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Testing the Validity of Non-Parametric Value Estimates in Treasury Bill Auctions Using Top-Up Auction Data

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Abstract

This note uses data from top-up auctions to test the validity of value functions I derive using the Hortaçsu and McAdams (2010) methodology in my paper on Polish Treasury Bills (Marszalec, 2017). The testing procedure assumes that bidders have the same value function across both auctions; in this setting, since the top-up phase price is fixed, a presence of the top-up bid can be used to pin down the position where the value function ought to lie. The test I propose rejects in over 70% of the bidding data when a top-up bid is observed, indicating that a bias may occur in the estimation method that does not model top-up auctions explicitly. The current note doesn't find a bound on the magnitude of the bias - but finding such a bound for both the non-parametric models of Hortaçsu and McAdams (2010) as well as the semi-parametric model of Fevrier et al. (2004)is now work in progress.

JEL Classification: D44, C57, G23

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1. An Introduction to top-up auctions and similar mechanisms

The main feature of top-up auctions and related mechanisms in multi-unit auctions is the linking of the average price from a competitive stage of the auction with a possibility for (certain) bidders to buy goods at this average price. More precisely, a "top-up" auction, or a "top-up phase of an auction" occurs as a second stage of a two-stage auction process. In the context of Polish Treasury bills and Bonds the first stage consists of a discriminatory auction for a pre-announced quantity of goods (bonds or bills). After this stage is over and the stop-out price is found, the auctioneer calculates the (weighted) average price. Depending on the specifics of the auction, the auctioneer then decides which bidders can participate in the top-up stage - usually, only bidders who have won non-zero quantities in the base-auction are allowed into the second stage. During the top-up stage, each bidder is allowed to submit a single

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"quantity only bid," indicating how much of the goods for offer they would like to buy at the average price. Frequently, the maximum share allowed to each top-up stage bidder is restricted by the auctioneer, and often will depend on the amount of goods won in the base-auction itself. The exact way in which the presence of a top-up phase in Polish bond auctions influences bidding in the base-auction is discussed in Section 3, but broadly speaking each bidder will face a (first-phase) trade-off between wanting to bid less aggressively (to reduce top-up phase price), and more aggressively (to get a chance for bidding for a higher share in the second phase).

The facility of allowing "non-competitive bids" is closely related to a two-stage auction with a top-up phase. In auctions where non-competitive offers are allowed, bidders are permitted to submit quantity-only bids, which will always be allocated "at the (weighted) average price." In many cases, such as in France (Fevrier et al. (2004)), the total allocated quantity of the goods sold is pre-announced, as is the maximum amount allocated to non-competitive bids. As in top-up auctions, when non-competitive offers are allowed, bidders also face individual bidding restrictions on how large quantities they can request non-competitively - and this level is usually dependent on quantities won in past auctions. What differs here from a two-phase mechanism with a top-up stage is that the non-competitive bids reduce the (effectively) available quantity in the first phase: the amount of non-competitive bids is (from an individual bidder's point of view) ex-ante random.[2]

In the context of non-competitive bids, bidders also face two opposing incentives: on the one hand, if a bidder has submitted a non-competitive bid, he has an incentive to bid less aggressively in the competitive stage to depress the price on the goods he wins non-competitively. Yet there is an incentive to bid "more aggressively" too, since this will increase the bidder's allowance for non-competitive bids in the future.

In what follows, I focus my attention at analysing the influence of top-up mechanisms on first–phase bidding, since my dataset on Polish bonds features this mechanism. However, given a suitable dataset, a precisely analogous analysis could be carried out in the context of auctions with non-competitive bidding.[3]

2. The significance of top-up auctions

The possibility of the Ministry of Finance running a top-up auction is an interesting feature of the Polish auction system, but it introduces further theoretical complications into the modeling of share auctions. Previous econometric work, including Fevrier et al. (2004), ignore

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[2]It could be argued that in fact even in the presence of non-competitive bids, the competitively allocated quantity is "perfectly known" if the amount of non-competitive bids always meets the limit set by the auctioneer. In practice, however, these limits are frequently not met, so some uncertainty remains.

[3]I am in the process of developing a model to accommodate non-competitive bids, and will aim to test it on data from France or the Philippines.

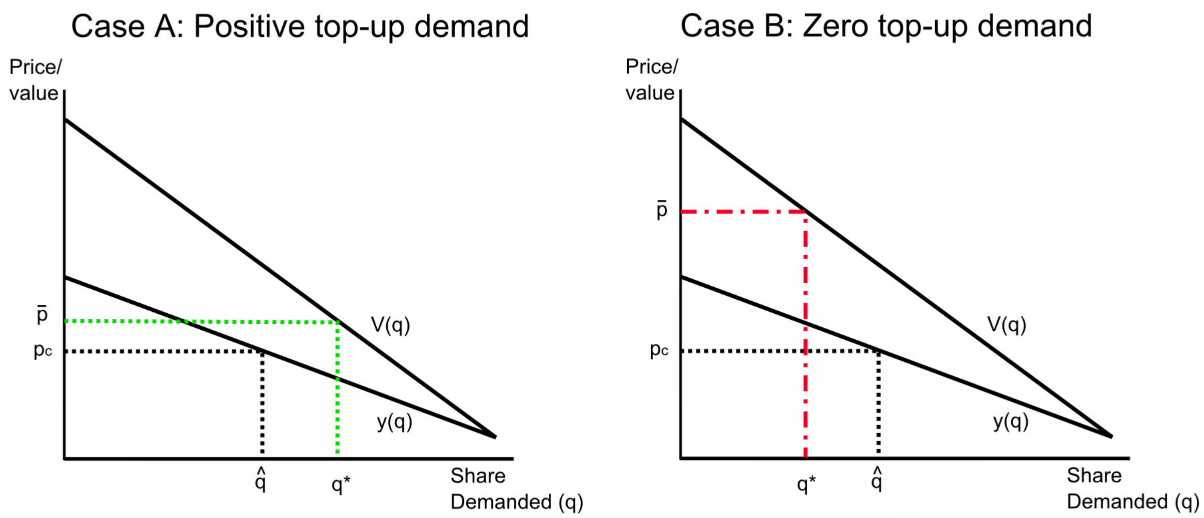

Figure 1: Demand in the top-up auctions, depending on average base auction price

the modeling of top-up auctions altogether. [4]

Since the first-stage auction is a discriminatory auction, the bidder (say, bidder i) has an incentive to 'shade' his bid, and submit a bidding function $y(q)$ that lies (strictly) below his true valuation function, $v(q)$. Suppose that given this bidding function, the stop-out price in the auction is p_c, so that the bidder is allocated an amount $\hat{\mathrm{q}}$, and the (weighted) average price of all sold bids is at $\bar{\mathrm{p}}$. How much should i bid in the top-up auction? The price in the second round is fixed at $\bar{\mathrm{p}}$, so we would expect i to request a (total) amount of bonds q^* such that $\bar{p} = v(q^*)$, or equivalently $q^* = v^{-1}(\bar{p})$. Looking at Figure 1, there are two cases that can now occur: either $q^* \geq \hat{q}$ (case A, i wants more bonds, at the average price), or $q^* < \hat{q}$ (case B, average price is too high - i doesn't want extra bonds). In practice, there is often an additional complication: the amount of second-round demand is restricted, often as a function of past (or first-round) winning amounts. This constraint is added to induce more aggressive bidding in the first round - intuitively, if second-round demand were unlimited, all bidders could submit low bids at first round, induce a low $\bar{\mathrm{p}}$ and request most their demand at that price.

For my stylised model of the top-up auction, suppose that a top-up auction occurs with a fixed and exogenous probability ρ. In this top-up phase, the each bidder can demand up to $r(q_1^i)$ of bills at the average price $\bar{p}$, where q_1^i is the first-round amount won by bidder i. I will assume $r(q)' > 0$, so that the top-up phase entitlement is increasing in base-auction quantity won. Given this setup, the "unconstrained" amount demanded at the second stage would be:

[4]Castellanos and Oviedo (2006) also mention the importance of top-up auctions in their Mexican data. Similarly to my paper, however, after setting up the optimization program, they cannot solve it out for use in estimation. Similarly to that paper, I am not yet able, to fully solve out a model with this complication, but the likely bias on base-auction bidding due to the presence of top-up auctions is discussed.

$\max\left(v^{-1}(\bar{p}) - q_1, 0\right)$. So the actual second stage demand will be:

$$\min\left(r(q_1), \max\left(v^{-1}(\bar{p}) - q_1, 0\right)\right).$$

Since $q_1 = y(p)$, I can rewrite the previous expression as

$$a(p) = \min\left(r(y(p)), \max\left(v^{-1}(\bar{p}) - y(p), 0\right)\right)$$

- this is the (constrained) optimal demand of bidder i, given a stop-out price of p.

Constructing the first-period expected payoff, thus yields:

$$\begin{aligned} ES_{with\ top-up} &= \int_0^\infty \left(\int_0^{y(p)} \left(v(s) - y^{-1}(s)\right) ds\right) dH(p|y) \\ &\quad + \rho \begin{pmatrix} \int_0^\infty \left(\int_{y(p)}^{y(p)+a(p)} v(s)\, ds\right) dH(p|y) \\ - \int_0^\infty a(p)\, E(\bar{p}|p)\, dH(p|y) \end{pmatrix} \end{aligned} \tag{1}$$

I have not yet been able to obtain a solution for this optimisation problem, since the $a(p)$ function is not differentiable. However, it appears that when a top-up auction is possible, there will be two new effects that influence bidding in the base auction itself. Firstly, there will be an incentive to shade more than before, since this will depress the price offered in the top-up auction. But since $\frac{dr}{dq} > 0$ – so that winning more in the first round increases a bidder's allocation in the top-up phase – then there will be an incentive to shade less and bid more aggressively, in order to obtain a higher top-up allowance. It is ex-ante ambiguous in which direction the incentives will go overall.

The dealers participating on the Polish market presented diverging views on the significance of the top-up auctions. For most, the primary response was that the top-up auction does not matter, and does not at all feature in their considerations for first-round bidding, since the actual occurrence of a top-up auction was uncertain. Yet many dealers subsequently admitted that they could frequently form "an informed guess" by looking at the state of the market before the base-auction as to whether a top-up auction was likely. Furthermore, a few dealers admitted that they sometimes submitted small quantity bids at high price in the base auction, in the hope that a top-up auction may occur and they could win a disproportionately large portion of the top-up supply (at the average price), in case some of the other first-round winners decided not to participate in the top-up auction. Irrespective of the explicit comments of the dealers, then, it looks likely that the top-up auctions may influence base-auction bidding at least implicitly.

3. Using Top-up Auctions for Testing Validity of Base Auction Valuations

I have outlined the theory of top-up auctions in Section 2, and I now show how information from such auctions can be used in the Polish context to test the validity of recovered valuation functions. Recall from Figure (1), there are two things that can happen when a top-up auction is present: either $q^* \geq \hat{q}$ (case A, i wants more bonds, at the average price),[5] or $q^* < \hat{q}$ (case B, average price is too high - i doesn't want extra bonds). In either case, the bids submitted in the top-up stage tells us where we should expect the base-auction valuation function to lie. If we have a method for estimating the bidders' base-auction valuation functions, and confidence intervals around them, we can use the data on top-up bids to check whether those are consistent with the estimated base-auction valuations. The Hortaçsu and McAdams (2010) model discussed in provides a feasible method of estimating the required kind of individual valuation functions (and bounds), and that is the model I will use below for further analysis; the value functions and confidence intervals are already calculated in Marszalec (2017).

In the ideal case we could just check whether at q^*, the value $\bar{p}$ lies within the error bounds on $v\left(q^*\right)$, and accept the validity of the estimates of v if it does, and reject otherwise. What makes this test attractive is that it sits well within the non-parametric paradigm, and does not require much further calculation. The major drawback of this testing procedure is that it is a weak test of the validity of v in general, since it only checks the validity of the v–function estimates at a single point. However, unless we make parametric assumptions as to the shape of v, we cannot make stronger inferences.

In the Polish top-up auctions, the bidders' top-up demands are constrained twofold. Firstly, the maximum amount of bonds available is 20% of the base-auction supply - so it makes no sense for a bidder to submit bids for more than this amount. Secondly, the bidder is constrained by the demands submitted by other bidders, both in the base auction and the top-up phase. Here the "allocation rule" for the top-up phase becomes important, and it is useful to describe it in fuller detail:

- Each bidder has a preliminary allocation equal to a proportion of the top-up supply which equals the proportion of bonds allocated to that bidder in the base-auction.[6]
- If every bidder submits a demand equal to, or exceeding , their preliminary allocation and all bidders who won something in the base auction participate - then everyone is given their preliminary allocation
- If not all first-round winners participate in the top-up phase, or any bidder bids less than their "preliminary allocation", the residual amount is allocated proportionately to those

[5] Recall the notation here: $q^* = v^{-1}\left(\bar{p}\right)$, and $\hat{q}$ is the actual base-auction quantity won.

[6] So a bidder winning 10% of base auction supply has a preliminary allocation of 10% of the top-up supply.

first-round winners who submitted demand in excess of their preliminary allocation. If after this step there is still some supply left, the re-allocation algorithm is run again.

- The iteration continues until either all supply has been allocated, or all submitted bids have been fulfilled.

Given the twofold constraint on the top-up phase bidding, I can interpret the second-stage bids as bounds on the values estimated from the first phase of the auction. Depending on precisely what quantity is demanded, there are three possible relationships that can be established (illustration follows in Figure 2):

- Case 1. Top-up bid not submitted: this implies $\hat{q} > q^*$. Hence at q̂, p̄ should be an upper bound on $v(\hat{q})$. I will take my test to "reject" when p̄ lies below the error-bars of v, at q̂.
- Case 2. Top-up bid submitted, in excess of "preliminary allowance", q_i^{prelim}: this suggests $\hat{q} < q^*$, and possibly $q^* > q_i^{\text{prelim}} + \hat{q}$. Thus at $\left(q^{\text{prelim}} + \hat{q}\right)$, p̄ is the lower bound on the valuation function. Thus if p̄ lies above the error-bounds on v at $q^* + q^{\text{prelim}}$, treat this as a rejection.
- Case 3. Top-up bid submitted, but below "preliminary allowance" amount. This tells us that $v(q^*) = \bar{p}$ should hold exactly . I will 'accept' this test if, at q^*, p̄ is inside the error bounds around $v(q^*)$, reject otherwise.

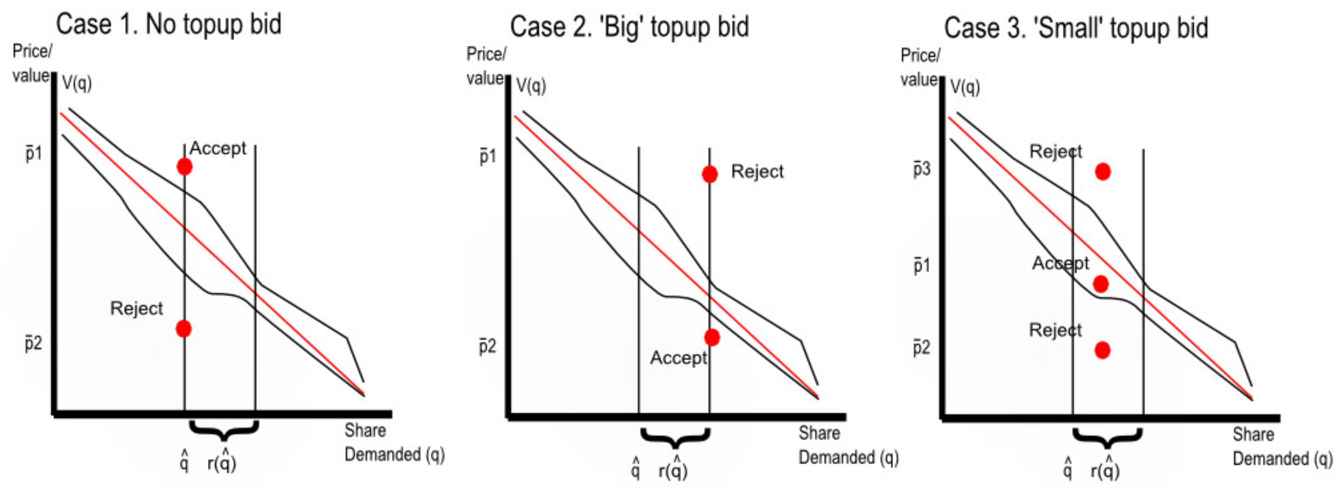

Figure 2: The three test cases

One more practical issue needs to be mentioned before commencing testing: namely, I will only have estimates of $v(q)$ over the support of q where bids were submitted in the first round - but $\hat{q}$ may fall in between two submitted quantities, or indeed outside the entire support. In the former case, I use linear interpolation on the error bounds, while in the latter case my test does not, strictly speaking, apply. This limitation is potentially serious, since roughly 75%

bidders in the top-up auction bid above their provisional allocation, while only around 25% of these bidders win the full submitted amount. Some of the bids, however, which are not fully allocated at the second stage do not appear reasonable: when the price is particularly attractive, every bidder submits demand for 100% of the overall additional supply, even though it is extremely unlikely they should win this amount. To evaluate the performance of my test over a broader set of (admissible) bid/valuation-function pairs, I will also apply my test at the *actually won* second-stage quantity. So suppose the requested top-up amount is q^*, while only q^a is allocated - I apply my test at both of these points. I would expect more valuation-functions to be admissible for testing at q^a than at q^*, and I should expect to reject (proportionately) less times at q^a. Results from the tests are presented below, in Table 3.

	Accept	Reject
Case 1.	38	21
Case 2. at q^a	21	63
Case2. at $\hat{q}$	5	55
Case 3.	0	24
Total	64	163

Table 1: Test of consistency of (v(p),y(p)) pairs

The error bounds on my inferred valuations are usually very small (as discussed in Marszalec (2017)), so it is unsurprising that all the Case 3. $(v(p), y(p))$ pairs produce a rejection. For Case 1, I find that my test rejects 21 out of 59, or 36% applicable $(v(p), y(p))$ pairs. The rejection ratio is much higher for case 2, with 75% pairs being rejected when testing at q^a, and 92% rejecting when tested at $\hat{q}$. There are two ways of interpreting this finding. Firstly, the Hortaçsu and McAdams (2010) method could be under-estimating the true valuation. But, secondly, this could indicate that ignoring the top-up auction when modeling base-auction demand is indeed introducing a significant bias. In Section 2 I noted that the balance of incentives from the top-up auction affecting first-round bidding is ambiguous. My results here would suggest that the "price incentive" is dominating the "share incentive". Whichever interpretation I follow, it seems likely that the top-up auctions is influencing my conclusions on the base-auction valuation functions - and for this reason the results of my estimates for the 2-year bonds may be less accurate than my conclusions for the 52-week bills.[7]

As of yet I don't have an estimate for the magnitude of possible bias, or the magnitude of its influence on the uniform-price upper-bound. This aspect will be explored further in subsequent versions of this paper. One possibility of getting more leverage from this "one-point" test would be to impose some (parametric) structure on the v–function; this fits well into the semi-parametric approach of Fevrier et al. (2004). Alternatively, to stay within the

[7]Since there has never been a top-up auction for 52w bills, it is uncontroversial to assume that in the context of our Equation 1, $\rho = 0$, so the optimisation programme is as before, and no bias should be expected.

non-parametric paradigm of Hortaçsu and McAdams (2010), it may be more straightforward to make reduced-form assumptions regarding the relative shading amount, based on the one-point estimate available from the present test.[8] My current work is pursuing both approaches.

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[8]The results of Ausubel et al. (2014) can then be used to argue about the relative changes in shading in discriminatory and uniform-price auctions, for quantities higher than what the bidder was allocated in the actual auction.