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Do Consumption Externalities Correspond to the Indivisible Tax Rates on Consumpiton?

Yoichi Gokan
Ritsumeikan University

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Abstract

This paper puts the apperantly different distortions of consumption externalities and endogenous consumption taxes to work in the one-sector Ramsey model without any distortions. We will prove that the two distortions can have similar or possibly exact same dynamic impacts on the aggregate economy, only if we use very familiar preferences in macro-dynamic literature. These two distortions analytically have a very close similarity as the obstacle distorting a market equilibrium path and consumption externalities seem the invisible tax rates (transefer rates) on consumption for positive (negative) external effects .

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*Faculty of Economics, Ritsumeikan university. 1-1-1, Noji-higashi, Kusatsu, Shiga, Japan. Tel: 81-77-561-4837. Fax: 81-77-561-4837.

1 Introduction

As asserted in a basic economic text, a market equilibrium allocation could deviate from its socially optimal one, if an economic distortion is introduced in a perfect competitive equilibrium model without any economic distortions. This paper picks up the apparently disparate distortions of consumption externalities and endogenous consumption taxes in the one-sector Ramsey growth model without any other distortions. We analyze how the two distortions distort an intertemporal allocation in equilibrium from its socially optimal one.

The purpose of this paper is to prove the property theoretically that consumption externalities can be very similar to endogenous consumption taxes in that consumption externalities and taxes have very similar impacts on a growth path in the one-sector Ramsey model. Only if we construct the theoretical growth model including the two economic distortions, we can′t help but feeling that consumption externalities precisely correspond to the invisible tax rates (subsidy rates) on consumption for positive (negative) external effects.

The analytical results are mildly influenced by how an agent's preference is specified, but the results are independent of how production technology is specified. For the analysis, this paper mostly picks up the three types of very familiar utility functions of leisure and consumption, the non-separable preferences à la King, Plosser and Rebelo (1988) and à la Greenwood, Hercovitz and Huffman (1988), and the separable preference used in much macro-dynamic literature. As long as we focus only on local dynamics near a steady state, it will be revealed that the two different distortions can have the similar or exactly the same impacts on a market equilibrium path. Though the definition of the two distortions falls behind, let us explain how different the two economic distoritons can be defined.

The activities of others' consumption could have some impacts on an agent's activety of consumption. How does an agent feel, when he/she perceives others' consumption activities ? Depending on an agen't personality, his/her own utility increases or decreases. An increase (a decline) in his/her own utility reflects an instance of the agent's feeling reverence (envy) toward others.

In contrast, endogenous consumption taxes are defined in that consumption tax rate endogenously adjusts to obtain a certain amount of the necessary revenue. If we would like to analyze the outcomes of countries relying strongly on consumption taxes as a source of necessary revenue like EU countries, it is very useful to seize endogenous consumption taxes in a theoretical model. It will proved that these two economic distortions can theoretically bear strong resemblance as the obstacle altering a market equilibrium growth path.

We would like to cite Giannitsarou (2007) and Alonso-Carrera *et al* (2008) as relatively associated literature with this paper. Using the separable preference regarded as the very special case of this paper, Giannitsarou (2007) revealed that the regular saddle point is always guaranteed in the one-sector Ramsey model, if consumption tax rate endogenously adjusts by contrast with income tax rate. Alonso-Carrera *et al* (2008) also illustrated that the steady state is a saddle, if they focus on the familiar homothetic utility of private and social consumption viewed as one example of the present paper.[1] This paper will be also able to clarify the close link that exists between the two papers.

This paper proceeds as follows. In Section 2, we describe the integrated Ramsey model with only the two distortions of consumption externalities and taxes. Section 3 (4) picks only up consumption externalities (endogenous consumption externalities) and the non-linear differential equations are linearly approximated around steady states. In Section 5, we define the conditions that the two economic distortions can have identical impacts on local dynamics. Section 6 clarifies that the conditions of Section 5 are easily satisfied, only if we use the very familiar non-separable utility function à la King, Plosser and Rebelo (1988) seen in RBC literature. Section 7 picks up the separable utility functions used in much macro-dynamic literature and Section 8 thinks of the alternatively familiar non-preference à la Greenwood, Hercovitz and Huffman (1988) that is lack of the income effect on the demand for leisure. By resorting to numerical simulations, these two sections get the similar implication as in Section 5. Section 9 considers that there are simultaneously the two distortions in the one-sector growth model and the robustness of results are checked. In Section 10, the conclusions are briefly summarized.

2 Framework

This paper considers the standard one-sector Ramsey growth model with the two kinds of economic distortions of consumption externalities and consumption taxes. The economy composes the three types of agents, identical infinitely lived households, identical competitive firms and the government. Let us specify their behaviors.

2.1 Household

We pick up an infinitely lived household and he/she is referred to as the representative agent. The representative agent supplies his/her labor to the market and consumes the final good at every period to maximize the sum of his/her discounted utility function:

$$\int_0^\infty U\left(1-l, c, \bar{c}\right) e^{-\rho t} dt, \tag{2-1}$$

where l denotes the amount of labor, c denotes his own consumption, $\bar{c}$ denotes the average amount of consumption in the economy, and ρ denotes the discount rate. We assume that the instantaneous utility function U of (2-1) is concave with respect to leisure $1-l$ and private consumption c. Apart from the standard properties: $U_1\left(1-l, c, \bar{c}\right) > 0$, $U_{11}\left(1-l, c, \bar{c}\right) < 0$, $U_2\left(1-l, c, \bar{c}\right) > 0$ and $U_{22}\left(1-l, c, \bar{c}\right) < 0$,[2] we additionally assume

Assumption 1 $U_{11}\left(1-l, c, \bar{c}\right) \cdot U_{22}\left(1-l, c, \bar{c}\right) > \left[U_{12}\left(1-l, c, \bar{c}\right)\right]^2$.

[1] Alonso-Carrera *et al* also showed that indeterminacy can arise because of consumption externalities, if they use the very unfamiliar utility function with respect to leisure, private and social consumption.

[2] Herefrom, the suffix in a function shows what number of the arguments in a function the partial derivative is taken.

By introducing the social consumption $\bar{c}$ in the utility function, we can consider the external effects derived from others' consumption. An increase in the utility level U following an increase in $\bar{c}$, i.e., $U_3(1-l, c, \bar{c}) > 0$ implies a feeling of reverence toward the others; i.e., positive consumption externalities from the other agents. A decrease in the utility level U following an increase in $\bar{c}$, i.e., $U_3(1-l, c, \bar{c}) < 0$ implies that a feeling of envy toward the others is shown; this corresponds to negative consumption externalities from the other agents.

The representative agent maximizes (2-1) subject to the usual budget constraint:

$$\dot{k} + \delta k + (1+\tau_c)\, c = rk + wl, \tag{2-2}$$

where k represents capital stock that depreciates at the rate δ, r represents the rental price of capital, w represents the real wage rate, and τ_c represents the consumption tax rate, which is specified more clearly in Sections 2-3.

We can easily solve the dynamic optimization problem above. Defining the coastate variable attached to the budget (2-2) as μ, the first order conditions are derived as follows:

$$-e^{-\rho t} U_2(1-l, c, \bar{c},) = (1+\tau_c)\,\mu, \tag{2-3}$$

$$\frac{U_1(1-l, c, \bar{c})}{U_2(1-l, c, \bar{c})} = \frac{w}{1+\tau_c}. \tag{2-4}$$

$$r = -\frac{\dot{\mu}}{\mu} \tag{2-5}$$

To impose the non-Ponzi game, the transversality condition is needed:

$$\lim_{t\to\infty} a \cdot \mu = 0.$$

Equation (2-3) shows that the tax-adjusted coastate variable equals the discounted marginal utility of private consumption. Equation (2-4) corresponds to the labor supply function, which determines the intratemporal choice between leisure and consumption. Equation (2-5) is the Keynes-Ramsey equation determining the intertemporal choice of consumption.

2.2 Firms

There are innumerably identical firms, but we consider only the representative firm. By combining capital and labor, the firm produces the final goods y that are consumed or invested. The production technology $y(k, l)$ is a constant return in the two inputs. Thus, it can be expressed in the intensive form: $y(k, l) = lf\,(a)$, where $f\,(a)$ is the per-capita production function defined for the capital-labor ratio, $a \equiv k/l$.

Let us impose the very standard assumption on its properties:

Assumption 2 *The per-capita production function $f\,(a)$ is continuous for $a \geq 0$ and has the derivatives $f^r(a)$ for large enough r, with $f'(a) > 0$ and $f''(a) < 0$.*

To maximize the profit, this firm demands the production factors such that the marginal products of capital and labor equal the rental price and the real wage rate, respectively. Defining $\omega(a) \equiv f(a) - af'(a)$, the competitive equilibrium real prices in this period are:

$$r = f'(a) \text{ and } w = \omega(a). \tag{2-6}$$

2.3 Government

Finally, let us specify the government behavior. As this paper is to identify the similarity or equivalence between consumption taxes and consumption externalities, we focus only on the annual revenue originated in consumption taxes. Then, the government budget is simply:

$$g = \tau_c \cdot c. \tag{2-7}$$

The consumption tax rate τ_c endogenously adjusts to remove the budget deficit for the preset level of government expenditure g. In other words, the tax rate τ_c is the time-dependent variable like consumption c and physical capital k *etc.*, while the expenditure g is the time-independent variable fixed by the government. This budget constraint is identical to the one in Giannitsarou (2007).

3 Consumption externalities

Section 3 concentrates only on the case of consumption externalities. If we set government expenditure equal to zero, i.e., $g = 0$, the consumption tax rate is determined at $\tau_c = 0$ over time.

3.1 Equilibrium dynamics

Using the equations in the previous section, we derive the equilibrium differential equations determining how the aggregate variables (k, l) evolve over time.

Let us consider (2-4) evaluated at the symmetric equilibrium $c = \bar{c}$. Substituting (2-6) in (2-4) yields consumption c as a function of capital stock k and labor supply l, i.e., $c = c_E(k, l)$. In terms of the percentage change, the equation $c = c_E(l, k)$ satisfies

$$\left[\varepsilon_E(k,l) - \eta_E(k,l) + \frac{\omega'(a)}{\omega(a)}a\right]\frac{\dot{l}}{l} + \left[\gamma_E(k,l) + \phi_E(k,l)\right]\frac{\dot{c}}{c} = \frac{\omega'(a)}{\omega(a)}a\frac{\dot{k}}{k}, \tag{3-1}$$

where

$$\gamma_E(k,l) \equiv -\left(\frac{U_{22} + U_{23}}{U_2}\right)c_E(k,l) > 0^3,$$

$$\varepsilon_E(k,l) \equiv \frac{U_{21}}{U_2}l,$$

[3] We consider that social consumption does not distort the property that the marginal utility of private consumption decreases with consumption, i.e., $U_{22} + U_{23} < 0$.

$$\phi_E(k,l) \equiv \left(\frac{U_{21}+U_{31}}{U_1}\right) c_E(k,l),$$

and

$$\eta_E(k,l) \equiv \frac{U_{11}}{U_1} l < 0.$$

Considering (2-3) and (2-5), we can get

$$\gamma_E(k,l)\frac{\dot{c}}{c} + \varepsilon_E(k,l)\frac{\dot{l}}{l} = \left[f'(a) - \delta - \rho\right]. \tag{3-2}$$

Defining the elasticity of capital-labor substitution in production as $\sigma(a)$, we can obtain $a\omega'(a)/\omega(a) = s(a)/\sigma(a)$, where $s(a)$ denote the capital share in total income, i.e., $s(a) \equiv af'(a)/f(a)$. To eliminate $\dot{c}/c$, the use of this relation, (3-1) and (3-2) yields

$$\frac{\dot{l}}{l} = \frac{-\frac{\phi_E(k,l)+\gamma_E(k,l)}{\gamma_E(k,l)}\left[f'(a)-\delta-\rho\right] + \frac{s(a)}{\sigma(a)}\frac{\dot{k}}{k}}{\frac{s(a)}{\sigma(a)} - \eta_E(k,l) - \frac{\phi_E(k,l)}{\gamma_E(k,l)}\varepsilon_E(k,l)}. \tag{3-3}$$

From (2-2) and (2-6), the good market equilibrium can be derived as

$$\dot{k} + \delta k + c(k,l) = lf(a) \tag{3-4}$$

and this equation indicates that final good can be consumed or invested.

Noting the definition of $a \equiv k/l$, the equilibrium path (k,l) is definitely determined by (3-3) and (3-4), which are identical to those in Alonso *et al* (2008) except for $\sigma(a) \neq 1$.

3.2 Steady state equilibrium

In the steady state equilibrium, all the economic variables are constant, i.e., $\dot{k} = \dot{l} = 0$. In the case of consumption externalities, the steady state value of x is defined as x_E^*. Using the above equations evaluated at the steady state, let us consider the existence of positive steady state value (k_E^*, l_E^*).

From Assumption 2, (3-2) or (3-3), the steady state capital-labor ratio $a^* (= k_E^*/l_E^*)$ is uniquely obtained by solving[4]

$$f'(a^*) = \delta + \rho, \tag{3-5}$$

if $\lim_{a\to\infty} f'(a) < \delta + \rho < \lim_{a\to 0} f'(a)$. The use of (2-4) and (2-6) yields

$$\frac{U_1(1-l_E^*, c_E^*, c_E^*)}{U_2(1-l_E^*, c_E^*, c_E^*)} = \omega(a^*). \tag{3-6}$$

[4] As will be evident, the steady state value of a^* in this case is the same as in endogenous consumption taxes. Thus, we omit the subscript E from a^*.

Noting $l_E^* f\ (a^*) = \frac{\delta+\rho}{s^*} k_E^*$, eq.(3-4) becomes

$$\frac{(1-s^*)\,\delta+\rho}{s^*} = \frac{c_E^*}{k_E^*}. \tag{3-7}$$

Eqs. (3-5)-(3-7) determine the steady state values of (c_E^*, k_E^*, l_E^*). However, we cannot argue the point any more, if we do not specify the feature of utility function U. Sections 6,7 and 8 consider the various types of utility functions which are extensively used in growth and RBC literature. Until then, let us assume the existence of positive steady state (k_E^*, l_E^*).

3.3 Local dynamics

This section explores the local stability near the steady state (k_E^*, l_E^*). Considering (3-1), let us linearize the differential equations (3-3) and (3-4) around the steady state. For the convenience of exposition, let us write the steady state values as $\gamma_E^* \equiv \gamma_E(k_E^*, l_E^*)$, $\varepsilon_E^* \equiv \varepsilon_E(k_E^*, l_E^*)$, $\phi_E^* \equiv \phi_E(k_E^*, l_E^*)$, $\eta_E^* \equiv \eta_E(k_E^*, l_E^*)$, $c_E^* \equiv c_E(k_E^*, l_E^*)$, $\sigma^* \equiv \sigma(a^*)$, and $s^* \equiv s(a^*)$. Eq. (3-1) implies the partial derivatives of $c_E(k_E^*, l_E^*)$ with respect to l and k. Considering them, we can get

$$\begin{bmatrix} \dot{l} \\ \dot{k} \end{bmatrix} = J_E \begin{bmatrix} l - l_E^* \\ k - k_E^* \end{bmatrix}, \tag{3-8}$$

where

$$J_E \equiv \begin{bmatrix} \dfrac{-\frac{\phi_E^*}{\gamma_E^*}\frac{(1-s^*)(\delta+\rho)}{\sigma^*} + \frac{s^*}{\sigma^*}\frac{\varepsilon_E^*-\eta_E^*+\frac{s^*}{\sigma^*}}{\phi_E^*+\gamma_E^*}\left(\frac{c_E^*}{k_E^*}\right)}{\frac{s^*}{\sigma^*}-\eta_E^*-\frac{\phi_E^*}{\gamma_E^*}\varepsilon_E^*} & \dfrac{\left(\frac{l_E^*}{k_E^*}\right)\left[-\frac{\phi_E^*+\gamma_E^*}{\gamma_E^*}\frac{(1-s^*)(\delta+\rho)}{\sigma^*} + \frac{s^*}{\sigma^*}\left(\rho - \frac{\frac{s^*}{\sigma^*}}{\phi_E^*+\gamma_E^*}\left(\frac{c_E^*}{k_E^*}\right)\right)\right]}{\frac{s^*}{\sigma^*}-\eta_E^*-\frac{\phi_E^*}{\gamma_E^*}\varepsilon_E^*} \\ \left(\frac{k_E^*}{l_E^*}\right)\left[(\delta+\rho)\frac{1-s^*}{s^*} + \frac{\varepsilon_E^*-\eta_E^*+\frac{s^*}{\sigma^*}}{\phi_E^*+\gamma_E^*}\left(\frac{c_E^*}{k_E^*}\right)\right] & \rho - \frac{\frac{s^*}{\sigma^*}}{\phi_E^*+\gamma_E^*}\left(\frac{c_E^*}{k_E^*}\right) \end{bmatrix}.$$

Using (3-8), we can compute the determinant D_E and trace T_E as

$$D_E = \frac{-\frac{\phi_E^*+\gamma_E^*}{\gamma_E^*}\frac{(1-s^*)(\delta+\rho)}{s^*}\frac{s^*}{\sigma^*}\left[\frac{(1-s^*)\delta+\rho}{s^*} + \frac{\varepsilon_E^*-\eta_E^*}{\phi_E^*+\gamma_E^*}\left(\frac{c_E^*}{k_E^*}\right)\right]}{\frac{s^*}{\sigma^*}-\eta_E^*-\frac{\phi_E^*}{\gamma_E^*}\varepsilon_E^*}, \tag{3-9}$$

$$T_E = \frac{-1}{\frac{s^*}{\sigma^*}-\eta_E^*-\frac{\phi_E^*}{\gamma_E^*}\varepsilon_E^*}\left[\frac{\phi_E^*}{\gamma_E^*}\frac{s^*}{\sigma^*}\frac{(1-s^*)(\delta+\rho)}{s^*} - \frac{s^*}{\sigma^*}\frac{\varepsilon_E^*}{\gamma_E^*}\left(\frac{c_E^*}{k_E^*}\right)\right] + \rho. \tag{3-10}$$

Needless to say, the determinant D_E is equal to the product of the two eigenvalues in the Jacobian (3-8) and the trace T_E equals the sum of the eigenvalues.

Noting $k_E^* = a^* l_E^*$ and (3-7), the differentiation of c_E^* with respect to l_E^* is $dc_E^*/dl_E^* =$

$\frac{(1-s^*)\delta+\rho}{s^*}a^*$, while the total differentiation of (3-6) leads to $dc_E^*/dl_E^* = -\frac{\varepsilon_E^*-\eta_E^*}{\phi_E^*+\gamma_E^*}\left(\frac{c_E^*}{l_E^*}\right)$.[5] It is easily understood that the relative size of the slopes of (3-6) and (3-7) in the plane (l_E^*, c_E^*) can determine the sign of the part of the numerator in (3-9). Noting that $dc_E/dl_E = -\frac{\varepsilon_E}{\gamma_E}\left(\frac{c_E}{l_E}\right)$ must be satisfied to keep U_2 constant in (3-6), the slope of Frisch labor supply curve[6] can be derived as $\frac{d\ln w}{d\ln l_E} = -\eta_E - \phi_E\frac{\varepsilon_E}{\gamma_E}$, while (2-6) means that the slope of labor demand curve is $-\frac{s(a)}{\sigma(a)}$. Thus, the sign of $\frac{s^*}{\sigma^*} - \eta_E^* - \frac{\phi_E^*}{\gamma_E^*}\varepsilon_E^*$ is positive if the two labor curves cross with normal slopes. Using the Ramsey model without any economic distortions, Hintermaier (2003) proved that $\frac{s^*}{\sigma^*} - \eta_E^* - \frac{\phi_E^*}{\gamma_E^*}\varepsilon_E^* > 0$ is satisfied, only if the utility function is concave with respect to leisure and consumption.

4 Endogenous consumption taxes

Section 4 focuses only on the case of endogenous consumption taxes, i.e., $g \neq 0$. In other words, we consider that consumption externalities are absent in the utility function of (2-1) and thus $U_3 = 0$ and $U_{3j} = 0$ are satisfied for $j = 1, 2$ and 3.

4.1 Equilibrium dynamics

Let us derive the equilibrium dynamic equations governing the behaviors of economic variables, when the consumption tax rate τ_c endogenously adjusts to satisfy the budget (2-6) for a given level of government expenditure g.[7]

Using the same method as in the case of consumption externalities, the combination of (2-4) and (2-6) yields consumption c that is a function of capital stock k and labor supply l, i.e., $c = c_g(k,l)$. Differentiating the equation $c = c_g(l,k)$ with respect to time and using $d\tau_c/\tau_c = -dc/c$ from (2-7), eq.(3-1) is rewritten as

$$\left[\varepsilon_g(k,l) - \eta_g(k,l) + \frac{\omega'(a)}{\omega(a)}a\right]\frac{\dot{l}}{l} + \left[\gamma_g(k,l) - \frac{g}{g+c} + \phi_g(k,l)\right]\frac{\dot{c}}{c} = \frac{\omega'(a)}{\omega(a)}a\frac{\dot{k}}{k}, \quad (4\text{-}1)$$

where

$$\gamma_g(k,l) \equiv -\frac{U_{22}}{U_2}c_g(k,l) > 0,$$

$$\varepsilon_g(k,l) \equiv \frac{U_{21}}{U_2}l$$

$$\phi_g(k,l) \equiv \frac{U_{21}}{U_1}c_g(k,l)$$

[5] The steady state capital-labor ratio a^* is exclusively determined by (3-5). If we substitute this value into (3-6) and (3-7), the steady state values of (c_E^*, l_E^*) are determined by (3-6) and (3-7).

[6] Frisch labor supply curve is defined as a combination of labor supply and the real wage, when the marginal utility of private consumption U_2 is kept constant.

[7] Giannitsarou (2007) used the discrete time-version of the Ramsey growth model, but the property of local stability is almost the same as in the continuous time-version as studied here.

and

$$\eta_g(k,l) \equiv \frac{U_{11}}{U_1} l < 0$$

Noting $U_3 = 0$ in (2-4) and using the budget (2-7), we can rewrite the Keynes-Ramsey equation (3-2) in this case as

$$\left[\gamma_g(k,l) - \frac{g}{g+c}\right]\frac{\dot{c}}{c} + \varepsilon_g(k,l)\frac{\dot{l}}{l} = \left[f'(a) - \delta - \rho\right]. \tag{4-2}$$

If we recall $a\omega'(a)/\omega(a) = s(a)/\sigma(a)$, combining (4-1) and (4-2) leads to

$$\frac{\dot{l}}{l} = \frac{-\frac{\phi_g(k,l)+\gamma_g(k,l)-\frac{g}{g+c}}{\gamma_g(k,l)-\frac{g}{g+c}}\left[f'(a)-\delta-\rho\right] + \frac{s(a)}{\sigma(a)}\frac{\dot{k}}{k}}{\frac{s(a)}{\sigma(a)} - \eta_g(k,l) - \frac{\phi_g(k,l)}{\gamma_g(k,l)-\frac{g}{g+c}}\varepsilon_g(k,l)}, \tag{4-3}$$

Using (2-2), (2-6) and (2-7), the good market equilibrium (3-4) can be replaced by

$$\dot{k} + \delta k + c + g = lf(a). \tag{4-4}$$

Government expenditure g is fixed. Noting $a \equiv k/l$ and $c = c_g(k,l)$, eqs.(4-3) and (4-4) determine the equilibrium path (k,l) for an initial value of state variable k.

4.2 Steady state equilibrium

Noting that the subscripts in variables denote the case of endogenous consumption taxes, let us consider the existence of positive steady state $\left(k_g^*, l_g^*\right)$, where all the economic variables are constant over time. Use of (4-3) leads to

$$f'(a^*) = \delta + \rho, \tag{4-5}$$

which verifies that the capital-labor ratio a^* takes the same value as in the case of consumption externalities and equivalently $k_E^*/l_E^* = k_g^*/l_g^*$ is satisfied. Compare (3-5) with (4-5). From (2-4) and (2-6), the intratemporal condition in this case is

$$\frac{U_1\left(1-l_g^*, c_g^*\right)}{U_2\left(1-l_g^*, c_g^*\right)} = \frac{\omega(a^*)}{1+\tau_c^*}. \tag{4-6}$$

Noting (4-3) and $\tau_c^* c_g^* = g$, the good market equilibrium becomes

$$\frac{(1-s^*)\delta + \rho}{s^*} = (1+\tau_c^*)\frac{c_g^*}{k_g^*} \tag{4-7}$$

The steady state values $\left(c_g^*, k_g^*, l_g^*\right)$ are determined by $\tau_c^* c_g^* = g$ and (4-5)-(4-7) for a given value of g. As stated in the case of consumption externalities, we cannot describe any more, if the function in (2-1) is not specified. Sections 6,7 and 8 prove that the steady states exist for the very familiar utility functions. Until then, we assume the existence of positive steady states $\left(k_g^*, l_g^*\right)$.

4.3 Local dynamics

In this section, the local stability is examined. As in consumption externalities, let us define the steady state values as $\gamma_g^* \equiv \gamma_g(k_g^*, l_g^*)$, $\varepsilon_g^* \equiv \varepsilon_g(k_g^*, l_g^*)$, $\phi_g^* \equiv \phi_g(k_g^*, l_g^*)$, $\eta_g^* \equiv \eta_g(k_g^*, l_g^*)$, $c_g^* \equiv c_g(k_g^*, l_g^*)$. Considering $c = c_g(k, l)$ and (4-1), the linearized system of (4-3) and (4-4) evaluated at the steady states becomes

$$\begin{bmatrix} \dot{l} \\ \dot{k} \end{bmatrix} = J_g \begin{bmatrix} l - l_g^* \\ k - k_g^* \end{bmatrix}, \tag{4-8}$$

where

$$J_g \equiv \begin{bmatrix} \dfrac{-\frac{\phi_g^*}{\gamma_g^* - \frac{g}{g+c^*}} \frac{(1-s^*)(\delta+\rho)}{\sigma^*} + \frac{s^*}{\sigma^*} \frac{\varepsilon_g^* - \eta_g^* + \frac{s^*}{\sigma^*}}{\phi_g^* + \gamma_g^* - \frac{g}{g+c^*}} \left(\frac{c_g^*}{k_g^*}\right)}{\frac{s^*}{\sigma^*} - \eta_g^* - \frac{\phi_g^*}{\gamma_g^*} \varepsilon_g^*} & \dfrac{\left(\frac{l_g^*}{k_g^*}\right) \left[\frac{\phi_g^* + \gamma_g^* - \frac{g}{g+c^*}}{\gamma_g^* - \frac{g}{g+c^*}} \frac{(1-s^*)(\delta+\rho)}{\sigma^*} + \frac{s^*}{\sigma^*} \left(\rho - \frac{\frac{s^*}{\sigma^*}}{\phi_g^* + \gamma_g^* - \frac{g}{g+c^*}} \left(\frac{c_g^*}{k_g^*}\right)\right)\right]}{\frac{s^*}{\sigma^*} - \eta_g^* - \frac{\phi_g^*}{\gamma_g^*} \varepsilon_g^*} \\ \left(\frac{k_g^*}{l_g^*}\right) \left[(\delta+\rho) \frac{1-s^*}{s^*} + \frac{\varepsilon_g^* - \eta_g^* + \frac{s^*}{\sigma^*}}{\phi_g^* + \gamma_g^* - \frac{g}{g+c^*}} \left(\frac{c_g^*}{k_g^*}\right)\right] & \rho - \frac{\frac{s^*}{\sigma^*}}{\phi_g^* + \gamma_g^* - \frac{g}{g+c^*}} \left(\frac{c_g^*}{k_g^*}\right) \end{bmatrix}.$$

From (4-8), we can obtain the determinant D_g and Trace T_g as

$$D_g = \frac{-\frac{\phi_g^* + \gamma_g^* - \frac{g}{g+c^*}}{\gamma_g^* - \frac{g}{g+c^*}} \frac{(1-s^*)(\delta+\rho)}{s^*} \frac{s^*}{\sigma^*}}{\frac{s^*}{\sigma^*} - \eta_g^* - \frac{\phi_g^*}{\gamma_g^* - \frac{g}{g+c^*}} \varepsilon_g^*} \left[\frac{(1-s^*)\delta + \rho}{s^*} + \frac{\varepsilon_g^* - \eta_g^*}{\phi_g^* + \gamma_g^* - \frac{g}{g+c^*}} \left(\frac{c_g^*}{k_g^*}\right)\right], \tag{4-9}$$

$$T_g = \frac{-1}{\frac{s^*}{\sigma^*} - \eta_g^* - \frac{\phi_g^*}{\gamma_g^* - \frac{g}{g+c^*}} \varepsilon_g^*} \left[\frac{\phi_g^*}{\gamma_g^* - \frac{g}{g+c^*}} \frac{s^*}{\sigma^*} \frac{(1-s^*)(\delta+\rho)}{s^*} - \frac{s^*}{\sigma^*} \frac{\varepsilon_g^*}{\gamma_g^* - \frac{g}{g+c^*}} \left(\frac{c_g^*}{k_g^*}\right)\right] + \rho. \tag{4-10}$$

The product of two eigenvalues in the Jacobian J_g corresponds to (4-9), while the sum of two eigenvalues is equal to (4-10).

Noting that a^* is already determined by (4-5), the slope of (4-6) in $\left(l_g^*, c_g^*\right)$ is $dc_g^*/dl_g^* = -\frac{\varepsilon_g^* - \eta_g^*}{\phi_g^* + \gamma_g^* - \frac{g}{g+c^*}} \left(\frac{c_g^*}{l_g^*}\right)$. Considering $k_g^* = a^* l_g^*$ and $\tau_c^* c_g^* = g$, the slope of (4-7) is $dc_g^*/dl_g^* = \frac{(1-s^*)\delta + \rho}{s^*} a^*$. If we know the relative size of these two slopes, the sign of the case arc in (4-9) can be known. Let us consider the sign of the denominator in (4-9) and (4-10). As for the labor demand curve (2-6), the slope $\frac{d \ln w}{d \ln l_g}$ is $-\frac{s(a)}{\sigma(a)} (< 0)$, which is the same as in the case of consumption externalities. Noting that the third argument $\bar{c}$ is absent in (2-4), eq.(2-4) defines the tax-adjusted Frisch labor supply curve expressing the combinations of (l, w) to keep the value of $\frac{U_2(1-l,c)}{1+\tau_c}$ constant. As $dc_g^*/dl_g^* = -\frac{\varepsilon_g^*}{\gamma_g^* - \frac{g}{g+c^*}} \frac{c_g}{l_g}$ must be satisfied to keep the value of $\frac{U_2(1-l,c)}{1+\tau_c}$ constant, the slope of tax-adjusted Frisch labor supply $\frac{d \ln w}{d \ln l_g}$ is $-\eta_g^* - \frac{\phi_g^*}{\gamma_g^* - \frac{g}{g+c^*}} \varepsilon_g^*$. The sigh of the denominator in (4-9) and (4-

10) can be known by the magnitude relation of the slopes of labor supply and demand curves. Assumption 1 declaring the concave utility function of (2-1) is also expressed as $\eta_g(k,l) + \frac{\varepsilon_g(k,l)}{\gamma_g(k,l)}\phi_g(k,l) < 0$. If government expenditure is infinitely close to zero, i.e., $g \to 0$, the slope of tax-adjusted Frisch labor supply is positive from this assumption. Then, the denominator in (4-9) and (4-10) is necessarily positive.

5 Comparison

This section clarifies the conditions under which the equilibrium dynamics of (3-8) are perfectly identical to those of (4-8). Phrased differently, we consider how the local dynamics of labor and capital are quite the same between the cases of consumption externalities and endogenous consumption taxes, only if the two economic distortions have a same size.

Let us substitute the good market equilibrium of (3-7) $\frac{c_E^*}{k_E^*} = \frac{(1-s^*)\delta+\rho}{s^*}$ in (3-9) and (3-10), while we substitute the equilibrium condition of (4-7) $\frac{c_g^*}{k_g^*} = \frac{1}{1+\tau_c^*}\frac{(1-s^*)\delta+\rho}{s^*}$ in (4-9) and (4-10). As stated above, the steady state capital-labor ratio a^* is identical between the cases of consumption externalities and endogenous consumption taxes. Thus, the capital share in total income $s(a^*)$ and the elasticity of capital-labor substitution $\sigma(a^*)$ are also equivalent between the two cases. We can also verify $k_E^* = k_g^*$, only if $l_E^* = l_g^*$ is satisfied. In (4-9) and (4-10), $\frac{g}{g+c^*} = \frac{\tau_c^*}{1+\tau_c^*}$ is satisfied. Considering these facts, we can get:

Proposition 1 *If $\gamma_E(k_E^*, l_E^*) = (1+\tau_c^*)\gamma_g(k_g^*, l_g^*) - \tau_c^*$, $\phi_E(k_E^*, l_E^*) = (1+\tau_c^*)\phi_g(k_g^*, l_g^*)$, $\eta_E(k_E^*, l_E^*) = \eta_g(k_g^*, l_g^*)$, $\varepsilon_E(k_E^*, l_E^*) = \varepsilon_g(k_g^*, l_g^*)$ and $l_E^* = l_g^*$, the local dynamics of (3-8) and (4-8) are completely identical. Phrased differently, the dynamic effects of consumption externalities are equivalent to the ones of endogenous consumption taxes, if we restrict attention to local dynamics near a steady state.*

Proof. Considering (3-7) and (4-7), compare (3-9) and (3-10) with (4-9) and (4-10), respectively. ■

As proved in Appendix A, we can also show that the sum of private and public consumption in endogenous taxes, i.e., $g + c_g$ locally behave in the same way as private consumption in consumption externalities, i.e., c_E, if Proposition 1 is realized.

To clarify how the conditions stated in Proposition 1 are satisfied, we impose the following restriction on the utility function in the case of consumption externalities.

Remark 1 *In the case of consumption externalities, we consider the feature of utility function, $U(1-l, c, \bar{c}) \equiv V(1-l, \xi(c,\bar{c}))$, in which $\xi(c,\bar{c})$ is homogeneous function of degree $1+\psi$ for private and social consumption.*

As for this function ξ, $(1+\psi)\xi(c,\bar{c}) = \xi_1(c,c)c + \xi_2(c,c)c$ and $\psi\xi_1(c,c) = \xi_{11}(c,c)c + \xi_{12}(c,c)c$ are satisfied at the symmetric equilibrium, $c = \bar{c}$. Using the two relations, the elasticities of marginal utitilities in consumption externalities can be rewritten as $\gamma_E^* = -(1+\psi)\frac{V_{22}\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)}{V_2\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)}\xi(c_E^*, c_E^*) - \psi$, $\phi_E^* = (1+\psi)\frac{V_{12}\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)}{V_1\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)}\xi(c_E^*, c_E^*)$,

$\varepsilon_E^* = \frac{V_{21}\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)}{V_2\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)} l_E^*$ and $\eta_E^* = \frac{V_{11}\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)}{V_1\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)} l_E^*$, where $c_E^* = c_E\left(l_E^*, k_E^*\right)$. As will be evident below, the value of ψ corresponds to the size of consumption externalities. Recall that the two kinds of economic distortions are set at a same size, i.e., $\psi = \tau_c^*$. Noting the elasticities just mentioned above and the definitions of γ_g^*, ϕ_g^*, ε_g^* and η_g^* in Proposition 1, Proposition 1 can be rewritten as follows:

Proposition 2 *If* $\gamma_g^* = \frac{V_{22}\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)}{V_2\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)} \xi\left(c_E^*, c_E^*\right)$, $\phi_g^* = \frac{V_{12}\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)}{V_1\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)} \xi\left(c_E^*, c_E^*\right)$, $\varepsilon_g^* = \frac{V_{21}\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)}{V_2\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)} l_E^*$, $\eta_g^* = \frac{V_{11}\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)}{V_1\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)} l_E^*$ *and* $l_E^* = l_g^*$, *the local dynamics of (3-8) and (4-8) are completely identical.*

Proof. From the properties stated in *Remark* 1 and the specification of $\psi = \tau_c^*$, Proposition 2 can be proved. ■

Proposition 2 is easily realized, if we use the non-separable utility functions à la King, Plosser and Rebelo (1988) and Bennett and Farmer (2000), which are extensively used in growth and RBC theories. Let us prove this fact in the next section.

6 Specification of utility function: King *et al* (1988)

This section considers the non-separable preference comprehending the non-separable utilities as specified by King, Plosser and Rebelo (1988) and Bennett and Farmer (2000). At first, let us specify the function $\xi(c, \bar{c})$ satisfying the property in Remark 1 as follows:

$$\begin{aligned} \xi(c, \bar{c}) &= \left[\frac{1}{1+\psi} c^{-\varphi} + \frac{\psi}{1+\psi} \bar{c}^{-\varphi}\right]^{-\frac{1+\psi}{\varphi}} \quad \text{for } \varphi \neq 0 \qquad \text{(6-1)} \\ &= c\bar{c}^{\psi} \text{ for } \varphi = 0, \end{aligned}$$

which are used also in Sections 7 and 8. As shown in the equations below, the transitional dynamics as well as the steady states are independent of the degree of φ determining the elasticity of substitution between c and $\bar{c}$, i.e., $\frac{1}{1+\varphi}$.

Noting that $\Gamma(1-l)$ is the utility function of leisure, let us pick up the preference:

$$V(1-l, \xi(c, \bar{c})) = \Gamma(1-l) \cdot [\xi(c, \bar{c})]^{1-\gamma}, \qquad \text{(6-2)}$$

where $\Gamma(1-l) = \frac{(1-l)^{\theta(1-\gamma)}}{1-\gamma}$ in Case 1[8], $\Gamma(1-l) = \frac{[\exp\theta(1-l)]^{1-\gamma}}{1-\gamma}$ in Case 2 and $\Gamma(1-l) = \left[\exp \frac{(1-l)^{1-\chi}}{1-\chi}\right]^{1-\gamma} / (1-\gamma)$ in Case 3[9].

Case 1 corresponds to the constant relative risk aversion (CRRA), Case 2 is the constant absolute risk aversion (CARA) and Case 3 does not belong to these cases, as long

[8] Case 1 coresponds to the preference à la King, Plosser and Rebelo (1988) for $\psi = 0$.

[9] For $\psi = 0$, Case 3 is utilized in Bennett and Farmer (2000).

as $\gamma \neq 1$.[10] The signs of γ, θ and χ are positive, while the degree of externalities ψ takes positive or negative signs. The case of $\psi = 0$ in (6-2) expresses the preferences used in the case of endogenous consumption taxes, $U(1-l, c)$.

It is convenient to define $\tilde{\gamma}_E^* \equiv -\frac{V_{22}\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)}{V_2\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)} \xi\left(c_E^*, c_E^*\right)$ and $\tilde{\phi}_E^* \equiv \frac{V_{12}\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)}{V_1\left(1-l_E^*,\, \xi\left(c_E^*, c_E^*\right)\right)} \xi\left(c_E^*, c_E^*\right)$. As proved in Appendix B, $l_E^* = l_g^*$ is satisfied. Defining the steady state value as l^*, we can get:

Case 1: $\frac{l_E^*}{1-l_E{}^*} = \frac{l_g^*}{1-l_g{}^*} = \frac{1}{\theta} \frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}$. Moreover, we can get $\gamma_g^* = \tilde{\gamma}_E^* = \gamma$, $\phi_g^* = \tilde{\phi}_E^* = 1-\gamma$, $\varepsilon_g^* = \varepsilon_E^* = \theta(1-\gamma)\frac{l^*}{1-l^*}$ and $\eta_E^* = \eta_g^* = \left[\theta(1-\gamma)-1\right]\frac{l^*}{1-l^*}$.

Case 2: $l_E^* = l_g^* = \frac{1}{\theta} \frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}$. Moreover, we can get $\gamma_g^* = \tilde{\gamma}_E^* = \gamma$, $\phi_g^* = \tilde{\phi}_E^* = 1-\gamma$, $\varepsilon_g^* = \varepsilon_E^* = \theta(1-\gamma) l^*$ and $\eta_E^* = \eta_g^* = \theta(1-\gamma) l^*$.

Case 3: $\frac{l_E^*}{(1-l_E{}^*)^\chi} = \frac{l_g^*}{(1-l_g{}^*)^\chi} = \frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}$. Moreover, we can get $\gamma_g^* = \tilde{\gamma}_E^* = \gamma$, $\phi_g^* = \tilde{\phi}_E^* = 1-\gamma$, $\varepsilon_g^* = \varepsilon_E^* = (1-\gamma)\frac{l^*}{(1-l^*)^\chi}$ and $\eta_E^* = \eta_g^* = \left[(1-\gamma) - \chi(1-l^*)^{\chi-1}\right]\frac{l^*}{(1-l^*)^\chi}$.

Proof. See Appendix B. ■

We could show that all the conditions stated in Proposition 2 are completely satisfied only if we use the preferences above. From Lemma 1, the parameters ψ and τ_c^* do not have any distortionary effects on the steady state values of l^* and k^*, but have the quite the same impacts on the transitional dynamics to the steady state equilibrium. Consumption taxation and the external effect are qualitatively quite the same in the one-sector Ramsey model, and we can say:

Proposition 3 *The extent to which a growth path in the market equilibrium deviates from a socially optimal one in the economy without any distortions makes no difference between the two cases of economic distortions, only if we specify the very familiar non-separable preferences in Cases 1-3.*

Proof. See Propositions 1-2 and Lemma 1. ■

Based on the familiar preferences above, these economic distortions have quite the same impacts on the local dynamics, when we set the size of consumption externality equal to a consumption tax rate, i.e., $\psi = \tau_c^*$. Let us express Assumption 1 in terms of Cases 1-3. The direct calculations lead to :

Remark 2 *Case 1:* $\theta\gamma + \gamma - \theta > 0$. *Case 2:* $(\gamma-1)\theta l^* > 0 \ (\Leftrightarrow \gamma > 1)$. *Case3:* $(\gamma-1)(1-l^*)^{-\chi} + \chi\frac{l^*}{1-l^*}\gamma > 0$.

Under the condition of $\psi = \tau_c^* (\equiv x)$, we characterize the local stability of (3-8) and (4-8). The detailed definition of x_1^{Cj} appearing in Lemma 2 is handed over to Appendix C. Noting that the parameter γ is the elasticity of intertemporal substitution in private consumption, the analytical results are:

[10] Following Carroll and Kimball (1996), if the magnitude of $\frac{\Gamma'''\Gamma'}{(\Gamma'')^2}$ is equal to one for any values of l, $\Gamma(1-l)$ is the case of constant absolute risk aversion. In contrast, if its magnitude is higher than one for any values of l, $\Gamma(1-l)$ is the constant relative risk averse.

Lemma 1 *If $\gamma \geq 1$ in all the Cases, there exists only a market equilibrium path approaching to the steady state (i.e., the steady state is a saddle). In contrast, suppose that $\gamma < 1$ is satisfied in Case 1 and 3. The steady state is a saddle for $x < x_1^{C_j}$, while it is a source for $x_1^{Cj} < x$ (C_j is the short for Case j, $j = 1$ and 3).*

Proof. See Appendix C. ■

Suppose that $\gamma \to 1$ in Case 3, when $\psi = 0$ and $g \neq 0$. Then, the preference collapses to the one used in Giannitsarou (2007). She showed that endogenous consumption taxes cannot be a source of indeterminacy unlike endogenous income taxes. If we consider the case of consumption externalities with $\gamma \to 1$ (i.e., $\psi \neq 0$ and $g = 0$), it is easily justified that the parameter ψ has no impact on the local dynamics near the steady state and the steady state is a saddle for any values of ψ. As shown above, these two economic distortions make no difference, as long as we use the preference. From this equivalence, therefore, the well-known result in Giannitsarou are very plausible.

We investigate how the economic distortions affect the speed of convergence to the steady state by using the preferences, when the steady state is a saddle, *i.e.*, $\gamma > 1$ or $x < x_1^{Cj}$ for $\gamma < 1$. Defining the stable root in Case j as $\hat{\lambda}_j$, the convergent speed becomes $-\hat{\lambda}_j$. Irrespective of the preferences, the relations between x and $-\hat{\lambda}_j$ can be summarized as follows, depending on the value of γ :

Proposition 4 *We can verify $\frac{d(-\hat{\lambda}_j)}{dx} \lesseqgtr 0$ for $\gamma \gtreqless 1$.*[11] *The economic distortions have negative (positive) impacts on the convergent speed, if $\gamma > 1$ $(\gamma < 1)$, while the distortions have no impact on the convergent speed, if $\gamma = 1$.*
Proof. *See Appendix D.* ■

Let us consider the intuition behind Proposition 4. As endogenous consumption taxes are theoretically identical to consumption externalities near the steady state, we pick up the case of consumption taxes in Case 1[12]. Suppose that τ_c^* increases due to the rise in g. If g increases, the pressure of an increase in the interest rate is stronger by prevent capital stock from accumulating. When the elasticity of intertemporal substitution in consumption $-\frac{U_2}{U_{22}c^*}$ is low, i.e., $\gamma^{-1} < 1$, consumption does not decrease greatly. Noting that the elasticity of intertemporal substitution in leisure $-\frac{U_2}{U_{21}(1-l^*)}$ is $[\theta(\gamma-1)]^{-1}$, labor supply must decrease to satisfy the intratemporal condition of (2-4).[13] Then, the equilibrium labor decreases, because the labor supply and demand curves cross with standard slopes. This reduces the productivity of capital and the convergent speed to the steady state capital stock. In contrast, when $\gamma^{-1} > 1$, labor supply increases to satisfy (2-4), because consumption decreases greatly. Thus, the speed of convergence is higher, if τ_c^* is higher.

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[11] To be precise, we must rule out $\gamma < 1$ in Case 2, as described in Remark 2.

[12] Needless to say, the same ariguments are equally ture of Cases 2 and 3.

[13] As for Case 3, we can get $-\frac{U_2}{U_{21}(1-l)} = \frac{(1-l^*)^{\chi-1}}{\gamma-1}$, which equals $-l^*\varepsilon_g^{-1}$. This relation can be also confirmed in Case 2.

Finally, we provide the more comprehensive characterization about the dynamic effects. For this purpose, we definitely derive the saddle path to the steady state equilibrium in Case 1 by using (4-8):

$$-a^{*}\left[(\delta+\rho)\frac{1-s^{*}}{s^{*}}+\left(\frac{l^{*}}{1-l^{*}}+\frac{s^{*}}{\sigma^{*}}\right)\frac{(1-s^{*})\,\delta+\rho}{s^{*}}\right](l-l^{*})$$
$$=\left[-\hat{\lambda}_{1}+\rho-\frac{(1-s^{*})\,\delta+\rho}{\sigma^{*}}\right](k-k^{*})\,, \tag{6-3}$$

where the part $\frac{l^*}{1-l^*}+\frac{s^*}{\sigma^*}$ and $\hat{\lambda}_1$ in (6-3) is respectively replaced by $\frac{s^*}{\sigma^*}$ and $\hat{\lambda}_2$ $\left(\chi\frac{l^*}{1-l^*}+\frac{s^*}{\sigma^*}\text{ and }\hat{\lambda}_3\right)$ in Case 2 (3). If $\gamma>1$ $(\gamma<1)$, eq.(6-3) implies that an increase in x reduces (raises) the level of labor l for a given value of capital stock k, as stated above. See Figure 1. We assume the case of $-\hat{\lambda}_i+\rho-\frac{(1-s^*)\delta+\rho}{\sigma^*}>0$ in drawing this figure.[14] Even if we consider the positively sloped saddle path unlike Fig.1, the essence of arguments does not change at all.

7 Separable utility function

Noting the definition of (6-1), we consider the very familiar separable utility function unlike in Section 6:

$$U(1-l,c,\bar{c})=\frac{(1-l)^{1-\chi}}{1-\chi}+\frac{[\xi(c,\bar{c})]^{1-\gamma}}{1-\gamma}, \tag{7-1}$$

which satisfies *Assumption* 1 and *Remark* 1. As for the utility function of leisure $\Gamma(1-l)$, we focus on Case 1 at $\theta=1$ in Section 6[15], because the main implications do not change even if the alternative preferences of labor are chosen. This utility function of (7-1) is also used in much macroeconomic dynamics literature.This section investigates how the results in Section 6 are modified.

According to the results compiled by Appendix E, there exists only a steady state value of l_E^* in consumption externalities for any values of γ, while in endogenous consumption taxes, the uniqueness of steady state l_g^* prevails for $\gamma\geq 1$, but there exist at most two steady states l_g^* for $\gamma<1$. This result also means that when $\gamma<1$, the Laffer curve effect exists between tax rate and revenue, while the Laffer curve effect is absent for $\gamma\geq 1$. The intuition behind the result is as follows. If $\gamma<1$, an agent does not have a strong incentive to smooth his intertemporal levels of consumption when τ_c^* increases. As c^* greatly decrease with τ_c^*, the high consumption and low tax rate can coexist with the low consumption and high tax rate. If $\gamma>1$, the intuition can be easily imagined from the case of $\gamma<1$.

From $\phi_i^*=0$ and $\varepsilon_i^*=0$ in (7-1) $(i=g$ and $E)$, eqs.(3-9) and (3-10) or (4-9) and

[14] If the substitutability σ^* is relatively high (low), this assumption is (is not) satisfied and thus the saddle path is negatively (positively) sloped. If σ^* is large, labor is substituted by capital as capital stock grows during the transition. If σ^* is small, labor must grow with capital stock during the transition.

[15] Needless to say, the results below do not change even if $\theta\neq 1$.

(4-10) are rewritten as

$$D_i = \frac{-\frac{(1-s^*)(\delta+\rho)}{s^*}\frac{s^*}{\sigma^*}}{\frac{s^*}{\sigma^*}+\gamma\frac{l_i^*}{1-l_i{}^*}}\frac{(1-s^*)\,\delta+\rho}{s^*}\left[1+\frac{\gamma\frac{l_i^*}{1-l_i{}^*}}{(1+x)\,\gamma-x}\right], \tag{7-2}$$

$$T_i = \rho, \tag{7-3}$$

where $x=\psi\left(\tau_c^*\right)$ for $i=E\left(g\right)$. As shown in eqs.(7-2) and (7-3), the expressions seem to be the same between the two cases. Unfortuantaly, the steady state values of labor are not identical unlike the previous section, *i.e.*, $l_E^* \neq l_g^*$ even for a same size of the two economic distortions, *i.e.*, $\psi=\tau_c^*$.

Noting that the assumption of $U_{22}+U_{23}<0$ in Footnote 2 corresponds to $(1+\psi)\,\gamma-\psi>0$ for $i=E$, the steady state is always a saddle in consumption externalities, because we can verify $D_E<0$. For $i=g$, $(1+\tau_c^*)\,\gamma-\tau_c^*>0$ is satisfied, when $\gamma\geq 1$. Then, eq.(7-2) means that the steady state is necessarily a saddle. When $\gamma<1$, in contrast, Appendix E proves that we can always get $(1+\tau_c^*)\,\gamma-\tau_c^*>0$ in the low steady state, while the sign of $(1+\tau_c^*)\,\gamma-\tau_c^*$ depends on the value of government expenditure g in the high steady state. In other words, $(1+\tau_c^*)\,\gamma-\tau_c^*>0$ is (is not) satisfied in the high steady state, if g is relatively small (large). Defining the value satisfying $(1+\tau_c^*)\,\gamma-\tau_c^*=0$ as $\bar{g}$, the high steady state is a saddle only for $0<g<\bar{g}$.

When $\gamma>1$, Appendix E indicates that the unique value of l_i^* can be necessarily obtained, but $l_E^*\neq l_g^*$ is satisfied even if $\psi=\tau_c^*$. Needless to say, the disparity between l_E^* and l_g^* can be considered as low, if the size of $\psi=\tau_c^*$ is sufficiently small. Then, the distortions ψ and τ_c^* have very similar effects also on the transitional dynamics to the steady state equilibrium. By restoring to numerical simulations, let us prove that the distortionary effects of the two economic distortions are quantitatively very similar, even if the economic distortions increase from zero to a relatively large size.

Before the anaysis, let us state the particular case of $\gamma=1$. As clarified in Appendix E, then, $l_E^*=l_g^*$ is satisfied and the variables ψ and τ_c^* have no impact on the transitional dynamics as well as the steady state. The reason behind the result is as follows. Eq.(2-4) shows that the allocation between leisure and consumption is chosen such that the marginal utility of consumption a dollar $U_2/\left(1+\tau_c^*\right)$ equals the marginal utility of leisure a dollar $U_1/\omega\left(a^*\right)$. When g increases, the price of consumption good $\left(1+\tau_c^*\right)$ increases and the marginal utility U_2 increases due to the decline in consumption. Noting that the level of $\omega\left(a^*\right)$ is independent of g, the level of l_g^* is unaffected, as the positive and negative effects on $U_2/\left(1+\tau_c^*\right)$ exactly cancel out at $\gamma=1$. As for consumption externalities in the case of $\gamma=1$, the reason can be easily understood, because (7-1) reduces to the separable preference between private and social consumption. Now, let us conduct the numerical simulations for $\gamma\neq 1$.

The value added tax rates in European countries are relatively high. As for the members of G10 in European countries, the average of consumption tax rates is 21%. Let us set the size of two economic distortions equal to 0.21.[16] The parameter values in Table

[16] Unlike the value of τ_c^*, we cannot precisely understand the empirical sizes of ψ, but the least upper limit of ψ will not be above the high value of 0.21.

Parameter	Value	Description
σ	1	Elasticity of capital-labor substitution
s^*	0.3	Share of capital in total income
A	0.165	Total factor productivity
ρ	0.065	The rate of time preference
δ	0.10	Capital depreciation rate
$1/\gamma$	2/3	Intertemporal elasticity of substitution
χ	6	The inverse of labor supply elasticity
τ_c^* or ψ	0.21 or 0	Economic Distortions

Table 1: Parameter Values

1 are relatively standard and almost the same as in existing literature. In endogenous consumption taxes, government spending is set at $g = 0.0297(0)$ to obtain $\tau_c^* = 0.21(0)$.

The estimates of intertemporal elasticity of substitution $1/\gamma$ are more variable throughout existing literature, but our choice 2/3 is well in the range of the empirical estimates, which with few exceptions lies in the range $(0, 1)$. Let us show that the dynamic effects of τ_c^* and ψ are relatively similar, even if the value of γ deviates from $\gamma = 1$ by the degree of 50%. As for the parameter χ, it is the inverse of Frisch labor supply elasticity and we choose the value of $\chi = 6$ that fall well within the estimates of Pencavel (1986), in which labor supply elasticity is relatively small. From the empirical time allocation studies, households tend to allocate about one-third of their time to market activity. See Cooley (1995). By choosing the value of $\chi = 6$, the steady state value of l_E^* and l_g^* are respectively 0.309 and 0.300, which are also compatible with the empirical time allocation studies. As for production technology, we use the unit-elasticity of capital-labor substitution,i.e., $\sigma^* = 1 : y(k, l) = Ak^{s^*} l^{1-s^*}$.[17]

Figures 2 summarize how the competitive equilibrium paths are quantitatively altered from the socially optimal ones in the economy without any distortions, when the sizes of both ψ and τ_c^* rise from 0 to 0.21. From these figures, we can easily see that the extents to which the competitive equilibrium paths of capital (labor) divert from the socially optimal ones of capital and labor are relatively small between consumption externalities and endogenous consumption taxes. Even if we choose the lower (higher) vallue of $\gamma = 0.5$ (2.0) than $\gamma = 1.5$, the conclusion is unchanged. See Figures 3 and 4.

Compared with the non-separable utilities (6-1), the implication in the preference (7-1) is somewhat weak, but it is partly the same. Even if the quantatively large differences exist in the dynamic effects, we cannot deny the close similarity between consumption taxes and externalities, as clarified in Section 9.

[17] The consideration of general technology is much less important than the consideration of various types of preferences. The main implication remains unchanged, even if we consider the CES technology.

8 No income effect on the demand for leisure

Noting the definition of $\xi(c, \bar{c})$ in (6-1), this section considers the different non-separable utility function[18] from in Section 6 in that there is no income effect on the demand for leisure:

$$U(1-l, c, \bar{c}) = \frac{\left[\frac{(1-l)^{1-\chi}}{1-\chi} + \xi(c, \bar{c})\right]^{1-\zeta}}{1-\zeta}, \tag{8-1}$$

that was initiated in Greenwood, Hercovitz and Huffman (1988), in which $\psi = 0$. Eq.(8-1) corresponds to the case of endogenous consumption taxes (consumption externalities) when $\psi = 0$ ($\psi \neq 0$). To see no income effect, let us express the left-hand side in (2-4) by using (8-1):

$$\frac{U_1(1-l, c, \bar{c})}{U_2(1-l, c, \bar{c})} = (1-l)^{-\chi}\,\bar{c}^{-\psi},$$

which evidently implies the absence of income effect on the demand for leisure, because the level of leisure is independent of private consumption c. Regarding the utility of leisure $\Gamma(1-l)$, we focus on Case 1 at $\theta = 1$, because of the same reason in Section 7.

As in the other preferences, the steady state value of capital-labor ratio a^* is identical between the two economic distortions. Irrespective of the types of economic distortions, Appendix F proves that there exist at most two steady states. Unfortunately, $(l_E^*, k_E^*) = (l_g^*, k_g^*)$ is not satisfied for $\psi = \tau_c^*$ also in this preference, even if we compare the high (low) steady state in endogenous taxes with the high (low) one in consumption externalities. See Appendix F. Noting that $\tilde{\gamma}_E^*$ and $\tilde{\phi}_E^*$ are defined in Section 6 and $l_E^* \neq l_g^*$ is satisfied, we can obtain:

Lemma 2 *For* $i \equiv E,\ g$, $\eta_i = \frac{-\zeta\omega(a^*)l_i^*}{\frac{1-l_i^*}{1-\chi}\omega(a^*) + \frac{(1-s)\delta+\rho}{s}a^*l_i^*} - \chi\frac{l_i^*}{1-l_i^*}$, $\varepsilon_i = \frac{-\zeta\omega(a^*)l_i^*}{\frac{1-l_i^*}{1-\chi}\omega(a^*) + \frac{(1-s)\delta+\rho}{s}a^*l_i^*}$, $\gamma_g^* = \tilde{\gamma}_E^* = \frac{\zeta\cdot\frac{(1-s)\delta+\rho}{s}a^*l_i^*}{\frac{1-l_i^*}{1-\chi}\omega(a^*) + \frac{(1-s)\delta+\rho}{s}a^*l_i^*}\ (\equiv \hat{\gamma}_i^*)$, *and* $\phi_g^* = \tilde{\phi}_E^* = \frac{-\zeta\cdot\frac{(1-s)\delta+\rho}{s}a^*l_i^*}{\frac{1-l_i^*}{1-\chi}\omega(a^*) + \frac{(1-s)\delta+\rho}{s}a^*l_i^*}\left(\equiv \hat{\phi}_i^*\right)$.

Proof. *See Appendix F.* ■

Noting $x = \tau_c^*$ (ψ) for $i = g$ (E) and Lemma 2, we can get:

$$D_i = \frac{-\frac{(1+x)\hat{\phi}_i^* + (1+x)\hat{\gamma}_i^* - x}{(1+x)\hat{\gamma}_i^* - x}\frac{(1-s^*)(\delta+\rho)}{s^*}\frac{s^*}{\sigma^*}}{\frac{s^*}{\sigma^*} - \eta_i^* - \frac{\hat{\phi}_i^*}{(1+x)\hat{\gamma}_i^* - x}\varepsilon_i^*}\left[1 + \frac{\varepsilon_i^* - \eta_i^*}{(1+x)\tilde{\phi}_i^* + (1+x)\tilde{\gamma}_i^* - x}\right]\frac{(1-s^*)\delta + \rho}{s^*}, \tag{8-2}$$

$$T_i = \frac{-1}{\frac{s^*}{\sigma^*} - \eta_i^* - \frac{\hat{\phi}_i^*}{(1+x)\hat{\gamma}_i^* - x}\varepsilon_i^*}\left[\frac{(1+x)\tilde{\phi}_i^*}{(1+x)\tilde{\gamma}_i^* - x}\frac{(1-s^*)(\delta+\rho)}{s^*}\frac{s^*}{\sigma^*} - \frac{\varepsilon_i^*}{(1+x)\hat{\gamma}_i^* - x}\frac{(1-s^*)\delta + \rho}{s^*}\frac{s^*}{\sigma^*}\right] + \rho. \tag{8-3}$$

[18] This preference is often used in sunspot-driven RBC literature: Jaimovich (2008), Meng and Yip (2008), Guo and Harrison (2010), Nourry, Seegmuller and Venditti (2013) and Dufourt, Nishimura and Venditti (2015).

Parameter	Value	Description
σ	1	Elasticity of capital-labor substitution
s^*	0.3	Share of capital in total income
A	0.144	Total factor productivity
ρ	0.065	The rate of time preference
δ	0.1	Capital depreciation rate
ξ	8.0	Degree of relative risk averse
χ	0.85	The inverse of labor supply elasticity
τ_c^* or ψ	0.21 or 0	Economic Distortions

Table 2: Parameter Values

Eqs.(8-2) and (8-3) means that the larger (smaller) eigenvalues in (3-8) and (4-8) are different, because $l_E^* \neq l_g^*$ is satisfied for $\psi = \tau_c^*$. By restoring to numerical simulations, however, we can easily obtain the same implication as in Section 7, where the dynamic effects of two economic distortions are similar because the divergence of labor between the two cases is relatively small. Table 2 shows the values of parameters that are used in numerical simulations.

A few parameters in Table 2 take the different values from in Table 1. The reasons can be stated as follows. Noting $\omega(a^*) = \frac{(1-s^*)(\delta+\rho)}{s^*} a^*$, eq.(F-3) means that $\omega(a^*) > 1$ must be satisfied to guarantee the existence of the value of $l_g^* \in (0,1)$. Noting that the values of s^*, ρ and δ remain the same, we replace the value of TFP in Table 1 by $A = 0.1091$ to obtain $\omega(a^*) > 1$. Because $\gamma_i > 0$ and $\eta_i < 0$ are less likely for $\chi \geq 1$ as implied in Lemma 2, it is more convenient to set $\chi < 1$. As a result, labor supply elasticity is much higher than in Section 7. More importantly, we assume that an agent is relatively risk-averse for the composite good of leisure and consumption. Phrased differently, an agent hates the situation that the compositie good is more volatile. This assumption is required for the steady state not to be locally indeterminate.[19] Under this set of parameter values, there is a unique equilibrium path to a steady state. As for the time allocation, we get $l^* = 0.3936$, when $\tau_c^* = \psi = 0$. Noting that we choose the higher value of labor supply elasticity, the time allocation is greatly decreased to $l_g^* = 0.241$ ($l_E^* = 0.282$) at $\tau_c^* = 0.21$ $(\psi = 0.21)$.

The quantitative results are shown in Figures 5. The two distortions of τ_c^* and ψ have almost the same distortionary effects on the transitional dynamics. Even if we decrease the degree of relative risk-averse from $\xi = 8.5$ to $\xi = 6$, the quantitative consequences are unchanged. See Figures 6. In contrast, if $\chi = 0.85$ is decreased from to $\chi = 0.65$, we get moderte differences in the dynamic effects. See Figures 7. As stated in Section 7, however, Figures 7 cannot depress the importance of the link between the two distortions, as emphasized in Section 9.

[19] Using the preference (8-1) with $\psi = 0$, Nourry, Seegmuller and Venditti (2013) proved that endogenous consumption can easily lead to the occurrence of indeterminacy.

9 General case

To peculiarize the theoretical similarity of the two distortions clearly, the previous sections separately pick up consumption taxes and consumption externalities in the one-sector Ramsey model. However, this section explores how the results in Sections 6-8 are modefied if there are simultaneously the two kinds of economic distortions in the model. Unlike the above, we consider the case of $U_3 \neq 0$, $U_{3j} \neq 0$ $(j = 1, 2$ and $3)$ and $\tau_c^* \neq 0$ $(\Leftarrow g \neq 0)$. Note $U(1-l, c, \bar{c}) \equiv V(1-l, \xi(c, \bar{c}))$, in which $\xi(c, \bar{c})$ is the homogeneous function of degree $1+\psi$ for c and $\bar{c}$ as specified in (6-1). Appendix G analyzes the existence of steady states (k^*, l^*) in the mixed cases of the two distortions. Using the Jacobian matrix associated with the linearized dynamics obtained here, we can get:

Lemma 3 *If $\tau_c^* \times \psi \to 0$, the determinant D and trace T can be expressed as:*

$$D = \frac{-\frac{(1+\tau_c^*+\psi)(\tilde{\phi}^*+\tilde{\gamma}^*)-(\tau_c^*+\psi)}{(1+\tau_c^*+\psi)\tilde{\gamma}^*-(\tau_c^*+\psi)}\frac{(1-s^*)(\delta+\rho)}{s^*}\frac{s^*}{\sigma^*}}{\frac{s^*}{\sigma^*}-\eta^*-\frac{(1+\tau_c^*+\psi)\tilde{\phi}^*}{(1+\tau_c^*+\psi)\tilde{\gamma}^*-(\tau_c^*+\psi)}\varepsilon^*}\left[1+\frac{\varepsilon^*-\eta^*}{(1+\tau_c^*+\psi)(\tilde{\phi}^*+\tilde{\gamma}^*)-(\tau_c^*+\psi)}\right]$$

$$\times\frac{(1-s^*)\,\delta+\rho}{s^*}, \tag{9-1}$$

$$T = \frac{-1}{\frac{s^*}{\sigma^*}-\eta^*-\frac{(1+\tau_c^*+\psi)\tilde{\phi}^*}{(1+\tau_c^*+\psi)\tilde{\gamma}^*-(\tau_c^*+\psi)}\varepsilon^*}\left[\frac{(1+\tau_c^*+\psi)\tilde{\phi}^*}{(1+\tau_c^*+\psi)\tilde{\gamma}^*-(\tau_c^*+\psi)}\frac{s^*}{\sigma^*}\frac{(1-s^*)(\delta+\rho)}{s^*}\right.$$

$$\left.-\frac{s^*}{\sigma^*}\frac{\varepsilon^*}{(1+\tau_c^*+\psi)\tilde{\gamma}^*-(\tau_c^*+\psi)}\frac{(1-s^*)\,\delta+\rho}{s^*}\right]+\rho, \tag{9-2}$$

where $\tilde{\gamma}^* \equiv -\frac{V_{22}(1-l^*,\ \xi(c^*,c^*))}{V_2(1-l^*,\ \xi(c^*,c^*))}\xi(c^*, c^*)$, $\tilde{\phi}^* \equiv \frac{V_{12}(1-l^*,\ \xi(c^*,c^*))}{V_1(1-l^*,\ \xi(c^*,c^*))}\xi(c^*, c^*)$, $\varepsilon^* \equiv \frac{V_{21}(1-l^*,\ \xi(c^*,c^*))}{V_2(1-l^*,\ \xi(c^*,c^*))}l^*$ *and* $\eta^* \equiv \frac{V_{11}(1-l^*,\ \xi(c^*,c^*))}{V_1(1-l^*,\ \xi(c^*,c^*))}l^*$, *where* $c^* = c(l^*, k^*)$.

Proof. See Apendix G. ■

As described in Section 7, there are many countries where the consumption tax rates τ_c^* are relatively high. However, the external effects from others' consumption ψ might be significantly small. Thus, it might be plausible to assume $\tau_c^* \cdot \psi \to 0$ in Lemma 3. Unlike Sections 6-8, eqs.(9-1) and (9-2) imply that we need not to pay an attention to the difference of steady state values of labor between the two cases of distortions.

In Sections 6-8, we defined $x = \tau_c^*$ or ψ and $l_i^* \equiv l_g^*$ or l_E^*. Note that l^* (k^*) is defined as the steady state value of labor (capital) in the mixd case of $\tau_c^* \neq 0$ and $\psi \neq 0$. Only if Section 9 replaces these definitions in Section 6 by $x \equiv \tau_c^* + \psi$, $l_i^* \equiv l^*$ and $k_i^* \equiv k^*$, the analytical properties in section 6 are equally true of this section. Therefore, we can regard that consumption externalities are perfectly the invisible tax rates (subsidy rates) on consumption if $\psi > 0$ $(\psi < 0)$. As long as the preference a là King, Plosser and Rebelo (1988) is used, the visible tax rate τ_c^* has more (less) quantitative impacts on the real economy, *i.e.*,the transitional dynamics to the setady state if $\psi > 0$ $(\psi < 0)$. It is very important to estimate the degree of external effects from consumption, when we want to examine the more appropriate tax rate on consumption.

Defining $x \equiv \tau_c^* + \psi$, $l_i^* \equiv l^*$ and $k_i^* \equiv k^*$ in Sections 7-8 as in the above, we can obtain the same Determinant and Trace as (7-2), (7-3), (8-2) and (8-3). See Appendix G. Unlike Sections 7-8, we need not to pay an attension to the difference between the values of l_g^* and l_E^*. Considering (G-3),(G-4) and (G-5), a change in τ_c^* has the quantitatively different impact on the value of l^* from a change in ψ. Therefore, we cannot regard that consumption externalities are quite the same distortions as consumption taxes. However, eqs.(9-1) and (9-2) imply that consumption externalities seem to be the indivisible tax or subsidy rates on consumption, depending on the sign of ψ. We have already shown that τ_c^* and ψ have similar impacts in quantity on the economic dynamics, when we increase either of the two distortions from zero to a certain large size. Therefore, it is very reasonable to expect that the dynamic effects of τ_c^* and ψ are very similar if we mildly increase $x\,(\equiv \tau_c^* + \psi)$ from a relatively large size of $x\,(< 0.21)$.[20]

10 Conclusion

This paper allows for only consumption externalities and endogenous consumption taxes regarded as distinctly different distortions in the one-sector Ramsey growth model without any other distortions. Based on the very familiar preferences, we compare how the two economic distortions influence the transitional dynamics to a steady state.

When we use the very familiar non-separable utility function as specified by King, Plosser and Rebelo (1988), consumption externalities and consumption taxes work in the same way as the obstacle distorting a market equilibrium path, because these dynamic impacts are quantitatively the same.

Even if we use the separable utility function used in an infinite number of macro-dynamics literature and the non-separable utility lack of an income effect on leisure à la Greenwood, Hercovitz and Huffman (1988), the similar implication can be obtained, because consumption externalities seems to be theoretically tax rates (subsidy rates) on consumption for positive (negative) external effects on consumption.

Irrespective of an agent's prferences, there exists a theoretically strong similarity between consumption externalites and consumption taxes. Therefore, if we would like to derive the more appropriate tax rates on consumption, it is very important to estimate the empirical values of external effects on consumption. Based on the empirical estimates of consumption externalities, we must consider the appropriate tax rate on consumption for the economy.

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[20] The analysis becomes almost the same as in Sections 7-8 if we greatly increase x from a relatively small size of x through a rise in either τ_c^* or ψ.

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Appendix A

Let us differentiate private consumption $c = c(l, k)$ in endogenous consumption taxes and evaluate it at the steady state value. From (4-1), the following can be obtained:

$$c = c_g^* + \frac{s^*/\sigma^*}{\phi_g^* + \gamma_g^* - \frac{\tau_c^*}{1+\tau_c^*}} \left(\frac{c_g^*}{k_g^*}\right)(k - k_g^*) - \frac{\varepsilon_g^* - \eta_g^* + s^*/\sigma^*}{\phi_g^* + \gamma_g^* - \frac{\tau_c^*}{1+\tau_c^*}} \left(\frac{c_g^*}{l_g^*}\right)(l - l_g^*)$$

Comparing (3-7) with (4-7), we can see $c_E^* = c_g^* + g$. If we substitute $\frac{c_g^*}{k_g^*} = \frac{1}{1+\tau_c^*} \frac{(1-s^*)\delta+\rho}{s^*}$ and $c_E^* = c_g^* + g$ in the above, the local dynamics of the sum of private and public consumption can be derived as:

$$c+g = c_E^* + \Pi \left[\frac{s^*/\sigma^*}{(1+\tau_c^*)\gamma_g^* - \tau_c^* + (1+\tau_c^*)\phi_g^*}(k - k_g^*) - \frac{\varepsilon_g^* - \eta_g^* + s^*/\sigma^*}{(1+\tau_c^*)\gamma_g^* - \tau_c^* + (1+\tau_c^*)\phi_g^*} a^*(l - l_g^*)\right] \tag{A-1}$$

where $\Pi \equiv \frac{(1-s^*)\delta+\rho}{s^*}$.

In consumption externalities, differentiating $c = c(l, k)$ and evaluating it at the steady state yields

$$c = c_E^* + \Pi \left[\frac{s^*/\sigma^*}{\gamma_E^* + \phi_E^*}(k - k_E^*) - \frac{\varepsilon_E^* - \eta_E^* + s^*/\sigma^*}{\gamma_E^* + \phi_E^*} a^*(l - l_E^*)\right] \tag{A-2}$$

If Proposition 1 is realized, the righthand side of (A-1) coincides with the righthand side of (A-2). Therefore, the local dynamics of the sum of private and public consumption in endogenous taxes are wholly equal to the dynamics of private consumption in consumption externalities. Then, the local behaviors of all the economic variables are quite the same in the two cases of consumption externalities and endogenous consumption taxes.

Appendix B

Let us prove that consumption externalities have exactly the same impacts on local dynamics as endogenous consumption taxes in each case categorized by the three types of non-separable preferences as specified in Cases 1-3.

Case 1: The utility function is $U(1-l, c, \bar{c}) = \frac{(1-l)^{\theta(1-\gamma)} \cdot (\xi(c,\bar{c}))^{1-\gamma}}{1-\gamma}$. Note that $\psi = 0$ is satisfied only in endogenous consumption taxes.

In the case of consumption externalities, let us derive the steady state value of (l_E^*, c_E^*). Noting that a^* is exclusively determined by (3-5), eqs.(3-6) and (3-7) can be rewritten as:

$$\theta c_E^* = \omega(a^*)(1 - l_E^*) \tag{3-6$'$}$$

$$c_E^* = \frac{(1-s^*)\delta + \rho}{s^*} a^* l_E^* \tag{3-7$'$}$$

Next, we consider the case of endogenous consumption taxes. Then, the consumption externalities are set to zero, i.e., $\psi = 0$. The steady state value of $\left(l_g^*, c_g^*\right)$ can be obtained by solving (4-6) and (4-7), which are rewritten as:

$$\theta\left(c_g^*+g\right)=\omega\left(a^*\right)\left(1-l_g^*\right), \tag{4-6$'$}$$

$$c_g^*+g=\frac{\left(1-s^*\right)\delta+\rho}{s^*}a^*l_E^*. \tag{4-7$'$}$$

If eqs.(3-6$'$) and (3-7$'$) are respectively compared with (4-6$'$) and (4-7$'$), we can easily see that $l_E^*=l_g^*$ and $c_E^*=c_g^*+g$ are satisfied. From $\omega\left(a^*\right)=\frac{(1-s^*)(\delta+\rho)}{s^*}a^*$, the identical value of labor l^* satisfies $\frac{l^*}{1-l^*}=\frac{1}{\theta}\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}$. As the capital-labor ratio a^* is the same between the two distortions, $k_E^*=k_g^*$ is also satisfied.

Based on $U\left(1-l,c,\bar{c}\right)=\frac{(1-l)^{\theta(1-\gamma)}\cdot(\xi(c,\bar{c}))^{1-\gamma}}{1-\gamma}$, the direct calculations lead to $\gamma_g^*=\tilde{\gamma}_E^*$, $\phi_g^*=\tilde{\phi}_E^*$, $\varepsilon_g^*=\varepsilon_E^*$ and $\eta_E^*=\eta_g^*$. Therefore, Proposition 1 is completely satisfied in this preference.

Case 2: $U\left(1-l,c,\bar{c}\right)=\frac{[\exp\theta(1-l)]^{1-\gamma}\cdot(\xi(c,\bar{c}))^{1-\gamma}}{1-\gamma}$.

In Case 2, eqs.(3-7$'$) and (4-7$'$) remain unchanged, but (3-6$'$) and (4-6$'$) are respectively replaced by:

$$\theta c_E^*=\omega\left(a^*\right), \tag{3-6$''$}$$

$$\theta\left(c_g^*+g\right)=\omega\left(a^*\right). \tag{4-6$''$}$$

Thus, we can find the same arguments as in Case 1 except that the identical value of labor satisfies $l^*=\frac{1}{\theta}\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}$.

Case 3: $U\left(1-l,c,\bar{c}\right)=\frac{\left[\exp\frac{(1-l)^{1-\chi}}{1-\chi}\right]^{1-\gamma}\cdot(\xi(c,\bar{c}))^{1-\gamma}}{1-\gamma}$.

Eqs.(3-7$'$) and (4-7$'$) are the same also in Case 3, but (3-6$'$) and (4-6$'$) must be respectively replaced by:

$$\theta c_E^*=\omega\left(a^*\right)\left(1-l_E^*\right)^{\chi}, \tag{3-6$'''$}$$

$$\theta\left(c_g^*+g\right)=\omega\left(a^*\right)\left(1-l_g^*\right)^{\chi}. \tag{4-6$'''$}$$

In Case 3, the identical value of labor l^* is obtained by solving $\frac{l^*}{(1-l^*)^{\chi}}=\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}$, but the other arguments are the same as in Cases 1 and 2.

Appendix C

Let us prove Lemma 2.

Case 1: The preference is $\frac{(1-l)^{\theta(1-\gamma)}(\xi(c,\bar{c}))^{1-\gamma}}{1-\gamma}$.

Note that $\psi=0$ ($\psi\neq 0$) corresponds to the case of endogenous consumption taxes (consumption externalities). Firstly, let us investigate the sign of determinant expressed as (3-9) and (4-9). Noting $x\equiv\tau_c^*$ or ψ as defined in Lemma 2, the denominator in (3-9) or (4-9) is

$$\frac{1}{\theta\left[\gamma-x\left(1-\gamma\right)\right]}\left[\left(\gamma-1\right)\left\{\frac{\left(1-s^{*}\right)\left(\delta+\rho\right)}{\left(1-s^{*}\right)\delta+\rho}+\frac{s^{*}}{\sigma^{*}}\theta\right\}x+\frac{\left(1-s^{*}\right)\left(\delta+\rho\right)}{\left(1-s^{*}\right)\delta+\rho}\left(\theta\gamma+\gamma-\theta\right)+\frac{s^{*}}{\sigma^{*}}\theta\gamma\right]. \tag{C-1}$$

From *Remark* 2 revealing the concave utility in Case 1, we can easily see that the sign of (C-1) is positive if $\gamma \geq 1$. Next, suppose that $\gamma < 1$ is satisfied. As Footnote 3 means $\gamma_E^* = \gamma - \psi\left(1-\gamma\right) > 0 \left(\Leftrightarrow \psi < \frac{\gamma}{1-\gamma}\right)$, the denominator in (C-1) is positive.[21] Moreover, the sign of (C-1) is positive also in the case of $\gamma < 1$ if $x < \frac{\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}(\theta\gamma+\gamma-\theta)+\frac{s\theta\gamma}{\sigma^*}}{\left[\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}+\frac{s\theta}{\sigma^*}\right](1-\gamma)} \left(\equiv x_1^{C1}\right)$, where $x_1^{C1} < \frac{\gamma}{1-\gamma}$ can be analytically verified. If $\frac{\gamma}{1-\gamma} > x > x_1^{C1}$, the sign of (C-1) is negative. In short, if $\gamma \geq 1$ or $x\left(\equiv \psi \text{ or } \tau_c^*\right) < x_1^{C1}$ in the case of $\gamma < 1$, the labor supply and demand curves cross with a standard slope. If $\frac{\gamma}{1-\gamma} > x > x_1^{C1}$ in the case of $\gamma < 1$, the labor supply and demand curves cross with a wrong slope.

The numerator in (3-9) or (4-9) is

$$-\frac{1}{\gamma-x\left(1-\gamma\right)}\frac{\left(1-s^{*}\right)\left(\delta+\rho\right)}{\sigma^{*}}\frac{\left(1-s^{*}\right)\delta+\rho}{s^{*}}\frac{1}{1-l^{*}}<0 \tag{C-2}$$

Considering the above, if $\gamma \geq 1$ or $x < x_1^{C1}$ in the case of $\gamma < 1$, the steady state is a saddle and then, the equilibrium path converging to the steady state is uniquely determined.

As for the sign of Trace shown in (3-10) and (4-10), the denominator in (3-10) or (4-10) is the same as in Determinant, while the numerator is expressed as:

$$\frac{\rho}{\theta\left[\gamma-x\left(1-\gamma\right)\right]}\left[\left(\gamma-1\right)\left\{\frac{\left(1-s^{*}\right)\left(\delta+\rho\right)}{\left(1-s^{*}\right)\delta+\rho}+\frac{s^{*}}{\sigma^{*}}\theta+\frac{\left(1-s^{*}\right)\left(\delta+\rho\right)}{\sigma^{*}}\frac{\theta}{\rho}\right\}x \right. \tag{C-3}$$

$$\left. +\frac{\left(1-s^{*}\right)\left(\delta+\rho\right)}{\left(1-s^{*}\right)\delta+\rho}\left(\theta\gamma+\gamma-\theta\right)+\frac{s^{*}}{\sigma^{*}}\theta\gamma\right]$$

From Case 1 in *Remark* 2, the sign of (C-3) is positive, if $\gamma > 1$. In contrast, suppose that $\gamma < 1$ is satisfied. Taking account of Footnote 3, the sign of (C-3) is positive, if $x < \frac{\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}(\theta\gamma+\gamma-\theta)+\frac{s\theta\gamma}{\sigma^*}}{\left[\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}+\frac{s\theta}{\sigma^*}+\frac{(1-s^*)(\delta+\rho)}{\sigma^*}\frac{\theta}{\rho}\right](1-\gamma)} \left(\equiv x_2^{C1}\right)$, where $0 < x_2^{C1} < x_1^{C1}$ can be analytically proved. If $\frac{\gamma}{1-\gamma} > x > x_2^{C1}$, the sign of (C-3) is negative. Noting $x_2^{C1} < x_1^{C1}$, the steady state is a source for $\frac{\gamma}{1-\gamma} > x > x_1^{C1}$, if $\gamma < 1$.

Case 2: The preference is $U\left(1-l, c, \bar{c}\right) = \frac{\left[\exp\theta(1-l)\right]^{1-\gamma}\cdot\left(\xi(c,\bar{c})\right)^{1-\gamma}}{1-\gamma}$.

When we consider Case 2, eq.(C-1) (i.e., the denominator of determinant) can be

[21] As for endgenous consumption taxes, we restrict attention to the range of $\tau_c^* < \frac{\gamma}{1-\gamma}$. However, the steady state is a source for $\tau_c^* > \frac{\gamma}{1-\gamma}$.

rewritten as:

$$\frac{1}{\theta\left[\gamma-x\left(1-\gamma\right)\right]}\left[\left(\gamma-1\right)\frac{s^*}{\sigma^*}\theta\cdot x-\left(1-\gamma\right)\theta l^*+\frac{s^*}{\sigma^*}\theta\gamma\right]. \tag{C-1$'$}$$

From Case 2 in *Remark* 2, the concavity of utility function rules out the case of $\gamma<1$. Thus, the sign of (C-1$'$) is necessarily positive. Moreover, eq. (C-2), (i.e., the numerator of determinant) is replaced by

$$-\frac{1}{\gamma-x\left(1-\gamma\right)}\frac{\left(1-s^*\right)\left(\delta+\rho\right)}{\sigma^*}\frac{\left(1-s^*\right)\delta+\rho}{s^*}<0. \tag{C-2$'$}$$

In Case 2, therefore, the steady state is a saddle for any values of these two economic distortions.

Case 3: The preference is $U\left(1-l,c,\bar{c}\right)=\left[\exp\frac{(1-l)^{1-\chi}}{1-\chi}\right]^{1-\gamma}\left(\xi\left(c,\bar{c}\right)\right)^{1-\gamma}/\left(1-\gamma\right).$

Let us consider the sign of determinant shown in (3-9) and (4-9). Eq.(C-1) is rewritten as:

$$\frac{1}{\left[\gamma-x\left(1-\gamma\right)\right]}\left[\left(\gamma-1\right)\left\{\chi\frac{l^*}{1-l^*}+\frac{s^*}{\sigma^*}\right\}x-\left(1-\gamma\right)l^*\left(1-l^*\right)^{-\chi}+\chi\frac{l^*}{1-l^*}\gamma+\frac{s^*}{\sigma^*}\gamma\right]. \tag{C-1$''$}$$

From Case 3 in *Remark* 2, the sign of (C-1$''$) are positive if $\gamma\geq 1$. Suppose that $\gamma<1$ is satisfied. Noting Footnote 3, (C-1$''$) is positive for $x<\frac{-(1-\gamma)l^*(1-l^*)^{-\chi}+\chi\frac{l^*}{1-l^*}\gamma+\frac{s^*}{\sigma^*}\gamma}{(1-\gamma)\left\{\chi\frac{l^*}{1-l^*}+\frac{s^*}{\sigma^*}\right\}}\ \left(\equiv x_1^{C3}\right)$, where $x_1^{C3}<\frac{\gamma}{1-\gamma}$ can be easily proved. As for (C-2), it becomes:

$$-\frac{1}{\gamma-x\left(1-\gamma\right)}\frac{\left(1-s^*\right)\left(\delta+\rho\right)}{\sigma^*}\frac{\left(1-s^*\right)\delta+\rho}{s^*}\left(1+\chi\frac{l^*}{1-l^*}\right)<0. \tag{C-2$''$}$$

Thus, the steady state is a saddle, if $\gamma\geq 1$ or $x<x_1^{C3}$ in the case of $\gamma<1$.

To consider the sign of Trace shown as (3-10) and (4-10), we must rewrite (C-3) as:

$$\frac{\rho}{\left[\gamma-x\left(1-\gamma\right)\right]}\left[\left(\gamma-1\right)\left\{\chi\frac{l^*}{1-l^*}+\frac{s^*}{\sigma^*}+\frac{\left(1-s^*\right)\left(\delta+\rho\right)}{\sigma^*}\frac{1}{\rho}\right\}x\right. \tag{C-3$''$}$$

$$\left.-\left(1-\gamma\right)l^*\left(1-l^*\right)^{-\chi}+\chi\frac{l^*}{1-l^*}\gamma+\frac{s^*}{\sigma^*}\gamma\right].$$

Noting Case 3 in *Remark* 2, the sign of (C-3$''$) is positive, if $\gamma\geq 1$. If $\gamma<1$, Footnote 3 means that (C-3$''$) is positive for $x<\frac{-(1-\gamma)l^*(1-l^*)^{-\chi}+\chi\frac{l^*}{1-l^*}\gamma+\frac{s^*}{\sigma^*}\gamma}{(1-\gamma)\left\{\chi\frac{l^*}{1-l^*}+\frac{s^*}{\sigma^*}+\frac{(1-s^*)(\delta+\rho)}{\sigma^*}\frac{1}{\rho}\right\}}\ \left(\equiv x_2^{C3}\right)$, while it is negative for $x>x_2^{C3}$, where $0<x_2^{C3}<x_1^{C3}$ can be analytically proved. Therefore, the steady state is a source, if $\frac{1}{1-\gamma}>x>x_1^{C3}$ in the case of $\gamma<1$.[22]

[22] In Cases 1 and 3, the steady state is a source, even if we consider the range of $x>\frac{\gamma}{1-\gamma}$, because the term of $\gamma-x\left(1-\gamma\right)$ cancels out in the denominator and numerator of D_i and T_i.

Appendix D

Let us prove that Proposition 4 is realized in Cases 1-3.

Case 1: The preference is $\frac{(1-l)^{\theta(1-\gamma)}(\xi(c,\bar{c}))^{1-\gamma}}{1-\gamma}$.

In this case, the determinant, (3-9) or (4-9) is

$$D_i = \frac{-\frac{(1-s^*)(\delta+\rho)}{\sigma^*}\frac{(1-s^*)\delta+\rho}{s^*}\frac{1}{1-l^*}}{\frac{1}{\theta}\left[\left\{\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}+\frac{s^*}{\sigma^*}\theta\right\}(\gamma-1)\,x+\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}(\theta\gamma+\gamma-\theta)+\frac{s^*}{\sigma^*}\theta\gamma\right]}. \tag{D-1}$$

Because the steady state level of labor l^* is independent of the parameter $x\,(=\psi$ or $g)$ as clarified in (3-6′) and (3-7′) or (4-6′) and (4-7′), eq.(D-1) means

$$\frac{dD_i}{dx}\gtreqless 0 \text{ for } \gamma \gtreqless 1. \tag{D-2}$$

The trace of (3-10) or (4-10) is

$$T_i = \frac{\frac{(1-s^*)(\delta+\rho)}{\sigma^*}}{\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}+\frac{s^*}{\sigma^*}\theta+\left\{\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}(\theta\gamma+\gamma-\theta)+\frac{s^*}{\sigma^*}\theta\gamma\right\}\frac{1}{(\gamma-1)x}}+\rho, \tag{D-3}$$

which leads to

$$\frac{dT_i}{dx}\gtreqless 0 \text{ for } \gamma \gtreqless 1. \tag{D-4}$$

Proposition 4 considers that the steady state is a saddle. Defining $\hat{\lambda}$ as the stable root in the matrix in (3-8) or (4-8), $[\Psi(\hat{\lambda},x)\equiv]\hat{\lambda}^2-T_i\cdot\hat{\lambda}+D_i=0$ is satisfied. The total differentiation of the quadratic equation $\Psi(\hat{\lambda},x)=0$ with respect to $\hat{\lambda}$ and x yields:

$$\frac{d\hat{\lambda}}{dx}=-\frac{\partial\Psi/\partial x}{\partial\Psi/\partial\hat{\lambda}}. \tag{D-5}$$

As the denominator of the right-hand in (D-5) is negative, the sign of $d\hat{\lambda}/dx$ coincides with the sign of $\partial\Psi/\partial x$. Considering (D-2), (D-4) and $\hat{\lambda}<0$, the sign of $\partial\Psi/\partial x$ is equal to the sign of $(\gamma-1)$. Thus, we can prove Proposition 4 in Case 1.

Let us consider the movement of consumption c_i. Because we focus on the negatively sloped saddle path as stated in Section 6, $l>l_i^*$ is satisfied for $k<k_i^*$. (A-1) and (A-2) means that $c<c_i^*$ is satisfied when $k<k_i^*$. Needless to say, we can show the same thing also in Cases 2 and 3.

Case 2: The preference is $U\,(1-l,c,\bar{c})=\frac{[\exp\theta(1-l)]^{1-\gamma}\cdot(\xi(c,\bar{c}))^{1-\gamma}}{1-\gamma}$.

In this case, eq.(D-1) can be rewritten as

$$D_i = \frac{-\frac{(1-s^*)(\delta+\rho)}{\sigma^*}\frac{(1-s^*)\delta+\rho}{s^*}}{\frac{s^*}{\sigma^*}\cdot(\gamma-1)\,x-(1-\gamma)\,l^*+\frac{s^*}{\sigma^*}\gamma}. \tag{D-1$'$}$$

For the utility function of Case 2 to be concave, $\gamma > 1$ must be satisfied In Case 2 as stated in Remark 2. Thus, we can get $dD/dx > 0$.

Eq.(D-3) can be replaced by

$$T_i = \frac{\frac{(1-s^*)(\delta+\rho)}{\sigma^*}}{\frac{s^*}{\sigma^*} + \left[-\left(1-\gamma\right) l^* + \frac{s^*}{\sigma^*}\gamma\right] \frac{1}{(\gamma-1)x}} + \rho \tag{D-3$'$}$$

From $\gamma > 1$, $dT/dx > 0$ is satisfied. Considering (D-5), we can verify $d\hat{\lambda}/dx > 0$, because Case 2 must restrict attention to the range of $\gamma > 1$. If we ignore the case of concave utility function, i.e., the case of $\gamma < 1$ is considered, $d\hat{\lambda}/dx < 0$ is satisfied.

Case 3: The preference is $U\left(1-l, c, \bar{c}\right) = \left[\exp \frac{(1-l)^{1-\chi}}{1-\chi}\right]^{1-\gamma} \left(\xi\left(c, \bar{c}\right)\right)^{1-\gamma} / \left(1-\gamma\right).$

Here, we must rewrite (D-1) as:

$$D_i = \frac{-\frac{(1-s^*)(\delta+\rho)}{\sigma^*} \frac{(1-s^*)\delta+\rho}{s^*} \left(1 + \chi\frac{l^*}{1-l^*}\right)}{\left[\chi\frac{l^*}{1-l^*} + \frac{s^*}{\sigma^*}\right]\left(\gamma-1\right)x + \left(\gamma-1\right) l^* \left(1-l^*\right)^{-\chi} + \chi\frac{l^*}{1-l^*}\gamma + \frac{s^*}{\sigma^*}\gamma}. \tag{D-1$'''$}$$

Noting that the value of l^* is independent of the parameter x also in this case, eq.(D-2) is equally true of this case.

Eq.(D-3) must be replaced by:

$$T_i = \frac{\frac{(1-s^*)(\delta+\rho)}{\sigma^*}}{\chi\frac{l^*}{1-l^*} + \frac{s^*}{\sigma^*} + \left[\left(\gamma-1\right) l^* \left(1-l^*\right)^{-\chi} + \chi\frac{l^*}{1-l^*}\gamma + \frac{s^*}{\sigma^*}\gamma\right] \frac{1}{(\gamma-1)x}} + \rho. \tag{D-3$'''$}$$

From (D-3$'''$), eq.(D-4) is equally true of this case. Thus, Proposition 4 is proved also in Case 3.

Appendix E

Section 7 thinks of the very familiar separable utility function with respect to consumption and leisure:

$$U\left(1-l, c, \bar{c}\right) = \frac{\left(1-l\right)^{1-\chi}}{1-\chi} + \frac{\left(\xi\left(c, \bar{c}\right)\right)^{1-\gamma}}{1-\gamma}, \tag{E-1}$$

which is used in much macro-dynamic literature. This preference indicates the case of consumption externalities, when $\psi \neq 0$ and $g = 0$, while it applies to the case of endogenous consumption taxes, when $\psi = 0$ and $g \neq 0$.

In consumption externalities, eq.(3-6) is:

$$\left(1-l_E^*\right)^{-\chi} = \omega\left(a^*\right)\left(c_E^*\right)^{-\gamma+\psi(1-\gamma)}. \tag{E-2}$$

The lefthand in (E-2) is the marginal utility of leisure U_1, that decreases with leisure $1-l_E^*$. The assumption in Footnote 3 is identical to $-\gamma+\psi\left(1-\gamma\right) < 0 \left(\Leftrightarrow U_{22} + U_{23} < 0\right)$. Thus, the combination of $\left(l_E^*, c_E^*\right)$ satisfying (E-2) is negatively sloped as illustrated in Fig.8. As eq.(3-7) remains unchanged in the case of this preference, Fig.8 shows that the steady state uniquely exists.

Secondly, let us consider the case of endogenous consumption taxes, i.e., $\psi = 0$. As stated in Section 4-3, the slope of (4-6) is $dc_g^*/dl_g^* = -\frac{\varepsilon_g^* - \eta_g^*}{\phi_g^* + \gamma_g^* - \frac{g}{g+c^*}} \left(\frac{c_g^*}{l_g^*}\right)$. Since eq.(E-1) means that $\varepsilon_g^* = 0$, $\varepsilon_g^* = 0$, $\gamma_g^* = \gamma$ and $\eta_g^* = -\chi \frac{l_g^*}{1-l_g^*} < 0$, we can easily establish $dc_g^*/dl_g^* = \frac{-\chi \frac{l_g^*}{1-l_g^*}}{\gamma - \frac{g}{g+c^*}} \left(\frac{c_g^*}{l_g^*}\right) < 0$ for $\gamma \geq 1$. Noting $\tau_c^* c_g^* = g$, eq.(4-7) does not change. When $\gamma \geq 1$, the uniqueness of steady state is guaranteed, if g is not extremely large. This is because the same figure as Fig.8 can be illustrated except that the upward straight line has an ordinate intercept with a negative value.

When $\gamma < 1$, let us prove that there exist at most two steady states. From (4-7), we can get:

$$\frac{(1-s^*)\,\delta + \rho}{s^*} a^* l_g^* = c_g^* + g. \tag{E-3}$$

Noting $\tau_c^* c_g^* = g$ and $\omega(a^*) = \frac{(1-s^*)(\delta+\rho)}{s^*} a^*$, we substitute (E-3) in (4-6) and then get:

$$\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\,\delta + \rho} \left(c_g^*\right)^{1-\gamma} = \frac{l_g^*}{\left(1-l_g^*\right)^{\chi}} \tag{E-4}$$

As the righthand of (E-4) is increases with l_g^* with an increasing returns, we can illustrate (E-4) as in Fig.9. Considering (E-3) and (E-4), Fig.9 shows that there exist at most two steady states, depending on the value of g. However, Fig.9 does not conveys any informations about the sign of $(1+\tau_c^*)\gamma - \tau_c^*$ and thus we cannot define the sign of D_g in (7-2).

To verify the sign of $(1+\tau_c^*)\gamma - \tau_c^*$, we must consider how the feature of (4-6) is illustrated if we do not substitute (E-3) in (4-6). Then, we can express (4-6) as

$$\omega(a^*) \left(c_g^*\right)^{1-\gamma} = \left(1-l_g^*\right)^{-\chi} \left(g + c_g^*\right). \tag{E-5}$$

When g is fixed at a certain value, for example, g_0, we realize that there exist the two values of c_g^* for a given value of l_g^* except for the value of a saddle node bifurcation $l_{g_0}^S$. As a result, the combination of $\left(l_g^*, c_g^*\right)$ satisfying (E-5) can be represented as Fig.10.[23] Suppose that g is increased from g_0. Then, eq.(E-3) goes down, while the combinations of $\left(l_g^*, c_g^*\right)$ satisfying (E-5) move inward in Fig.10. There continue to exist the two steady states until a saddle node bifurcation occurs. Since the slope of (E-5) is $\frac{-\chi \frac{l_g^*}{1-l_g^*}}{\gamma - \frac{\tau_c^*}{1+\tau_c^*}} \left(\frac{c_g^*}{l_g^*}\right)$, $(1+\tau_c^*)\gamma - \tau_c^* < 0$ is always satisfied at the low steady state. This relation is satisfied also at the high steady state, as long as eqs.(E-3) intersects with (E-5) in the range of negative slopes of (E-5). Then, the two steady states are saddle. Otherwise, eq.(7-3) implies that the high steady state changes to be a source.

[23] In Fig.12, the combination of $\left(l_g^*, c_g^*\right)$ satisfying (E-5) is smoothly illustrated. However, we can completely eliminate the curve with an strong irregularity leading to more than two steady state equilibria.

The low steady state is necessarily a saddle, while the high steady state is a saddle if g is relatively low, but the high steady state is a source if g is relatively large and is below a saddle node bifurcation value. Moreover, we can easily assert $l_E^* \neq l_g^*$ and $k_E^* \neq k_g^*$.

Finally, we briefly state the ralation between the two cases, when $\gamma = 1$. Noting that $c_E^* = \frac{(1-s^*)\delta+\rho}{s^*} a^* l_E^*$ and $\omega(a^*) = \frac{(1-s^*)\delta+\rho}{s^*} a^*$, then, $l_E^* = l_g^*$ is satisfied and the transitional dynamic as well as the steady state are unaffected by changes in ψ and τ_c^*. See (7-2), (7-3), (E-2) and (E-4).

Appendix F

Let us consider the existence of steady state, when $U(1-l, c, \bar{c}) = \frac{\left(\frac{(1-l)^{1-\chi}}{1-\chi} + \xi(c,\bar{c})\right)^{1-\zeta}}{1-\zeta}$. Recall that $\psi = 0$ ($\psi \neq 0$) corresponds to the case of endogenous consumption taxes (consumption externalities).

In the case of consumption externalities, the steady state value of a^* is exclusively determined by (3-5). Eqs.(3-6) and (3-7) are respectively expressed as:

$$(1-l_E^*)^{-\chi} c_E^{*-\psi} = \omega(a^*), \tag{F-1}$$

$$c_E^* = \frac{(1-s^*)\delta+\rho}{s^*} a^* l_E^*. \tag{F-2}$$

As for the combination of (l_E^*, c_E^*) satisfying (F-1), we can obtain $\frac{dc_E^*}{dl_E^*} = \frac{\chi}{\psi} \frac{c_E^*}{1-l_E^*} > 0$ and $\frac{d^2 c_E^*}{dl_E^{*2}} > 0$ for $\psi > 0$. The similar figure to Fig.7 can be obtained and thus, there eixst at most two steady state solutions (l_E^*, c_E^*) by solving (F-1) and (F-2), depending on the parameter values.

In the case of endogenous consumption taxes, the value of a^* is determined only by (4-5). Eqs.(4-6) and (4-7) are respectively:

$$\left(1-l_g^*\right)^{-\chi} = \frac{\omega(a^*)}{1+\tau_c^*}, \tag{F-3}$$

$$(1+\tau_c^*) c_g^* = \frac{(1-s^*)\delta+\rho}{s^*} a^* l_g^*. \tag{F-4}$$

Noting $\omega(a^*) = \frac{(1-s^*)(\delta+\rho)}{s^*} a^*$, let us substitute (F-4) and the budget $\tau_c^* c_g^* = g$ in (F-3). Then, we can get:

$$\frac{l_g^*}{\left(1-l_g^*\right)^{\chi}} = \frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho} c_g^* \tag{F-5}$$

As for (F-5), we can verify that $dl_g^*/dc_g^* > 0$, $d^2 l_g^*/dc_g^{*2} > 0$ and $\lim_{l_g^* \to 0} \frac{dl_g^*}{dc_g^*} = \frac{(1-s^*)\delta+\rho}{(1-s^*)(\delta+\rho)}$. Thus, if $\frac{(1-s^*)\delta+\rho}{(1-s^*)(\delta+\rho)} > \frac{(1-s^*)\delta+\rho}{s^*} a^* \Leftrightarrow \omega(a^*) > 1$, there are necessarily two steady states for $0 < g < g_S$ by solving eqs.(F-4) and (F-5). Needless to say, g_S is the government expenditure generating the two steady states that are infinitely close to each other.

Eqs.(F-1)-(F-4) mean that the values of labor and capital in the high (low) steady state of consumption extrnalities are respectively different from the values of those in the high (low) steady state of endogenous consumption taxes, i.e., $l_E^* \neq l_g^*$ and $c_E^* \neq c_g^*$.

Next, let us prove Lemma 2. When $x = \psi$, the direct calculation leads to $\eta_E = \frac{-\zeta(1-l_E^*)^{-\chi} \cdot l_E^*}{\frac{(1-l_E^*)^{1-\chi}}{1-\chi} + c_E^{*1+\psi}} - \chi \frac{l_E^*}{1-l_E^*}$. Substituting (F-1) into η_E yields $\eta_E = \frac{-\zeta\omega(a^*) l_E^* \cdot c_E^{*\psi}}{\frac{1-l_E^*}{1-\chi}\omega(a^*) \cdot c_E^{*\psi} + c_E^{*1+\Psi}} - \chi \frac{l_E^*}{1-l_E^*}$. Cosidering (F-2), $c_E^* = \frac{(1-s^*)\delta+\rho}{s^*} a^* l_E^*$ in the equation, we can get $\eta_E = \frac{-\zeta\omega(a^*) l_E^*}{\frac{1-l_E^*}{1-\chi}\omega(a^*) + \frac{(1-s^*)\delta+\rho}{s^*} a^* l_E^*} - \chi \frac{l_E^*}{1-l_E^*}$. In the case of $x = g$, we can derive $\eta_g = \frac{-\zeta(1-l_g^*)^{-\chi} \cdot l_g^*}{\frac{(1-l_g^*)^{1-\chi}}{1-\chi} + c_g^*} - \chi \frac{l_g^*}{1-l_g^*}$. Substituting (F-3) into η_g yields $\eta_g = \frac{-\gamma\omega(a^*) l_g^*}{\frac{1-l_g^*}{1-\chi}\omega(a^*) + (1+\tau_c^*) c_g^*} - \chi \frac{l_g^*}{1-l_g^*}$. Considering (F-4), $(1+\tau_c^*)\, c_g^* = \frac{(1-s^*)\delta+\rho}{s^*} a^* l_g^*$ in the equation, we can get $\eta_g = \frac{-\zeta\omega(a^*) l_g^*}{\frac{1-l_g^*}{1-\chi}\omega(a^*) + \frac{(1-s^*)\delta+\rho}{s^*} a^* l_g^*} - \chi \frac{l_g^*}{1-l_g^*}$. The other elasticities ε_i, ϕ_i and γ_i in Lemma 2 can be also proved by using the similar ways.

In this paper, we do not characterize the local stabilities around the steady states by using (8-2) and (8-3). Nourry et al (2013) recently proved that the steady state can be locally indeterminate in the case of endogenous consumption taxes by using the almost same preference as (8-1) at $\psi = 0$. We can show that consumption externalites can produce an indeterminacy of equilibria in the same mechanism as the endogenous taxes.

Because the analytical comparison is relatively complicated and the large space is required here, we would like the detailed analysis to be transferred to our another paper, in which local stability analysis is completely done. As seen in Section 8, therefore, this paper focuses on the range of paprameter values producing the very stable case that there exists a monotonically converging equilibrium to a steady state.

Appendix G

Let us derive (9-1) and (9-2). In the mixed case of the two distortions, the steady state conditions correspond to (3-5), (3-6) and (4-7) if l_E^* and l_g^* (k_E^* and k_g^*) in these equations are replaced by l^* (k^*). Using almost the same ways used in Sections 3-3 and 4-3, we can get the determinant D and trace T:

$$D = \frac{-\frac{(1+\psi)\tilde{\phi}^* + [(1+\psi)\tilde{\gamma}^* - \psi] - \frac{\tau_c^*}{1+\tau_c^*}}{[(1+\psi)\tilde{\gamma}^* - \psi] - \frac{\tau_c^*}{1+\tau_c^*}} \frac{(1-s^*)(\delta+\rho)}{s^*} \frac{s^*}{\sigma^*}}{\frac{s^*}{\sigma^*} - \eta^* - \frac{(1+\psi)\tilde{\phi}^*}{[(1+\psi)\tilde{\gamma}^* - \psi] - \frac{\tau_c^*}{1+\tau_c^*}} \varepsilon^*} \left[\frac{(1-s^*)\,\delta + \rho}{s^*} \right. \tag{G-1}$$

$$\left. + \frac{\varepsilon^* - \eta^*}{(1+\psi)\tilde{\phi}^* + [(1+\psi)\,\tilde{\gamma}^* - \psi] - \frac{\tau_c^*}{1+\tau_c^*}} \left(\frac{c^*}{k^*} \right) \right],$$

$$T = \frac{-1}{\frac{s^*}{\sigma^*} - \eta^* - \frac{+(1+\psi)\tilde{\phi}^*}{[(1+\psi)\tilde{\gamma}^* - \psi] - \frac{\tau_c^*}{1+\tau_c^*}} \varepsilon^*} \left[\frac{(1+\psi)\tilde{\phi}^*}{[(1+\psi)\,\tilde{\gamma}^* - \psi] - \frac{\tau_c^*}{1+\tau_c^*}} \frac{s^*}{\sigma^*} \frac{(1-s^*)\,(\delta+\rho)}{s^*} \right.$$

$$-\frac{s^*}{\sigma^*}\frac{\varepsilon^*}{[(1+\psi)\tilde{\gamma}^*-\psi]-\frac{\tau_c^*}{1+\tau_c^*}}\left(\frac{c^*}{k^*}\right)\Bigg]+\rho. \tag{G-2}$$

If $\psi\times\tau_c^*\to 0$, we can clarify

$$\frac{(1+\psi)\tilde{\phi}^*}{[(1+\psi)\tilde{\gamma}^*-\psi]-\frac{\tau_c^*}{1+\tau_c^*}}$$

$$=\frac{(1+\psi)(1+\tau_c^*)\tilde{\phi}^*}{(1+\psi)(1+\tau_c^*)\tilde{\gamma}^*-\psi(1+\tau_c^*)-\tau_c^*}$$

$$=\frac{(1+\psi+\tau_c^*)\tilde{\phi}^*}{(1+\psi+\tau_c^*)\tilde{\gamma}^*-(\psi+\tau_c^*)}.$$

In addition, substituting $\frac{c^*}{k^*}=\frac{(1-s^*)\delta+\rho}{s^*}\frac{1}{1+\tau_c^*}$ of (3-7) in (G-1) and (G-2) yields (9-1) and (9-2), only if $\psi\times\tau_c^*\to 0$.

In the preference of (6-2) of Section 6, it is easily proved that the steady state value of l^* is independent of the sizes of τ_c^* and ψ. Moreover, all the arguments in Section 6 are equally true of this mixed case of two distortions if we redefine $x\equiv\tau_c^*+\psi$. In the separable preference of (7-1) in Section 7, we can clarify $\tilde{\gamma}^*=\gamma$, $\tilde{\phi}^*=0$, $\eta^*=-\chi\frac{l^*}{1-l^*}$ and $\varepsilon^*=0$. Substituting these elasticities and (3-7) in (9-1) and (9-2), and replacing $x=\tau_c^*$ or ψ by $x\equiv\tau_c^*+\psi$, we can get the same expressions as (7-2) and (7-3). If we define $x\equiv\tau_c^*+\psi$ also in the non-separable preference of (8-1) and substitute (3-7) in (G-1) and (G-2), we can obtain (8-2) and (8-3), in which the elasticities are denoted in Lemma 2.

Let us prove that the steady state effects of τ_c^* and ψ on labor l^* are different in quantity except for the non-separable preference of (6-2). Using (3-6) and (4-7) in the separable preference of (7-1) in Section 7, the steady state values of labor and consumption satisfy:

$$\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}(c^*)^{(1-\gamma)(1+\psi)}=\frac{l^*}{(1-l^*)^{\chi}}, \tag{G-3}$$

$$c^*=\frac{(1-s^*)\delta+\rho}{s^*}a^*l^*-g. \tag{G-4}$$

We can easily see that the combination of l^* and c^* satisfying (G-3) has positive (negative) slopes, i.e., $dc^*/dl^*>0$ (<0), if $\gamma<1$ $(\gamma>1)$. Considering $-\gamma+\psi(1-\gamma)<0$ $(\Leftrightarrow U_{22}+U_{23}<0)$, we can clarify $d^2c^*/dl^{*2}>0$. Thus, the analytical arguments in the case of $g\neq 0$ and $\psi=0$ of Appendix E are equally ture of this case. Eqs.(G-3) and (G-4) clearly show that the steady state effects on l^* of a rise in τ_c^* through an increase in g differ from the steady state effects on l^* of a rise in ψ.

In the case of non-separable preference of (8-1), eq.(G-3) is replaced by

$$\frac{(1-s^*)(\delta+\rho)}{(1-s^*)\delta+\rho}(c^*)^{1+\psi}=\frac{l^*}{(1-l^*)^{\chi}}. \tag{G-5}$$

We can understand that the slope of (G-5) is positive, i.e., $dc^*/dl^*>0$. Unfortunately, we cannot clarify the sign of d^2c^*/dl^{*2}, but we can analytically show $\lim_{l^*\to 0}\frac{d^2c^*}{dl^{*2}} < 0$ and $\lim_{l^*\to 1}\frac{d^2c^*}{dl^{*2}} > 0$. Thus, we can find out the range of values of parametrs that generate the multiple steady state values of l^* and c^*. Eqs.(G-4) and (G-5) clearly show that changes in τ_c^* and ψ have different impacts on the values of l^* and c^*.

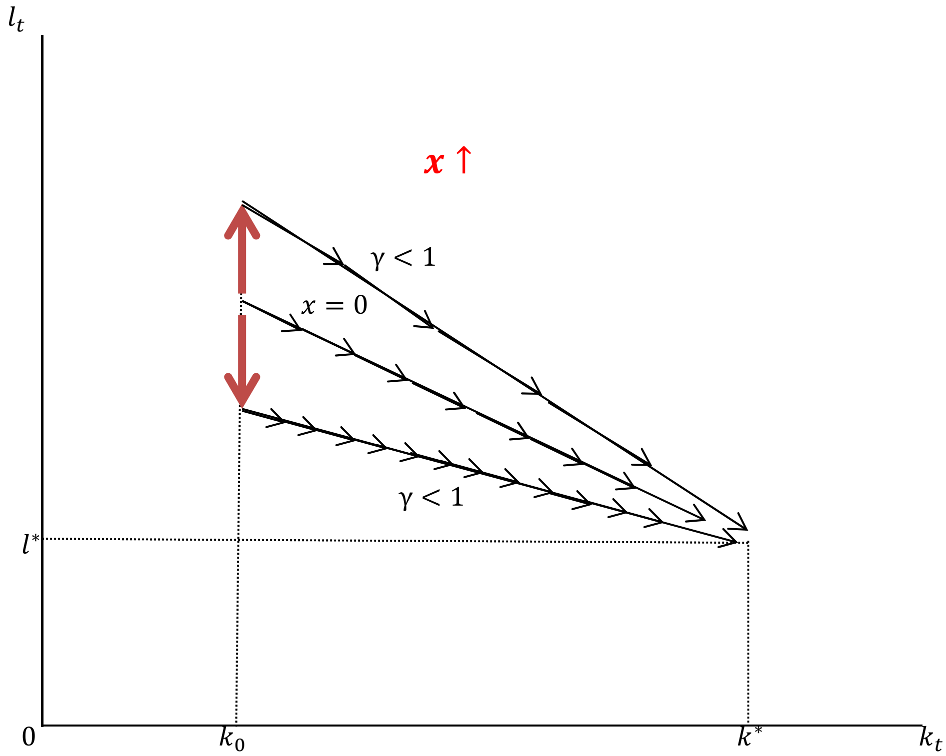


Fig.1: The deviation from the socially optimal path when x increases

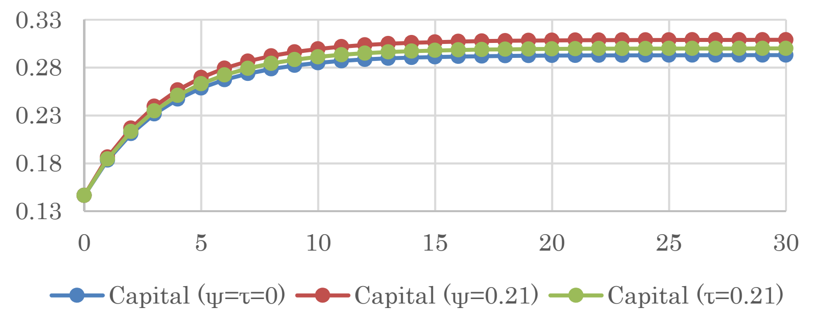


Figure 2-1: Capital (γ=1.5)

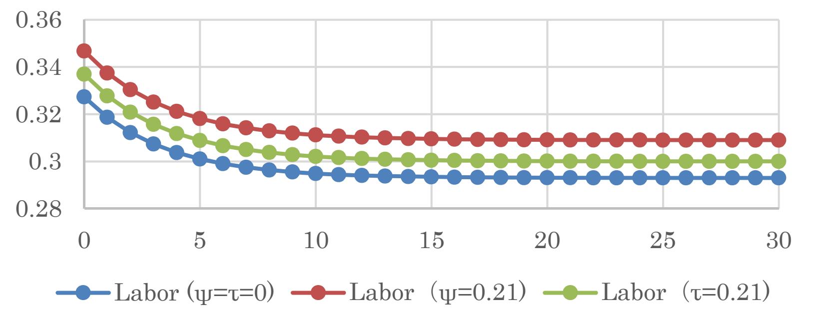


Figure 2-2: Labor (γ=1.5)

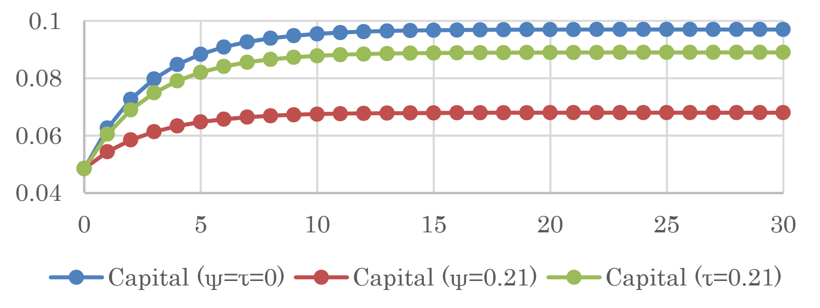


Figure 3-1: Capital (γ=0.5)

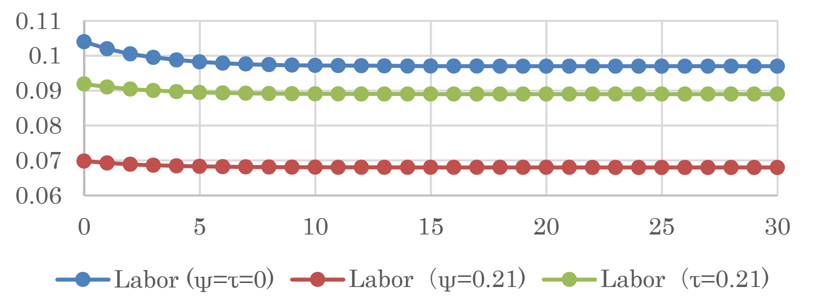


Figure 3-2: Labor (γ=0.5)

Figure 4-1: Capital (γ=2.0)

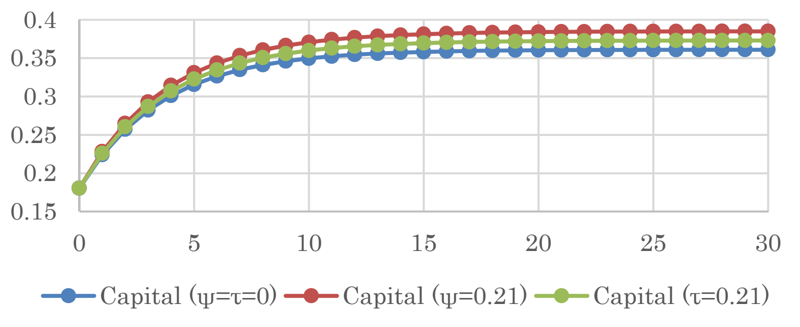


Figure 4-2: Labor (γ=2.0)

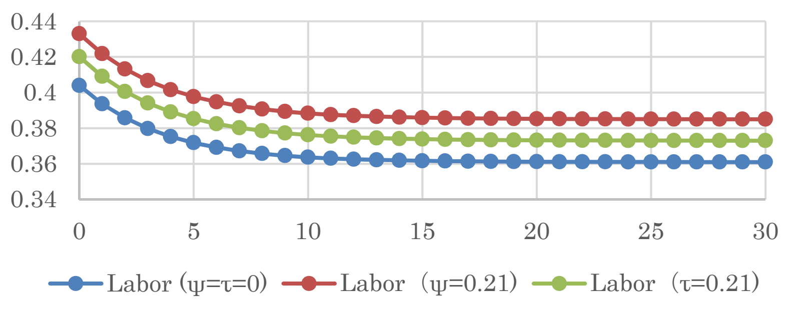


Figure 5-1: Capital (ξ=8 and χ=0.85)

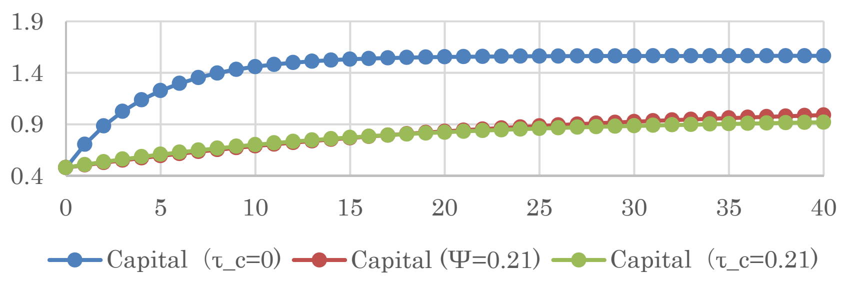


Figure 5-2: Labor (ξ=8 and χ=0.85)

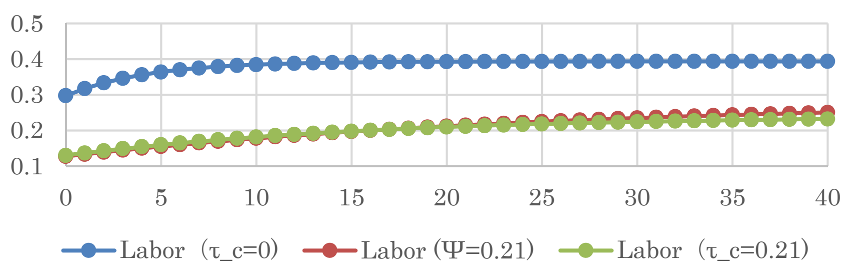


Figure 6-1 :Capital (ξ=6 and χ=0.85)

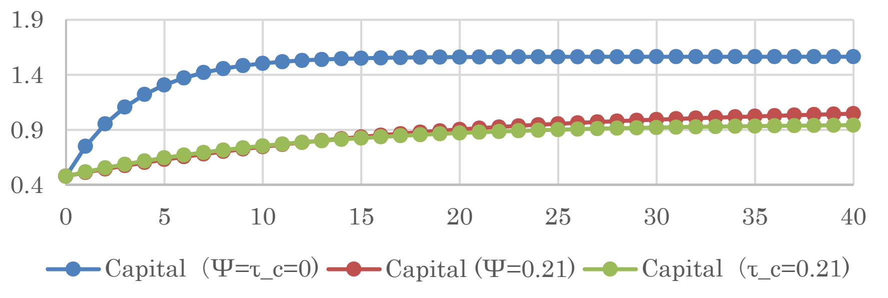


Figure 6-2 :Labor (ξ=6 and χ=0.85)

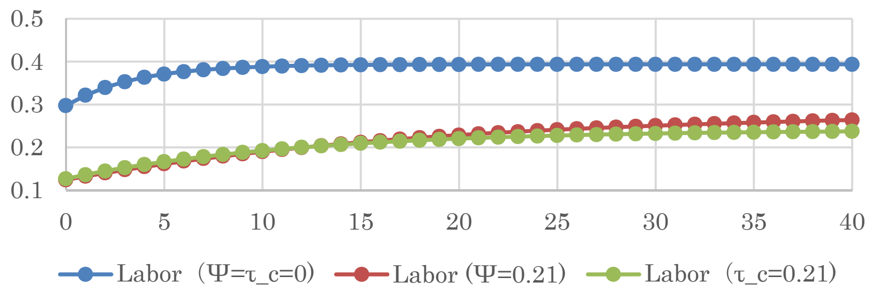


Figure 7-1: Capital (ξ=8 and χ=0.65)

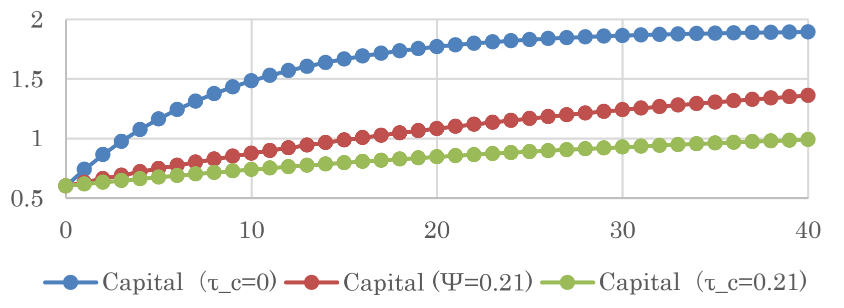
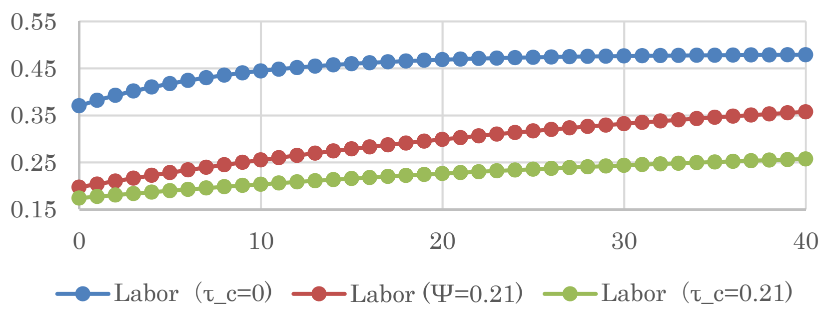


Figure 7-2: Labor (ξ=8 and χ=0.65)



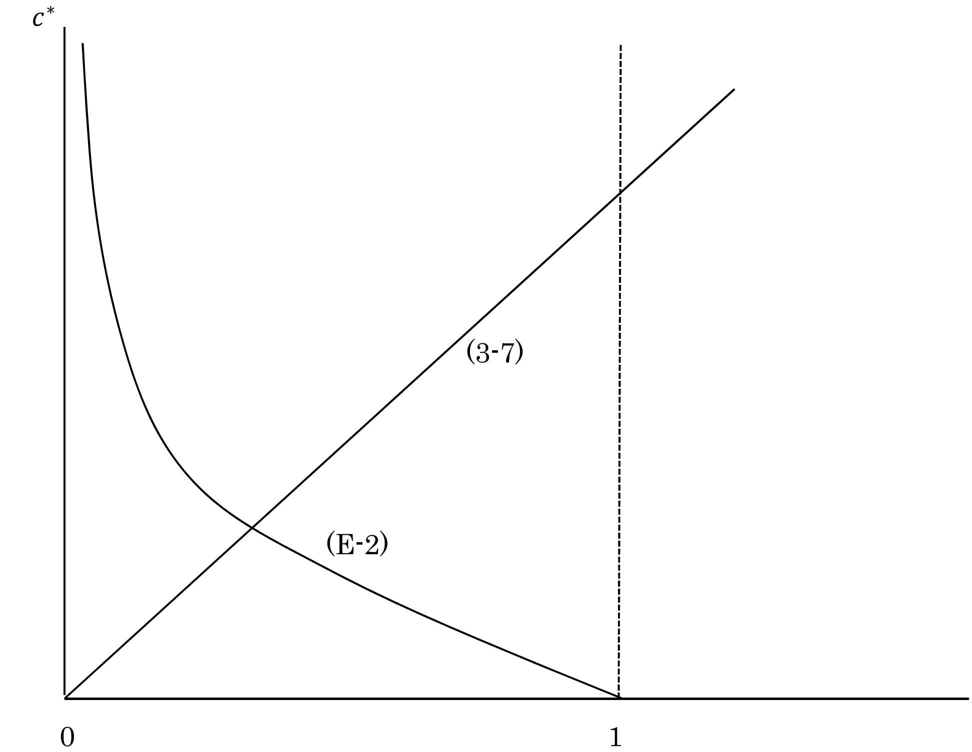


Fig.8: Endogenous consumption taxes, $\gamma < 1$

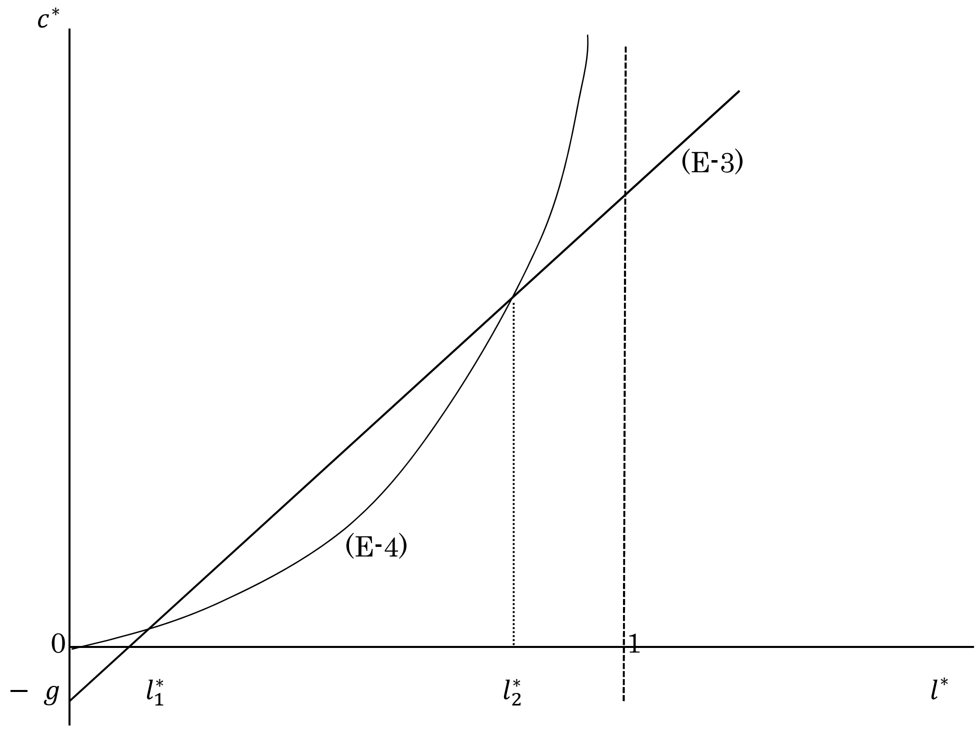


Fig.9: Endogenous consumption taxes, $\gamma < 1$

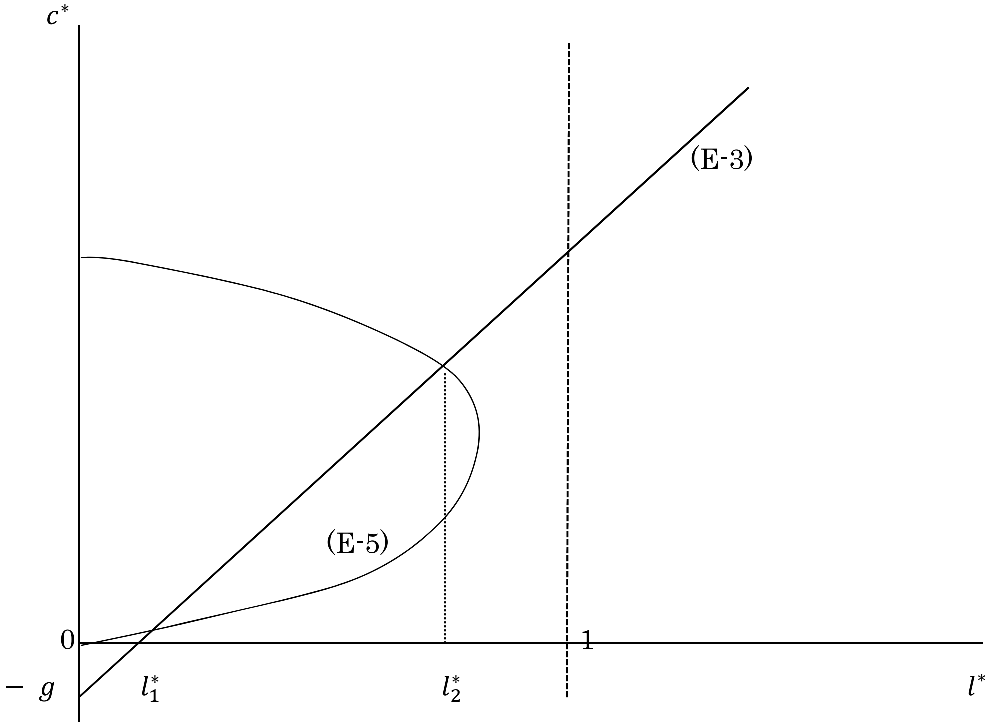


Figure 10: Information about the sign of D_g