small surplus for the firm under a set of plausible parameters. Through this channel, the Frisch elasticity also influences the magnitudes of fluctuations in unemployment and vacancies.

In the large-firm model with intra-firm bargaining and concave production, the steady-state allocation is not constrained efficient even under the Hosios condition because the firms can cut the wage rate by hiring excessively, known as the overhiring effect (Smith, 1999), and the resulting hours of work are too low (Kudoh and Sasaki, 2011). However, little is known about efficiency in the model's out-of-steady-state allocation. Thus, we characterize a planner's allocation and compare it with our model's. We show that the equilibrium level of hours per employee is below its efficient level. Interestingly, if hours of work are determined in bargaining or chosen by workers, then the equilibrium hours are even less than those in the basic model.

Although our model replicates the observed cyclical behavior of the Japanese labor market fairly well, the magnitude of fluctuations in employment is about a half of the magnitude of employment fluctuations in the data. This could partly be explained by the absence of the other extensive margin, namely, job creation by entrants. To assess the importance of this additional margin, we extend our model to include stochastic firm entry. We find that this extended model replicates the data strikingly well. While the textbook search-matching model makes no distinction between jobs created by incumbents and entrants, our large-firm model clearly distinguishes between them. Our exercise suggests the importance of exploring this line of research.

We intend our empirical analysis to be a contribution to the growing literature on re-examination of hours of work using new models and new datasets. This includes Rogerson (2006) and Ohanian and Raffo (2012), to name a few. Particularly relevant is Ohanian and Raffo (2012), who find that the intensive margin is increasingly more important for labor adjustment in 14 OECD countries they studied. Using the dataset we built for Japan, our empirical analysis confirms the importance of the intensive margin.

In terms of the structure of the model, particularly related to our study are Cooper *et al.* (2007) and Kudoh and Sasaki (2011), in which determination of hours of work is considered in the context of a search-matching model with large firms.[5] Cooper *et al.* (2007) study both employment and hours of work over the business cycle using a model similar to ours. They emphasize the importance of nonlinear cost of posting vacancies, and as a result, wage determination is simplified by assuming a take-it-or-leave-it offer protocol. While we restrict our analysis to a linear vacancy cost, we employ a more general bargaining problem.

Our model is also closely related to the model developed by Fang and Rogerson (2009), who study the intensive and extensive margins using the framework of Merz (1995) and Andolfatto (1996). In this model, the production unit is a matched worker-job pair, and this rules out the issue of intra-firm bargaining. In addition, the Merz-Andolfatto paradigm has a utility-maximizing household, who optimally chooses the level of consumption. Fang and Rogerson (2009) show that, with a concave utility function, there is a rich interaction between employment and hours through consumption, and as a result, an increase in the vacancy cost decreases employment and increases hours of work. This rich interaction comes from the labor-supply side. In contrast, we focus on the labor demand and especially on its composition over the business cycle.

The remainder of the paper is organized as follows. In Section 2, we empirically examine how labor inputs are adjusted over the business cycle in Japan, and present some other cyclical characteristics of the labor market in Japan. Section 3 describes our basic model, followed by characterization of equilibrium of the model in Section 4. Section 5 introduces a planner to define the efficient allocation, and discusses efficiency of the allocations in the basic model and its variants. In Section 6, we calibrate the model parameters and present the business cycle properties of our model. Section 7 explores firm entry over the business cycle and Section 8 concludes. Proofs and some additional results are found in the Appendix.

[5] In a recent, independent contribution, Trapeznikova (2017) takes up the issue similar to ours. The model developed in Trapeznikova (2017) introduces on-the-job search, and is calibrated to match the Danish labor market to replicate the firm-level facts.

2 Labor Market Facts

This section presents some empirical facts on the Japanese labor market. We primarily focus on the cyclical behaviors of total labor input, employment, and hours of work.

2.1 Data

We obtain the series of the number of employed workers and the number of the labor force from the Labour Force Survey (LFS), conducted by the Statistics Bureau and the Director-General for Policy Planning. The series of the average monthly hours worked per worker are obtained from the Monthly Labour Survey (MLS) conducted by the Ministry of Health, Labour and Welfare (MHLW). We construct our measure of total labor input as the product of the average monthly hours worked per worker and the number of employed workers, normalized by the labor force. These measures are consistent with those used in Ohanian and Raffo (2012).

The TFP series are from Braun *et al.* (2006).[6] We obtain the unemployment rate series from the LFS. The vacancy rate series are obtained from the Monthly Report on Employment Service (*Shokugyo Antei Gyomu Tokei*) conducted by the MHLW. Following Miyamoto (2011) and Lin and Miyamoto (2012), we construct the job finding and separation rates from the LFS. The series of real earnings and real wages are constructed from the nominal earnings series from the MLS conducted by the MHLW. Following the convention, we use the consumer price index (CPI) to obtain the real series.

Our data are quarterly, which, when necessary, are obtained by averaging or aggregating the corresponding monthly series. The sample covers 1980Q2–2010Q4. All series are seasonally adjusted using the Census Bureau's X12 ARIMA procedure and transformed by taking natural logarithms. Since our focus is on cyclical fluctuations in the series, the low-frequency movements in the data are filtered out by using the Hodrick-Prescott (HP) filter with smoothing parameter 1600.

[6]The TFP series are extended to 2010 by Nao Sudo, and we use the extended series.

2.2 Labor Market Fluctuations in Japan

Figure 1 plots the cyclical components of total labor input, hours of work per worker, and the number of employed workers. The figure shows that total labor input and its components fluctuate significantly over the business cycle, and both hours and employment comove with total labor input. It also indicates that total labor input comoves more closely with hours of work per worker than with employment, and that employment is less volatile than hours per worker.

Table 1: Summary statistics, quarterly Japanese data, 1980-2010

Variables	$\hat{u}$	$\hat{v}$	$\hat{f}$	$\hat{s}$	$\hat{W}$	$\hat{w}$	$\hat{t}$	$\hat{h}$	$\hat{l}$	$\hat{A}$	$\hat{y}$
Standard deviation	0.059	0.096	0.091	0.092	0.012	0.010	0.009	0.008	0.004	0.011	0.016
Correlation matrix											
$\hat{u}$	1	-0.777	-0.420	0.449	-0.317	-0.084	-0.526	-0.325	-0.558	-0.281	-0.734
$\hat{v}$	-	1	0.320	-0.517	0.656	0.113	0.728	0.644	0.378	0.470	0.781
$\hat{f}$	-	-	1	-0.344	0.192	0.123	0.201	0.139	0.185	0.005	0.215
$\hat{s}$	-	-	-	1	-0.365	0.035	-0.416	-0.422	-0.108	-0.371	-0.501
$\hat{W}$	-	-	-	-	1	0.751	0.554	0.507	0.253	0.287	0.436
$\hat{w}$	-	-	-	-	-	1	-0.023	-0.121	0.193	0.002	0.003
$\hat{t}$	-	-	-	-	-	-	1	0.902	0.485	0.359	0.741
$\hat{h}$	-	-	-	-	-	-	-	1	0.060	0.392	0.602
$\hat{l}$	-	-	-	-	-	-	-	-	1	0.038	0.493
$\hat{A}$	-	-	-	-	-	-	-	-	-	1	0.680
$\hat{y}$	-	-	-	-	-	-	-	-	-	-	1

Table 1 quantifies what we see in Figure 1 and summarizes cyclical characteristics of the key labor market variables such as the unemployment rate (u), the vacancy rate (v), the job-finding rate (f), the separation rate (s), total earnings (W), the real wage rate (w), total hours of work or total labor input (t), hours of work per employee (h), the number of employed workers (l), TFP (A), and output (y). All variables are with hat, meaning that they are in percentage deviations from their trend levels. Since the series are in natural logarithms, the standard deviations can be interpreted as mean percentage deviations from their trend levels.

2.2.1 Hours and Employment

Table 1 shows a strong positive relationship between total labor input and hours worked per worker, in line with Figure 1. The correlation between them is 0.90. We also find a positive relationship between total hours and employment, with a correlation of 0.49. This is significantly smaller than the correlation between total hours and hours per worker. As Braun *et al.* (2006) pointed out, there is no strong correlation between hours worked per worker and employment. The correlation between them is 0.06.

Total labor input and its components comove positively with TFP and output, and comove negatively with the unemployment rate, indicating that they are all pro-cyclical. Interestingly, the correlation between employment and TFP is 0.04 while the correlation between hours per worker and TFP is 0.39. The standard deviation of total labor input is 0.9 percent, that of hours worked per worker is 0.8 percent, and that of employment is 0.4 percent. Thus, hours of work per worker are twice as volatile as employment. Since the standard deviations of TFP and output are 1.1 percent and 1.6 percent, respectively, total labor input and its components are less volatile than TFP or output.

2.2.2 Unemployment and Vacancy

The unemployment rate is counter-cyclical and the vacancy rate is pro-cyclical. The correlation between the unemployment rate and TFP is -0.28 and the correlation between the unemployment rate and output is -0.73. The correlation between the vacancy rate and TFP is 0.47 and the correlation between the vacancy rate and output is 0.78. Since the unemployment rate is counter-cyclical and the vacancy rate is procyclical, these two series comove negatively. The correlation between them is -0.78, which implies the Beveridge curve.

Both the unemployment rate and the vacancy rate are significantly more volatile than TFP or output. While the standard deviations of TFP and output is 1.1 percent and 1.6 percent, respectively, the standard deviations of the unemployment and vacancy rates are 5.9 percent and 9.6 percent, respectively.

2.2.3 Job Finding Rate and Separation Rate

The job finding rate is acyclical or procyclical and the separation rate is counter-cyclical. The correlation between the job finding rate and TFP, 0.005, is very weak. However, the correlation between the job finding rate and output is 0.22. The correlation between the separation rate and TFP is -0.37 and the correlation between the separation rate and output is -0.50. The standard deviations of the job finding rate and separation rate are 9.1 percent and 9.2 percent, respectively. Both the job finding rate and separation rate are more volatile than TFP or output.

2.2.4 Real Earnings and Real Wage Rate

In this paper, we clearly distinguish between earnings and the hourly wage rate. While real earnings are procyclical, the real hourly wage rate is acyclical. The correlation between real earnings and TFP is 0.29 and the correlation between real earnings and output is 0.44, indicating that real earnings are procyclical. However, the correlation between the real hourly wage rate and TFP is 0.002 and the correlation between the real hourly wage rate and output is 0.003. Real earnings and the real hourly wage rate are both positively correlated with employment.

Interestingly, while real earnings are positively correlated with hours of work, the real hourly wage rate is *negatively* correlated with hours. This negative correlation between hours and the hourly wage rate might look inconsistent with the presence of overtime payment. A possible explanation of the result is that the compensation scheme has a constant term and a convex function of hours of work so that the *average* hourly wage rate is deceasing while the *marginal* hourly wage rate is increasing in hours. In fact, we can only compute the series of the *average* hourly wage rate from the data and the series of the *marginal* hourly wage rate are not available. Another possible explanation for the negative correlation between hours and the hourly wage rate is cyclical changes in the composition of jobs. Suppose that there are high-wage jobs with inflexible hours of work and low-wage jobs with flexible hours. It is then easy to show that the average hours of work and the average wage rate are negatively correlated.[7]

2.3 Decomposition of Fluctuations

We now study the relative contributions of the intensive and extensive margins to fluctuations in the total labor input. With $\hat{t} = \hat{h} + \hat{l}$, variance of total labor input can be decomposed as

$$Var(\hat{t}) = Var(\hat{h}) + Var(\hat{l}) + 2Cov(\hat{h}, \hat{l}) = Cov(\hat{t}, \hat{h}) + Cov(\hat{t}, \hat{l}). \quad (1)$$

The term $Cov(\hat{t}, \hat{h})$ gives the amount of variations in $\hat{t}$ that derived from variations in $\hat{h}$ and through its comovement with $\hat{l}$. Similarly, the term $Cov(\hat{t}, \hat{l})$ is the amount of variations in $\hat{t}$ that derived from variations in $\hat{l}$ and through its comovement with $\hat{h}$. By dividing the both sides of (1) by $Var(\hat{t})$, we obtain

$$1 = \frac{Cov(\hat{t}, \hat{h})}{Var(\hat{t})} + \frac{Cov(\hat{t}, \hat{l})}{Var(\hat{t})} = \beta^h + \beta^l, \quad (2)$$

where β^h and β^l are the relative contributions of variations in $\hat{h}$ and $\hat{l}$ to variations in $\hat{t}$. These measures are an application of the "beta value" in finance.[8]

From the data, we find that $\beta^h = 0.79$ and $\beta^l = 0.21$. In other words, the intensive margin explains 79 percent of variations in total labor input and the extensive margin accounts for 21 percent of the variations. This implies that over the business cycle, Japanese firms adjust labor inputs both by the intensive and extensive margins, but they use the intensive margin more heavily than the extensive margin.[9]

[7]We thank an anonymous referee for this interpretation.

[8]Petrongolo and Pissarides (2008) and Fujita and Ramey (2009) apply this measure to decompose unemployment fluctuations into inflow and outflow fluctuations.

[9]This result is in sharp contrast with what we find from the US data. We utilize the dataset constructed by Ohanian and Raffo (2012) and find that $\beta^h = 0.21$ and $\beta^l = 0.79$ for the US labor market. Thus, US firms adjust labor inputs mainly through the extensive margin.

3 The Model

This section presents our basic model for explaining cyclical behaviors of employment and hours of work. The key feature of the model is that while hours of work per worker can change within a period, firms need to open (costly) vacancies in a frictional labor market to hire new employees. To focus on the composition of the labor demand at each firm, we build a search-matching model with large firms.

3.1 Environment

Consider an economy consisting of a large number of homogeneous workers and homogeneous firms. The measures of workers and firms are both normalized to unity. The issue of firm entry and exit is discussed in Section 7. Time is discrete and all agents discount the future at the common discount rate r.

All workers are risk neutral, and maximize the expected lifetime utility, given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \left[I_t - e\left(h_t\right) \right],$$

where $\delta \equiv 1/(1+r)$ is the discount factor, I_t denotes income, and $e(h_t)$ represents disutility from working for h_t hours. We assume that $e'(\cdot) > 0$, $e''(\cdot) > 0$ and $\lim_{h\to\infty} e(h) = \infty$. Our specification of disutility function is

$$e\left(h\right) = e_0 \frac{h^{1+\mu}}{1+\mu}, \tag{3}$$

where $e_0 > 0$ and $1/\mu$ is the Frisch elasticity.

The production technology for each firm is given by $A_t L_t^{\alpha} k_t^{1-\alpha}$, where $0 < \alpha < 1$, A_t denotes the level of TFP, k_t denotes the stock of capital at each firm, and L_t denotes total labor input. The level of TFP is stochastic. Assuming homogeneous labor, we postulate that $L_t \equiv h_t l_t$, where l_t is the number of employees at each firm. Thus, output y_t is given by $y_t = A_t h_t^{\alpha} l_t^{\alpha} k_t^{1-\alpha}$.

Each firm possesses a technology that converts one unit of the final consumption good into a unit of investment good. Let i_t be the level of investment made in period t. The stock of capital then evolves according to $k_{t+1} = (1-\delta_k)k_t + i_t$, where δ_k is the rate of capital depreciation.

As part of our robustness analysis, we consider the possibility of costly capital adjustment. Specifically, we assume that $C(k_t, k_{t+1})$ units of the final good must be spent in order to change the stock of capital from k_t to k_{t+1}. The cost function takes the following standard form:

$$C(k_t, k_{t+1}) = \frac{\kappa_0}{2}\left(\frac{i_t}{k_t}\right)^2 k_t = \frac{\kappa_0}{2}\left(\frac{k_{t+1} - k_t}{k_t} + \delta_k\right)^2 k_t.$$

Evidently, the cost of capital adjustment is zero when i_t is zero. Note, however, that the cost is positive even in a steady state because capital investment $i = \delta_k k$ must be positive to maintain the steady state level of capital. We include costly capital adjustment only to assess whether flexible capital adjustment will suppress the labor adjustment channel. Thus, in what follows we will drop this component by setting $\kappa_0 = 0$ whenever possible.

The labor market is frictional. The number of matches in period t is determined by the matching technology $m_0 U^{\xi} V^{1-\xi}$, where $m_0 > 0$ and $0 < \xi < 1$ are parameters, U_t is the total number of job seekers, and V_t is the number of aggregate job vacancies. Let $V_t / U_t \equiv \theta_t$ denote the labor market tightness. A vacancy is matched to a worker during a period with probability q_t, where

$$q_t = m_0 U_t^{\xi} V_t^{1-\xi} / V_t = m_0 \theta_t^{-\xi} \equiv q(\theta_t). \tag{4}$$

This is referred to as the vacancy filling rate. It is easy to verify that an increase in labor market tightness θ_t decreases this probability. Similarly, the probability that a worker is matched with a vacancy, or the job finding rate, is given by $m_0 U_t^{\xi} V_t^{1-\xi} / U_t = m_0 \theta_t^{1-\xi} = \theta_t q(\theta_t)$. This probability is increasing in θ_t.

We assume exogenous separations. At the end of each period, a fraction λ_t of the current employees are assumed to leave the firm, where λ_t may be stochastic. Since the firm creates v_t units of vacancies, the number of new employees for the next period is $q(\theta_t) v_t$. These new employees are not hit by the separation shock. In addition, we assume that there is no job-to-job transition of workers.[10] Thus, the number of employees at each firm evolves according to

[10] Fujita and Nakajima (2016) develop a related model with job-to-job transitions of workers and calibrate it to match the US labor market facts. While job-to-job transitions of workers are important for understanding the US labor market dynamics, they are less important in Japan. The *annual* turnover rate is 5.3 percent during 1990-2000 in Japan, whereas in the U.S, the *monthly* job-to-job transition rate is on average 2.4 percent during 1994-2010.

$l_{t+1} = (1 - \lambda_t) l_t + q(\theta_t) v_t$. Throughout, we will treat λ_t as a constant whenever possible.

3.2 Indivisible Labor

In the basic model, we assume that labor is *indivisible* from each worker's point of view, while the firm optimally chooses the requirement of hours of work. In contrast to the existing models with indivisible labor, the level of hours of work requirement varies over the business cycle.[11] Although direct evidence is hard to come by, we argue that the indivisible labor view is realistic. For instance, Fitzgerald (1998, p.810) reported that "[r]oughly 85% of all full-time wage and salary workers in the United States report having very little flexibility in their work schedules."

Firms are not simply given the right to manage by assumption.[12] While implicit, our assumption is closely related to the notion of team production proposed by Fitzgerald (1998). Rogerson (2006, p.406) made this point clear by arguing that "many organizations require a significant amount of coordination of working hours among employees, so that for most individuals their existing wage comes from a single possibility for hours of work." Thus, in a model with large firms, it is natural to assume that each worker takes hours of work as given.

We also note that there is a large discrepancy between the estimates of the labor supply elasticity. Namely, the micro elasticity, which primarily reflects the intensive margin of labor supply, is much smaller than the macro elasticity, which reflects both the intensive and extensive margins of labor supply. We interpret this fact as indirect evidence that while labor market participation (extensive margin of labor supply) is chosen by each individual, labor supply conditional on employment (i.e., hours of work per employee) is driven by the demand side.

While we prefer the indivisible labor view, in Section 5.2 we also present models with alternative assumptions on how hours of work are determined. In Section 5.1, we show that the equilibrium allocation under indivisible labor is more efficient than that under divisible labor

[11] An important related contribution in this line of research is Nakajima (2005), in which the level of hours requirement is high when the firm is active and low when it is inactive.

[12] The right-to-manage assumption is often considered (and tested) in the labor union literature. In sharp contrast with the model in this literature, workers are *not* unionized in our model.

with bargained hours of work. Interestingly, as we show in Section 5.3, such a distinction does not arise in the textbook search-matching model.

3.3 Timing

Let $S_t = (A_t, l_t, k_t, U_t)$ be the set of state variables in period t. Among the state variables, the level of A_t is revealed at the beginning of each period. Given the state variables, each firm and its employees bargain over a state-contingent *contract* that specifies an *earnings schedule*, which maps h into an amount of compensation W (Kudoh and Sasaki, 2011). At this point, the level of earnings W is not realized because hours of work are determined only after the earnings schedule is agreed upon. The bargaining outcome is summarized by $W(h_t; S_t)$. With this schedule, the firm chooses hours of work per employee (h_t), vacancies to create (v_t), and the level of capital investment (i_t). Then, production takes place and output y_t is realized. Finally, $\lambda_t l_t$ of the current employees leave the firm, and $q(\theta_t)v_t$ workers are newly employed. Note that a firm cannot choose hours of work before the bargaining stage because the marginal hourly wage rate is realized only after the earnings schedule is agreed upon.[13]

3.4 Firms

We solve the firm's optimization problem by stationary dynamic programming, taking this period's bargaining outcome $W(h; S)$ as given, which reflects the fact that the set of state variables S and the state-contingent contract $W(h; S)$ are known at the time of the firm's optimization. The instantaneous payoff to a firm is given by $y - W(h; S)l - cv - i$, where $c > 0$ is the (constant) cost of posting a vacancy. Let l_{+1} and k_{+1} denote the levels of employment and capital in the next period, respectively. The value of a firm $J(S)$ satisfies the following Bellman equation:

$$J(S) = \max_{h,v,i} \left\{ Ah^{\alpha} l^{\alpha} k^{1-\alpha} - W(h; S)l - cv - i - C(k, k_{+1}) + \delta \mathbb{E} J(S_{+1}) \right\}, \tag{5}$$

[13] In addition, a firm has *no incentive* to choose h in advance because doing so cannot manipulate the bargaining outcome. In contrast, a firm does have an incentive to manipulate the bargaining outcome by choosing a large l, generating the well-known overhiring result (Smith, 1999).

and the maximization is subject to $l_{+1} = (1-\lambda)l + q(\theta)v$ and $k_{+1} = (1-\delta_k)\,k + i$. The first-order conditions with respect to h, v, and i imply

$$\alpha A h^{\alpha-1} l^{\alpha-1} k^{1-\alpha} = W_h(h;S), \tag{6}$$

$$\delta \mathbb{E} J_l\left(S_{+1}\right) = \frac{c}{q(\theta)}, \tag{7}$$

$$\delta \mathbb{E} J_k\left(S_{+1}\right) = 1 + C_2\left(k, k_{+1}\right). \tag{8}$$

The envelope conditions yield

$$J_l\left(S\right) = \alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} - W(h;S) - W_l(h;S)l + (1-\lambda)\,\delta \mathbb{E} J_l\left(S_{+1}\right), \tag{9}$$

$$J_k\left(S\right) = (1-\alpha)\,A h^{\alpha} l^{\alpha} k^{-\alpha} - W_k(h;S)l - C_1\left(k, k_{+1}\right) + (1-\delta_k)\left[\delta \mathbb{E} J_k\left(S_{+1}\right) - C_2\left(k, k_{+1}\right)\right]. \tag{10}$$

3.5 Workers

The value of being employed, $J^E(S)$, satisfies

$$J^E\left(S\right) = W(h;S) - e\left(h\right) + \lambda \delta \mathbb{E} J^U\left(S_{+1}\right) + (1-\lambda)\delta \mathbb{E} J^E\left(S_{+1}\right), \tag{11}$$

where $J^U(S)$ is the value of being unemployed. As hours of work are determined by the firm, the worker takes h and the level of disutility $e(h)$ as given. The value of being unemployed can be written as

$$J^U\left(S\right) = b + \theta q(\theta)\delta \mathbb{E} J^E\left(S_{+1}\right) + (1-\theta q(\theta))\delta \mathbb{E} J^U\left(S_{+1}\right), \tag{12}$$

where b is the unemployment benefit. Since disutility from long working hours is captured by the disutility function, our b does not include the value of leisure. Thus, it primarily reflects the unemployment insurance provided by the government.

3.6 Earnings Schedule

We assume that at the beginning of each period, workers and a firm bargain over a state-contingent *contract*, which takes in the form of an *earnings schedule* $W(h;S)$ that maps hours of work into earnings. From the earnings schedule, we can compute the *average* hourly wage rate $w(h) = W(h;S)/h$

and the *marginal* hourly wage rate $W_h(h;S)$. We emphasize that what we observe in the data is the average hourly wage rate while what matters for decision making is the marginal hourly wage rate, which is not directly observable. We also note that the exact amount of earnings W and hours of work h are not determined in the bargaining stage.

As in Stole and Zwiebel (1996), we assume that workers are not unionized and each worker is treated as the *marginal worker* in the bargaining stage. Consider a bargaining between a firm and a group of workers of measure Δ. The threat point for the firm is $J(A, l-\Delta, k, U)$ because failing to agree on a contract implies losing the workers. The total match surplus is therefore $J(A,l,k,U) - J(A, l-\Delta, k, U) + \Delta(J^E(S) - J^U(S))$. If the firm's bargaining power is given by $1-\beta \in [0,1]$, then the bargaining outcome must satisfy $\beta[J(A,l,k,U) - J(A,l-\Delta,k,U)] = (1-\beta)\Delta[J^E(S) - J^U(S)]$. In the limit as $\Delta \to 0$,

$$\beta J_l(S) = (1-\beta)\left[J^E(S) - J^U(S)\right]. \tag{13}$$

This is the key equation for rent sharing.

While providing a full strategic foundation for this bargaining outcome is beyond the scope of this paper, a few comments are in order. First, this bargaining outcome amounts to maximizing the asymmetric Nash product $[J_l(S)]^{1-\beta}[J^E(S) - J^U(S)]^{\beta}$ with respect to $W(h;S)$. The relationship between Nash bargaining and alternating-offer bargaining is provided by Binmore *et al.* (1986). Second, when determining the threat point for the firm, $J(A, l-\Delta, k, U)$, it is assumed that the remaining $l-\Delta$ workers accept the contract $W(h;S)$. In other words, we rule out the off-the-equilibrium possibility of re-bargaining between the firm and the remaining $l-\Delta$ workers over a new contract that might take place when the original contract is rejected by the Δ workers. Third, we assume that the disagreement payoff and the outside-option payoff are the same.[14]

Proposition 1 *The earnings schedule is given by*

$$W(h;S) = \frac{\alpha\beta A h^{\alpha} l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + (1-\beta)\left[e(h) + b\right] + \beta c\theta. \tag{14}$$

[14]See Hall and Milgrom (2008) on this important issue. Christiano et al. (2016) develop a medium-scale DSGE model with alternating offer wage bargaining of Hall-Milgrom type.

Proof. See Appendix A. ■

The earnings schedule (14) is a natural extension of the one derived in Kudoh and Sasaki (2011). If the worker's bargaining power β is zero, then (14) takes a very simple form: $W(h;S) = e(h) + b$. This makes the worker indifferent between the states of employment and unemployment. Similarly, if the firm's bargaining power is zero ($\beta = 1$), then we obtain $W(h;S)l = y + c\theta l$, forcing the firm to pay more than what it produces. This implies $\beta < 1$.

From (14), we obtain

$$W_h(h;S) = \frac{\alpha^2 \beta A h^{\alpha-1} l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + (1-\beta)\, e'(h) > 0, \tag{15}$$

$$W_{hh}(h;S) = -(1-\alpha)\frac{\alpha^2 \beta A h^{\alpha-2} l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + (1-\beta)\, e''(h), \tag{16}$$

$$W_l(h;S) = -(1-\alpha)\frac{\alpha \beta A h^{\alpha} l^{\alpha-2} k^{1-\alpha}}{\alpha\beta + 1 - \beta} < 0, \tag{17}$$

$$W_k(h;S) = (1-\alpha)\frac{\alpha \beta A h^{\alpha} l^{\alpha-1} k^{-\alpha}}{\alpha\beta + 1 - \beta} > 0. \tag{18}$$

The key result here is that the marginal hourly wage rate given by (15) is nonlinear in hours of work, and is influenced by the marginal product of hours per worker ($\alpha A h^{\alpha-1} l^{\alpha} k^{1-\alpha}/l$) and the marginal disutility from longer hours of work. The influence of the former (latter) increases (decreases) with β. Expression (16) suggests that, when the disutility function is sufficiently convex and the worker's bargaining power β is sufficiently small, the earnings schedule itself becomes convex in hours of work ($W_{hh} > 0$).

Another key result is that the level of earnings is decreasing in the number of employees ($W_l < 0$). This property induces the firm to employ too many workers in order to cut the wage rate, known as the overhiring effect (Smith, 1999). It is interesting to observe from (14) and (15) that labor market tightness θ has no effect on the marginal hourly wage rate, while it influences the level of earnings. This will imply that the labor market conditions summarized by θ has no direct impact on the choice of hours of work.

Substitute (7), (8), and (15)–(18), into (9) and (10) to obtain

$$J_l(S) = \frac{(1-\beta)\,\alpha}{\alpha\beta+1-\beta} A h^{\alpha} l^{\alpha-1} k^{1-\alpha} - (1-\beta)\left[e(h)+b\right] - \beta c\theta + \frac{(1-\lambda)\,c}{q(\theta)}, \tag{19}$$

$$J_k(S) = \frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta} A h^{\alpha} l^{\alpha} k^{-\alpha} - C_1(k, k_{+1}) + 1 - \delta_k. \tag{20}$$

4 Equilibrium

4.1 Definition

We look for a rational expectations equilibrium in which TFP follows an exogenous stochastic process. Below, we define equilibrium of the model as a system of stochastic difference equations. From (6) and (15), we obtain the equation governing hours of work:

$$\frac{\alpha}{\alpha\beta+1-\beta} A_t K_t^{1-\alpha} = e'(h_t), \tag{21}$$

where $K_t \equiv k_t / h_t l_t$ is the capital-labor ratio.

Substitute (19) and (21) into (7) to obtain

$$\frac{(1+r)\,c}{q(\theta_t)} = \mathbb{E}_t\left\{(1-\beta)\left[e'(h_{t+1})\,h_{t+1} - e(h_{t+1}) - b\right] - \beta c\theta_{t+1} + \frac{(1-\lambda)\,c}{q(\theta_{t+1})}\right\}. \tag{22}$$

This is the job creation condition which determines the demand for l_t. The evolution of employment follows $l_{t+1} = (1-\lambda) l_t + q(\theta_t) v_t$, which determines the the firm's vacancy v_t.

Similarly, substitute (20) into (8) to obtain

$$\mathbb{E}_t\left[\frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta} A_{t+1} K_{t+1}^{-\alpha}\right] = r + \delta_k + \kappa_0\left[r\delta_k + \delta_k^2/2\right]. \tag{23}$$

This determines the demand for capital. The evolution of capital stock is given by $k_{t+1} = (1-\delta_k)k_t + i_t$, which determines investment i_t.

The aggregate variables are determined as follows. In this economy, the number of the unemployed U_t equals the the rate of unemployment because the labor force is normalized to unity. In each period, $\theta_t q(\theta_t) U_t$ job seekers find jobs. Similarly, the aggregate number of employees is $1 - U_t$, from which the aggregate number of separations is $\lambda(1-U_t)$. Thus, the number of the unemployed evolves according to

$$U_{t+1} - U_t = \lambda \left(1 - U_t\right) - \theta_t q \left(\theta_t\right) U_t. \tag{24}$$

In any steady state, the flow into employment $\theta q(\theta) U$ must equal the flow into unemployment $\lambda(1 - U)$, or $m(U, V) = \lambda(1 - U)$, which defines the Beveridge curve. Labor market tightness is given by $\theta_t = V_t / U_t$.

Since the number of firms is normalized to unity, we obtain

$$\frac{1 - U_t}{l_t} = 1, \tag{25}$$

where the numerator is the aggregate number of employees and the denominator is the number of employees at each firm, so the ratio defines the number of firms in the economy. For the same reason, the aggregate number of vacancies equals the number of vacancies created by each firm, or $V_t = v_t$. Similarly, the aggregate output, or GDP of the economy Y_t, is given by $Y_t = y_t$.

4.2 Steady State Equilibrium

From (21)-(25), we obtain the equations that determine a non-stochastic steady state:

$$\frac{(1-\beta)(1-\alpha)}{\alpha\beta + 1 - \beta} A K^{-\alpha} = r + \delta_k + \kappa_0 \left[r\delta_k + \delta_k^2/2 \right], \tag{26}$$

$$\frac{\alpha}{\alpha\beta + 1 - \beta} A K^{1-\alpha} = e'(h), \tag{27}$$

$$\frac{(r+\lambda) c}{q(\theta)} + \beta c \theta = (1-\beta) \left[e'(h) h - e(h) - b \right], \tag{28}$$

$$U = \frac{\lambda}{\lambda + \theta q(\theta)} = 1 - l, \tag{29}$$

where $K = k/hl$. For existence of a steady-state equilibrium, β must be strictly less than one, otherwise total compensation exceeds output. Since the right-hand side of (28) must be positive, parameters must be chosen to satisfy $e'(h)h - e(h) - b > 0$ (or $\mu(1+\mu)^{-1} e_0 h^{1+\mu} > b$) and $\beta < 1$.

Uniqueness of the steady state is easily verified as follows. First, the steady-state capital-labor ratio K is determined by (26). Given K, the steady-state level of hours of work h is determined by (27). Given h, (28) determines θ. Given θ, the steady state level of l is determined by (29). Finally, given the values of K, h, and l, we can derive the value of k by $k = Khl$.

Note that in this economy the number of firms is normalized to unity. In many environments with fixed measure of firms, there is a possibility of strategic interactions among firms (Kudoh and Sasaki, 2010). It is therefore important to verify whether the assumption of a unit measure of firms in this economy is harmless. The following result establishes that in the model with endogenous number of firms, in any steady state, a firm is indifferent between entry and exit, leaving the number of firms indeterminate.

Proposition 2 *In the model without costly capital adjustment ($\kappa_0 = 0$), the steady-state number of firms is indeterminate.*

Proof. In Appendix B. ■

Thus, we can safely normalize the measure of firms to be unity as in the in the neoclassical economy, in which each competitive firm makes zero profit. In general, for this result to hold, the model need to have a constant-returns-to-scale production technology and linear adjustment costs for both labor and capital.[15] This result does not arise in Smith (1999) or Kudoh and Sasaki (2011) because the production technology exhibits decreasing returns to scale in these models.

5 Efficiency

This section discusses efficiency of the economy. We first define the efficient allocation by solving a planner's allocation problem subject to search-matching frictions. We then compare the levels of hours determined by the planner, chosen by the firm, and determined in bargaining. Finally, we present a version of the textbook search-matching model to show that our efficiency results are unique to the large-firm economy.

5.1 Planner's Allocation

To assess the efficiency of the equilibrium allocation, we now consider a social planner's problem. The key is that while the planner cannot remove search-matching frictions, he or she does have

[15]With costly capital adjustment, there will be a nondegenerate capital distribution among firms.

the ability to internalize the externality arising from the matching process and the inefficiency arising from bargaining. The planner's allocation problem is given by

$$\max_{\{h_t, l_{t+1}, k_{t+1}, i_t, U_t, V_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \left[I_t - (1 - U_t)\, e\,(h_t) \right]$$

subject to

$$l_{t+1} = (1-\lambda) l_t + M\,(U_t, V_t)\,, \tag{30}$$

$$k_{t+1} = (1-\delta_k)\, k_t + i_t, \tag{31}$$

$$I_t = A_t h_t^{\alpha} l_t^{\alpha} k_t^{1-\alpha} - cV_t - i_t + U_t b \tag{32}$$

$$U_{t+1} - U_t = \lambda\,(1 - U_t) - M\,(U_t, V_t)\,, \tag{33}$$

$$l_t = 1 - U_t. \tag{34}$$

Proposition 3 *If the Hosios condition* $(-q'(\theta)\theta/q(\theta) = \beta)$ *is satisfied, then the planner's allocation satisfies*

$$\mathbb{E}_t \left[(1-\alpha)\, A_{t+1} K_{t+1}^{-\alpha} \right] = r + \delta_k, \tag{35}$$

$$\alpha A_t K_t^{1-\alpha} = e'\,(h_t)\,, \tag{36}$$

$$\frac{c\,(1+r)}{q\,(\theta_t)} = \mathbb{E}_t \left\{ (1-\beta) \left[e'\,(h_{t+1})\, h_{t+1} - e\,(h_{t+1}) - b \right] + \frac{c\,(1-\lambda)}{q\,(\theta_{t+1})} - c\beta\theta_{t+1} \right\}, \tag{37}$$

$$l_{t+1} = (1-\lambda)\, l_t + M\,(U_t, V_t) \tag{38}$$

Proof. In Appendix C. ■

We shall refer to the allocation defined by (35)–(38) as the efficient allocation. With $U_t = 1 - l_t$, the expressions (24) and (38) are identical. In addition, if the Hosios condition ($\xi = \beta$) is satisfied, the equation governing the equilibrium job creation (22) becomes identical to (37). Proposition 3 establishes that the equilibrium allocation is not efficient even under the Hosios condition, found first in a related stationary environment by Kudoh and Sasaki (2011). In what follows, we shall focus on the environment without the inefficiency that arises from search-matching frictions by imposing the Hosios condition. What remains is the inefficiency that arises from the intra-firm bargaining.

5.2 Divisible Labor with Bargained Hours of Work

Our basic model assumes that the firm and its employees bargain over an earnings schedule and that the firm chooses hours of work per employee from the state-contingent contract $W(h;S)$. A possible criticism on this setup is that, from a theoretical perspective, it is more natural to assume that both hours of work and earnings are determined in bargaining because the level of hours of work that maximizes the Nash product is efficient, given a match surplus. We will demonstrate, however, that the level of bargained hours of work is *less* efficient than the level of hours of work chosen by the firm.

We consider an alternative model in which hours of work per employee are determined by bargaining. Specifically, we assume that the *level* of compensation W (not the earnings schedule) and hours per employee h are determined so as to maximize the asymmetric Nash product: $[J_l(S)]^{1-\beta}[J^E(S) - J^U(S)]^{\beta}$. The first-order conditions with respect to W and h imply $\beta J_l(S) = (1-\beta)[J^E(S) - J^U(S)]$, which generates (14), and

$$\frac{\partial[J_l(S)]}{\partial h} + \frac{\partial[J^E(S)]}{\partial h} = 0, \tag{39}$$

where $\partial[J_l(S)]/\partial h = [(1-\beta)\alpha^2/(\alpha\beta+1-\beta)]Ah^{\alpha-1}l^{\alpha-1}k^{1-\alpha} - (1-\beta)e'(h)$ from (19) and $\partial[J^E(S)]/\partial h = [\beta\alpha^2/(\alpha\beta+1-\beta)]Ah^{\alpha-1}l^{\alpha-1}k^{1-\alpha} - \beta e'(h)$ from (11).

Thus, (21) is replaced with

$$\frac{\alpha^2}{\alpha\beta+1-\beta}A_t K_t^{1-\alpha} = e'(h_t). \tag{40}$$

As a result, the job-creation condition (22) is now

$$\frac{(1+r)\,c}{q(\theta_t)} = \mathbb{E}_t\left\{(1-\beta)\left[\frac{1}{\alpha}e'(h_{t+1})\,h_{t+1} - e(h_{t+1}) - b\right] - \beta c\theta_{t+1} + \frac{(1-\lambda)\,c}{q(\theta_{t+1})}\right\}. \tag{41}$$

Equations (40) and (41) are nearly identical to (21) and (22), making the qualitative properties of this model similar to those of the basic model. However, the quantitative properties of these two models are significantly different. We shall come back to this issue in Section 6.

Hours of Work Chosen Cooperatively Suppose that the level of hours of work is chosen at the beginning of each bargaining stage to maximize the joint surplus. Under this timing assumption, wage bargaining simply distributes the maximized joint surplus to the firm and the marginal worker. Consider the second stage in which the maximized surplus is divided. The level of total compensation W must satisfy $\beta J_l(S) = (1-\beta)[J^E(S) - J^U(S)]$ as in the basic model. In the first stage, the level of hours of work is chosen to maximize the joint surplus $J_l(S) + [J^E(S) - J^U(S)]$, from which we obtain $\partial[J_l(S)]/\partial h + \partial[J^E(S)]/\partial h = 0$, which is the same condition as (39). Thus, the model in which h is bargained and the model in which h is chosen to maximize the joint surplus are equivalent.

Hours of Work Chosen by Workers For completeness, we consider yet another scenario in which the level of hours of work per employee is chosen by each worker. As in the basic model, each firm and its employees bargain over an earnings schedule at the beginning of each period. Given the earnings schedule $W(h;S)$, each employee chooses the level of hours of work in each period. In this case, we replace (11) with

$$J^E(S) = \max_h \left\{ W(h;S) - e(h) + \lambda\delta\mathbb{E}J^U(S_{+1}) + (1-\lambda)\delta\mathbb{E}J^E(S_{+1}) \right\}.$$

The first-order condition requires $W_h(h;S) = e'(h)$, from which we obtain the expression identical to (40). Thus, the model in which hours of work are chosen by workers and the model in which hours of work are bargained generate the same equilibrium conditions, as in a related steady-state economy studied by Kudoh and Sasaki (2011).

Proposition 4 *Let h_t^p, h_t^f, and h_t^b denote hours of work chosen by the planner, set by the firm, and determined in bargaining, respectively. For the same set of parameter values, under the Hosios condition,*

$$h_t^b < h_t^f < h_t^p \tag{42}$$

holds for all t.

Proof. In Appendix D. ■

Proposition 4 establishes that the level of hours of work in our basic model is below the efficient level. Interestingly, the level of hours of work determined in bargaining is even less efficient than the level of hours chosen by the firm. When hours are bargained, given a match surplus, hours of work are efficient. However, with intra-firm bargaining and concave production, the match surplus itself is not efficient. As a result, the level of hours is below the level in the basic model. We obtain $h_t^b = h_t^f = h_t^p$ when $\alpha = 1$.

5.3 An Economy with Small Firms

To understand the role of the large-firm setting, this section presents the textbook search-matching model, modified to include capital and variable hours of work. To make the model comparable to our basic model and the planner's problem in the preceding sections, we adopt a version of Marimon and Zilibotti (2000), in which each production unit employs at most one employee and produces and sells homogeneous intermediate good to the final good producer who possesses the entire stock of capital. While somewhat ad-hoc, this market structure helps us introduce capital into the textbook model.

The final consumption good is produced by the representative final output firm with production technology, $y_t = A_t X_t^{\alpha} k_t^{1-\alpha}$, where X_t is the amount of intermediate input. We assume that the final good market and the intermediate good market are both perfectly competitive. The price of the intermediate good is p_t. The final output firm's problem is given by

$$\max_{X_t, i_t} \mathbb{E} \sum_{t=0}^{\infty} \delta^t \left[A_t X_t^{\alpha} k_t^{1-\alpha} - p_t X_t - i_t \right],$$

subject to $k_{t+1} = (1 - \delta_k) k_t + i_t$. Let $F(k)$ be the value of the firm. Thus, the Bellman equation for this problem is

$$F(k) = \max_{X, k'} \left[A X^{\alpha} k^{1-\alpha} - pX - k' + (1 - \delta_k) k + \delta \mathbb{E} F(k') \right],$$

from which we obtain $\alpha A_t X_t^{\alpha-1} k_t^{1-\alpha} = p_t$ and $\mathbb{E}_t[(1-\alpha) A_{t+1} X_{t+1}^{\alpha} k_{t+1}^{-\alpha}] = r + \delta_k$.

There is a continuum of intermediate firms. Free entry determines the aggregate number of intermediate firms (and jobs). Each intermediate firm employs at most one worker and produces the intermediate good using a linear technology. Thus, the value of a filled job is

$$J_t^F = \max_{h_t} \left\{ p_t h_t - W(h_t) + \lambda \delta \mathbb{E}_t J_{t+1}^V + (1 - \lambda) \delta \mathbb{E}_t J_{t+1}^F \right\},$$

from which the demand for hours satisfies $p_t = W'(h_t)$. The value of a vacancy is standard: $J_t^V = -c + q(\theta_t)\delta\mathbb{E}_t J_{t+1}^F + [1 - q(\theta_t)]\delta\mathbb{E}_t J_{t+1}^V$. Free entry of jobs implies $J_t^V = 0$. Thus, we obtain the standard job-creation condition:

$$\frac{(1+r)\,c}{q\,(\theta_t)} = \mathbb{E}_t\left[p_{t+1}h_{t+1} - W\,(h_{t+1}) + \frac{(1-\lambda)\,c}{q\,(\theta_{t+1})}\right].$$

As in our basic model, we assume that the firm and the worker bargain over the contract that specifies the earnings schedule $W(h)$. Thus, $W(h)$ is the solution to the following problem:

$$\max_{W(h)} \left(J_t^F - J_t^V\right)^{1-\beta}\left(J_t^E - J_t^U\right)^{\beta},$$

where the value functions for workers are standard: $J_t^E = W(h_t) - e(h_t) + \lambda\delta\mathbb{E}_t J_{t+1}^U + (1-\lambda)\delta\mathbb{E}_t J_{t+1}^E$ and $J_t^U = b + \theta_t q(\theta_t)\delta\mathbb{E}_t J_{t+1}^E + [1 - \theta_t q(\theta_t)]\delta\mathbb{E}_t J_{t+1}^U$. We obtain the wage schedule as $W(h_t) = \beta p_t h_t + (1-\beta)[e(h_t) + b] + \beta c\theta_t$, from which $W'(h_t) = \beta p_t + (1-\beta)e'(h_t)$. In any equilibrium, $X_t = h_t l_t$ holds. Thus, $\alpha A_t (h_t l_t)^{\alpha-1} k_t^{1-\alpha} = e'(h_t)$. Finally, worker flows imply $U_{t+1} = U_t - \theta_t q\,(\theta_t)\,U_t + \lambda\,(1 - U_t)$.

Proposition 5 *For the same set of parameter values, under the Hosios condition,*

$$h_t^b = h_t^f = h_t^p$$

for all t.

Proof. In Appendix E. ■

In sharp contrast with our basic model, under the Hosios condition, the textbook search-matching model replicates the planner's allocation both in and out of the steady state. This result is in line with the property of the plane vanilla DMP model (Pissarides, 2000).

6 Quantitative Analysis

In this section, we study a quantitative version of the basic model. We first calibrate the model to match the selected long-run Japanese labor market facts. We then solve the quantitative model by approximating the equilibrium conditions around the non-stochastic steady state, and simulate it to obtain the model's cyclical properties. We also explore the role of the intensive margin in the model and present some robustness analyses for our results.

6.1 Calibration

We choose the model period to be a quarter and set the discount rate to be $r = 0.01$, which implies the discount factor to be $\delta = 1/(1+r) = 0.99$. This choice of the parameter is somewhat a priori, but is consistent with other studies such as Braun *et al.* (2006). In the production function, we set $\alpha = 2/3$ to target the labor share.[16] Following Esteban-Pretel *et al.* (2010), we set the deprecation rate to be $\delta_k = 0.028$.

The matching function is Cobb-Douglas, given by $m(U,V) = m_0 U^{\xi} V^{1-\xi}$, where m_0 is the matching constant and ξ is the matching elasticity with respect to the number of job-seekers. Lin and Miyamoto (2014) estimate the elasticity ξ to be 0.6. We adopt their estimate to set $\xi = 0.6$. This value lies in the plausible range of 0.5–0.7, which is reported by Petrongolo and Pissarides (2001). Following the convention, we use the Hosios (1990) condition to pin down the worker's bargaining power, so $\beta = \xi = 0.6$. Note that under the Hosios condition, the inefficiency arising from job creation is neutralized, helping us focus on the inefficiency that arises from intra-firm bargaining.

Using the panel property of the monthly LFS, Miyamoto (2011) and Lin and Miyamoto (2012) construct the job-finding rate and the separation rate in Japan. Miyamoto (2011) also reports the mean value of the vacancy-unemployment ratio to be 0.78. Given this, we target the vacancy-unemployment ratio to be $\theta = 0.78$. We use the monthly job-finding rate 0.142 and the vacancy-unemployment ratio to pin down the scale parameter m_0. In particular, m_0 is the solution to $m_0(0.78)^{1-0.6} = 3 \times 0.142$. We also set the exogenous separation rate $\lambda = 0.014 = 3 \times 0.0048$ from Miyamoto (2011) and Lin and Miyamoto (2012).

We choose $\mu = 1.8$ or the Frisch elasticity is $1/\mu = 0.56$, which is consistent with the micro

[16] We aware that in our frictional economy, α is not necessarily the labor's share of national income. Nonetheless, we assume α to take the same value as in the perfectly competitive economy.

evidence that the Frisch elasticity is less than one.[17] Our parameter value is also consistent with the evidence that the Frisch elasticity for males in Japan is in the range of 0.2–0.7 (Kuroda and Yamamoto, 2008). We will discuss the sensitivity of the model to the choice of μ.[18]

We target the steady-state value of hours worked to be $1/3$.[19] With $h = 1/3$, (26) and (27) jointly determine the implied value of e_0, which is 12.576.

In our model, the value of b reflects mostly the unemployment benefit provided by the government because the value of leisure is captured by the disutility from work $e(h)$. According to Nickell, (1997), the benefit replacement rate in Japan is about 60 percent.[20] We thus adopt this estimate to target the unemployment benefit b to satisfy $b = 0.6W$. Given this, we determine b and c by solving (14) and (28) with targets $\theta = 0.78$ and $h = 1/3$. The implied values are $b = 0.348$ and $c = 0.020$.

Finally, we assume that TFP follows a first order autoregressive process. Specifically, $\log A_t$ satisfies $\log A_t - \log A = \rho (\log A_{t-1} - \log A) + \varepsilon_t$,where $0 < \rho < 0$ and $\varepsilon_t \sim N(0, \sigma^2)$. We set $\rho = 0.612$ and $\sigma = 0.009$ to match the first-order autocorrelation and standard deviation of TFP in the data.[21]

The parameter values for the benchmark analysis are summarized in Table 2. Note that the values of parameters m_0, e_0, b, and c are endogenous in the sense that the values of these parameters are re-calibrated to match the target moments for each set of purely exogenous parameters.

Before proceeding to our main results, in Table 3 we report the steady-state levels of selected endogenous variables. The values of θ and h equal to their calibration targets. The model generates the unemployment rate of 3.3 percent, which is close to the observed average unemployment

[17]In a similar environment to ours, Cooper *et al.* (2007) calibrated the value of μ to be 1.9 for the US economy.

[18]To generate a realistic magnitude of fluctuations in hours of work per employee, we need a small μ (or, a large Frisch elasticity). At the same time, we need to constrain our choice of μ to be within the range of the set of parameters that supports a steady state equilibrium to exists (namely, $\mu(1+\mu)^{-1} e_0 h^{1+\mu} > b$), which requires a large μ.

[19]In other words, we target h to be 8 hours per day or 40 hours per week (or 5 business days). However, our quantitative results are independent of the choice of the target level for h.

[20]See also Martin (2000).

[21]Thus, for all simulations, the standard deviation of the percentage deviation of TFP from its steady-state level is 0.011, as in the data.

Table 2: Parameter values

Parameter	Description	Value	Source/Target
r	Interest rate	0.01	Esteban-Pretel *et al.* (2010)
δ	Discount rate $1/(1+r)$	0.99	-
α	Parameter in production function	2/3	Labor share
m_0	Matching efficiency	0.471	Monthly job-finding rate
ξ	Matching elasticity	0.6	Lin and Miyamoto (2014)
λ	Exogenous separation rate	0.014	Monthly separation rate
e_0	Parameter in disutility function	12.576	$h = 1/3$
δ_k	Depreciation rate	0.028	Esteban-Pretel *et al.* (2010)
μ	Inverse of Frisch elasticity	1.8	Kuroda and Yamamoto (2008)
b	Unemployment benefits	0.348	Replacement rate = 60 percent
β	Worker's bargaining power	0.6	$\beta = \xi$ (Hosios condition)
c	Vacancy cost	0.020	$v - u$ ratio = 0.78
A	Productivity	1.0	Normalization
ρ	AR-coefficient of shock	0.612	Data
σ	Standard deviation of the shock	0.0085	Data

rate of 3.4 percent. Other steady-state values from our model are harder to compare with the associated values of the data for two reasons. One reason is because we choose to normalize the level of quarterly hours of work per employee to be 1/3. The other reason is because we abstract savings decisions of households from the model and the supply of capital is assumed to be virtually abundant as in a small open economy.[22]

[22]Introducing savings decisions of households allows one to pin down the supply of capital. We choose to abstract this issue from our model because we introduce capital only to define TFP and explaining capital is beyond the scope of this paper.

Table 3: Steady State Values

U	V	θ	y	t	h	l	k	W	c/y	k/y	i
0.033	0.026	0.78	0.67	0.75	0.33	0.97	2.94	0.58	0.03	4.36	0.083

6.2 Main Results

We now compare the selected business cycle statistics from the simulated series with their empirical counterparts. To this end, we shall primarily focus on the magnitude of fluctuations in each variable measured by the standard deviation. Table 4 reports the standard deviations of the unemployment rate, the vacancy rate, earnings, the (average) hourly wage rate, total labor input, hours of work per employee, and employment, scaled by the standard deviation of TFP. We also report the relative contributions of the intensive and extensive margins calculated by formula (2).

Table 4: Hours of Work and Labor Market Fluctuations

	Relative standard deviations								
	$\hat{U}$	$\hat{V}$	$\hat{W}$	$\hat{w}$	$\hat{t}$	$\hat{h}$	$\hat{l}$	β^h	β^l
Data	**5.45**	**8.92**	**1.16**	**0.91**	**0.80**	**0.70**	**0.35**	**0.79**	**0.21**
Basic model	4.33	12.81	1.55	0.91	0.75	0.63	0.15	0.84	0.19
Model w/o hours	0.96	2.85	1.22	1.22	0.03	-	0.03	-	-
Model with $h = 1/3$	4.33	12.81	0.96	0.96	0.15	-	0.15	-	-
Bargained hours	1.36	4.02	1.51	0.88	0.67	0.63	0.05	0.95	0.07
Planner	4.33	12.81	1.55	0.91	0.75	0.63	0.15	0.84	0.19

For all variables listed in Table 4, the standard deviations obtained from our basic model are close to those obtained from the data. The relative standard deviations generated by the model of total labor input, hours per employee, and employment are 0.75, 0.63, and 0.15, respectively. The corresponding relative standard deviations from the data are 0.80, 0.70, and 0.35. While the magnitude of employment fluctuations in the model is about a half of that in the data, the model replicates much of fluctuations in the labor demand and its compositions. In particular, the model captures the observation that much of fluctuations in total labor input is accounted for by fluctuations in hours of work per employee. Indeed, by applying (2) to decompose the variations in total labor input, we find that $\beta^h = 0.84$ and $\beta^l = 0.19$.

The model's inability in generating enough extensive margin fluctuations can be partly explained by the absence of the *other* extensive margin, namely, changes in employment driven by entry and exit of firms over the business cycle. With this limitation in mind, we conclude that our model captures the labor market dynamics in Japan fairly well.[23] While developing a full-fledged model of firm entry over the business cycle is beyond the scope of this paper, in Section 7 we explore the role of firm's entry and exit.

It is well recognized that the textbook search-matching model fails to account for the observed level of volatility in the unemployment and vacancy rates, often referred to as the unemployment volatility puzzle (Shimer, 2005).[24] Interestingly, Table 4 shows that our model does replicate fluctuations in the unemployment rate and the vacancy rate with realistic or even greater magnitudes. The relative standard deviation of the unemployment rate from the model is 4.33 while that from the data is 5.45, and the relative standard deviation of the vacancy rate from the model is 12.81 while that from the data is 8.92. The relative standard deviation of the labor market tightness, the ratio of the vacancy rate to the unemployment rate, for the model is 14.66 while that for the data is 12.92. We emphasize that these results are obtained without wage rigidity or additional shocks. In addition, the model generates persistence in unemployment. The autocorrelation between U_t and U_{t-1} is 0.88 in the model while it is 0.96 in the data.

Although the allocation in the basic model differs from the planner's, the magnitudes of fluctuations in all variables are the same for the two economies.[25] This occurs because we impose the

[23] We also note that there is a sizable proportion of employees under short-term contracts in Japan, referred to as non-regular employment (Miyamoto, 2016). Labor adjustment along the extensive margin for non-regular employment is considered to be less frictional. This partly explains why fluctuations along the extensive margin in the data are greater than those implied by the model.

[24] Esteban-Pretel *et al.* (2011) and Miyamoto (2011) show that the unemployment volatility puzzle holds for the Japanese economy.

[25] A caveat is that while the levels of working hours are different under the same parameter set, in our numerical

Hosios condition, making the job-creation conditions for the two economies identical. An important implication of this result is that our basic model and the small-firms economy presented in Section 5.3 have the same business cycle properties.

6.2.1 Model without Hours of Work

To clarify the importance of the presence of hours of work, we consider a polar case in which the intensive margin is completely shut down. To be more concrete, we study a model in which h is *removed*. Thus, the level of labor input is now $L_t = l_t$ so that the production function is replaced with $A_t l_t^{\alpha} k_t^{1-\alpha}$. We also remove disutility term $e(h_t)$ from the model.[26]

Results in Table 4 show the importance of modeling hours of work. Without hours of work, the relative standard deviation of total labor input is 0.03, which is 1/25 of the corresponding value in the basic model and far from the data. This means that the extensive margin alone cannot account for fluctuations in total labor input. Further, introduction of the intensive margin helps magnify labor adjustment along the extensive margin. In the model without hours of work, the relative standard deviations of unemployment and vacancies are 0.96 and 2.85, respectively. The corresponding values from the basic model are respectively 4.33 and 12.81, which are about 4 times as large as those from the model without hours of work.

6.2.2 Model with Fixed Hours of Work

To understand the impact of *variable* hours of work on the magnitude of extensive margin fluctuations, we report an additional result in Table 4. Instead of removing hours of work from the model, we study the model in which hours of work are fixed at our calibration target $h = 1/3$. Thus, in the steady state, this model is identical to the basic model, implying that workers incur

analysis, we choose the parameters so that $h = 1/3$ for *all models*.

[26] This corresponds to the model studied by Krause and Lubik (2013), in which the cyclical properties of a model with large firms are compared to those of a textbook search-matching model to clarify whether the multi-worker paradigm helps resolve the unemployment volatility puzzle. They conclude that while the large-firm paradigm helps increase the magnitudes of fluctuations in unemployment and vacancies, there is no sizable quantitative improvement.

disutility from long hours of work as in the basic model. However, the intensive margin is shut down in the sense that firms cannot change hours per employee in response to shocks.

Interestingly, in this model, unemployment and vacancies fluctuate as much as those in the basic model. Thus, we conclude that introduction of *disutility* from long hours of work per se helps generate high magnitudes of fluctuations in unemployment and vacancies, irrespective of whether the intensive margin is active or not. This is consistent with studies that emphasize the importance of including the value of leisure in calibrating the flow value of unemployment (Hagedorn and Manovskii, 2008; Hall and Milgrom, 2008). As is evident from (14), the level of disutility $e(h)$ works in the same direction as b does. Thus, what matters is the level of $z = b + e(h)$: it increases the equilibrium level of total compensation, increasing the sensitivity of firm's response to external shocks (Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2016).

6.2.3 Bargained Hours of Work

By comparing (40) with (21), one might argue that our basic model generates a small employment volatility because it overestimates the marginal benefit of an additional hour of work (because hours of work are chosen only to maximize the firm's value), making the extensive margin relatively less attractive.

Interestingly, Table 4 shows that the standard deviations of unemployment, vacancies, and employment obtained from the model with bargained hours of work are about 1/3 of those from the basic model. The marginal product of an additional hour is reduced by factor of α, which reduces the firm's incentive to utilize the intensive margin. However, for the model to match the target steady-state hours of work per employee, $h = 1/3$, parameters e_0 and c need to be re-calibrated. The implied values are now $e_0 = 8.384$ and $c = 0.075$. The smaller e_0 offsets the impact of the reduction in the marginal product, while the increase in c increases the marginal cost of posting a vacancy. As a result, the magnitudes of fluctuations along the extensive margin, namely, fluctuations in employment, vacancies, and unemployment, are all reduced, keeping the standard deviation of hours of work per employee virtually unchanged.

6.3 Robustness

This section considers robustness of our results in two dimensions. First, we assess whether our basic model is "too small." Specifically, we check whether introduction of additional components, namely, costly capital adjustment and separation shocks, improves the model's performance.[27] Second, we provide some sensitivity analyses in terms of the labor supply elasticity, the unemployment benefit, and the bargaining power. We take these parameters for our sensitivity analyses as they are known to have a range of estimates and calibrated values.

Table 5: Alternative Assumptions and Parameter Values

	Relative standard deviations								
	$\hat{U}$	$\hat{V}$	$\hat{W}$	$\hat{w}$	$\hat{t}$	$\hat{h}$	$\hat{l}$	β^h	β^l
Data	**5.45**	**8.92**	**1.16**	**0.91**	**0.80**	**0.70**	**0.35**	**0.79**	**0.21**
Basic model	4.33	12.81	1.55	0.91	0.75	0.63	0.15	0.84	0.19
Capital adjustment	3.01	8.22	1.18	0.69	0.56	0.49	0.10	0.87	0.17
Stochastic separations	7.39	13.04	1.55	0.91	0.81	0.63	0.25	0.76	0.26
$\mu = 1.9$	3.36	9.95	1.52	0.92	0.69	0.60	0.11	0.87	0.16
$\mu = 1.7$	6.26	18.53	1.58	0.91	0.83	0.67	0.21	0.79	0.25
$\mu = 1.613$ $(1/\mu = 0.62)$	10.74	31.77	1.61	0.91	1.00	0.70	0.36	0.68	0.36
$b/\bar{W} = 0.4$	0.77	2.28	1.56	0.92	0.65	0.63	0.03	0.97	0.04
$\beta = 0.5$	3.89	11.50	1.46	0.83	0.73	0.63	0.13	0.86	0.18
$\beta = 0.7$	4.72	13.96	1.62	0.99	0.76	0.63	0.16	0.83	0.21

6.3.1 Capital Adjustment

We choose to work with TFP as the driver of labor market fluctuations. This requires capital as an input of production. A possible side effect of introducing capital is that, if capital adjustment is perfectly flexible (*i.e.*, $\kappa_0 = 0$), it might make adjustment in labor relatively more costly. This

[27] In Appendix F, we provide additional results from "even bigger" models.

could generate too little fluctuations in total labor input, which could explain the low volatility of employment in the basic model. It is therefore important to assess whether flexible capital adjustment will suppress the labor adjustment channel.

Table 5 shows that, with costly capital adjustment ($\kappa_0 = 1$), the magnitudes of fluctuations in *all* variables are *smaller* (rather than larger) than those in the basic model as well as those in the data. This is because the cost of capital adjustment reduces the steady-state level of capital. With a smaller stock of capital, the marginal product of hours of work per employee decreases, and this dominates the reduction in the marginal hourly wage rate. As a result, the firm's incentive to utilize the intensive margin declines. Similarly, a reduction in capital reduces the marginal product of employment, reducing the firm's incentive to utilize the extensive margin.

Overall, introduction of costly capital adjustment reduces the magnitudes of fluctuations in both the intensive and extensive margins. Thus, the presence of perfectly flexible capital does not make labor adjustment more costly. We also note that, in the model without costly capital adjustment, there is little room for improving the magnitude of fluctuations in capital. In the data, the standard deviation of capital is 0.012 while that of the basic model is 0.017. We therefore conclude that we can drop this component by setting $\kappa_0 = 0$.

6.3.2 Separations

The basic model assumes that there is no firing decision and the separation rate is constant over time. However, recent empirical studies show that both unemployment inflow and outflow significantly contribute to the unemployment dynamics in Japan (Miyamoto, 2011; Lin and Miyamoto, 2012). Furthermore, in the data, TFP and the separation rate are negatively correlated. This could amplify fluctuations in the unemployment rate. It is therefore important to verify whether the assumption of a constant separation rate is responsible for the low employment volatility in the basic model. There is a caveat. Shimer (2005) finds that while introduction of separation shocks helps increase the magnitudes of fluctuations in unemployment and vacancies, it destroys the negative correlation between the unemployment rate and the vacancy rate, or the Beveridge curve.

To assess the importance of the unemployment inflow channel in generating fluctuations in employment and unemployment within our framework and to see if the Beveridge curve is preserved, we study a model in which the separation rate follows some stochastic process. Specifically, we assume that the separation rate follows a first-order autoregressive process of $\log \lambda_t - \log \lambda = \rho_\lambda (\log \lambda_{t-1} - \log \lambda) + \varepsilon_{\lambda,t}$, where $0 < \rho_\lambda < 1$ and $\varepsilon_{\lambda,t} \sim N(0, \sigma_\lambda^2)$. From the data, we set $\rho_\lambda = 0.158$, $\sigma_\lambda = 0.091$. and $corr(\hat{A}_t, \hat{\lambda}_t) = -0.371$.

Our results are mixed. Table 5 shows that introduction of stochastic separations significantly increases the standard deviations of employment, unemployment, and vacancies. In particular, the model with separation shocks accounts for much of the observed fluctuations in employment. The standard deviation of employment (relative to that of TFP) is 0.25 while that from the data is 0.35. Although the model is successful along this dimension, the magnitudes of fluctuations in unemployment and vacancies are too large to match the data. Further, as in the literature, stochastic separations weaken the negative correlation between unemployment and vacancies. The correlation between unemployment and vacancies in this economy is -0.41, in contrast to -0.72 in the basic model. From this exercise, we conclude that, while our model precludes the firing margin of labor adjustment, this may not be a serious omission.

6.3.3 Frisch Elasticity

It is well-known that the Frisch labor supply elasticity, $1/\mu$, has a range of estimates. The empirical literature shows that the elasticity from the micro data is much smaller than that from the macro data because the macro elasticity includes variations in labor market participation (or the extensive margin of labor supply). Further, the micro evidence suggests that the Frisch elasticity is less than one. Our benchmark model employs $\mu = 1.8$. Table 5 presents the results under alternative values.

For $\mu = 1.9$ (or, $1/\mu = 0.53$) the values of e_0 is re-calibrated to match the target $h = 1/3$, to obtain $e_0 = 14.036$. Similarly, the values of c is re-calibrated to be 0.026. These values are greater than those under $\mu = 1.8$. While a higher μ implies a greater marginal hourly wage rate, a higher c implies a greater marginal cost of posting a vacancy. Thus, fluctuations in total labor input, hours per employee, and employment are all less than those from the basic model because the marginal costs for the both margins increased. Overall, the effect on the extensive margin dominates the other, and the relative importance of the intensive margin increases to 0.87. For $\mu = 1.7$ (or, $1/\mu = 0.59$), the mechanism is reversed, and the magnitudes of fluctuations in all measures are greater than those for the benchmark case.

Finally, we choose μ to target the observed relative standard deviation in hours of work, which is 0.70. The implied Frisch elasticity is 0.62, which implies $\mu = 1.613$. This value is within the range of estimate of the Frisch elasticity, 0.2-0.7, reported in Kuroda and Yamamoto (2008) and clearly far below unity. Table 5 shows that under this Frisch elasticity, the model's relative standard deviation in employment is also very close to that in the data. In sharp contrast with the standard real business cycle model, which typically requires a counterfactually high Frisch elasticity to generate realistic magnitudes of fluctuations in hours, our model requires a very small elasticity to generate large fluctuations. However, under this Frisch elasticity, unemployment and vacancies fluctuate too much to explain the data.

6.3.4 Replacement Rate

In our benchmark calibration, we choose the value of b to target the actual replacement rate of 60 percent in Japan. The choice of the parameter value for b has been the subject of discussion in the literature. For the US labor market, Shimer (2005) sets b so that the replacement rate is 40 percent to target the actual replacement rate in the U.S, while Hagedorn and Manovskii (2008) argue that Shimer's b is too low and that with a much higher b, search-matching models can replicate unemployment and vacancy fluctuations with realistic magnitudes. With this debate in mind, we consider an alternative level of b that corresponds to the replacement rate of 40 percent.

Under the replacement rate of 40 percent, the corresponding value of b is $b = 0.230$, and the implied value for the vacancy cost is $c = 0.112$. Since the vacancy cost is 5.6 times as large as that for the basic model, the incentive to utilize the extensive margin shrinks significantly. As a result, the magnitude of employment fluctuations decreases significantly, keeping the magnitude of fluctuations in hours per employee unchanged, as shown in Table 5. Consequently, the relative importance of the intensive margin increases from 0.84 to 0.97. Further, Table 5 suggests that a smaller b causes unemployment and vacancies to be much less volatile. These results are in line with Hagedorn and Manovskii (2008).[28]

6.3.5 Bargaining Power

In the basic model, we follow the convention to choose $\beta = \xi$ to meet the Hosios condition. Another useful benchmark is to assume the symmetric Nash product, which implies $\beta = 0.5$ in our framework. With this value, the implied parameter values are $e_0 = 13.225$ and $c = 0.031$. As is clear from Table 5, this case is somewhat similar to the case under a lower replacement rate. The implied values of e_0 and c are both greater than those of the basic model, making both the intensive and extensive margins more costly. However, the value of e_0 is only slightly greater than that for the basic model while the value of c is about 1.6 times as large as that for the basic model. This makes the extensive margin more costly for firms.

Table 5 also presents the results under $\beta = 0.7$, which is greater than the value for the basic model. The results show that the model under $\beta = 0.7$ fluctuates more along the extensive margin than under $\beta = 0.6$, keeping the magnitude of fluctuations in the intensive margin the same. The results are in line with the literature. The higher the worker's bargaining power is, the higher the resulting level of earnings becomes. As a result, the surplus for the firm is smaller, making the firm more sensitive to shocks.

[28]The relationship between the magnitude of fluctuations and the match surplus is clarified by Hornstein *et al.* (2005) and Elsby and Michaels (2013). Hagedorn and Manovskii (2008) argued that, given that individuals typically have access to alternative sources of income such as home production, the flow value of unemployment should be close to the flow value of employment.

7 Firm Entry and Exit

Our basic model assumes that there is a unit measure of firms. This rules out the extensive margin fluctuations driven by firm entry and exit. This could be a serious omission because the empirical counterpart of the model, studied in Section 2, contains labor market fluctuations driven by firm entry and exit. Indeed, Coles and Kelishomi (2011) show that much of the net job creation in the U.S. is attributed to firm entry. The purpose of this section is to quantify the importance of this additional margin.

Since any potential entrant is indifferent between entry and exit in any steady state, our modeling strategy is to introduce out-of-steady-state fluctuations in the number of firms while keeping the steady-state number of firms to be normalized to unity. In principle, this extension requires us to evaluate the expected value of entry over the business cycle, which is quite challenging. In the literature, two methods are known. One method is to numerically evaluate the value of entry (Clementi and Palazzo, 2016). Another way is to introduce the stock market to evaluate the value of each firm (Bilbiie *et al.*, 2012). Since building a full-fledged model of firm dynamics is beyond the scope of this paper, we instead introduce *stochastic firm entry* into our basic model.

Consider a version of the basic model with a constant separation rate and without costly capital adjustment. Let N_t denote the number of firms. Since the steady-state number of firms is indeterminate, as in the basic model we set $N = 1$. However, we allow N_t to vary over the business cycle. For simplicity, the firm exit rate is assumed to be constant. To be specific, at the end of each period, each entrant and incumbent firm is hit by an exogenous exit shock with probability $\phi \in [0, 1)$. Thus, the number of firms evolves according to

$$N_{t+1} = (1-\phi)\left(N_t + N_t^E\right) = (1-\phi)\left(1 + n_t^E\right) N_t, \tag{43}$$

where N_t^E is the number (i.e., measure) of entrants in period t and $n_t^E = N_t^E / N_t$ is the entry rate. For this to be consistent with the steady state equilibrium, $n^E = \phi/(1-\phi)$ must hold in any steady state. We set $\phi = 0.051/4$ to target the annual entry rate in Japan.[29]

[29] We take the average of the annual exit rate series taken from the Establishment and Enterprise Census (EEC). For a related study using the data, see Mukoyama (2009).

The key assumption is that n_t^E is exogenous and follows a stochastic process. Specifically, we assume that the entry rate follows a first-order autoregressive process, $\log n_t^E - \log n^E = \rho_n \left(\log n_{t-1}^E - \log n^E\right) + \varepsilon_{n,t}$, where $0 < \rho_n < 1$ and $\varepsilon_{n,t} \sim N(0, \sigma_n^2)$. From the data, we set $\rho_n = 0.532$, $\sigma_n = 0.004$, and $corr(\hat{n}_t^E, \hat{A}_t) = 0.503$.[30]

The value of entry in any period is given by

$$-\frac{cl_{t+1}}{q(\theta_t)} - k_{t+1} + (1-\phi)\,\delta \mathbb{E} J(S_{t+1}).$$

The linear vacancy cost implies that the optimal firm size is the same for the entrant and the incumbent. Thus, each entrant creates $l_{t+1}/q(\theta_t)$ vacancies in period t to become a firm with size l_{t+1}. The same is true for the optimal level of capital. Thus, the aggregate level of vacancies in each period satisfies

$$V_t = N_t v_t + N_t^E \frac{l_{t+1}}{q(\theta_t)}, \tag{44}$$

which is the sum of jobs created by incumbents and entrants.

The aggregate unemployment evolves according to $U_{t+1} = U_t + s(1-U_t) - \theta_t q(\theta_t)(1-\phi)U_t$, where $s = \lambda + (1-\lambda)\phi = \phi + (1-\phi)\lambda$ is the effective separation rate. The aggregate employment satisfies $l_t N_t = 1 - U_t$. The equilibrium conditions are nearly identical to those in the basic model, except for (43), (44), and that r is replaced with $r_\phi \equiv (r+\phi)/(1-\phi)$ to reflect the presence of exogenous exit of firms.

Table 6: Firm Entry

	Relative standard deviations								
	$\hat{U}$	$\hat{V}$	$\hat{W}$	$\hat{w}$	$\hat{t}$	$\hat{h}$	$\hat{l}$	β^h	β^l
Data	**5.45**	**8.92**	**1.16**	**0.91**	**0.80**	**0.70**	**0.35**	**0.79**	**0.21**
Basic model	4.33	12.81	1.55	0.91	0.75	0.63	0.15	0.84	0.19
Model with entry	4.26	13.10	1.97	1.13	1.03	0.83	0.34	0.77	0.29

We calibrate the model's parameter values using the same calibration procedure and targets

[30] We were able to obtain only the annual series for the entry rate in Japan from the Establishment and Enterprise Census (EEC). We therefore obtain these statistics from the HP-filtered annual series of the entry rate and TFP.

as those of the basic model. The results are presented in Table 6. With stochastic entry, our model replicates the magnitudes of fluctuations in employment without doing much harm to other variables. We therefore conclude that distinction between job creation by incumbent firms and entrants is important for understanding the labor market fluctuations over the business cycle.

8 Conclusion

In this paper, we introduced variable hours of work into a canonical search-matching model with large firms to investigate how firms utilize the intensive and extensive margins of labor adjustment over the business cycle. Although the existing search-matching models largely ignore the intensive margin, we showed that the intensive margin helps magnify labor market fluctuations even with a small Frisch elasticity. We demonstrated that our model is particularly successful in accounting for fluctuations in total labor input, hours of work, unemployment, and vacancies in Japan.

While the model's magnitude of fluctuations in employment is about a half of its empirical counterpart, we showed that the extended model with firm entry and exit fills this gap. This suggests that the well-known inability of the textbook search-matching model in generating large fluctuations in unemployment and vacancies at least partly comes from its inability in distinguishing between job creation and firm entry. This also suggests the importance of developing a richer model of firm entry for understanding labor market fluctuations over the business cycle.

An important line of future research is to identify the institutional characteristics across countries and provide a unified framework that captures the observed cross-country differences in the composition of labor demand over the business cycle. For this investigation, one may need a model with endogenous firing. With a micro-founded model of firing, one can study the impact of employment protection such as firing restriction on the importance of the intensive and extensive margins over the business cycle.[31] An important recent contribution along this line is Llosa

[31] It is certainly easy to introduce a separation cost into our model. However, we believe that such a model cannot approximate the reality of an economy with firing costs. A useful model of firing costs must possess both involuntary

et al. (2014), in which a frictionless model with firing costs is developed. Our framework, when modified accordingly, will provide a frictional counterpart of their model. A caveat is that, as we demonstrated, the model with separation fluctuations does not perform well; in particular, it does not produce a strong negative correlation between unemployment and vacancies.

Another important line of future research is to consider differences in types of employment. While we assumed a single employment contract for all workers, Japan's labor market is best understood as being polarized into two groups of workers, those with well-protected long-term contracts and those with less-paid, less-protected "non-regular" employment contracts, under which firms may terminate contracts at will and hours of work are more flexible. Workers under such non-regular employment contracts have been increasing, and they amount to 40 percent of total employees in Japan (Miyamoto, 2016). Our framework has an advantage in investigating this important issue because it is designed to study the composition of labor demand.

separations (i.e., firing) and involuntary labor hoarding (i.e. restricted firing).

Appendix

A Proof of Proposition 1

Substitute (7) into (9) to obtain

$$J_l(S) = \alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} - W(h;S) - W_l(h;S)l + \frac{(1-\lambda)c}{q(\theta)}, \tag{45}$$

which is the value of the firm at the marginal worker. Subtract (12) from (11) to obtain

$$J^E(S) - J^U(S) = W(h;S) - e(h) - b + [1-\lambda-\theta q(\theta)]\left[\delta \mathbb{E} J^E(S_{+1}) - \delta \mathbb{E} J^U(S_{+1})\right]. \tag{46}$$

Observe that, (7) and (13) imply

$$\frac{1-\beta}{\beta}\left[\delta \mathbb{E} J^E(S_{+1}) - \delta \mathbb{E} J^U(S_{+1})\right] = \frac{c}{q(\theta)}.$$

Use this to rewrite (46) as follows:

$$J^E(S) - J^U(S) = W(h;S) - e(h) - b + \frac{1-\lambda-\theta q(\theta)}{1-\beta}\frac{\beta c}{q(\theta)}. \tag{47}$$

Substitute (45) and (47) into (13) to obtain

$$\begin{aligned}
&\beta\left[\alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} - W(h;S) - W_l(h;S)l + \frac{(1-\lambda)c}{q(\theta)}\right] \\
&= (1-\beta)\left[W(h;S) - e(h) - b\right] + [1-\lambda-\theta q(\theta)]\frac{\beta c}{q(\theta)},
\end{aligned}$$

which reduces to $W(h;S) = \beta[\alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} - W_l(h;S)l] + (1-\beta)[e(h)+b] + \beta c\theta$, or

$$W_l(h;S)l + \frac{1}{\beta}W(h;S) = \alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} + \frac{1-\beta}{\beta}[e(h)+b] + c\theta. \tag{48}$$

This is a differential equation about the unknown earnings function. This equation satisfies for all $l \geq 0$, along with the condition that

$$W(h;S)l \leq A h^{\alpha} l^{\alpha} k^{1-\alpha}, \tag{49}$$

which requires that the total wage payment does not exceed the firm's revenue. It is useful to observe that

$$\begin{aligned}\frac{\partial}{\partial l}\left[W(h;S)l^{\frac{1}{\beta}}\right] &= \left[W_l(h;S)l + \frac{1}{\beta}W(h;S)\right] l^{\frac{1}{\beta}-1} \\ &= \left[\alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} + \frac{1-\beta}{\beta}\left[e\left(h\right)+b\right] + c\theta\right] l^{\frac{1}{\beta}-1} \\ &= \alpha A h^{\alpha} l^{\alpha+\frac{1}{\beta}-2} k^{1-\alpha} + \left[\frac{1-\beta}{\beta}\left[e\left(h\right)+b\right] + c\theta\right] l^{\frac{1}{\beta}-1}.\end{aligned}$$

Since (49) implies $W(h;S)l^{\frac{1}{\beta}} \leq Ah^{\alpha}l^{\alpha+\frac{1}{\beta}-1}k^{1-\alpha}$, we have $\lim_{l\to 0} W(h;S)l^{\frac{1}{\beta}} = 0$. Thus, it follows that

$$\begin{aligned}W(h;S)l^{\frac{1}{\beta}} &= \int_0^l \left\{\alpha A h^{\alpha} i^{\alpha+\frac{1}{\beta}-2} k^{1-\alpha} + \left[\frac{1-\beta}{\beta}\left[e\left(h\right)+b\right] + c\theta\right] i^{\frac{1}{\beta}-1}\right\} di \\ &= \frac{\alpha A h^{\alpha} k^{1-\alpha}}{\alpha+\frac{1}{\beta}-1} l^{\alpha+\frac{1}{\beta}-1} + \left[(1-\beta)\left[e\left(h\right)+b\right] + \beta c\theta\right] l^{\frac{1}{\beta}}.\end{aligned}$$

Thus, we finally obtain

$$W(h;S) = \frac{\alpha A h^{\alpha} k^{1-\alpha}}{\alpha+\frac{1}{\beta}-1} l^{\alpha-1} + (1-\beta)\left[e\left(h\right)+b\right] + \beta c\theta$$

as shown in the proposition.

B Proof of Proposition 2

We follow Smith (1999) and Kudoh and Sasaki (2011) to assume that each entrant creates vacancies and build capital so that it operates with the steady-state levels of employment l and capital k in the next period. With linear vacancy cost and without costly capital adjustment, doing so is indeed optimal for each entrant. This environment conveniently rules out the possibility of equilibrium size distribution of firms. Because the rate of filling a vacancy is $q(\theta)$, in order to achieve l_{+1} in the next period, the firm must create exactly $l_{+1}/q(\theta)$ vacancies in the current period. Thus, the value of entry is given by

$$J\left(0\right) = -\frac{cl_{+1}}{q(\theta)} - k_{+1} + \delta \mathbb{E} J\left(S_{+1}\right), \tag{50}$$

Therefore, the number of firms, N_t, is determined by $J(0) = 0$, or

$$\frac{cl_{+1}}{q(\theta)} + k_{+1} = \delta \mathbb{E} J\left(S_{+1}\right). \tag{51}$$

In any steady state, the firm's value of operation (without imposing (51)) is

$$\begin{aligned}(1-\delta)\,J\,(S) &= Ah^{\alpha}l^{\alpha}k^{1-\alpha} - W(h;S)l - cv - i \\ &= \left[\frac{1-\beta}{\alpha\beta+1-\beta}\right] Ah^{\alpha}l^{\alpha}k^{1-\alpha} - (1-\beta)\left[e\,(h)+b\right]l - \beta c\theta l - c\frac{\lambda l}{q(\theta)} - \delta_k k \\ &= \frac{(1-\beta)\,(1-\alpha)}{\alpha\beta+1-\beta}Ah^{\alpha}l^{\alpha}k^{1-\alpha} + \frac{rc}{q(\theta)}l - \delta_k k \\ &= r\frac{cl}{q(\theta)} + rk,\end{aligned}$$

from which we obtain

$$\delta J\,(S) = \frac{cl}{q(\theta)} + k.$$

Thus, in any steady state, the value of entry is

$$J\,(0) = -\frac{cl}{q(\theta)} - k + \delta J\,(S) = 0.$$

Thus, firms are indifferent between entry and exit. As a result, the number of firms will be *indeterminate* in this economy in the sense that a free entry condition cannot pin down the number of firms.

C Proof of Proposition 3

With (34), (30) and (33) imply each other. We can reduce the problem as follows.

$$\max_{\{h_t,l_{t+1},k_{t+1},V_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \left[A_t h_t^{\alpha} l_t^{\alpha} k_t^{1-\alpha} - cV_t - k_{t+1} + (1-\delta_k)\,k_t + (1-l_t)\,b - e\,(h_t)\,l_t\right] \tag{52}$$

subject to

$$l_{t+1} = (1-\lambda)\,l_t + M\,(1-l_t, V_t)\,. \tag{53}$$

Let ω_t be the Lagrange multiplier. The Lagrangian for this problem is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \left\{ \begin{array}{c} A_t h_t^{\alpha} l_t^{\alpha} k_t^{1-\alpha} - cV_t - k_{t+1} + (1-\delta_k)\,k_t + (1-l_t)\,b - e\,(h_t)\,l_t \\ +\omega_t\left[(1-\lambda)\,l_t + M\,(1-l_t, V_t) - l_{t+1}\right] \end{array} \right\}.$$

The first-order conditions are

$$\alpha A_t h_t^{\alpha-1} l_t^{\alpha} k_t^{1-\alpha} - e'(h_t)\, l_t = 0,$$

$$-\omega_t + \mathbb{E}_t \delta \left[\alpha A_{t+1} h_{t+1}^{\alpha} l_{t+1}^{\alpha-1} k_{t+1}^{1-\alpha} - b - e(h_{t+1})\right] + \mathbb{E}_t \delta \omega_{t+1} \left[(1-\lambda) - M_U(1-l_{t+1}, V_{t+1})\right] = 0,$$

$$-1 + \mathbb{E}_t \delta \left[(1-\alpha) A_{t+1} h_{t+1}^{\alpha} l_{t+1}^{\alpha} k_{t+1}^{-\alpha} + 1 - \delta_k\right] = 0,$$

$$-c + \omega_t M_V(1-l_t, V_t) = 0.$$

Eliminating ω_t, we establish that the planner's optimal allocation satisfies

$$\begin{aligned}
\mathbb{E}_t\left[(1-\alpha) A_{t+1} K_{t+1}^{-\alpha}\right] &= r + \delta_k, \\
\alpha A_t K_t^{1-\alpha} &= e'(h_t), \\
\frac{c(1+r)}{M_V(U_t, V_t)} &= \mathbb{E}_t\left\{\left[e'(h_{t+1})\, h_{t+1} - b - e(h_{t+1})\right] + \frac{c(1-\lambda)}{M_V(U_{t+1}, V_{t+1})} - c\frac{M_U(U_{t+1}, V_{t+1})}{M_V(U_{t+1}, V_{t+1})}\right\}, \quad (54) \\
l_{t+1} &= (1-\lambda)\, l_t + M(U_t, V_t),
\end{aligned}$$

where $K_t = k_t / h_t l_t$. With the Cobb-Douglas specification, $M(U_t, V_t) = m_0 U_t^{\xi} V_t^{1-\xi}$, we have $M_V(U_t, V_t) = (1-\xi)\, q(\theta_t)$ and $M_U(U_t, V_t) = \xi \theta_t q(\theta_t)$. Thus, we can rewrite (54) as

$$\frac{c(1+r)}{q(\theta_t)} = (1-\xi)\left[e'(h_{t+1})\, h_{t+1} - e(h_{t+1}) - b\right] + \frac{c(1-\lambda)}{q(\theta_{t+1})} - c\xi\theta_{t+1}.$$

This is the job-creation condition for the planner. Finally, we obtain (37) by imposing the Hosios condition ($\xi = \beta$).

D Proof of Proposition 4

Impose Hosios and consider the efficiency of hours. Divide (23) by (35) to obtain

$$\left(\frac{K_t^p}{K_t^f}\right)^{-\alpha} = \frac{1-\beta}{\alpha\beta + 1 - \beta} < 1 \Leftrightarrow K_t^f < K_t^p$$

for all t, where K_t^p and K_t^f are the capital-labor ratio chosen by the planner and the firm, respectively. Thus, the equilibrium level of the capital-labor ratio is below the efficient level. Similarly, divide (21) by (36) to obtain

$$\frac{e'\left(h_t^p\right)}{e'\left(h_t^f\right)} = (\alpha\beta + 1 - \beta)\left(\frac{K_t^p}{K_t^f}\right)^{1-\alpha} = (\alpha\beta + 1 - \beta)\left(\frac{1-\beta}{\alpha\beta + 1 - \beta}\right)^{-\frac{1-\alpha}{\alpha}}$$
$$= (1-\beta)\left[1 + \frac{\alpha\beta}{1-\beta}\right]^{\frac{1}{\alpha}} \equiv g\left(\beta\right).$$

Note that $g\left(0\right) = 1$ and

$$g'\left(\beta\right) = \frac{\beta\left(1-\alpha\right)}{\alpha\beta + 1 - \beta}\left[1 + \frac{\alpha\beta}{1-\beta}\right]^{\frac{1}{\alpha}} > 0.$$

By continuity, we establish that $g\left(\beta\right) > 1$ for $\beta \in (0, 1)$, which implies $h_t^f < h_t^p$ for all t.

We next compares h_t^b and h_t^f. With divisible labor, if hours of work are determined in bargaining or chosen by workers, we have (23) and (40). This immediately implies $K_t^b = K_t^f < K_t^p$, where K_t^b is the capital-labor ratio when hours of work are determined in bargaining or chosen by workers. Given $K_t^b = K_t^f$, we divide (40) by (21) to show that

$$\frac{e'\left(h_t^b\right)}{e'\left(h_t^f\right)} = \alpha < 1 \Leftrightarrow h_t^b < h_t^f.$$

This proves the proposition. For a related result, see Kudoh and Sasaki (2011).

E Proof of Proposition 5

We assume that both the level of compensation W_t and the level of hours of work h_t are determined so as to maximize the asymmetric Nash product: $[J_t^F - J_t^V]^{1-\beta}[J_t^E - J_t^U]^{\beta}$. The first-order conditions with respect to W_t and h_t imply $\beta[J_t^F - J_t^V] = (1-\beta)[J_t^E - J_t^U]$ and $\partial J_t^F/\partial h_t + \partial J_t^E/\partial h_t = 0$, where $\partial J_t^F/\partial h_t = p_t - W'(h_t)$ and $\partial J_t^E/\partial h_t = W'(h_t) - e'(h_t)$. The latter condition implies $p_t = e'(h_t)$, or $\alpha A_t h_t^{\alpha-1} l_t^{\alpha-1} k_t^{1-\alpha} = e'\left(h_t\right)$, or $\alpha A_t K_t^{1-\alpha} = e'\left(h_t\right)$.

F Additional Results

This section presents some additional results from our analysis by adding more components to the basic model. Specifically, we consider a version our model with decreasing efficiency in hours of work in production, wage rigidity, as well as costly capital adjustment ($\kappa_0 = 1$). We limit our analysis to a single-shock economy because models with many frictions and shocks belong to the medium-scale DSGE paradigm, which typically requires the Bayesian estimation method to determine their model parameters (Lin and Miyamoto, 2014).

In the benchmark model, we assume that while longer hours of work cause greater disutility, there is no loss in productivity. Using the UK micro data, Pencavel (2014) found that the relationship between output and hours of work per worker is concave. To capture the possibility that the productivity of hours per employee is concave, we modify our basic model such that the effective labor input satisfies $L = h^{\eta} l$, where $\eta \leq 1$ captures the efficiency of hours per employee. The model with $\eta = 1$ corresponds to the benchmark economy. Here, we adopt Pencavel's (2014) estimate, $\eta = 0.8$.

Shimer (2005) suggests that introduction of some wage rigidity helps increase the magnitudes of fluctuations in unemployment and vacancies. Although our basic model perfectly replicates the observed volatility in hourly wage rate, earnings in the model fluctuate more than those in the data. Thus, there is a (small) room for improvement. A caveat here is that, with variable hours of work and a nonlinear earnings schedule, the concept of wage rigidity is not well-defined, as it assumes to have a model in which the labor contract takes in the form of hourly wage rate. Thus, we instead study the model with *contract rigidity*.

Our modeling strategy is to modify the model as little as possible, rather than to write down a full-fledged micro-founded model of rigidity. To be more specific, we introduce an ad-hoc earnings schedule with rigidity that possesses the following two properties. One is that in any steady state, the rigid earnings schedule is identical to the earnings schedule without rigidity. In other words, while ad-hoc, the rigid earnings schedule does not alter the steady state of the model. The other is that the current contract does not fully reflect the current TFP (Pissarides, 2009). Let $S_t^P = (A_t^P, l_t, k_t, U_t)$ be the *perceived* state of the economy in period t, where A_t^P is the perceived level of TFP which is assumed to satisfy

$$A_t^P = \chi A_t + (1 - \chi) A,$$

where $1-\chi$ is the degree of contract rigidity. Thus, the perceived TFP is given by the weighted average of the true current TFP and its steady-state level.[32] We assume that in the bargaining stage, the true state is only partially verifiable, and as a result, the contract is conditional only on the perceived state. For our numerical analysis, we choose $1-\chi$ to be 0.359 from a structural estimation by Lin and Miyamoto (2014).

The value of a firm is given by

$$J(S) = \max_{h,v,i} \left\{ Ah^{\alpha\eta} l^{\alpha} k^{1-\alpha} - W(h;S^P)l - cv - i - C(k, k_{+1}) + \delta \mathbb{E} J(S_{+1}) \right\}.$$

and the optimization conditions need to be modified accordingly. The earnings function for this economy is

$$W(h;S^P) = \frac{\alpha\beta A^P h^{\alpha\eta} l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + (1-\beta)\left[e(h) + b\right] + \beta c\theta.$$

It is important to note that, while the contract is rigid, it does *not* guarantee that the wage rate is also rigid because changes in hours of work do change the level of earnings.

Table 7: Model Extensions

	Relative standard deviations							
	$\hat{U}$	$\hat{V}$	$\hat{W}$	$\hat{w}$	$\hat{k}$	$\hat{\imath}$	$\hat{h}$	$\hat{l}$
Data	**5.45**	**8.92**	**1.16**	**0.91**	**1.18**	**0.80**	**0.70**	**0.35**
Basic model with $\kappa_0 = 0$	4.33	12.81	1.55	0.91	1.56	0.75	0.63	0.15
Extended model with $\eta = 0.8$	1.51	4.23	1.02	0.41	0.35	0.65	0.62	0.05
Extended model with $\eta = 1.0$	3.85	10.71	1.14	0.48	0.42	0.74	0.66	0.13
Bargained hours with $\eta = 0.8$	0.93	2.61	0.89	0.46	0.31	0.45	0.43	0.03
Bargained hours with $\eta = 1.0$	1.21	3.39	0.96	0.51	0.34	0.48	0.46	0.04

Table 7 presents the results from four extended models. The first two models assume that hours of work are chosen by firms and the other two models assume that hours are chosen in

[32]Our formulation of wage rigidity is inspired by Hall (2005) and in particular Krause and Lubik (2007). These authors assume an ad-hoc wage equation in which the actual current wage rate is given by the weighted average of the Nash bargained wage rate and a reference wage rate, such as the past wage rate and the steady-state level.

bargaining. The fist model has all three components (i.e., costly capital adjustment, decreasing efficiency, and contract rigidity) while the second model drops the decreasing efficiency component from the first model. Similarly, the third model has all three components and the fourth model drops the decreasing efficiency component from the third model. It is evident from the table that while hours in the second model fluctuate more than those in the basic model, the magnitudes of fluctuations in employment, unemployment, and vacancies in the four models are all below those from in basic model. Moreover, while contract rigidity improves the magnitude of fluctuations in earnings, it implies conterfactually low levels of fluctuations in the hourly wage rates for all extended models. Thus, in this single-shock economy, these additional components are not essential for understanding labor market fluctuations over the business cycle.

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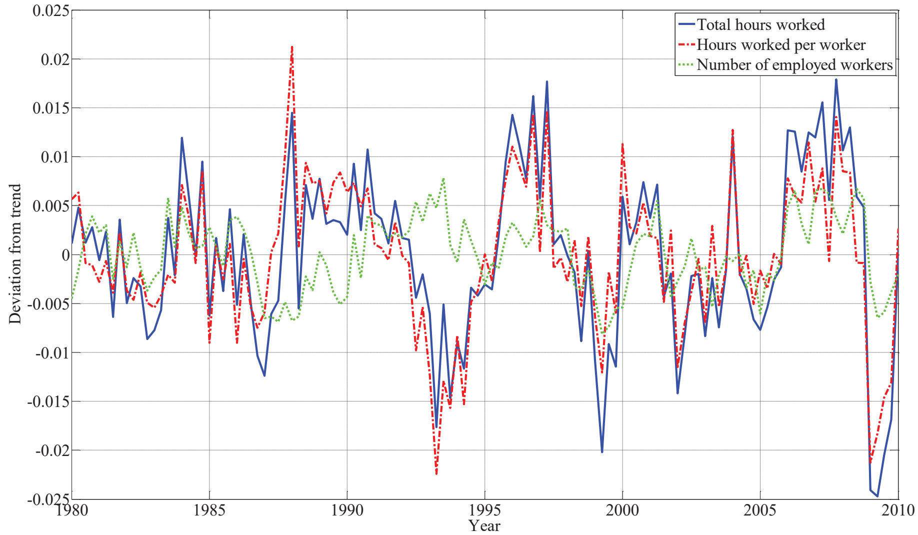

Figure 1: Total hours worked and its components over business cycles in Japan

Note: The solid line indicates the cyclical component of total hours worked. The dash-dotted line indicates the cyclical component of hours worked per worker. The dashed line indicates the cyclical components of employed workers. The cyclical components are obtained by using the HP filter with smoothing parameter 1600. Sample covers 1980Q2-2010Q4.