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Framing Game Theory

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Abstract

An economic agent (player) sometimes fails to correct hypothetical (contingent) thinking, which may increase the occurrence of anomalies in various economic situations. This paper demonstrates a method to encourage such a boundedly rational player to practice correct hypothetical thinking in strategic situations with imperfect information. We introduce a concept termed "frame" as a description of a synchronized cognitive procedure, through which a player decides multiple actions in a step-by-step manner, shaping his (or her) strategy selection as a whole. We could regard a frame as a supposedly irrelevant factor from the viewpoint of full rationality. However, this paper theoretically shows that in a multi-unit auction with private values, the ascending proxy auction has a significant advantage over the second-price auction in terms of the boundedly rational players' incentive to practice correct hypothetical thinking, because of the difference, not in physical rule, but in background frame, between these auction formats. By designing frames appropriately, we generally show that any static game that is solvable in iteratively undominated strategies is also solvable even if players cannot practice correct hypothetical thinking without the help of a well-designed frame.

Keywords: Hypothetical Thinking, Frame Design, Quasi-Obvious Dominance, Ascending Proxy Auction, Abreu-Matsushima Mechanism

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1. Introduction

An economic agent (player) sometimes fails to practice correct hypothetical thinking in strategic situations with imperfect information[3]. Since each player cannot observe the other players' strategies, it is inevitable that he makes a hypothesis about the other players' strategies, provided he wants to carry out rational behavior. Hypothetical thinking implies the "what-if" manner of strategic thought, such that a rational player first makes a hypothesis about the other players' strategies and then reasons about his best strategy from this hypothesis, where he (or she) does not recognize whether the hypothesis is true.

A real agent, however, sometimes avoids or mispractices such hypothetical thinking. Instead of thinking hypothetically, he (or she) incorrectly thinks: "I expect the other players to select a strategy profile if I intend to select a strategy, while I expect them to select another strategy profile if I intend to select another strategy." In other words, he incorrectly expects the other players' strategies to depend on which strategy he intends to select, even if he ought to recognize that they cannot observe his strategy selection.

In the prisoners' dilemma game, for instance, a boundedly rational player incorrectly thinks: "I expect the other player to select cooperation if I intend to select cooperation, while I expect him (or her) to select defection if I intend to select defection."

The failure of hypothetical thinking generally causes various anomalies in economics, such as the winner's curse, overbidding, non-pivotal voting, Ellsberg's paradox, and Allais' paradox. Players may fail to think hypothetically even in simple situations that have dominant strategies, such as the prisoners' dilemma and a second price auction. Hence, it is substantial in game theory to consider the possibility that players irrationally avoid correct hypothetical thinking. It is important to explore a method that encourages such boundedly rational players to practice more appropriate hypothetical thinking.

This paper argues that *frame design* serves to promote correct hypothetical thinking. We define a frame as a description of the players' cognitive procedure, which is a re-formulation of the static game that players face, as an extensive-form game with imperfect

[3] We have a rich literature in cognitive psychology that studied hypothetical thinking. See Evans (2007), for instance. See also Shafir and Tversky (1992).

information. We can regard a frame as a "supposedly irrelevant factor" from the viewpoint of fully rationality. We assume that the frame is common knowledge among all players.

A frame divides each player's strategy selection into multiple cognitive steps of decision-making. According to the frame, we regard each player's strategy as a combination of multiple actions that he sequentially decides through the cognitive procedure implied by the frame. A frame synchronizes a player's sequential decisions with each other. At each step, a player perceives that the other players have already decided the actions that the frame requires them to decide before this step, whereas they did not yet decide the actions that the frame requires them to decide at this and later steps. We show that this sequential and synchronized nature of a frame plays a significant role in helping players to practice correct hypothetical thinking.

We categorize hypothetical thinking, which a player ought to practice at each step about the other players' actions, into two types as follows: The first type concerns the actions that the other players have already decided before the current step. The second type concerns the actions that the other players will decide at the current and future steps. This paper does assume that players can correctly practice the first type of hypothetical thinking, whereas they fail to practice the second type of hypothetical thinking.

At each step, a player perceives the actions that the other players have already decided to take as *irreversible* ones. This perception serves to make the player correctly perceive that his action decision at the current step has no relation with these actions, whereas, on the other hand, the player does not perceive the actions that the other players will decide to take at the current and future steps as irreversible ones. This misperception prevents him from correctly perceiving that his action decision at the current step has no relation with these actions. Hence, a player can practice the first type of hypothetical thinking, whereas he does not necessarily practice the second type.

To overcome the failure of applying the second type of hypothetical thinking, we propose methods of designing a frame, and then show various permissive results in frame design. For instance, we can gain insights into the comparison between the second price auction and the ascending proxy auction, both of which are static games with imperfect information. The strategic situations implied by these auctions are logically equivalent, but their background frames are different. The second price auction accompanies a degenerate frame, whereas the detailed description of the open-bid ascending auction protocol frames the ascending proxy auction. The latter frame positions the decisions that are more suspected of causing the failure on the later steps. Because of this difference in background frame, players can practice correct hypothetical thinking more in the ascending proxy auction than in the second price auction. This observation is consistent with the evidence that people have historically hesitated to apply the second price auction, whereas people are willing to use proxy bids in online open-bid ascending auctions.

The failure to think hypothetically also badly influences the practice of higher-order reasoning and iterative elimination in strategic situations. Without the help of an appropriate frame design, a player cannot even expect the other players to play undominated strategies, because they do not necessarily practice correct hypothetical thinking. This significantly obstructs the practice of higher-order reasoning and iterative elimination of dominated strategies.

However, a well-designed frame can avoid such obstacles. We show that for any static game solvable in iteratively undominated strategies, there always exists a frame that motivates even boundedly rational players to practice both hypothetical thinking and higher-order reasoning, that is, to play the unique iteratively undominated strategy profile. To overcome the aforementioned difficulties, we design a frame that positions the strategies that can be eliminated in earlier stages of iteration on the later steps.

2. Literature Review and Contributions

Hypothetical thinking is a growing concern in experimental and theoretical economics. The failure of applying hypothetical thinking contains a clue for discovering the origin of various anomalies in laboratory experiments such as the winner's curse (Charness and Levin, 2009), non-pivotal voting (Esponda and Vespa, 2014), market failure caused by informational asymmetry (Ngangoue and Weizsacker, 2015), ambiguity, and loss aversion (Esponda and Vespa, 2016). There are various equilibrium analyses in game theory that considered bounded rationality in hypothetical thinking, such as Jehiel (2005), Eyster and Rabin (2005), Esponda (2008), and Li (2017).

The difficulty of applying hypothetical thinking in daily life sometimes justifies the advantage of dynamic mechanism design with perfect information over static mechanism design. For instance, by assuming perfect information about the players' previous decisions, we can replace hypothetical thinking with information extraction from observed data, which may be much easier to practice than hypothetical thinking. For this reason, Li (2017) emphasized that the open-bid ascending auction is a better mechanism design than the sealed-bid second-price auction.

In contrast with Li, this study does not consider such devices of mechanism design with perfect information. We instead focus on fixed static games with imperfect information. We do not cover dynamic games with perfect information, such as the game implied by open-bid ascending auctions. Instead of designing mechanisms, we fix a static game and design a frame as a description of the players' cognitive procedure, through which they sequentially decide multiple actions that shape their strategy selections as a whole. We should regard a frame as a "supposedly irrelevant factor" from the viewpoint of full rationality. However, a frame can "nudge" a boundedly rational player to think and behave more rationally. This paper shows that our frame design overcomes the difficulty in applying hypothetical thinking by encouraging boundedly rational players to practice it correctly. In this respect, the sealed-bid ascending proxy auction has a much better frame design than the sealed-bid second-price auction, even if both auctions have the same physical rules of strategic interaction.

Since players need hypothetical thinking even in playing dominant strategies, Li (2017) introduced a stronger solution concept than dominant strategy, which was termed obviously dominant strategy. This strategy totally excludes the practice of hypothetical thinking by regarding each player as being the most pessimistic about the other players' strategy selection. Friedman and Shenker (1996) and Friedman (2002) introduced a stronger version than dominated strategy in a way similar to Li, which this paper considers obviously dominated strategy. They further introduced solvability through iterative elimination of obviously undominated strategies. However, a static game with imperfect information generally has no obviously dominant strategy, or does not satisfy solvability in iteratively obviously undominated strategies, even if it has a dominant strategy.

Based on these observations, this study weakens the obviously dominant strategy and the solvability in iteratively obviously undominated strategies, by permitting a player to practice the first type of hypothetical thinking, which this study terms *quasi-obviously dominant strategy*, and *solvability in iteratively quasi-obviously undominated strategies*, respectively. We then show that for every static game that is solvable in iteratively undominated strategies, there always exists a frame that even makes the game solvable in iteratively quasi-obviously undominated strategies. This permissive result holds irrespectively of either complete or incomplete information environments.

We will apply this result to the robustness of the Abreu-Matsushima mechanisms in hypothetical thinking, where we consider general incomplete information environments. Several works, such as Abreu and Matsushima (1992a, 1992b, 1994) and Matsushima (2008a, 2008b, 2017), showed that every social choice function is uniquely implementable in iteratively undominated strategies (in the virtual or exact sense), whenever it satisfies a form of incentive compatibility. To prove this permissive result, we designed so-called Abreu-Matsushima mechanisms, which require players to announce multiple messages about their types and use only small monetary fines. In the Abreu-Matsushima mechanism, their truth-telling in multiple revelations is the unique iteratively undominated strategy profile, which always implements the value of the social choice function. This paper shows that there exists a frame, according to which their truth-telling is also the unique iteratively quasi-obviously undominated strategy profile. This implies that the Abreu-Matsushima mechanism is robust in hypothetical thinking.

This paper is related to the experimental work by Esponda and Vespa (2016), who conducted laboratory experiments for testing the Sure-Thing Principle and showed that subjects tend to fail to apply hypothetical thinking in various situations of single-person decision making. Esponda and Vesta compared a noncontingent treatment and a contingent treatment, where the contingent treatment added to the corresponding noncontingent treatment a device to guide a subject to hypothetical thinking. This study generalizes the devices of guidance in single-person problems to multi-person strategic problems as frames, that is, extensive-form games with imperfect information.

The problem with this generalization is that it is not always possible to provide all aspects of multi-person decision-making with the devices of guidance, because the order of decision-making across players matters in frame design. We need a careful frame design to divide each player's strategy selection into multiple steps of action decisions and then to specify the order of these action decisions for each player. Hence, this study formulates a frame as a single extensive-form game with imperfect information.

Game theory typically interprets an extensive-form game as a physical rule of strategic interaction. However, we should view a frame as a description of not a physical rule but as a cognitive procedure, through which players determine strategies in a step-by-step manner. Glazer and Rubinstein (1992) argued that an extensive-form game provides information about how to carry out iterative elimination in a normal form game. In contrast with Glazer and Rubinstein, this study emphasizes that as a frame, an extensive-form game removes the difficulty in hypothetical thinking. Because of the difference in role, the manner of designing extensive-form games in this study is substantially different from that by Glazer and Rubinstein. Extensive-form games in Glazer and Rubinstein assume perfect information and put the decisions in the order of elimination, whereas extensive-form games in this study assume imperfect information and put the decisions in the reverse order of elimination. For instance, in the Abreu-Matsushima mechanism, Glazer and Rubinstein design an extensive-form game that fines the first deviants, whereas this paper designs a frame that fines the last deviants.

There is a literature of level-k models in game theory, where a player has an exogenous limitation in the depth of cognitive hierarchy. The failure of rational play in the centipede game is a popular example. See Nagel (1995) and Crawford and Iriberri (2007). In contract, this study does not assume any exogenous limitation in the depth of cognitive hierarchy, and instead explains that each player's depth limitation is endogenously determined by the degree to which the other players fail to practice correct hypothetical thinking. In Section 8, we introduce a class termed competitive games, which includes a simultaneous version of centipede game. We can explain the failure of rational play in this game without the help of frame design, even if we assume unlimited depth of cognitive hierarchy.

This paper assumes that a player can practice correct hypothetical thinking concerning the actions that the other players have decided before. This assumption excludes the case that Shafir and Tversky (1992) investigated as a variant of the Newcomb Problem, where a decision maker behaves irrationally in front of a predictor with miraculous power irrespective of whether this predictor is the almighty or a high-quality artificial intelligence.

3. Outline

Section 4 defines standard notions, such as a static game with complete information, dominant strategy, and dominated strategy. We then define obviously dominant strategy, and obviously dominated strategy, according to the basic concepts by Li (2017).

Section 5 introduces a frame, defined as an extensive-form game, or multi-stage game, with imperfect information. Section 6 introduces quasi-obviously dominant strategy and quasi-obviously dominated strategy in a game with a frame, where we permit a player to practice the first type of hypothetical thinking correctly, but not the second type of hypothetical thinking. We show a necessary and sufficient condition for the existence of a frame such that the dominant strategy profile is also quasi-obviously dominant in a game with this frame.

As an example, we investigate “externality games,” which describe various aspects of a social dilemma. We further consider prisoner’s dilemma games as a special case of an externality game. The class of prisoner’s dilemma games that have quasi-obviously dominant strategy profiles is restrictive.

Section 7 introduces weak quasi-obviously dominant strategy and weak quasi-obviously dominated strategy by weakening the incentive requirements of strict inequalities in a quasi-obviously dominant strategy and a quasi-obviously dominated strategy, respectively. Importantly, we show that in a multi-unit ascending proxy auction, sincere bidding is the weakly quasi-obviously dominant strategy. This finding is in contrast with a second-price auction, which accompanies just a degenerate frame and fails to motivate bidders to play dominant strategies.

Section 8 introduces iteratively undominated strategy, iteratively obviously undominated strategy, and iteratively quasi-obviously undominated strategy. We show an important theorem of this study: whenever a game is solvable in iteratively undominated strategies, then we can design a frame such that the game with this frame is solvable in iteratively quasi-obviously undominated strategies. This implies that frame design can overcome the difficulty regarding hypothetical thinking. As an example, we investigate “competition games,” which are defined as a generalization of a Bertrand competition and a static version of the centipede game.

Section 9 considers static games in incomplete information environments. By replacing Bayesian games with the associated agent-normal form games, we can directly apply the arguments for static games with complete information to static games with incomplete information. However, there exists a drawback to this application to incomplete information environments in that we generally need to define a frame as an extensive-form game, not for the set of all players, but for the set of all type-dependent agents. This makes the issue of frame design in incomplete information environments much more complicated to implement than in complete information environments.

Despite this, we can show that the Abreu-Matsushima mechanism in implementation theory is robust in the practice of correct hypothetical thinking, even if we use just a simple frame that is defined, not for the set of type-dependent agents, but for the set of players. Section 10 concludes this paper.

4. Static Games with Complete Information

We consider a static game with complete information, which is described by a normal form game $G = (N, A, u)$, where $N \equiv \{1,...,n\}$ is the set of all players, $n \geq 2$, A is the set of all strategy profiles; $A \equiv \underset{i\in N}{\times} A_i$, where A_i is the set of all strategies for player $i \in N$; and $u \equiv (u_i)_{i\in N}$, where $u_i : A \to R$ is the payoff function for player i. Let $\hat{A}_i \subset A_i$ denote an arbitrary subset of strategies for player i. Let $\hat{A} \equiv \underset{i\in N}{\times} \hat{A}_i$ and $\hat{A}_{-i} \equiv \underset{j\in N\setminus\{i\}}{\times} \hat{A}_j$.

Definition 1: A strategy $a_i \in A_i$ for player i is said to be *dominated* for $\hat{A}$ in G if $a_i \in \hat{A}_i$, and there exists $\hat{a}_i \in \hat{A}_i \setminus \{a_i\}$ such that

$$u_i(a_i, \hat{a}_{-i}) < u_i(\hat{a}_i, \hat{a}_{-i}) \text{ for all } \hat{a}_{-i} \in \hat{A}_{-i}.$$

It is said to be *dominant* for $\hat{A}$ in G if $a_i \in \hat{A}_i$, and

$$u_i(a_i, \hat{a}_{-i}) > u_i(\hat{a}_i, \hat{a}_{-i}) \text{ for all } \hat{a}_i \in \hat{A}_i \setminus \{a_i\} \text{ and } \hat{a}_{-i} \in \hat{A}_{-i}.$$

It is said to be *weakly dominated* for $\hat{A}$ in G if $a_i \in \hat{A}_i$, and there exists $\hat{a}_i \in \hat{A}_i \setminus \{a_i\}$ such that

$$u_i(a_i, a_{-i}) \leq u_i(\hat{a}_i, a_{-i}) \text{ for all } a_{-i} \in A_{-i},$$

and the strict inequality holds for some $a_{-i} \in A_{-i}$. It is said to be *weakly dominant* for $\hat{A}$ in G if $a_i \in \hat{A}_i$, and for every $\hat{a}_i \in \hat{A}_i \setminus \{a_i\}$,

$$u_i(a_i, \hat{a}_{-i}) \geq u_i(\hat{a}_i, \hat{a}_{-i}) \text{ for all } \hat{a}_{-i} \in \hat{A}_{-i},$$

and the strict inequality holds for some $\hat{a}_{-i} \in \hat{A}_{-i}$.

If $\hat{A} = A$, we will simply say that a_i is dominated in G. We will say similarly for the other definitions

Note that a strategy for player i is dominant for $\hat{A}$ in G if and only if it is the unique undominated strategy for $\hat{A}$ in G. It is weakly dominant for $\hat{A}$ in G if and only if it is the unique weakly dominated strategy for $\hat{A}$ in G. If it is dominant for $\hat{A}$ in G, it is also weakly dominant for $\hat{A}$ in G. If it is dominated for $\hat{A}$ in G, it is also weakly dominated for $\hat{A}$ in G.

According to the basic concept in the seminal paper by Li (2017), we introduce obvious dominance as follows:[4][5]

Definition 2: A strategy $a_i \in A_i$ for player i is said to be *obviously dominated* for $\hat{A}$ in G if $a_i \in \hat{A}_i$, and there exists $\hat{a}_i \in \hat{A}_i$ such that

$$\max_{\hat{a}_{-i} \in \hat{A}_{-i}} u_i(a_i, \hat{a}_{-i}) < \min_{\hat{a}_{-i} \in \hat{A}_{-i}} u_i(\hat{a}_i, \hat{a}_{-i}).$$

It is said to be *obviously dominant* for $\hat{A}$ in G if $a_i \in \hat{A}_i$, and

$$\min_{\hat{a}_{-i} \in \hat{A}_{-i}} u_i(a_i, \hat{a}_{-i}) > \max_{\hat{a}_{-i} \in \hat{A}_{-i}} u_i(\hat{a}_i, \hat{a}_{-i}) \text{ for all } \hat{a}_i \in \hat{A}_i \setminus \{a_i\}.$$

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[4] See Friedman and Shenker (1996) and Friedman (2002) for the definition of obviously dominated strategy.

[5] Li (2017) defined obviously dominant strategy for dynamic games, while this paper defines it for static games.

Definition 2 permits each player's expectation about the other players' strategies to depend on his (or her) strategy selection. This permission implies that each player $i \in N$ fails to practice hypothetical thinking in the correct manner such that he selects a strategy a_i if the other players select a profile of strategies a_{-i}, whereas he selects another strategy a_i' if the other players select another profile of strategies a_{-i}'. Instead of practicing such hypothetical thinking, each player i incorrectly thinks about the other players' strategies in a strategy-dependent manner such that the other players select a profile of strategies a_{-i} if he selects a strategy a_i, while the other players select another profile of strategies a_{-i}' if he selects another strategy a_i.

Based on this incorrect manner, an obviously dominated strategy implies that a player hesitates to select a strategy even if he is the most optimistic in his strategy-dependent expectation about the other players' strategy selections. An obviously dominant strategy implies that a player prefers selecting a strategy even if he is the most pessimistic in his strategy-dependent expectation about the other players' strategy selections.

Note that if a strategy for player i is obviously dominant for $\hat{A}$ in G, then it is also dominant for $\hat{A}$ in G. If it is obviously dominated for $\hat{A}$ in G, then it is also dominated for $\hat{A}$ in G. Moreover, it is obviously dominant for $\hat{A}$ in G if and only if it is the unique obviously undominated strategy for $\hat{A}$ in G.

5. Frame

Associated with a game G, we introduce a concept that we term a *frame*, denoted by $\Gamma = (T, (A_{i,t}, \tilde{A}_{i,t}(\cdot))_{t\in T}, \delta_i)_{i\in N})$, in the following manner. Each player makes multiple action decisions sequentially through a discrete time horizon, that is, from step 1 to step T. At each step $t \in \{1,...,T\}$, each player i selects an action $a_{i,t}$ from a finite set $A_{i,t}$. Let $a_i^t \equiv (a_{i,1},...,a_{i,t})$ denote a sequence of player $i's$ action decisions from step 1 to step t. For every $t \in \{1,...,T\}$, we define the set of possible sequences of player $i's$

action decisions from step 1 to step t,denoted by $A_i^t \subset \underset{\tau=1}{\overset{t}{\times}} A_{i,\tau}$, and we also define the sequence-dependent set of actions at step t, denoted by a function $\tilde{A}_{i,t} : A_i^{t-1} \to 2^{A_{i,t}}$, where

$$\tilde{A}_{i,1}(a_i^0) = A_{i,1} = A_i^1,$$

and we assume that a possible sequence $a_i^t \in A_i^t$ must be *consistent with* $\tilde{A}_{i,t}$ in the sense that for every $t \in \{2,...,T\}$,

$$[a_i^t \in A_i^t] \Leftrightarrow [a_{i,\tau} \in \tilde{A}_{i,\tau}(a_i^{\tau-1}) \textit{ for all } \tau \in \{1,...,t\}].$$

At each step $t \in \{1,...,T\}$, where player i has determined the sequence of his decisions $a_i^{t-1} = (a_{i,1},...,a_{i,t-1})$ from step 1 to step $t-1$, he selects an action $a_{i,t}$ from the sequence-dependent subset $\tilde{A}_{i,t}(a_i^{t-1}) \subset A_{i,t}$.

Let $\delta_i : A_i \to A_i^T$ denote a one-to-one correspondence, where we regard a strategy $a_i \in A_i$ for player i in the game G as the complete sequence of player i's action decisions $\delta_i(a_i) \in A_i^T$. Hence, we will write

$$a_i = \delta_i(a_i) = (a_{i,1},...,a_{i,T}).$$

We interpret a frame as a description of players' cognitive procedure regarding how to determine their strategy selections. Each player i determines his strategy selection in the game G according to the cognitive procedure implied by frame Γ. We assume that not only the game Γ, but also the frame Γ is common knowledge among all players. Importantly, at each step $t \in \{1,...,T\}$, each player i perceives that any other player $j \neq i$ has already decided the sequence of actions $a_j^{t-1} = (a_{j,1},...,a_{j,t-1})$ from step 1 to step $t-1$, but has not yet decided $(a_{j,t},...,a_{j,T})$.

6. Quasi-Obvious Dominance

Because of the imperfect information assumption for the game G, each player i cannot observe the other players' action decisions during the cognitive procedure implied by the frame Γ. However, at each step $t \in \{1,...,T\}$, each player i perceives that the other players have already decided actions a_{-i}^{t-1} as *irreversible* ones. With the help of the perception of irreversibility, he can correctly recognize that his action decision $a_{i,t}$ at step t has no relation to a_{-i}^{t-1}, and can therefore practice correct hypothetical thinking for the other players' past decisions a_{-i}^{t-1}. On the other hand, he perceives that the other players have not yet decided $(a_{-i}^{t},...,a_{-i}^{T})$. This perception motivates him to expect, incorrectly, that his decision may influence the other players' future decisions, causing his failure to apply hypothetical thinking.

Based on these observations, we introduce a new concept that we term quasi-obvious dominance as follows. For each sequence of player $i's$ action decisions up to step t, $a_i^t \in A_i^t$, we define the set of all strategies for player i that are consistent with a_i^t by

$$A_i(a_i^t) \equiv \{\hat{a}_i \in A_i \mid \hat{a}_i^t = a_i^t\}.$$

Definition 3: A strategy $a_i \in A_i$ for player i is said to be *quasi-obviously dominated for* $\hat{A}$ *in a game with a frame* (G,Γ) if $a_i \in \hat{A}_i$, and there exist $t \in \{1,...,T\}$ and $\hat{a}_i \in \hat{A}_i$ such that

$$\hat{a}_i \in A_i(a_i^{t-1}),$$

$$\hat{a}_{i,t} \neq a_{i,t},$$

and

(1) $$\max_{\hat{a}_{-i} \in \times_{j\neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(a_i,\hat{a}_{-i}) < \min_{\hat{a}_{-i} \in \times_{j\neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(\hat{a}_i,\hat{a}_{-i}) \text{ for all } a_{-i}^{t-1} \in \hat{A}_{-i}^{t-1}.$$

The strategy is said to be *quasi-obviously dominant for* $\hat{A}$ *in* (G,Γ) if $a_i \in \hat{A}_i$, and for every $t \in \{1,...,T\}$ and $\hat{a}_i \in \hat{A}_i$, whenever $\hat{a}_i \in A_i(a_i^{t-1})$ and $\hat{a}_{i,t} \neq a_{i,t}$,

(2) $$\min_{\hat{a}_{-i} \in \times_{j\neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(a_i,\hat{a}_{-i}) > \max_{\hat{a}_{-i} \in \times_{j\neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(\hat{a}_i,\hat{a}_{-i}) \text{ for all } a_{-i}^{t-1} \in \hat{A}_{-i}^{t-1}.$$

According to a frame Γ, each player i determines whether he does not select a strategy a_i, and whether he does not select another strategy $\hat{a}_i$, at the first step that distinguishes these strategies, that is, at the step t, where

$$a_i^{t-1} = \hat{a}_i^t \text{ and } a_{i,t} \neq \hat{a}_{i,t}.$$

He perceives that the other players have already made the action decisions that the frame requires them to decide, from step 1 to step $t-1$, whereas they have not yet decided their future actions. A quasi-obviously dominated strategy implies that a player hesitates to select a strategy rather than another strategy even if, at the first step that distinguishes these strategies, he is the most optimistic in his strategy-dependent expectation about the other players' future decisions.

A quasi-obviously dominant strategy implies that a player prefers selecting a strategy over another strategy even if, at the first step that distinguishes these strategies, he is the most pessimistic in his strategy-dependent expectation about the other players' future decisions.

The main difference between obvious dominance and quasi-obvious dominance is that at each step during the procedure implied by the frame, any player can correctly practice hypothetical thinking regarding the other players' past action decisions. However, he fails to practice hypothetical thinking regarding their current and future action decisions.

Note that if a strategy for player i is quasi-obviously dominant for $\hat{A}$ in G, then it is dominant for $\hat{A}$ in G. If it is obviously dominant for $\hat{A}$ in G, then it is quasi-obviously dominant for $\hat{A}$ in G. If it is quasi-obviously dominated for $\hat{A}$ in G, then it is dominated for $\hat{A}$ in G. If it is obviously dominated for $\hat{A}$ in G, then it is quasi-obviously dominated for $\hat{A}$ in G. Moreover, it is quasi-obviously dominant for $\hat{A}$ in G, if and only if it is the unique quasi-obviously undominated strategy for $\hat{A}$ in G.

Consider an arbitrary strategy profile $a^* \in A$. We demonstrate a necessary and sufficient condition for the existence of a frame Γ such that a^* is quasi-obviously dominant in (G, Γ).

Fix an arbitrary strategy profile $\overline{a} \in A$. Let

$$\rho : \bigcup_{i \in N} A_i \setminus \{\overline{a}_i\} \to \{1, ..., \sum_{i \in N} |A_i| - n\}$$

denote an arbitrary one-to-one correspondence, which describes an order of all players' strategies except for $\{\overline{a}_1,...,\overline{a}_n\}$. Associated with ρ , we define $\mu(\cdot,\rho):\{1,...,\sum_{i\in N}|A_i|-n\}\to N$ by

$$[\rho(a_i)=h]\Rightarrow[\mu(h,\rho)=i] \quad \text{for all } i\in N,\ a_i\in A_i\setminus\{\overline{a}_i\}, \text{ and } h\in\{1,...,\sum_{i\in N}|A_i|-n\}.$$

Hence, a player $i's$ strategy $a_i\in A_i\setminus\{\overline{a}_i\}$ is placed in the position $\rho(a_i)\in\{1,...,\sum_{i\in N}|A_i|-n\}$, and $\mu(\rho(a_i),\rho)=i$ identifies the player who occupies the position $\rho(a_i)$.

Associated with ρ, we specify a frame $\Gamma^{\rho}=(T,(A_{i,t},\tilde{A}_{i,t}(\cdot))_{t\in T},\delta_i)_{i\in N})$ such that

$$T=\sum_{i\in N}|A_i|-n,$$

$$A_{i,t}=\{0,1\},$$

$$\tilde{A}_{i,t}(a_i^{t-1})=\{0\} \quad \text{if either } \mu(t,\rho)\neq i \text{ or } a_{i,\tau}=1 \text{ for some } \tau\in\{1,...,t-1\},$$

$$\tilde{A}_{i,t}(a_i^{t-1})=\{0,1\} \quad \text{otherwise,}$$

and

$$\delta_i(a_i)=(\delta_{i,t}(a_i))_{t=1}^{\sum_{i\in N}|A_i|-n},$$

where

$$\delta_{i,t}(\overline{a}_i)=0 \quad \text{for all } t\in\{1,...,T\},$$

and for every $a_i\in A_i\setminus\{\overline{a}_i\}$,

$$\delta_{i,\rho(a_i)}(a_i)=1,$$

and

$$\delta_{i,t}(a_i)=0 \quad \text{for all } t\neq\rho(a_i).$$

At each step $t\in\{1,...,T\}$, only player $\mu(t,\rho)$ is active, who decides whether to select strategy $\rho^{-1}(t)\in A_i\setminus\{\overline{a}_i\}$ (i.e., decide action "1"), or not (i.e., decide action "0"). By deciding "0" at all steps, player i can select strategy $\overline{a}_i$. By deciding "1" at the step of $\rho(a_i)$, player i can select strategy $a_i \in A_i \setminus \{\overline{a}_i\}$. Note that each player can choose "1" at most once during the cognitive procedure implied by the frame Γ^ρ.

For each $a_{-i} \in A_{-i}$ and $h \in \{1,...,T\}$, we define the set of all players $j \neq i$ who select strategy a_j after step h by

$$C(a_{-i}, i, h, \rho) \equiv \{j \in N \setminus \{i\} \mid \rho(a_j) > h\}.$$

We also define the set of all strategies that player i can select after step h by

$$A_i(h, \rho) \equiv \{a_i \in A_i \mid \rho(a_i) > h\}.$$

It is clear from the specification of Γ^ρ and the definitions of $C(a_{-i}, i, h, \rho)$ and $A_i(h, \rho)$ that the inequalities (2) for $a_i = a_i^*$, which is the necessary and sufficient condition for a_i^* to be quasi-obviously dominant, are equivalent to the following inequalities: for every $a_i \neq a_i^*$ and $a_{-i} \in A_{-i}$,

(3)
$$\min_{\tilde{a}_{C(a_{-i},i,h,\rho)} \in A_{C(a_{-i},i,h,\rho)}(h,\rho)} u_i(a_i^*, a_{-i-C(a_{-i},i,h,\rho)}, \tilde{a}_{C(a_{-i},i,h,\rho)})$$

$$> \max_{\tilde{a}_{C(a_{-i},i,h,\rho)} \in A_{C(a_{-i},i,h,\rho)}(h,\rho)} u_i(a_i, a_{-i-C(a_{-i},i,h,\rho)}, \tilde{a}_{C(a_{-i},i,h,\rho)}),$$

where we denote $h = \min[\rho(a_i^*), \rho(a_i)]$.

We show that if there exists no ρ such that a^* is quasi-obviously dominant in (G, Γ^ρ), then there generally exists no frame Γ such that it is quasi-obviously dominant in (G, Γ).[6]

Theorem 1: *There exists a frame* Γ *such that a strategy profile* a^* *is quasi-obviously dominant in* (G, Γ) *if and only if there exists* ρ *such that* a^* *is quasi-obviously dominant in* (G, Γ^ρ)*, that is, the inequalities (3) hold.*

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[6] Note that a dominant strategy is not necessarily quasi-obviously dominant. However, if we permit frames to differ across players, we can generally regard any dominant strategy as a quasi-obviously dominant strategy. That is, each player i is assigned a frame within which player i moves at the last step. Hence, any player perceives that the other players move before he decides. This perception eliminates the failure of hypothetical thinking.

Proof: Suppose that a^* is quasi-obviously dominant in (G,Γ). For each $i \in N$ and $t \in \{1,...,T\}$, we define

$$\bar{A}_i(t) \equiv \{a_i \in A_i \mid a_i \in A_i(a_i^{*t-1}) \quad \textit{and} \quad a_i^t \neq a_i^{*t}\},$$

which is the set of all strategies a_i such that step t distinguishes a_i and a_i^*. Let $\bar{a} = a^*$. We specify ρ such that for every $(i,j) \in N^2$, $(t,t') \in \{1,...,T\}^2$, and $(a_i, a'_j) \in \bar{A}_i(t) \times \bar{A}_j(t)$,

$$[t > t'] \Rightarrow [\rho(a_i) > \rho(a'_j)].$$

Fix arbitrary $i \in N$ and $a_i \in A_i \setminus \{a_i^*\}$. Let $t \in \{1,...,T\}$ denote the step in the frame Γ such that $a_i^{t-1} = a_i^{*t-1}$ and $a_i^t \neq a_i^{*t}$, that is, $a_i \in \bar{A}_i(t)$. From (2), it follows that for every $a_{-i} \in A_{-i}$,

(4) $$\min_{\hat{a}_{-i} \in A_{-i}(a_{-i}^{t-1})} u_i(a_i^*, \hat{a}_{-i}) > \max_{\hat{a}_{-i} \in A_{-i}(a_{-i}^{t-1})} u_i(a_i, \hat{a}_{-i}).$$

From the specification of ρ, it follows that for every $j \in C(a_{-i}, i, h, \rho)$,

$$A_j(h,\rho) \subset A_j(a_j^{t-1}),$$

where we denote $h = \min[\rho(a_i^*), \rho(a_i)]$. From this inclusion, $a_j \in A_j(a_j^{t-1})$, and (4), it follows that for every $a_i \neq a_i^*$ and $a_{-i} \in A_{-i}$,

$$\min_{\tilde{a}_{C(a_{-i},i,h,\rho)} \in A_{C(a_{-i},i,h,\rho)}(h,\rho)} u_i(a_i^*, a_{-i-C(a_{-i},i,h,\rho)}, \tilde{a}_{C(a_{-i},i,h,\rho)})$$

$$> \max_{\tilde{a}_{C(a_{-i},i,h,\rho)} \in A_{C(a_{-i},i,h,\rho)}(h,\rho)} u_i(a_i, a_{-i-C(a_{-i},i,h,\rho)}, \tilde{a}_{C(a_{-i},i,h,\rho)}),$$

that is, the inequalities (3) hold.

Q.E.D.

In the proof of Theorem 1, to make a^* obviously dominant, we designed the frame that positions the action decisions, which are more likely the cause of the failure, on the later steps.

Example 1 (Externality Game): Consider the following game, which this study terms an *externality game*: For every $i \in N$,

$$A_i = \{0,1\},$$

and

$$u_i(a) = a_i - v_i(a_{-i}) \quad \text{for all} \quad a \in A.$$

Note that $a^* = (1,...,1)$ is the dominant strategy profile. Without loss of generality, we assume $\bar{a} = (0,...,0)$. Since each player has only two strategies, we can regard any function ρ as equivalent to a permutation on N. According to the frame Γ^ρ, at each step $t \in \{1,...,n\}$, player $\mu(t,\rho) \in N$ selects his strategy between 0 and 1.

Let

$$C(t,\rho) \equiv \{i \in N \mid i = \mu(\tau,\rho) \ \textit{for some}\ \tau > t\},$$

which is the set of all players who select strategies after step t. The following proposition shows a necessary and sufficient condition for a^* to be quasi-obviously dominant in (G,Γ^ρ).

Proposition 1: *In an externality game* G, a^* *is the quasi-obviously dominant strategy profile in* (G,Γ^ρ) *if and only if for every* $t \in \{1,...,n\}$,

$$1 > \max_{\substack{\tilde{a}_{-\mu(t,\rho)} \in A_{-\mu(t,\rho)} \\ a_{C(t,\rho)} \in A_{C(t,\rho)}}} \{v_{\mu(t,\rho)}(\tilde{a}_{-\mu(t,\rho)}) - v_{\mu(t,\rho)}(\tilde{a}_{-\mu(t,\rho)-C(t,\rho)}, a_{C(t,\rho)})\}.$$

Proof: At each step $t \in \{1,...,n\}$, player $\mu(t,\rho)$ fails to practice the hypothetical thinking regarding all players who move after this step, that is, all players who belong to $C(t,\rho)$. This implies that a^* is the quasi-obviously dominant strategy profile in (G,Γ^ρ) if and only if for every $t \in \{1,...,n\}$ and $a_{-\mu(t,\rho)} \in A_{-\mu(t,\rho)}$,

$$1 - \min_{\tilde{a}_{C(t,\rho)} \in A_{C(t,\rho)}} v_{\mu(t,\rho)}(a_{-\mu(t,\rho)-C(t,\rho)}\tilde{a}_{C(t,\rho)})$$

$$> 0 - \max_{\tilde{a}_{C(t,\rho)} \in A_{C(t,\rho)}} v_{\mu(t,\rho)}(a_{-\mu(t,\rho)-C(t,\rho)}\tilde{a}_{C(t,\rho)}),$$

that is,

$$1 > \max_{\substack{\tilde{a}_{-\mu(t,\rho)} \in A_{-\mu(t,\rho)} \\ a_{C(t,\rho)} \in A_{C(t,\rho)}}} \{v_{\mu(t,\rho)}(\tilde{a}_{-\mu(t,\rho)}) - v_{\mu(t,\rho)}(\tilde{a}_{-\mu(t,\rho)-C(t,\rho)}, a_{C(t,\rho)})\}.$$

Q.E.D.

Example 2 (Prisoners' Dilemma): Figure 1 describes a *prisoners' dilemma*, which is a special case of externality games. Clearly, strategy 1 is a dominant strategy for each player, it is obviously dominant for player 2, but it is not obviously dominant for player 1. However, by designing a frame that assigns the first move to player 2, we can make the strategy profile $(1,1)$ quasi-obviously dominant.

		player 2: 0	player 2: 1
Player 1	0	1 -3	-1 0
	1	2 -3	0 -2

Figure 1

Figure 2 describes another prisoners' dilemma, where strategy 1 is a dominant strategy, but not obviously dominant, for each player. In this case, irrespective of which frame we design, strategy 1 fails to be quasi-obviously dominant for the first mover.

		player 2: 0	player 2: 1
Player 1	0	1 1	-1 2
	1	2 -1	0 0

Figure 2

7. Weak Quasi-Obvious Dominance

The following is a weaker version of quasi-obvious dominance, where we replace the strict inequalities with weak inequalities.

Definition 4: A strategy $a_i \in A_i$ for player i is said to be *weakly quasi-obviously dominated for* $\hat{A}$ *in* (G,Γ) if $a_i \in \hat{A}_i$, and there exist $t \in \{1,...,T\}$ and $\hat{a}_i \in \hat{A}_i$ such that

$$\hat{a}_i \in A_i(a_i^{t-1}),$$

$$\hat{a}_{i,t} \neq a_{i,t},$$

$$\max_{\hat{a}_{-i} \in \times_{j \neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(a_i, \hat{a}_{-i}) \leq \min_{\hat{a}_{-i} \in \times_{j \neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(\hat{a}_i, \hat{a}_{-i}) \text{ for all } a_j^{t-1} \in \hat{A}_{-i}^{t-1},$$

and the strict inequality holds for some $a_j^{t-1} \in \hat{A}_{-i}^{t-1}$. It is said to be *weakly quasi-obviously dominant for* $\hat{A}$ *in* (G, Γ) if $a_i \in \hat{A}_i$, and for every $t \in \{1,...,T\}$ and $\hat{a}_i \in \hat{A}_i$, whenever $\hat{a}_i \in A_i(a_i^{t-1})$ and $\hat{a}_{i,t} \neq a_{i,t}$, then

$$\min_{\hat{a}_{-i} \in \times_{j \neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(a_i, \hat{a}_{-i}) \geq \max_{\hat{a}_{-i} \in \times_{j \neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(\hat{a}_i, \hat{a}_{-i}) \text{ for all } a_j^{t-1} \in \hat{A}_{-i}^{t-1},$$

and the strict inequality holds for some $a_j^{t-1} \in \hat{A}_{-i}^{t-1}$.

The following theorem parallels Theorem 1, which demonstrates a necessary and sufficient condition for the existence of a frame Γ such that a^* is weakly quasi-obviously dominant in (G, Γ).

Theorem 2: *There exists a frame such that* a^* *is weakly quasi-obviously dominant if and only if there exists* ρ *such that it is weakly quasi-obviously dominant in* (G, Γ^ρ), *that is, for every* $a_i \neq a_i^*$,

$$\min_{\tilde{a}_{C(a_{-i},i,h,\rho)} \in A_{C(a_{-i},i,h,\rho)}(h,\rho)} u_i(a_i^*, a_{-i-C(a_{-i},i,h,\rho)}, \tilde{a}_{C(a_{-i},i,h,\rho)})$$

$$\geq \max_{\tilde{a}_{C(a_{-i},i,h,\rho)} \in A_{C(a_{-i},i,h,\rho)}(h,\rho)} u_i(a_i, a_{-i-C(a_{-i},i,h,\rho)}, \tilde{a}_{C(a_{-i},i,h,\rho)}) \text{ for all } a_{-i} \in A_{-i},$$

and the strict inequality holds for some $a_{-i} \in A_{-i}$, *where we denote* $h = \min[\rho(a_i^*), \rho(a_i)]$.

Proof: We can prove this theorem in the same way as Theorem 1 by replacing the strict inequalities with weak inequalities.

Q.E.D.

Example 3 (Proxy Game): Consider the game $G=(N,A,u)$, which we term a *proxy game*. Let A_i denote a nonempty finite set of integers. We assume that

$$a_i \neq a_j \text{ for all } i \neq j \text{ and } (a_i, a_j) \in A_i \times A_j .$$

We assume that for every $i \in N$, $a \in A$, and $a' \in A$, whenever

$$[a_j < \min[a_i, a_i']] \Rightarrow [a_j = a_j'] \text{ and}$$

$$[a_j > \min[a_i, a_i']] \Rightarrow [a_j' > \min[a_i, a_i']] \text{ for all } j \in N \setminus \{i\},$$

then

$$[u_i(a) \geq u_i(a_i', a_{-i})] \Rightarrow [u_i(a) \geq u_i(a_i', a_{-i}')].$$

This assumption implies that whether $u_i(a)$ or $u_i(a_i', a_{-i}')$ is greater is irrelevant to (a_{-i}, a_{-i}'), provided that for every $j \in N \setminus \{i\}$,

$$\text{either } [a_j = a_j'] \text{ or } [\min[a_j, a_j'] > \min[a_i, a_i']].$$

Let $\bar{a} = (\max a_1, ..., \max a_n)$. Consider a frame Γ^ρ, where all strategies except for $\{\bar{a}_i\}_{i \in N}$ are arranged in descending order, that is,

$$\rho^{-1}(t) < \rho^{-1}(t+1) \text{ for all } t \in \{1, ..., \sum_{i \in N} |A_i| - n\}.$$

According to Γ^ρ, at each step $t \in \{1, ..., \sum_{i \in N} |A_i| - n\}$, player $\mu(t,\rho)$ decides whether or not to select the strategy $\rho^{-1}(t) \in A_{\mu(t,\rho)}$. He perceives that any other player has already decided whether or not to select any strategy that is smaller than $\rho^{-1}(t)$. From Theorem 2, it follows that if a^* is a weakly dominant strategy profile in the proxy game G, then it is also weakly quasi-obviously dominant in (G, Γ^ρ).

Example 4 (Multi-unit Ascending Proxy Auction): A special case of the proxy games is a *multi-unit unit-demand ascending proxy auction*, where we regard a strategy as a proxy bid, and

$$u_i(a) = v_i - p(a,m) \qquad \text{if } a_i \geq p(a,m),$$

$$u_i(a) = 0 \qquad \text{otherwise.}$$

Here, $p(a,m)$ denotes the $m-th$ highest proxy bid. Each player i simultaneously selects a proxy bid. He obtains a single unit of the commodity if and only if his proxy bid is equal to, or greater than, the $m-th$ highest proxy bid. We assume $v_i \in A_i$ for all $i \in N$, and let a_i^* be the truthful strategy, that is, $a_i^* = v_i$. Clearly, it is a weakly dominant strategy, and therefore, is weakly quasi-obviously dominant, because of the arguments in Example 3. However, it is not obviously dominant.

With the assumption of single-unit demands, boundedly rational bidders fail to play rationally in the Vickrey multi-unit auction (equivalently, the uniform price auction), whereas they can successfully play rationally in the multi-unit ascending proxy auction, even if the physical aspect is logically equivalent in both protocols.

8. Iterative Quasi-Obvious Dominance

We define iterative dominance in G as follows: Let

$$A_i(0) = A_i.$$

For every $k \geq 1$, we define $A_i(k) \subset A_i$ by

$$[\, a_i \in A_i(k)\,]$$

$$\Leftrightarrow [\, a_i \in A_i(k-1), \text{ and } a_i \text{ is undominated for } A(k-1) \text{ in } G\,],$$

where we denote $A(k-1) \equiv \underset{i \in N}{\times} A_i(k-1)$. Let $A_i(\infty) \equiv \bigcap_{k=0}^{\infty} A_i(k)$.

Definition 5: A strategy $a_i \in A_i$ for player i is said to be *iteratively undominated* in G if $a_i \in A_i(\infty)$.

We define iterative obvious dominance in G by replacing 'undominated' in Definition 5 with 'obviously undominated' as follows:

Definition 6: A strategy $a_i \in A_i$ for player i is said to be *iteratively obviously undominated* in G if $a_i \in A_i^*(\infty)$, where we define $A_i^*(\infty)$ similarly to $A_i(\infty)$, replacing "undominated" with "obviously undominated."

Note that if a strategy for player i is the unique iteratively obviously undominated strategy, then it is the unique iteratively undominated strategy. However, even if it is the unique iteratively undominated strategy, it is not necessarily the unique iteratively obviously undominated strategy.[7]

We define iterative quasi-obvious dominance in (G,Γ) by replacing "dominated in G" with "quasi-obviously dominated in (G,Γ)."

Definition 7: A strategy $a_i \in A_i$ for player i is said to be *iteratively quasi-obviously undominated* in (G,Γ) if $a_i \in A_i^*(\infty,\Gamma)$, where we define $A_i^*(\infty,\Gamma)$ similarly to $A_i(\infty)$, replacing "dominated in G" with "quasi-obviously dominated in (G,Γ)."

The following proposition shows that *the dominant strategy profile is always the unique iteratively quasi-obviously undominated strategy profile*. This is in contrast with Proposition 1, which implies that the class of games that equalize the dominant strategy and quasi-obviously dominant strategy is substantially restricted.

For every permutation μ on N, we define the frame Γ^{μ} with $T = n$ steps, according to which, at each step $t \in \{1,...,n\}$, player $\mu(t)$ selects a strategy from the set of all his strategies, A_i.

Proposition 2: *If $a^* \in A$ is the dominant strategy profile in G, then, irrespective of the specification of permutation μ, it is the unique iteratively quasi-obviously dominant strategy profile in (G,Γ^{μ}).*

Proof: At the last step $T = n$ in Γ^{μ}, player $\mu(n)$ is willing to select the strategy $a^*_{\mu(n)}$ because it is the dominant strategy in G and he can practice hypothetical thinking

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[7] Unique iteratively obviously undominated strategy corresponds to the notion of O-solvability in Friedman and Shenker (1996) and Friedman (2002).

correctly as the last person to move in Γ^{μ}. Consider an arbitrary step $t \in \{1,...,n-1\}$. Suppose that at every subsequent step $t' \in \{t+1,...,n\}$, player $\mu(t')$ selects $a^{*}_{\mu(t')}$ as the unique iteratively quasi-obviously undominated strategy in (G,Γ^{μ}). Then, player $\mu(t)$ can also select $a^{*}_{\mu(t)}$ as the unique quasi-obviously dominant strategy, because he perceives that any subsequent movers will play a^{*}, and he can practice correct hypothetical thinking about the previous movers. These observations imply that a^{*} is the unique iteratively quasi-obviously undominated strategy profile in (G,Γ^{μ}).

Q.E.D.

We show that *whenever a strategy profile is the unique iteratively undominated strategy profile in* G*, then there always exists a frame* Γ *such that it is the unique iteratively quasi-obviously undominated strategy profile in* (G,Γ). Hence, with the help of frame design, we can make the solvability in iterative dominance equivalent to the solvability in iterative quasi-obvious dominance. To prove the theorem, we design a frame that positions the strategies that can be eliminated in earlier stages of iteration on the later steps.

Theorem 3: *There exists a frame* Γ *such that a strategy profile* a^{*} *is the unique iteratively quasi-iteratively undominated strategy profile in* (G,Γ) *if and only if it is the unique iteratively undominated strategy profile in* G.

Proof: Let $\bar{a} = a^{*}$. Let $A_i(t)$ be the subset of strategies for player i that survives through the $t-time$ iterative eliminations of dominated strategies. We specify ρ so that for every $(t,t') \in T^2$, $(i,j) \in N^2$, $a_i \in A_i(t) \setminus A_i(t-1)$, and $a'_j \in A_j(t') \setminus A_j(t'-1)$,

$$\rho(a_i) < \rho(a'_j) \quad \text{whenever} \quad t > t',$$

where we have $T = \sum_{i \in N} |A_i| - n$. Note that ρ positions the strategies that can be eliminated in earlier stages of iteration on the later steps.

It is clear that at the last step $T = \sum_{i \in N} |A_i| - n$, player $\mu(T,\rho)$ decides action "0," that is, eliminates the strategy $\rho^{-1}(T) \in A_{\mu(T,\rho)}$, because $\rho^{-1}(T)$ is quasi-obviously dominated for A. Consider an arbitrary $t \in \{1,...,T-1\}$. Note that there exists an integer k such that $\rho^{-1}(t) \in A_{\mu(t,\rho)}(k) \setminus A_{\mu(t,\rho)}(k-1)$. Suppose that at any later step $t' \in \{t+1,...,T\}$, player $\mu(t',\rho)$ decides action "0," that is, eliminates the strategy $\rho^{-1}(t') \in A_{\mu(t',\rho)}$. Then, from the specification of ρ, it follows that every strategy in $A_i \setminus A_i(k-1)$ is eliminated at the later steps. This implies that player $\mu(t,\rho)$ is willing to choose action "0," that is, eliminate strategy $\rho^{-1}(t) \in A_{\mu(t,\rho)}$, because $\rho^{-1}(t)$ is quasi-obviously dominated for $A(k-1)$.

From these observations, all players decide actions "0" during the cognitive procedure implied by the frame Γ^{ρ}. Hence, we have proved that a^*, which corresponds to the complete sequence of action "0" decisions, is the unique iteratively quasi-obviously undominated strategy profile in (G, Γ^{ρ}).

Q.E.D.

Example 5 (Competition Game): Consider a two-player game that we term a *competition game*. Let

$$n = \{1,2\},$$

$$A_1 = \{1,3,5,...,2L-1\},$$

$$A_2 = \{2,4,6,...,2L\},$$

and for every $a \in A$ and $a_i' \in A_i$,

$$u_i(a) > u_i(a_i', a_j) \qquad \text{if either } a_i' > a_i \geq a_j - 1 \text{ or } a_i < a_i' \leq a_j - 1,$$

and

$$u_i(a) > u_i(a_i', a_j) \qquad \text{if } a_i = a_j - 1 \text{ and } a_i' = a_j + 1.$$

Any player prefers selecting the strategy that is smaller than the other player's strategy for one point. This nature of competition game corresponds to price competition a la Bertrand, where a firm prefers to set its price slightly lower than that of its rival, the firm's profit decreases as its price increases whenever it is greater than the rival's price, and the firm's profit increases as its price increases whenever it is less than the rival's price. We can also see the competitive game as a simultaneous version of the centipede game.

Note that the strategy profile $a^* = (1,2) \in A$ is the unique iteratively undominated strategy profile. However, it is not the unique iteratively obviously undominated strategy profile. In order to navigate boundedly rational players to play rationally, we need a carful frame design as follows. Let us consider the frame Γ^ρ, where

$$T = 2L,$$

and

$$\rho(t) = t \quad \text{for all} \quad t \in \{1,...,2L\}.$$

According to Γ^ρ, we can iteratively eliminate strategies in descending order of high value, which implies that the price vector $(1,2)$ is the unique iteratively quasi-obviously undominated strategy profile in (G,Γ^ρ).

In the Appendix, we define iterative weak dominance as a weaker version of iterative dominance, and show various results that parallel the results in this section.

9. Static Games with Incomplete Information

This section investigates a static game with incomplete information. Consider a Bayesian game $\Lambda \equiv (N, A, u, \Omega, p)$, where $\Omega \equiv \underset{i\in N}{\times} \Omega_i$, Ω_i is the set of all types for player i, $u \equiv (u_i)_{i\in N}$, $u_i : A \times \Omega \to R$, $p \equiv ((p_i(\cdot\,|\,\omega_i))_{\omega_i\in\Omega_i})_{i\in N}$ is a belief system, $p_i(\cdot\,|\,\omega_i) : \Omega_{-i} \to [0,1]$, and $p_i(\omega_{-i}\,|\,\omega_i)$ is the probability of the occurrence of the other players' type profile ω_{-i} conditional on player $i's$ type ω_i. Let $s_i : \Omega_i \to A_i$ denote a (type-dependent) strategy rule for player i. Let S_i denote the set of all strategy rules for player i. Let $s \equiv (s_i)_{i\in N}$ and $S \equiv \underset{i\in N}{\times} S_i$.

It is implicit to assume in this study that the state ω is determined before the Bayesian game starts, and each player i therefore has no trouble with hypothetical thinking concerning ω_{-i}.

By treating each type as an individual agent, we can regard the Bayesian game Λ as equivalent to the agent-normal form game defined as a static game with complete information, which is denoted by $G(\Lambda) \equiv (M, B, w)$, where M is the set of all agents, that is,

$$M = \{1, 2, ..., \sum_{i \in N} |\Omega_i|\},$$

$B \equiv \underset{m \in M}{\times} B_m$, B_m is the set of all strategies for agent $m \in M$, $w \equiv (w_m)_{m \in M}$, and $w_m : B \rightarrow R$ is the payoff function for agent m.

In this case, there exists a one-to-one correspondence $\eta : \underset{i \in N}{\cup} \Omega_i \rightarrow M$ such that for every $i \in N$ and $\omega_i \in \Omega_i$,

$$B_{\eta(\omega_i)} = A_i,$$

and

$$w_{\eta(\omega_i)}(b) \equiv E[u_i((b_{\eta(\omega_j)})_{j \in N}, \omega) \mid \omega_i] \text{ for all } b \in B.$$

Hence, we treat each type ω_i in the Bayesian game Λ as an agent $\eta(\omega_i) \in M$ in the agent-normal form game $G(\Lambda)$. Note that a Bayesian Nash equilibrium s in the Bayesian game Λ is a Nash equilibrium b in the associated agent-normal form game $G(\Lambda)$, where we specify b as

$$b_{\eta(\omega_i)} = s_i(\omega_i) \text{ for all } i \in N \text{ and } \omega_i \in \Omega_i.$$

Hence, by replacing a Bayesian game Λ with the associated agent-normal form game $G(\Lambda)$, we can directly apply the arguments for the complete information environments in the previous sections to the incomplete information environments in this section.

We denote by $\Psi = (T, (B_{m,t}, \tilde{B}_{m,t}(\cdot))_{t \in T}, \iota_m)_{m \in M})$ a frame associated with the agent-normal form game $G(\Lambda)$, which we define in the same way as Γ, by replacing N, A, and δ with M, B, and ι, respectively.

Definition 8: A strategy rule profile $s \in S$ is said to be *iteratively quasi-obviously undominated in a Bayesian game with a frame* (Λ, Ψ) if the strategy profile $b \in B$ is iteratively quasi-obviously undominated in $(G(\Lambda), \Psi)$, where we assume that

$$b_{\eta(\omega_i)} = s_i(\omega_i) \text{ for all } i \in N \text{ and } \omega_i \in \Omega_i .$$

However, a frame Ψ generally has a non-negligible complexity in that it is a cognitive procedure not for the set of all (real) players N but for the set of all type-contingent (fake) agents M. To address such cognitive complexity, we should investigate the solvability in iterative quasi-obvious dominance by using a simple frame that is defined not for M, but for N. Here, we can regard Γ as a simple case of Ψ, where for every $i \in N$,

$$(B_{m,t}, \tilde{B}_{m,t}(\cdot))_{t \in T} = (B_{m',t}, \tilde{B}_{m',t}(\cdot))_{t \in T} \text{ if } m \in \Omega_i \text{ and } m' \in \Omega_i ,$$

and

$$(B_{m,t}, \tilde{B}_{m,t}(\cdot))_{t \in T} = (A_{i,t}, \tilde{A}_{i,t}(\cdot))_{t \in T} .$$

Let us consider a special case of a Bayesian game Λ where we assume that

$$A_i = A_{i,1} \times \cdots \times A_{i,T} \text{ for all } i \in N .$$

Each player i sequentially makes multiple action decisions from step 1 to step T, where we assume imperfect information. We denote by $s_i = (s_{i,t})_{t=1}^{T}$ a strategy rule for player i, where we denote $s_{i,t} : \Omega_i \to A_{i,t}$.

Fix an arbitrary strategy rule profile $s^* = (s_i^*)_{i \in N}$. For every $t \in \{1,...,T\}$, let

$$S_i(t) \equiv \{s_i \in S_i \mid s_{i,\tau} = s_{i,\tau}^* \quad \textit{for} \quad \textit{all} \quad \tau \in \{t+1,...,T\}\} .$$

We will assume that for every $t \in \{1,...,T\}$, $i \in N$, and $\omega_i \in \Omega_i$,

$$a_{\eta(\omega_i),t} = s_{i,t}^*(\omega_i) \text{, whenever } a_{\eta(\omega_i)} \text{ is undominated for } S(t).$$

This assumption guarantees that s^* is the unique iteratively undominated strategy rule profile.

We specify a frame Γ^* for the set of all players N so that at each step $t \in \{1,...,T\}$, each player i selects an action $a_{i,t}$ from $A_{i,t}$. We can show that s^* is the unique iteratively quasi-obviously undominated strategy rule profile in (Λ, Γ^*).

Proposition 3: *In the above-mentioned Bayesian game* Λ, s^* *is the unique quasi-obviously iteratively undominated strategy rule profile in* (Λ, Γ^*).

Proof: From the assumption regarding the Bayesian game Λ in this section, it follows that at the last step T, each player $i \in N$ with each type $\omega_i \in \Omega_i$ is willing to select $s_{i,T}(\omega_i) = s^*_{i,T}(\omega_i)$ as being undominated for $S = S(T)$ in $(G(\Lambda), \Gamma^*)$. Fix an arbitrary step $t \in \{1,...,T-1\}$, and suppose that at each step $\tau \in \{t+1,...,T\}$ after step t, each player $i \in N$ with each type $\omega_i \in \Omega_i$ is willing to select $s_{i,\tau}(\omega_i) = s^*_{i,\tau}(\omega_i)$ as being quasi-obviously undominated for $S(\tau)$ in $(G(\Lambda), \Gamma^*)$. Then, each player i expects that any other player $j \neq i$ selects $s_{j,\tau}(\omega_j) = s^*_{j,\tau}(\omega_j)$ for all $\omega_i \in \Omega_i$ and $\tau \in \{t+1,...,T\}$. This along with the assumption regarding the Bayesian game Λ implies that each player $i \in N$ with each type ω_i is willing to select $s_{i,t}(\omega_i) = s^*_{i,t}(\omega_i)$ as being quasi-obviously undominated for $S(t)$ in $(G(\Lambda), \Gamma^*)$. From these observations, we have proved the proposition.

Q.E.D.

Example 6 (Abreu-Matsushima Mechanism): We consider the allocation problem with incomplete information, where C denotes the set of possible allocations, and $f : \underset{i \in N}{\times} \Omega_i \to C$ denotes a social choice function.

For simplicity of the argument, we will assume that the realization of the type profile (state) $\omega \in \Omega$ is contractible after the determination of the allocation[8]. Because of this assumption, we can define a mechanism as (A, g, x), where $A = \underset{i \in N}{\times} A_i$, A_i is the set of possible messages for player i, $\Delta(C)$ denotes the set of all lotteries over allocations, and $g : A \to \Delta(C)$ and $x = (x_i)_{i \in N} : A \times \Omega \to R^n$ denote an allocation rule and a

[8] We make this assumption for convenience of arguments. Many works in the literature did not make this assumption and investigated virtual implementation instead of exact implementation like this paper. In this respect, Matsushima (2017) is closely related to this paper, because it assumed the ex-post verification. However, we do not need substantial changes in the argument of this paper even if we delete the assumption.

payment rule, respectively. Note that the side payment vector $x(a,\omega)$ can depend on the true state ω, but the allocation $g(a)$ cannot depend on it.

Each player's valuation function is defined as $v_i : \Delta(C) \times \Omega \to R$, where we assume expected utility and quasi-linearity. The associated payoff function $u_i : A \times \Omega \to R$ for player i is given by

$$u_i(a,\omega) = v_i(g(a),\omega) + x_i(a,\omega) \text{ for all } a \in A \text{ and } \omega \in \Omega .$$

Importantly, we permit only a small monetary fine $\varepsilon \geq 0$, that is, for every $i \in N$, $a \in A$, and $\omega \in \Omega$,

$$|x_i(a,\omega)| \leq \varepsilon .$$

According to the basic concept in Abreu and Matsushima (1992a, 1992b, 1994) and Matsushima (2017), we define the Abreu-Matsushima mechanism (A,g,x) in the following manner[9]. Let $A_i = A_{i,1} \times \cdots \times A_{i,T}$, and we specify

$$A_{i\tau} = \Omega_i \text{ for all } t \in \{1,...,T\} \text{, that is, } A_i = \Omega_i^T \text{ and } A = \Omega^T .$$

At each step $t \in \{1,...,T\}$, each player $i \in N$ reports his type, which is denoted by $a_{i,t} \in A_{i,t} = \Omega_i$. We specify the allocation rule g by

$$g(a) = \frac{\sum_{t=1}^{T} f(a_t)}{T} .$$

The central planner selects an integer $t \in \{1,...,T\}$ and determines the allocation $f(a_t) \in C$. Based on the ex-post verification, we specify the payment rule x by

$x_i(a,\omega) = -\varepsilon$ if player i is the last person who tells a lie, that is, there exists $t \in \{1,...,T\}$ such that $a_{i,t} \neq \omega_i$ and $a_{j,\tau} = \omega_j$ for all $\tau > t$ and $j \neq i$,

$x_i(a,\omega) = 0$ otherwise.

[9] This section assumes imperfect information in the Abreu-Matsushima mechanism. This assumption is crucial for the incomplete information environments, because otherwise players have incentive to make informational manipulation at earlier steps. In contrast, we can safely replace static mechanisms with dynamic mechanisms with perfect information in the complete information environments. See Glazer and Perry (1996) and Glazer and Rubinstein (1996).

The central planner fines the last deviant from the truthful revelation.

Consider the Bayesian game induced by the above-specified Abreu-Matsushima mechanism, which is denoted by $\Lambda^* = (N, A, u, \Omega, p)$. Let $s^* = (s_i^*)_{i \in N}$ denote the truthful strategy rule profile, where

$$s_{i,t}^*(\omega_i) = \omega_i \text{ for all } i \in N,\ \omega_i \in \Omega_i, \text{ and } t \in \{1,...,T\}.$$

By playing the truthful strategy rule profile s^*, the players always achieve the value of the social choice function $f(\omega)$ without paying monetary fines.

Fix an arbitrary real number $\varepsilon > 0$, which is positive but close to zero. Let us select a positive integer T that is large enough to satisfy the following inequality:

$$\varepsilon > \frac{1}{T} \max_{(\omega, c, c', i) \in \Omega \times C^2 \times N} \left| v_i(c, \omega) - v_i(c', \omega) \right|.$$

From the previous works related to the allocation problem in this study, it is clear that if the social choice function f satisfies Bayesian incentive compatibility in a strict sense, then the truthful strategy rule profile s^* is the unique iteratively undominated strategy rule profile in the Bayesian game Λ^* associated with the Abreu-Matsushima mechanism.

The following theorem shows that s^* is also the unique iteratively quasi-obviously undominated strategy rule profile in (Λ^*, Γ^*), where Γ^* is the frame for the set of players N that this section introduced for the incomplete information environments.

Theorem 4: *Suppose that the social choice function f satisfies strict Bayesian incentive compatibility in the sense that for every $i \in N$ and $\omega_i \in \Omega_i$,*

$$E[v_i(f(\omega), \omega) \mid \omega_i] > E[v_i(f(\omega_i', \omega_{-i}), \omega) \mid \omega_i] \text{ for all } \omega_i' \in \Omega_i \setminus \{\omega_i\}.$$

Then, the truthful strategy rule profile s^* is the unique quasi-obviously iteratively undominated strategy rule profile in the Bayesian game Λ^* with the frame Γ^*.

Proof: Note that s^* is the unique iteratively undominated strategy rule profile in the Bayesian game Λ^*. Note also that the Bayesian game Λ^* corresponds to the case investigated in Proposition 3. Hence, from Proposition 3, it is clear that s^* is the unique obviously iteratively undominated strategy rule profile in (Λ^*, Γ^*).

Q.E.D.

10. Conclusion

This study investigated the possibility that even boundedly rational players employs rational behavior in a static game with imperfect information. We assumed that each player fails to practice hypothetical thinking regarding the present and future actions of other players, but not the previous actions of the other players. We proposed the method of frame design that induces such players to practice correct hypothetical thinking as much as possible.

With the help of frame design, we generally showed that the solvability in iterative undominated strategies is equivalent to the solvability in iteratively quasi-obviously undominated strategies. This implies that a well-designed frame successfully motivates boundedly rational players to employ rational behavior. This study also provided a cogent explanation about why the ascending proxy auction has more popularity than the second price auction even if both have the same physical rule.

We further extended the method of frame design to general incomplete information environments. We showed that the Abreu-Matsushima mechanism satisfies the solvability in iteratively quasi-obviously undominated strategies, that is, it has robustness in hypothetical thinking.

Frame design bridges the gap between rationality and bounded rationality. It might be anticipated that frame design avoids the obstruction caused by various aspects of bounded rationality besides the failure of hypothetical thinking that this study intensively investigated. We are concerned about the case that we need different frame designs to overcome different aspects of bounded rationality. It would be an important future research topic to consider how to design frame systems that can solve multiple bounded rationality issues at once.

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Appendix: Iterative Weak Quasi-Obvious Dominance

We define iterative weak dominance as a weaker version of iterative dominance. Consider an arbitrary sequence of subsets $(\hat{A}(k))_{k=0}^{\infty}$, where we assume that

$$\hat{A}_i(0) = A_i,$$

for every $k \geq 1$,

$$[a_i \in \hat{A}_i(k-1) \text{ but } a_i \notin \hat{A}_i(k)]$$

$$\Rightarrow [a_i \text{ is weakly dominated for } \hat{A}(k-1) \text{ in } G],$$

and

$$[\hat{A}_i(k) = \hat{A}_i(k-1)]$$

$$\Rightarrow [\text{There is no strategy } a_i \in \hat{A}_i(k-1) \text{ that is weakly dominated for } \hat{A}(k-1) \text{ in } G],$$

where we denote $\hat{A}(k) \equiv \underset{i \in N}{\times} \hat{A}_i(k)$. Let $\hat{A}_i(\infty) \equiv \bigcap_{k=0}^{\infty} \hat{A}_i(k)$.

Definition A-1: A strategy $a_i \in A_i$ for player i is said to be *iteratively weakly undominated* in G and $(\hat{A}(k))_{k=0}^{\infty}$ if $a_i \in \hat{A}_i(\infty)$.

The difference from iterative dominance is that the definition of iteratively weakly undominated strategy depends on the specification of $(\hat{A}(k))_{k=0}^{\infty}$, that is, depends on the order of iterative elimination of dominated strategies. We define iterative weak obvious dominance by replacing "weakly dominated" with "weakly obviously dominated."

Definition A-2: A strategy $a_i \in A_i$ for player i is said to be *iteratively weakly obviously undominated* in G and $(\hat{A}^*(k))_{k=0}^{\infty}$ if $a_i \in \hat{A}_i^*(\infty)$, where we define $\hat{A}_i^*(\infty)$ similarly to $\hat{A}_i(\infty)$, by replacing "weakly dominated" with "weakly obviously dominated."

If a strategy for player i is the unique iteratively weakly obviously undominated strategy in G and $(\hat{A}^*(k))_{k=0}^{\infty}$, then it is the unique iteratively weakly undominated strategy in G and $(\hat{A}^*(k))_{k=0}^{\infty}$. However, even if it is the unique iteratively weakly undominated strategy in G and $(\hat{A}(k))_{k=0}^{\infty}$, there does not necessarily exist $(\hat{A}^*(k))_{k=0}^{\infty}$ such that it is the unique iteratively weakly obviously undominated strategy in G and $(\hat{A}^*(k))_{k=0}^{\infty}$. We define iterative weak quasi-obvious dominance by replacing "weakly dominated in G" with "weakly quasi-obviously dominated in (G,Γ)."

Definition A-3: A strategy $a_i \in A_i$ for player i is said to be *iteratively weakly quasi-obviously undominated* in (G,Γ) and $(\hat{A}^*(k,\Gamma))_{k=0}^{\infty}$ if $a_i \in \hat{A}_i^*(\infty,\Gamma)$, where we define $\hat{A}_i^*(\infty,\Gamma)$ similarly to $\hat{A}_i(\infty)$, by replacing "weakly dominated in G" with "weakly quasi-obviously dominated in (G,Γ)."

Proposition A-1: *If $a^* \in A$ is the weakly dominant strategy profile in G, then, irrespective of the specification of μ, there exists $(\hat{A}^*(k,\Gamma^{\rho}))_{k=0}^{\infty}$ such that it is the unique iteratively weakly quasi-obviously dominant strategy profile in (G,Γ^{ρ}) and $(\hat{A}^*(k,\Gamma^{\rho}))_{k=0}^{\infty}$.*

Proof: We can prove this proposition in the same way as Proposition 2 by replacing "quasi-obviously" with "weakly quasi-obviously."

Q.E.D.

Theorem A-1: *There exists a frame Γ and $(\hat{A}^*(k,\Gamma))_{k=0}^{\infty}$ such that a strategy profile a^* is the unique iteratively weakly quasi-iteratively undominated strategy profile in (G,Γ) and $(\hat{A}^*(k,\Gamma))_{k=0}^{\infty}$ if and only if it is the unique iteratively undominated strategy profile in G.*

Proof: We can prove this theorem in the same way as Theorem 3 by replacing "quasi-obviously" with "weakly quasi-obviously."

Q.E.D.

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