CIRJE-F-1015

Mechanism Design in Hidden Action and Hidden Information: Richness and Pure Groves

Hitoshi Matsushima
University of Tokyo

Shunya Noda
Stanford University

May 2016

Mechanism Design in Hidden Action and Hidden Information: Richness and Pure Groves[1]

Hitoshi Matsushima[2]
University of Tokyo

Shunya Noda[3]
Stanford University

May 30, 2016

Abstract

We investigate general collective decision problems related to hidden action and hidden information. We assume that each agent has a wide availability of action choices at an early stage, which provides significant externality effects on the other agent's valuations in all directions. We characterize the class of all mechanisms that solve the hidden action problem, and demonstrate equivalence properties in the ex-post term. Importantly, we find that pure Groves mechanisms, defined as the simplest form of canonical Groves mechanisms, are the only efficient mechanisms that solve such hidden action problems. We argue that the resolution of the hidden action problem automatically resolves the hidden information problem.

JEL Classification Numbers: C72, D44, D82, D86
Keywords: Hidden Action, Hidden Information, Externalities, Richness, Pure Groves, Equivalence.

---

[1] This study is supported by a grant-in-aid for scientific research (KAKENHI 25285059, 26J10076, 16H02009) from the Japan Society for the Promotion of Science (JSPS) and the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of the Japanese government. This paper is also supported by the Funai Overseas Scholarship, the E. K. Potter Fellowship, the JSPS research fellowship (DC1), and the CARF research fund at University of Tokyo. We thank Gabriel Carroll, Fuhito Kojima, Takuo Sugaya, Yuichi Yamamoto, and all the seminar participants in Hitotsubashi University, University of Tokyo, Osaka University, Kyoto University, and Stanford University. All errors are ours.

[2] Department of Economics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. (hitoshi[at]e.u-tokyo.ac.jp)

[3] Department of Economics, Stanford University, 579 Serra Mall, Stanford, CA 94305-6072, USA. (shunya.noda[at]gmail.com)

1. Introduction

This study investigates a general class of collective decision problems that includes issues of imperfect monitoring (hidden action) as well as incomplete information (hidden information), such as principal-agent relationships, partnerships, and general resource allocations (including auctions and public good provision), with the assumptions of quasi-linearity and risk-neutrality. Multiple agents independently make their action choices at an early stage before a state occurs, influencing their valuation functions through stochastic state determination. The central planner then determines an allocation that is relevant to the welfare of all agents as well as the central planner in a state-contingent manner.

We assume *hidden action* in that the central planner cannot observe the agents' action choices. The central planner, therefore, designs a state-contingent mechanism, according to which the central planner makes side payments to agents, as well as an allocation decision, incentivizing the agents to select the action profile that the central planner desires.

We first demonstrate a benchmark model that addresses only the hidden action problem. We then incorporate *hidden information* by assuming that the central planner can observe neither the state nor the agents' action choices, and therefore requires agents to announce their private information regarding the state (i.e., their types). The purpose of this study is to clarify whether, and how, the central planner overcomes the incentive problem in cases of both hidden action and hidden information.

This study substantially differs from previous research on mechanism design and contract theory in that each agent potentially has various aspects of activities such as information acquisition, R&D investment, patent control, standardization, M&A, rent-seeking, positive/negative campaigns, environmental concern, product differentiation, entry/exit decisions, preparation of infrastructure, and headhunting. The central planner generally lacks information about the breadth of these potential aspects, because of, for example, the separation of ownership and control. Accordingly, the central planner cannot know which aspects of agents' activities are actually relevant to the current problem. In such cases, the central planner must, inevitably, account for all of these aspects in mechanism design.

One example is a dynamic aspect of auctions for allocating business resources such as spectrum licenses, where each participant undertakes various profit-seeking activities to gain an advantage over rival companies in the upcoming auction event.[4] Whether, and to what degree, such activities distort the welfare crucially depends on the auction format design.

Specifically, the central planner worries that each agent's hidden action has significant *externality effects* on the other agents' valuation functions (i.e., each agent can smoothly change the distribution of the state in all directions via a unilateral deviation from the desired action profile). This broad potential for externalities, which this study terms *richness*, dramatically restricts the range of possible mechanisms that can incentivize agents to make the desired action choices (i.e., in this study's terminology, it restricts the range of possible mechanisms that can *induce* the desired action profile).

The main contribution of this study is to characterize mechanisms that can induce the desired action profile, even in the presence of such richness. Based on this characterization, we present *equivalence* properties in the ex-post term under the constraints of inducibility. That is, the ex-post payments, the ex-post revenue, and the ex-post payments are *unique* up to constants.

We further clarify the possibility that the central planner achieves an efficient action profile and efficient allocations without any trouble in liability. With the assumption of private values, we define *pure Groves* mechanisms as the simplest form of canonical Groves mechanisms, in which the central planner gives each agent the welfare of the other agents and the central planner and imposes on her a *fixed* monetary fee. We show that a mechanism induces an efficient action profile if and only if it is pure Groves. In other words, *pure Groves mechanisms are the only efficient mechanisms that solve the hidden action problem.*

---

[4] See Klemperer (2004) and Milgrom (2004), for example.

This theoretical finding has important implications not only in cases of hidden action but also in cases of hidden information. Suppose that the central planner cannot observe the state, and, therefore, requires agents to announce their private information regarding the state. The above-mentioned dynamic auction is an example of this case. Importantly, once the central planner designs any mechanism that induces the efficient action profile (i.e., solves the hidden action problem), then this mechanism *automatically* solves the hidden information problem, because of its internalization feature.

The generally accepted view in mechanism design is that in some environments, Groves mechanisms[5] are the only mechanisms that solve the *hidden information* problem. In contrast to this view, this study shows that pure Groves mechanisms (i.e., the special form of Groves mechanisms), are the only mechanisms that solve the *hidden action* problem. Moreover, the resolution of the hidden action problem in this manner generally and automatically can solve the hidden information problem.

Because the class of pure Groves mechanisms is a proper subclass of Groves mechanisms, it might be more difficult for the central planner to earn *non-negative* revenues with hidden action than without hidden action. In fact, the popular Vickrey–Clarke–Groves (VCG) mechanism, which aligns each agent's payoff with her marginal contribution, generally guarantees non-negative revenues, but it is not pure Groves (i.e., it fails to satisfy inducibility).

To be more precise, we show an impossibility result that with richness, there exists no pure Groves mechanism (i.e., no well-behaved mechanism) that satisfies the requirements of non-negative revenues and ex-post individual rationality, while the VCG mechanism satisfies both. Accordingly, our result suggests that the central planner, who wants to defeat richness in order to overcome the hidden action problem, should collect information in advance about which aspects of agents' activities are actually relevant.

An example without richness is a case of partnerships with *finite* action spaces, where the central planner can successfully tailor the side-payment rule to the detail of specifications, making inducibility compatible with non-negative revenues, or even

[5] See Vickrey (1961), Clarke (1971), and Groves (1973).

budget-balancing (Legros and Matsushima (1991), for example). In contrast, with richness, the central planner cannot remove agents' evasions when she replaces a pure Groves mechanism with any complicated mechanism.

The difficulty in liability depends on the presence of each agent's *externality* effect on the other agents' valuation functions. If any agent's action choice has no externality and the central planner recognizes this in advance, then we would obtain the equivalence result for *interim* payoffs, rather than ex-post payoffs. Specifically, to achieve efficiency, a wider class of mechanisms than the class of Groves mechanisms (*expectation-Groves* mechanisms) can solve the incentive problems of both hidden action and hidden information, without deficits, or even with budget-balancing.

The remainder of this paper is organized as follows. Section 2 reviews the literature. Section 3 presents the benchmark model, where we account for only hidden actions. Section 4 incorporates hidden information into the model. Section 5 focuses on the achievement of efficiency. Section 6 examines the central planner's revenues. Section 7 studies the case of no externality. Section 8 presents an alternative definition of richness from the viewpoint of full dimensionality. Section 9 concludes.

2. Related Literature

This study makes important contributions to the literature regarding the hidden action problem as follows. When an agent has a wide variety of action choices, a complicated contract design might motivate the agent to deviate from desired behavior. In this case, a simply designed contract could function better than a complicated one. For example, Holmström and Milgrom (1987) studied principal-agent relationships in a dynamic context, where randomly determined outputs are accumulated through time, and the agent flexibly adjusts the effort level depending on output-histories. They showed that the optimal incentive contract that maximizes the principal's revenue must be linear with respect to the output accumulated at the ending time. Carroll (2014) investigated optimal contract design in a static principal-agent relationship, where the principal experiences ambiguity about the range of activities that the agent can undertake. Carroll showed that the optimal contract must be linear with respect to the resultant output, provided that the principal follows the maxmin expected utility hypothesis.

This study examines the hidden action problem by introducing multiple agents, general state spaces, general allocation rules, and various criteria such as efficiency, instead of revenue optimization. We account deliberately for the wide range of externality effects of each agent's action choice on other agents' valuation functions by assuming richness. However, we do not assume ambiguity or behavioral modes such as maxmin utility. We then demonstrate the characterization result for well-behaved incentive mechanisms, implying that only a simple form of mechanism design (i.e., pure Groves) functions, and we further show equivalence properties in the ex-post term.

Any well-behaved efficient mechanism must be pure Groves. In this regard, Athey and Segal (2013) showed that with private values, but regardless of whether richness is present, pure Groves mechanisms induce efficiency in hidden action. We extend their study to show that, with richness, pure Groves mechanisms are the only mechanisms that can achieve efficiency in hidden action.

The literature regarding the hidden action problem has shown that without richness (i.e., when the scope of action spaces is sufficiently limited), we can design efficient incentive mechanisms by tailoring the payment rule to detailed specifications. We can even make the incentive constraints compatible with either full surplus extraction or budget-balancing. See Matsushima (1989), Legros and Matsushima (1991), and Williams and Radner (1995), for example. See also Obara (2008), whose model is close to this study's model, but without richness.

With richness, however, a mechanism's dependence on detail might encourage each agent to deviate, because such dependence inevitably results in a loophole that puts the agent in a more advantageous position. The poor functioning that results from complication in mechanism design makes it difficult to cope with incentives in hidden action as well as non-negativity of revenues.

This study makes important contributions to the literature on the hidden information problem. Green and Laffont (1977, 1979) and Holmström (1979) showed that in hidden information environments with differentiable valuation functions, smooth path-connectedness, and private values, Groves mechanisms are the only efficient mechanisms that satisfy incentive compatibility in the dominant strategy.

This study reconsiders Groves mechanisms from the viewpoint of hidden action, and shows that only pure Groves mechanisms resolve the hidden action problem. While the resolution of the hidden action problem automatically resolves the hidden information problem, the reverse is not true.[6]

This study also investigates the case in which externality effects are absent, showing that expectation-Groves mechanisms, which are more general than Groves and require each agent to pay the same amount as Groves in expectation, are the only mechanisms that can achieve efficiency. Hatfield, Kojima, and Kominers (2015) is relevant to this result, because they showed that when we confine our attention to efficient mechanisms that are detail-free (i.e., independent of detailed knowledge about the set of actions that the agents can take), the mechanisms must be Groves. We permit a mechanism to not be detail-free, and then show that expectation-Groves mechanisms successfully achieve efficiency. Importantly, the class of expectation-Groves mechanisms includes the AGV mechanism (see Arrow (1979) and D'Aspremont and Gerard-Varet (1979)), which satisfies budget-balancing. This result contrasts with that of Hatfield et al. because Groves mechanisms generally fail to satisfy budget-balancing.

It is worth noting that our results are irrelevant to the fine detail of the state space and valuation functions. Consequently, this study articulates the desirability of pure Groves mechanisms and expectation-Groves mechanisms, even if the prerequisites of Green–Laffont–Holmström fails to hold (e.g., even if type spaces are finite).

[6] Hausch and Li (1993) and Persico (2000) are related. They demonstrate that first-price and second-price auctions provide different incentives in information acquisition that make the other agents' valuation more accurate. We explain that this difference comes from the difference in ex-post payoffs between these auction formats.

3. Hidden Action

Let us consider a setting with one central planner and n agents indexed by $i \in N = \{1, 2, ..., n\}$. As a benchmark model of this study, we investigate an allocation problem that consists of the following four stages.

Stage 1: The central planner commits to a *mechanism* defined as (g, x), where $g : \Omega \to A$ and $x \equiv (x_i)_{i \in N} : \Omega \to R^n$. Here, Ω denotes the finite set of all states and A denotes the set of all allocations. We call g and x the *allocation rule* and the *payment rule*, respectively.[7]

Stage 2: Each agent $i \in N$ selects a hidden action $b_i \in B_i$, where B_i denotes the set of all actions for agent i. The cost function for the agent's action choice is given by $c_i : B_i \to R_+$. We assume that there is a no-effort option $b_i^0 \in B_i$ such that $c_i(b_i^0) = 0$. Let $B \equiv \times_{i \in N} B_i$ and $b \equiv (b_1, ..., b_n) \in B$.

Stage 3: The state $\omega \in \Omega$ is randomly drawn according to a conditional probability function $f(\cdot | b) \in \Delta(\Omega)$, where $b \in B$ is the action profile selected at stage 2, $\Delta(\Omega)$ denotes the set of all distributions (i.e., lotteries) over states, and $f(\omega | b)$ denotes the probability that the state ω occurs provided that the agents select the action profile b.

[7] This study assumes that the central planner commits to the mechanism *before* agents take action. Without this assumption, the desired outcomes, such as efficiency, may not be achievable. Consider the central planner who commits to a mechanism *after* the agents' action choices. In this case, the central planner prefers the VCG mechanism because it yields greater revenue than the mechanism that this study discusses. The VCG mechanism, however, fails to induce any efficient action profile; by anticipating that the central planner will set a VCG mechanism, agents are willing to select inefficient actions.

Stage 4: The central planner determines the allocation $g(\omega) \in A$ and the side payment vector paid to the central planner $x(\omega) = (x_i(\omega))_{i \in N} \in R^n$, where ω is the state that occurs at stage 3. The resultant payoff of each agent $i \in N$ is given by

(1) $$v_i(g(\omega), \omega) - x_i(\omega) - c_i(b_i),$$

where we assume that each agent's payoff function is quasi-linear and risk-neutral, and the cost of the agent’s action choice is additively separable.

Figure 1 describes the timeline of the benchmark model. This section intensively studies the incentives in hidden action at stage 2.

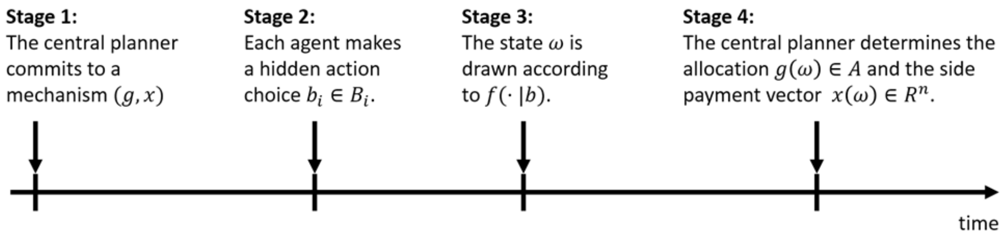

Figure 1: Timeline with Hidden Action

Definition 1 (Inducibility): A mechanism (g, x) is said to *induce* an action profile $b \in B$ if b is a Nash equilibrium in the game implied by the mechanism (g, x), i.e., for every $i \in N$,

(2) $$E[v_i(g(\omega), \omega) - x_i(\omega) \,|\, b] - c_i(b_i)$$
$$\geq E[v_i(g(\omega), \omega) - x_i(\omega) \,|\, b_i', b_{-i}] - c_i(b_i') \quad \text{for all } b_i' \in B_i,$$

where $E[\cdot \,|\, b]$ denotes the expectation operator conditional on b, i.e., for every function $\xi : \Omega \to R$,

$$E[\xi(\omega) \,|\, b] \equiv \sum_{\omega \in \Omega} \xi(\omega) f(\omega \,|\, b).$$

Lemma 1: *Consider an arbitrary combination of an action profile and an allocation rule* (b,g). *There exists a payment rule* x *such that* (g,x) *induces* b *if and only if there exists a function* $w=(w_i)_{i\in N}:\Omega\to R^n$ *such that*

(3) $$E[w_i(\omega)\,|\,b]-c_i(b_i)\geq E[w_i(\omega)\,|\,b_i',b_{-i}]-c_i(b_i') \text{ for all } i\in N \text{ and } b_i'\in B_i.$$

Proof: Consider an arbitrary $(b,(g,x))$. We specify w by

$$w_i(\omega)=v_i(g(\omega),\omega)-x_i(\omega) \text{ for all } i\in N \text{ and } \omega\in\Omega.$$

It is clear from Definition 1 that (g,x) induces b if and only if (3) holds.

Q.E.D.

Let $\operatorname{int}\Delta(\Omega)\subset\Delta(\Omega)$ denote the set of all full-support distributions over states. We introduce a condition on an action profile $b\in B$, namely *richness*, as follows.

Definition 2 (Richness): An action profile $b\in B$ is said to be *rich* if for every $i\in N$ and $\delta\in\Delta(\Omega)$, there exist $\bar{\alpha}>0$ and a path on B_i, $\beta_i(\delta,\cdot):[-\bar{\alpha},\bar{\alpha}]\to B_i$, such that

$$\beta_i(\delta,0)=b_i,$$

(4) $$\lim_{\alpha\to 0}\frac{f(\cdot\,|\,\beta_i(\delta,\alpha),b_{-i})-f(\cdot\,|\,b)}{\alpha}=\delta(\cdot)-f(\cdot\,|\,b),$$

and $c_i(\beta_i(\delta,\alpha))$ is differentiable in α at $\alpha=0$.[8]

Richness implies that each agent $i\in N$ has a variety of action choices around b_i that can smoothly and locally change the distribution over states in all directions from $f(\cdot\,|\,b)$ at a differentiable cost.[9]

[8] This study assumes that the state space is finite. We, however, can eliminate this assumption without substantial changes. Instead of the finite state space, we can assume that the state space is a subset of a multi-dimensional Euclidian space and the distribution over states is continuous. In fact, we can obtain the uniqueness results such as Lemmas 2 and 3 almost everywhere.

[9] Section 8 will replace Definition 2 with an alternative that concerns only finitely many directions.

Under the richness, if (g,x) induces b, i.e., b is a Nash equilibrium in the game implied by (g,x), the following *first order condition* holds for every $i \in N$ and $\delta \in \Delta(\Omega)$:

$$\frac{\partial}{\partial \alpha}\left\{E[v_i(g(\omega),\omega) - x_i(\omega) \mid \beta_i(\delta,\alpha), b_{-i}] - c_i(\beta_i(\delta,\alpha))\right\}\Big|_{\alpha=0} = 0.$$

Furthermore, the richness assures that each agent can change the expected value of any non-constant payment rules by making a unilateral deviation. Consider an arbitrary non-constant function $\xi : \Omega \to R$. Note that there exists $\bar{\omega} \in \Omega$ such that $\xi(\bar{\omega}) > E[\xi(\omega) \mid b]$. Let $\delta^{\bar{\omega}}$ denote the degenerate distribution where $\delta^{\bar{\omega}}(\bar{\omega}) = 1$. Taking $\beta_i(\delta^{\bar{\omega}},\cdot) : [-\bar{\alpha},\bar{\alpha}] \to B_i$ as defined in Definition 2, we have

$$\frac{\partial E[\xi(\omega) \mid \beta_i(\delta^{\bar{\omega}},\alpha), b_{-i}]}{\partial \alpha}\Big|_{\alpha=0} = \sum_{\omega\in\Omega} \xi(\omega) \cdot \lim_{\alpha\to 0} \frac{f(\cdot \mid \beta_i(\delta,\alpha), b_{-i}) - f(\cdot \mid b)}{\alpha}$$

$$= \sum_{\omega\in\Omega} \xi(\omega)\{\delta^{\bar{\omega}}(\omega) - f(\omega \mid b)\} = \xi(\bar{\omega}) - E[\xi(\omega) \mid b] > 0.$$

Hence, each agent can make a unilateral deviation to take advantage of non-constant ξ.

In the single-agent case, where we regard the set of all actions of agent 1 as the set of all full-support distributions over states, i.e., $B_1 = \operatorname{int}\Delta(\Omega)$, the well-known *directional differentiability* for arbitrary directions will be a tractable sufficient condition for the richness.[10][11]

Proposition 1: *In the single-agent case, an action $\delta \in B_1 = \operatorname{int}\Delta(\Omega)$ is rich if for every $\delta' \in \Delta(\Omega)$, the agent's cost function $c_1 : \operatorname{int}\Delta(\Omega) \to R$ has the directional derivative along $\delta' - \delta$:*

---

[10] The existence of directional derivatives of c_1 at δ for all directions is implied by the total differentiability of c_1 at δ.

[11] When Ω is a continuum, the directional differentiability is generalized to the Gâteaux differentiability.

$$\nabla_{\delta'-\delta} c_1(\delta) \equiv \lim_{\varepsilon \to 0} \frac{c_1((1-\varepsilon)\delta + \varepsilon\delta') - c(\delta)}{\varepsilon}.$$

Proof: For every $\delta' \in \Delta(\Omega)$, specify $\beta_1(\delta', \cdot) : [-\bar{\alpha}, \bar{\alpha}] \to B_i$ by

$$\beta_1(\delta', \alpha) = (1-\alpha)\delta + \alpha\delta' \text{ for all } \alpha \in [-\bar{\alpha}, \bar{\alpha}],$$

where $\bar{\alpha} > 0$ is selected sufficiently small so that $\beta_1(\delta', \alpha) \in \operatorname{int} \Delta(\Omega) = B_1$ holds for all $\alpha \in [-\bar{\alpha}, \bar{\alpha}]$ (since δ is a full-support distribution, such $\bar{\alpha} > 0$ always exists). Then, it follows from

$$\lim_{\alpha \to 0} \frac{c_1(\beta_1(\delta', \alpha)) - c_1(\delta)}{\alpha} = \lim_{\alpha \to 0} \frac{c_1((1-\alpha)\delta' + \alpha\delta) - c_1(\delta)}{\alpha} = \nabla_{\delta'-\delta} c_1(\delta)$$

that the directional differentiability of c_1 along $\delta' - \delta$ assures the differentiability of $c_1(\beta_1(\delta', \alpha))$ in α at $\alpha = 0$. Hence, δ is rich.

Q.E.D.

Note that the converse of Proposition 1 does not hold, because the directional differentiability does not account for the differentials along curves whose tangent at b_i satisfies (4). Note also that it follows from Proposition 1 that in this single-agent case, if c_1 is directionally differentiable everywhere on B_1, then *every action* $b_1 \in B_1$ *is rich.*

We further consider the following class of *multi-agent* problems. For every $i \in N$, specify

$$B_i = \operatorname{int} \Delta(\Omega) \times B_i^2,$$

where we denote $b_i = (b_i^1, b_i^2) \in \operatorname{int} \Delta(\Omega) \times B_i^2$. Each agent $i \in N$ makes a *recommendation* about the state distribution as the first component of her action, $b_i^1 \in \operatorname{int} \Delta(\Omega)$, and *lobbies* her recommendation as the second component of her action, $b_i^2 \in B_i^2$. Denote $b^1 = (b_i^1)_{i \in N}$ and $b^2 = (b_i^2)_{i \in N}$. Define

$$f(\cdot \mid b) = \sum_{i \in N} \gamma_i(b^2) b_i^1(\cdot),$$

where $\gamma_i(b^2) \in (0,1)$ and $\sum_{i \in N} \gamma_i(b^2) = 1$. The state distribution is determined as the *compromise* among the agents' recommendations, i.e., the average of their recommendations b^1 weighted by each agent's *influence* $(\gamma_i(b^2))_{i \in N}$, which is the consequent of the agents' lobbying activities. We assume that for every $i \in N$ and $b_i \in B_i$, $c_i(\cdot, b_i^2)$ is directionally differentiable.

In the same manner as Proposition 1, we can prove that *every action profile* $b \in B$ *is rich* in this case. Regardless of the action profile $b \in B$, each agent $i \in N$ can smoothly and locally change the distribution in all directions from $f(\cdot \mid b)$ by manipulating her recommendation (the first component) $b_i^1 \in \operatorname{int} \Delta(\Omega)$. Note that the class of problems discussed here includes substantially general multi-agent problems. In fact, it does not require any restriction on the lobbying action spaces $(B_i^2)_{i \in N}$, the influence functions $(\gamma_i)_{i \in N}$, and the cost functions $(c_i)_{i \in N}$, besides the directional differentiability of $(c_i(\cdot, b_i^2))_{i \in N}$.

The following lemma implies that with richness, the function w that satisfies (3) is *unique* up to constants.

Lemma 2: *Suppose that an action profile* b *is rich, and a function* w *satisfies (3). For every function* $\tilde{w} = (\tilde{w}_i)_{i \in N} : \Omega \to R^n$, $\tilde{w}$ *satisfies the properties implied by (3), i.e., for every* $i \in N$,

$$E[\tilde{w}_i(\omega) \mid b] - c_i(b_i) \geq E[\tilde{w}_i(\omega) \mid b_i', b_{-i}] - c_i(b_i') \quad \textit{for all} \quad b_i' \in B_i,$$

if and only if there exists a vector $z = (z_i)_{i \in N} \in R^n$ *such that*

$$\tilde{w}_i(\omega) = w_i(\omega) + z_i \quad \textit{for all} \quad i \in N \quad \textit{and} \quad \omega \in \Omega.$$

Proof: The proof of the sufficiency is straightforward. We present the proof of the necessity as follows. Take an arbitrary w which satisfy (3) and consider an arbitrary agent $i \in N$. Take an arbitrary $\tilde{w}_i$ such that $\xi \equiv \tilde{w}_i - w_i$ is a non-constant function.

Then, there exists $\varpi \in \Omega$ such that $\xi(\varpi) > E[\xi(\omega) | b]$. Let δ^{ϖ} denote the degenerate distribution where $\delta^{\varpi}(\varpi) = 1$. Due to richness, there exist $\bar{\alpha} > 0$ and $\beta_i(\delta^{\varpi}, \cdot) : [-\bar{\alpha}, \bar{\alpha}] \to B_i$ such that

$$\lim_{\alpha \to 0} \frac{f(\cdot | \beta_i(\delta^{\varpi}, \alpha), b_{-i}) - f(\cdot | b)}{\alpha} = \delta^{\varpi}(\cdot) - f(\cdot | b).$$

Since w satisfies (3), the first order condition along $\beta_i(\delta^{\varpi}, \cdot)$ must hold, i.e.,

(5) $$\frac{\partial}{\partial \alpha} \left\{ E[w_i(\omega) | \beta_i(\delta^{\varpi}, \alpha), b_{-i}] - c_i(\beta_i(\delta^{\varpi}, \alpha)) \right\} \Big|_{\alpha = 0} = 0.$$

On the other hand,

$$\frac{\partial}{\partial \alpha} \left\{ E[\tilde{w}_i(\omega) | \beta_i(\delta^{\varpi}, \alpha), b_{-i}] - c_i(\beta_i(\delta^{\varpi}, \alpha)) \right\} \Big|_{\alpha = 0}$$

$$= \frac{\partial}{\partial \alpha} \left\{ E[\xi(\omega) + w_i(\omega) | \beta_i(\delta^{\varpi}, \alpha), b_{-i}] - c_i(\beta_i(\delta^{\varpi}, \alpha)) \right\} \Big|_{\alpha = 0}$$

$$= \frac{\partial}{\partial \alpha} \left\{ E[\xi(\omega) | \beta_i(\delta^{\varpi}, \alpha), b_{-i}] \right\} \Big|_{\alpha = 0} = \xi(\varpi) - E[\xi(\omega) | b] > 0.$$

Hence, if $\xi \equiv \tilde{w}_i - w_i$ is non-constant, agent i has incentive to increase α along $\beta_i(\delta^{\varpi}, \cdot)$ from $\alpha = 0$. Accordingly, whenever w and $\tilde{w}$ satisfy (3), $\xi \equiv \tilde{w}_i - w_i$ is constant, i.e., there exists $z \in R^n$ such that $\tilde{w}(\omega) = w(\omega) + z$ for all $\omega \in \Omega$.

Q.E.D.

Based on Lemmas 1 and 2, we show the following theorem, which states that *the payment rule that guarantees inducibility is unique up to constants*.

Theorem 1: *Consider an arbitrary combination of an action profile and a mechanism* $(b, (g, x))$. *Suppose that* b *is rich and* (g, x) *induces* b. *Accordingly, for every payment rule* $\tilde{x}$, *the associated mechanism* $(g, \tilde{x})$ *induces* b *if and only if there exists a vector* $z \in R^n$ *such that*

$$x(\omega) - \tilde{x}(\omega) = z \quad \textit{for all} \quad \omega \in \Omega .$$

Proof: The proof of the sufficiency is straightforward. We present the proof of the necessity as follows. Suppose that (g, x) induces b. According to the proof of Lemma 1, we specify w by

(6) $$w_i(\omega) = v_i(g(\omega), \omega) - x_i(\omega) \text{ for all } i \in N \text{ and } \omega \in \Omega .$$

Suppose also that $(g, \tilde{x})$ induces b. Similarly, we specify $\tilde{w}$ by

(7) $$\tilde{w}_i(\omega) = v_i(g(\omega), \omega) - \tilde{x}_i(\omega) \text{ for all } i \in N \text{ and } \omega \in \Omega .$$

Lemma 2 implies that there exists $z \in R^n$ such that $\tilde{w}(\omega) - w(\omega) = z$ for all $\omega \in \Omega$. By subtracting (6) from (7), we have

$$x(\omega) - \tilde{x}(\omega) = z \quad \text{for all} \quad \omega \in \Omega .$$

Q.E.D.

Theorem 1 implies the following *equivalence properties in the ex-post term*. Consider an arbitrary combination of an action profile and an allocation rule (b, g). Consider two arbitrary payment rules x and $\tilde{x}$ such that both (g, x) and $(g, \tilde{x})$ induce b. Let $U_i \in R$ and $\tilde{U}_i \in R$ denote the respective ex-ante expected payoff for each agent $i \in N$:

$$U_i \equiv E[v_i(g(\omega), \omega) - x_i(\omega) \mid b] - c_i(b_i) ,$$

and

$$\tilde{U}_i \equiv E[v_i(g(\omega), \omega) - \tilde{x}_i(\omega) \mid b] - c_i(b_i) .$$

Accordingly, Theorem 1 exhibits that *the ex-post payment for each agent* i *is unique up to constants* in that

$$\tilde{x}_i(\omega) = x_i(\omega) - U_i + \tilde{U}_i \quad \text{for all} \quad \omega \in \Omega ,$$

the ex-post revenue for the central planner is unique up to constants in that

$$\sum_{i \in N} \tilde{x}_i(\omega) = \sum_{i \in N} x_i(\omega) - \sum_{i \in N} (U_i - \tilde{U}_i) \quad \text{for all} \quad \omega \in \Omega ,$$

and *the ex-post payoff for each agent* i *is unique up to constants* in that

$$v_i(g(\omega),\omega)-\tilde{x}_i(\omega)-c_i(b_i)=\{v_i(g(\omega),\omega)-x_i(\omega)-c_i(b_i)\}+U_i-\tilde{U}_i$$

for all $\omega\in\Omega$.

4. Hidden Information

Let us specify an information structure where the state $\omega\in\Omega$ is decomposed as

$$\omega=(\omega_0,\omega_1,...,\omega_n).$$

Here, we call ω_0 a *public signal* and ω_i a *type for each agent* $i\in N$. Let Ω_0 denote the set of all public signals and Ω_i denote the set of all types for each agent $i\in N$. Let $\Omega\equiv\underset{i\in N\cup\{0\}}{\times}\Omega_i$.

We assume that the public signal $\omega_0\in\Omega_0$ becomes observable to all agents as well as the central planner *just before* the central planner determines an allocation and side payments, and it is, therefore, contractible. The central planner, however, cannot observe the profile of all agents' types, which is denoted by $\omega_{-0}=(\omega_i)_{i\in N}\in\Omega_{-0}\equiv\underset{i\in N}{\times}\Omega_i$. Each agent $i\in N$ can observe her type $\omega_i\in\Omega_i$, but cannot observe the profile of the other agents' types, denoted by $\omega_{-i}=(\omega_j)_{j\in N\cup\{0\}\backslash\{i\}}\in\Omega_{-i}\equiv\underset{j\in N\cup\{0\}\backslash\{i\}}{\times}\Omega_j$.

Because of the above-mentioned hidden information structure, we replace stages 3 and 4 with the following stages (i.e., stages 3' and 4', respectively). Importantly, the central planner requires each agent to reveal her type, which the central planner cannot directly observe.[12]

[12] Because the central planner induces a pure action profile, we can safely focus on revelation mechanisms where each agent only reports her type. If the central planner attempts to induce a mixed action profile, we need to consider mechanisms where each agent reports not only her own type but also her selection of pure action. See Obara (2008). For further discussions, see Footnote 17.

Stage 3': The state $\omega = (\omega_0, \omega_1, \cdots, \omega_n) \in \Omega$ is randomly drawn according to the conditional probability function $f(\cdot \mid b) \in \Delta(\Omega)$, where $b \in B$ is the action profile selected at stage 2. Each agent $i \in N$ observes his type $\omega_i \in \Omega_i$, but cannot observe $\omega_{-i} \in \Omega_{-i}$ at this stage.

Stage 4': Each agent $i \in N$ announces $\tilde{\omega}_i \in \Omega_i$ about her type. Afterward, all agents, as well as the central planner, observe the public signal $\omega_0 \in \Omega_0$. According to the profile of the agents' announcements $\tilde{\omega}_{-0} = (\tilde{\omega}_i)_{i \in N} \in \Omega_{-0}$ and the observed public signal $\omega_0 \in \Omega_0$, the central planner determines the allocation $g(\omega_0, \tilde{\omega}_{-0}) \in A$ and the side payment vector $x(\omega_0, \tilde{\omega}_{-0}) \in R^n$. The resultant payoff of each agent i is given by

$$v_i(g(\omega_0, \tilde{\omega}_{-0}), \omega) - x_i(\omega_0, \tilde{\omega}_{-0}) - c_i(b_i).$$

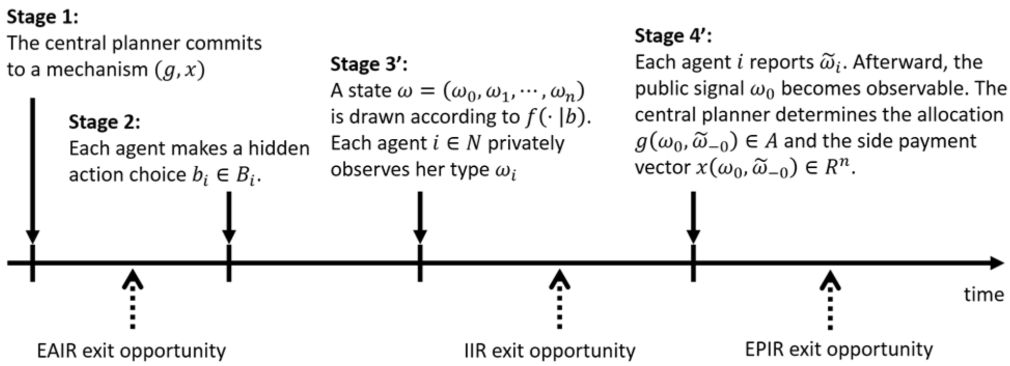

Figure 2: Timeline with Hidden Action and Hidden Information

Figure 2 describes the timeline of the model with hidden action and hidden information. We consider the agents' incentives in hidden information by introducing the concept of ex-post incentive compatibility.

Definition 3 (Ex-Post Incentive Compatibility): A mechanism (g,x) is said to be *ex-post incentive compatible (hereafter EPIC)* if truth-telling is an ex-post equilibrium; for every $i \in N$ and $\omega \in \Omega$,

$$v_i(g(\omega),\omega) - x_i(\omega) \geq v_i(g(\tilde{\omega}_i,\omega_{-i}),\omega) - x_i(\tilde{\omega}_i,\omega_{-i}) \text{ for all } \tilde{\omega}_i \in \Omega_i .$$[13]

EPIC is independent of the action profile, because of the additive separability of the cost of actions. We further introduce a weaker notion, namely *Bayesian Implementability*, as follows.

Definition 4 (Bayesian Implementability): A combination of an action profile and a mechanism $(b,(g,x))$ is said to be *Bayesian implementable (hereafter BI)* if the selection of the action profile b at stage 2 and the truthful revelation at stage 4' results in a perfect Bayesian equilibrium; for every $i \in N$, every $b_i' \in B_i$, and every function $\sigma_i : \Omega_i \to \Omega_i$,

$$\begin{aligned} & E[v_i(g(\omega),\omega) - x_i(\omega) \mid b] - c_i(b_i) \\ & \geq E[v_i(g(\sigma_i(\omega_i),\omega_{-i}),\omega_{-i}) - x_i(\sigma_i(\omega_i),\omega_{-i}) \mid b_i', b_{-i}] - c_i(b_i') . \end{aligned}$$

BI includes the inducibility of b in order to account for the possibility of contingent deviations; BI requires $(b,(g,x))$ to exclude the possibility that each agent i benefits by deviating from both the action choice b_i at stage 2 and the truthful revelation at stage 4'.

The following theorem states that if there is a mechanism that induces an action profile b but fails to be incentive compatible, then it is generally impossible to discover mechanisms that satisfy both inducibility and incentive compatibility.

[13] Because of additive separability, the incentive compatibility is irrelevant to the shape of the cost functions.

Theorem 2: *Consider an arbitrary combination of an allocation rule and an action profile* (b, g)*, where we assume that* b *is rich.*

1. *Suppose that there exists a payment rule* x *such that* (g, x) *induces* b *and satisfies EPIC. For every payment rule* $\tilde{x}$*, whenever* $(g, \tilde{x})$ *induces* b*, it satisfies EPIC.*
2. *Suppose that there exists a payment rule* x *such that* $(b, (g, x))$ *satisfies BI. For every payment rule* $\tilde{x}$*, whenever* $(g, \tilde{x})$ *induces* b*,* $(b, (g, \tilde{x}))$ *satisfies BI.*

Proof: Suppose that both (g, x) and $(g, \tilde{x})$ induce b. From Theorem 1, there exists $z \in R^n$ such that

$$x(\omega) - \tilde{x}(\omega) = z \quad \text{for all} \quad \omega \in \Omega,$$

which implies that (g, x) satisfies EPIC if and only if $(g, \tilde{x})$ satisfies EPIC. We can similarly prove that $(b, (g, x))$ satisfies BI if and only if $(b, (g, \tilde{x}))$ satisfies BI.

Q.E.D.

Theorem 2 implies that the following two statements are equivalent.

(i) There exists a mechanism that satisfies inducibility but fails to satisfy incentive compatibility.

(ii) There exists no mechanism that satisfies both inducibility and incentive compatibility.

5. Efficiency

We denote by $v_0 : A \times \Omega \to R$ the valuation function for the central planner. This section and the next intensively study allocation rules and action profiles that are *efficient* (i.e., maximize the welfare that includes the central planner's welfare as well as the agents' welfare); an allocation rule g is said to be *efficient* if

$$\sum_{i\in N\cup\{0\}} v_i(g(\omega),\omega) \geq \sum_{i\in N\cup\{0\}} v_i(a,\omega) \text{ for all } a\in A \text{ and } \omega\in\Omega.$$

A combination of an action profile and an allocation rule (b,g) is said to be *efficient* if g is efficient and the selection of b maximizes the expected welfare:

$$E[\sum_{i\in N\cup\{0\}} v_i(g(\omega),\omega)\,|\,b] - \sum_{i\in N\cup\{0\}} c_i(b_i)$$

$$\geq E[\sum_{i\in N\cup\{0\}} v_i(g(\omega),\omega)\,|\,\tilde{b}] - \sum_{i\in N\cup\{0\}} c_i(\tilde{b}_i) \text{ for all } \tilde{b}\in B.$$

A payment rule x is said to be *Groves* if there exists $y_i:\Omega_{-i}\to R$ for each $i\in N$ such that

$$x_i(\omega) = -\sum_{j\in N\cup\{0\}\setminus\{i\}} v_j(g(\omega),\omega) + y_i(\omega_{-i}) \text{ for all } i\in N \text{ and } \omega\in\Omega.$$

According to a Groves payment rule, the central planner gives each agent $i\in N$ the monetary amount that is equivalent to the other agents' welfare plus the central planner's welfare (i.e., $\sum_{j\in N\cup\{0\}\setminus\{i\}} v_j(g(\omega),\omega)$), and imposes on the agent the monetary payment $y_i(\omega_{-i})$ that is independent of ω_i.

A payment rule x is said to be *pure Groves* if it is Groves and $y_i(\cdot)$ is *constant* for each $i\in N$; there exists a vector $z=(z_i)_{i\in N}\in R^n$ such that

$$x_i(\omega) = -\sum_{j\in N\cup\{0\}\setminus\{i\}} v_j(g(\omega),\omega) + z_i \text{ for all } i\in N \text{ and } \omega\in\Omega.$$

Pure Grove payment rules are special cases of Grove payment rules where the central planner imposes a fixed amount z_i on each agent i as a non-incentive term.

The following theorem states that an efficient mechanism induces an efficient action profile if and only if the payment rule is pure Groves. Accordingly, *under efficiency and inducibility, without loss of generality, we can focus on pure Groves mechanisms (i.e., combinations of an efficient allocation rule and a pure Groves payment rule).*[14]

[14] This study makes a slight extension of the canonical definition of a Groves mechanism, by accounting for the valuations of the central planner.

Theorem 3: *Suppose that* (b,g) *is efficient. For every payment rule* x, (g,x) *induces* b *if* x *is pure Groves. Suppose that* b *is rich and* (b,g) *is efficient. For every payment rule* x, (g,x) *induces* b *if and only if* x *is pure Groves.*

Proof: If (b,g) is efficient, for every $i \in N$ and $b_i' \in B_i$,

$$E[v_i(g(\omega),\omega) - x_i(\omega) \mid b] - E[v_i(g(\omega),\omega) - x_i(\omega) \mid b_i', b_{-i}]$$
$$= E[\sum_{j \in N \cup \{0\}} v_j(g(\omega),\omega) \mid b] - E[\sum_{j \in N \cup \{0\}} v_j(g(\omega),\omega) \mid b_i', b_{-i}] \geq 0,$$

which implies the inducibility of b. If b is rich, it is clear from Theorem 2 and the definition of the pure Groves payment rule that any payment rule that guarantees inducibility must be pure Groves.

Q.E.D.

Note that we can construct any pure Groves mechanism without detailed knowledge of (f,B,c). Even if the central planner is allowed to utilize such knowledge, the central planner's best choice would be pure Groves.

The following theorem demonstrates a necessary and sufficient condition for the satisfaction of both inducibility and incentive compatibility on the assumption of efficiency and richness.

Theorem 4: *Suppose that* b *is rich and* (b,g) *is efficient. There exists a payment rule* x *such that* (g,x) *induces* b *and satisfies EPIC if and only if for every* $i \in N$, $\omega \in \Omega$, *and* $\tilde{\omega}_i \in \Omega_i$,

(8)
$$\sum_{j \in N \cup \{0\}} v_j(g(\omega),\omega) \geq v_i(g(\tilde{\omega}_i,\omega_{-i}),\omega) + \sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(g(\tilde{\omega}_i,\omega_{-i}),\tilde{\omega}_i,\omega_{-i}).$$

There exists a payment rule x *such that* $(b,(g,x))$ *satisfies BI if and only if for every* $i \in N$, $b_i' \in B_i$, *and* $\sigma_i : \Omega_i \to \Omega_i$,

(9) $$E[\sum_{j\in N\bigcup\{0\}} v_j(g(\omega),\omega)\,|\,b]-c_i(b_i)\geq E[v_i(g(\sigma_i(\omega_i),\omega_{-i}),\omega)$$
$$+\sum_{j\in N\bigcup\{0\}\backslash\{i\}} v_j(g(\sigma_i(\omega_i),\omega_{-i}),\sigma_i(\omega_i),\omega_{-i})\,|\,b_i',b_{-i}]-c_i(b_i').$$

Proof: Consider the proof for the case of EPIC. From Theorem 3, this proof only requires us to clarify a necessary and sufficient condition for a pure Grove payment rule to guarantee EPIC. Let x denote the pure Groves payment rule given by

$$x_i(\omega)=-\sum_{j\in N\bigcup\{0\}\backslash\{i\}} v_j(g(\omega),\omega) \text{ for all } i\in N \text{ and } \omega\in\Omega.$$

Accordingly, (g,x) satisfies EPIC if and only if for every $i\in N$, $\omega\in\Omega$, and $\tilde{\omega}_i\in\Omega_i$,

$$v_j(g(\omega),\omega)-x_i(\omega)=\sum_{j\in N\bigcup\{0\}} v_j(g(\omega),\omega)$$
$$\geq v_i(g(\tilde{\omega}_i,\omega_{-i}),\omega)+\sum_{j\in N\bigcup\{0\}\backslash\{i\}} v_j(g(\tilde{\omega}_i,\omega_{-i}),\tilde{\omega}_i,\omega_{-i})$$
$$=v_i(g(\tilde{\omega}_i,\omega_{-i}),\omega)-x_i(\tilde{\omega}_i,\omega),$$

which is equivalent to (8). We can prove the case of BI in a similar manner.

Q.E.D.

Note that (8) is a necessary and sufficient condition for a Groves mechanism to satisfy EPIC. However, if each agent's payoff function has interdependent values, any Groves mechanism does not satisfy EPIC, and, therefore, (8) generally fails.[15] Based on this outcome, we shall focus on the case of *private values*, where for every $i\in N$, the valuation $v_i(a,\omega)$ is independent of the profile of the other agents' types $\omega_{-0-i}\equiv(\omega_j)_{j\in N\backslash\{i\}}\in\Omega_{-0-i}\equiv\times_{j\in N\backslash\{i\}}\Omega_j$, and $v_0(a,\omega)$ is independent of ω_{-0}. With private values, any Groves mechanism satisfies (8) (i.e., EPIC). With private values, we write

[15] Maskin (1992), Dasgupta and Maskin (2000), and Bergemann and Välimäki (2002) proposed the generalized VCG mechanism and showed that it satisfies EPIC for some environments, even with interdependent values. The generalized VCG mechanism, however, fails to induce an efficient action profile, because it is not pure Groves. Mezzetti (2004), however, showed that if the realized valuation $v_i(g(\omega),\omega)$ is observable as an ex-post public signal and is contractible, we can achieve efficiency with EPIC even with interdependent values. See Noda (2016) for more general ex-post signals.

$v_i(a,\omega_0,\omega_i)$ instead of $v_i(a,\omega)$ for each $i\in N$, and $v_0(a,\omega_0)$ instead of $v_0(a,\omega)$.[16] With private values, EPIC is equivalent to *incentive compatibility in a dominant strategy (thereafter DIC)*, where for every $i\in N$, $\omega\in\Omega$, and $\tilde{\omega}_i\in\Omega_i$,

$$v_i(g(\omega),\omega_0,\omega_i)-x_i(\omega)\geq v_i(g(\tilde{\omega}_i,\omega_{-i}),\omega_0,\omega_i)-x_i(\tilde{\omega}_i,\omega_{-i}).$$

Theorem 5: *Suppose that (b,g) is efficient. With private values, for every payment rule x, (g,x) induces b and satisfies DIC if x is pure Groves. Suppose that b is rich and (b,g) is efficient. With private values, for every payment rule x, (g,x) induces b and satisfies DIC if and only if x is pure Groves.*

Proof: From efficiency, on the assumption of private values, inequalities (8) always hold. Because, with richness, inequalities (8) are necessary and sufficient for a Groves mechanism to satisfy DIC, Theorems 3 and 4 imply the latter part of Theorem 5.

Q.E.D.

With private values, any DIC mechanism that satisfies inducibility and efficiency must be pure Groves, although any Groves mechanism generally satisfies DIC and efficiency. For example, consider the *VCG mechanism* (g,x), which is defined as an efficient (Groves) mechanism specified by

$$x_i(\omega)=-\sum_{j\in N\cup\{0\}\setminus\{i\}}v_i(g(\omega),\omega_0,\omega_j)+\min_{\omega_i'\in\Omega_i}\sum_{j\in N\cup\{0\}}v_i(g(\omega_i',\omega_{-i}),\omega_0,\omega_j)$$

for all $i\in N$ and $\omega\in\Omega$.

Clearly, the VCG mechanism is not pure Groves; thus, it fails to satisfy the inducibility of efficient action profile.[17]

---

[16] We permit the valuations to depend on the public signal ω_0. For convenience, we sometimes write $v_0(a,\omega_0,\omega_0)$ instead of $v_0(a,\omega_0)$.

[17] Obara (2008) showed that, when the action space is finite, full surplus extraction is achievable in approximation, under a weaker condition than exact full surplus extraction. By carefully mixing

6. Revenues and Deficits

By assuming private values, this section determines whether the central planner can achieve efficiency even *without deficits*. We define the central planner's ex-post revenue by

(10) $$v_0(g(\omega),\omega_0)+\sum_{i\in N}x_i(\omega)\,,$$[18]

and the expected revenue in the ex-ante term by

$$E[v_0(g(\omega),\omega_0)+\sum_{i\in N}x_i(\omega)\,|\,b]\,.$$

We introduce three concepts of individual rationality as follows. The timings of exit opportunities are depicted in Figure 2.

Definition 5 (Ex-Ante Individual Rationality): A combination of an action profile and a mechanism $(b,(g,x))$ is said to satisfy *ex-ante individual rationality (hereafter EAIR)* if

$$E[v_i(g(\omega),\omega_0,\omega_i)-x_i(\omega)\,|\,b]-c_i(b_i)\geq 0 \quad \text{for all} \quad i\in N\,.$$

Definition 6 (Interim Individual Rationality): A combination of an action profile and a mechanism $(b,(g,x))$ is said to satisfy *interim individual rationality (hereafter IIR)* if

$$E_{\omega_{-i}}[v_i(g(\omega_i,\omega_{-i}),\omega_0,\omega_i)-x_i(\omega_i,\omega_{-i})\,|\,b,\omega_i]\geq 0$$

for all $i\in N$ and $\omega_i\in\Omega_i$,

where

$$E_{\omega_{-i}}[\xi(\tilde{\omega}_i,\omega_{-i})\,|\,b,\omega_i]\equiv\sum_{\omega_{-i}\in\Omega_{-i}}\xi(\tilde{\omega}_i,\omega_{-i})f_{-i}(\omega_{-i}\,|b,\omega_i)$$

---

agents' actions, Obara generated a correlation between (b_i,ω_i) and (b_{-i},ω_{-i}), and then applied the technique of Cremer and McLean (1985, 1988). Obara's argument, however, required extremely large side payments and very detailed knowledge about model specifications.

[18] Note that the central planner's revenue includes not only payments from agents, but also the valuation of the central planner. In the single agent case, where the allocation space is degenerate, the value of (10) corresponds to the principal's payoff in a standard principal-agent model with risk-neutrality.

denotes the expectation of a function $\xi(\tilde{\omega}_i, \cdot) : \Omega_{-i} \to R$ conditional on (b, ω_i).

Definition 7 (Ex-Post Individual Rationality): A mechanism (g, x) is said to satisfy *ex-post individual rationality (hereafter EPIR)* if

$$v_i(g(\omega), \omega_0, \omega_i) - x_i(\omega) \geq 0 \quad \text{for all} \quad i \in N \quad \text{and} \quad \omega \in \Omega .$$

EPIR automatically implies IIR. IIR, however, does not necessarily imply EAIR, because the cost for the action choice at stage 2 is a sunk cost. The following proposition shows that EPIR implies EAIR.

Proposition 2: *Suppose that* (g, x) *induces* b. *Whenever* (g, x) *satisfies EPIR,* $(b, (g, x))$ *satisfies EAIR.*

Proof: Because $c_i(b_i^0) = 0$, it follows from EPIR and inducibility that

$$E[v_i(g(\omega), \omega_0, \omega_i) - x_i(\omega) \mid b] - c_i(b_i)$$
$$\geq E[v_i(g(\omega), \omega_0, \omega_i) - x_i(\omega) \mid b_i^0, b_{-i}] - c_i(b_i^0) \geq 0 ,$$

which implies EAIR.

Q.E.D.

EPIR is the strongest requirement for voluntary participation among the above three concepts. The purpose of this section is to show that EPIR is not compatible with the non-negativity of revenue in expectation.

The following proposition calculates the maximal expected revenues (i.e., the least upper bounds of the central planner's expected revenues).

Proposition 3: *Suppose that* (b,g) *is efficient,* b *is rich, and the private value assumption is satisfied. Then, the maximal expected revenue from the mechanism* (g,x) *that induces* b *and satisfies EPIR is given by*

(11) $$R^{EPIR} \equiv n \min_{\omega\in\Omega} \sum_{j\in N\cup\{0\}} v_j(g(\omega),\omega_0,\omega_j) - (n-1)E\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega),\omega_0,\omega_j)\middle| b\right].$$

The maximal expected revenue from the mechanism (g,x) *that induces* b *and satisfies IIR is given by*

(12) $$R^{IIR} \equiv \sum_{i\in N} \min_{\omega_i\in\Omega_i} E_{\omega_{-i}}\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega_i,\omega_{-i}),\omega_0,\omega_j)\middle| b,\omega_i\right]$$
$$-(n-1)E\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega),\omega_0,\omega_j)\middle| b\right].$$

The maximal expected revenue from the mechanism (g,x) *that induces* b *and satisfies EAIR is given by*

(13) $$R^{EAIR} \equiv E\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega),\omega_0,\omega_j)\middle| b\right] - \sum_{j\in N} c_j(b_j).$$

The maximal expected revenue from the mechanism (g,x) *that induces* b *and satisfies IIR and EAIR is given by*

(14) $$R^{EAIR,IIR} \equiv E\left[v_0(g(\omega),\omega_0)\middle| b\right] + \sum_{i\in N} \min\Big\{E\left[v_i(g(\omega),\omega_0,\omega_i)\middle| b\right] - c_i(b_i),$$
$$\min_{\omega_i\in\Omega_i} E_{\omega_{-i}}\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega_i,\omega_{-i}),\omega_0,\omega_j)\middle| b,\omega_i\right] - E\left[\sum_{j\in N\cup\{0\}\setminus\{i\}} v_j(g(\omega),\omega_0,\omega_j)\middle| b\right]\Big\}.$$

Proof: See the Appendix.

Clearly, R^{EAIR} is equal to the *maximized expected social welfare*. From the relative strength of incentive compatibility constraints, it is also clear that

$$R^{EPIR} \le R^{IIR,EAIR} \le R^{IIR} \text{ and } R^{IIR,EAIR} \le R^{EAIR}.$$

However, which is greater between R^{IIR} and R^{EAIR} depends on specifications.

The following proposition indicates that *with the constraints of EPIR, it is generally difficult for the central planner to achieve efficiency without deficits.*

Proposition 4: *Suppose that (b, g) is efficient, b is rich, and the private value assumption is satisfied. Suppose also that $\sum_{i \in N} c_i(b_i) > 0$ and there exists a null state $\underline{\omega} = (\underline{\omega}_0, ..., \underline{\omega}_n) \in \Omega$ in the sense that*

$$v_0(a, \underline{\omega}_0) = 0 \quad \textit{for all} \quad a \in A,$$

and for every $i \in N$,

$$v_i(a, \omega_0, \underline{\omega}_i) = 0 \quad \textit{for all} \quad a \in A \quad \textit{and} \quad \omega_0 \in \Omega_0.$$

With EPIR, the central planner has a deficit in expectation: $R^{EPIR} < 0$.

Proof: See the Appendix.

If $R^{EAIR} < 0$, the conclusion is immediate from $R^{EPIR} \leq R^{EAIR}$. Suppose $R^{EAIR} \geq 0$. It follows from $\sum_{i \in N} c_i(b_i) > 0$ that the second term of R^{EPIR} in (11) is negative. Due to the presence of the null state, the first term of R^{EPIR} in (11) is non-positive. Accordingly, R^{EPIR} is negative.

With private values, the VCG mechanism generally satisfies EPIR (and DIC) and guarantees that the central planner will earn a non-negative ex-post revenue at all times. In contrast, once we require inducibility, the VCG mechanism does not function, and the central planner fails to earn non-negative revenue, even with an ex-ante expectation.

By replacing EPIR with weaker constraints, such as IIR and EAIR, and adding some restrictions, it becomes much easier for the central planner to achieve efficiency without deficits. The following proposition states that the maximal expected revenue of the central planner is irrelevant to whether inducibility is required.

Proposition 5: *Suppose that* (b, g) *is efficient,* b *is rich, and the private value assumption is satisfied. Suppose also that we have:*

Conditional Independence: *For every* $\tilde{b} \in B$ *and* $\omega \in \Omega$,

$$f(\omega \mid \tilde{b}) = \prod_{i \in N \cup \{0\}} f_i(\omega_i \mid \tilde{b}),$$

where $f_i(\omega_i \mid \tilde{b})$ *denotes the probability of* $\tilde{\omega}_i$ *occurring when the agents select the action profile* $\tilde{b}$. *The expected revenue achieved by any Groves mechanism that satisfies IIR and EAIR is less than or equal to* $R^{IIR,EAIR}$. *Furthermore, there exists a pure Groves mechanism that satisfies IIR and EAIR and achieves* $R^{IIR,EAIR}$.

Proof: See the Appendix.

We might expect that the central planner can receive larger expected revenue than $R^{IIR,EAIR}$ once we remove the requirement of inducibility. However, with conditional independence, no Grove mechanism is able to make the expected revenue greater than $R^{IIR,EAIR}$. This implies that $R^{IIR,EAIR}$ *is the upper bound of expected revenue regardless of whether we require inducibility*.

Proposition 5 is related to the observation from risk-sharing in a classical principal-agent model. When the principal is risk-averse but the agent is risk-neutral, the principal's best choice is to sell the company to the agent by giving the entire outcome (externalities to the principal) in exchange for a fixed constant fee.[19] By regarding pure Groves mechanisms as an extension of such selling-out contracts, Proposition 5 indicates that *with richness, selling the company is the best choice even if the principal is risk-neutral.*

It is well-accepted that efficiency is achievable by a Groves mechanism without running expected deficits if we do not require inducibility. With conditional

[19] See Harris and Raviv (1979), for example.

independence and some moderate restrictions, we can guarantee the non-negativity of $R^{IIR,EAIR}$ as follows.

Proposition 6: *Assume the suppositions in Proposition 4, conditional independence, and the following conditions.*

Non-Negative Valuation: *For every* $i \in N \bigcup \{0\}$ *and* $\omega \in \Omega$,

$$v_i(g(\omega), \omega_0, \omega_i) \geq 0 .$$

Non-Negative Expected Payoff: *For every* $i \in N$,

(15) $$E[v_i(g(\omega), \omega_0, \omega_j) \mid b] - c_i(b_i) \geq 0 .$$

With IIR and EAIR, the central planner has non-negative expected revenue: $R^{IIR,EAIR} \geq 0$.

Proof: See the Appendix.

Non-negative valuation excludes the case of bilateral bargaining addressed by Myerson and Satterthwaite (1983), where it is impossible for the central planner to achieve efficiency without deficits. Non-negative expected payoff excludes the case of opportunism in the hold-up problem, where the sunk cost $c_i(b_i)$ is so significant that it violates inequality (15). By eliminating these cases, and replacing EPIR with IIR and EAIR, we can derive the possibility result in liability implied by Proposition 6.

7. No Externality

So far, we have assumed richness, i.e., that each agent's action choice provides a significant externality effect on the other agents' types and the public signal. This section assumes that no such externality exists.

Specifically, this section assumes *independence* of the information structure, requiring conditional independence and requiring that each agent i's action choice

$b_i \in B_i$ only influences the marginal distribution of the agent's type ω_i; for every $\omega \in \Omega$ and $b \in B$,

$$f(\omega \mid b) = f_0(\omega_0)\prod_{i \in N} f_i(\omega_i \mid b_i).$$

Here, $f_i(\cdot \mid b_i)$ denotes the distribution of each agent i's type ω_i that is assumed to depend only on b_i, and $f_0(\cdot)$ denotes the distribution of the public signal ω_0 that is assumed to be independent of the action profile $b \in B$. When we have such independence, agent i's action space is equivalent to the set of available marginal distributions on agent i's type. Accordingly, this restriction expresses no externality. With independence, for every $\xi : \Omega \to R$ and $\xi_i : \Omega_i \to R$, we can simply write $E_{\omega_{-i}}[\xi(\tilde{\omega}_i, \omega_{-i}) \mid b_{-i}]$ and $E[\xi_i(\omega_i) \mid b_i]$ instead of $E_{\omega_{-i}}[\xi(\tilde{\omega}_i, \omega_{-i}) \mid b, \omega_i]$ and $E[\xi_i(\omega_i) \mid b]$, respectively.

On the action profile b, we introduce a condition of *private richness*, the no-externality version of richness, as follows.

Definition 8 (Private Richness): An action profile $b \in B$ is said to be *privately rich* if we have the independence of the information structure, and for every $i \in N$ and $\delta_i \in \Delta(\Omega_i)$, there exist $\bar{\alpha} > 0$ and a path on B_i, $\beta_i(\delta_i, \cdot) : [-\bar{\alpha}, \bar{\alpha}] \to B_i$, such that

$$\beta_i(\delta_i, 0) = b_i,$$

(16) $$\lim_{\alpha \to 0} \frac{f_i(\cdot \mid \beta_i(\delta_i, \alpha)) - f_i(\cdot \mid b_i)}{\alpha} = \delta_i(\cdot) - f_i(\cdot \mid b_i),$$

and $c_i(\beta_i(\delta_i, \alpha))$ is differentiable in α at $\alpha = 0$.

Private richness implies that each agent $i \in N$ has a wide variety of action choices that can smoothly and locally change the distribution of the agent's type ω_i (not of the entire state ω) in all directions from $f_i(\cdot \mid b_i)$. Since the action choice problem of each agent is separated by the independence of the information structure, Proposition 1 guarantees that an action profile $a \in A$ satisfies private richness if the cost function for each agent $i \in N$, c_i, has directional derivatives at b.

Private richness is a much weaker restriction than the original version of richness. Accordingly, we can expect a much wider class of payment rules than pure Groves to satisfy inducibility. The main purpose of this section is to characterize payment rules that guarantee inducibility under private richness.

Lemma 3: *Suppose that an action profile b is privately rich, and a function w satisfies (3). For every function $\tilde{w} = (\tilde{w}_i)_{i \in N} : \Omega \to R^n$, $\tilde{w}$ satisfies the properties implied by (3), i.e., for every $i \in N$,*

$$E[\tilde{w}_i(\omega) \,|\, b] - c_i(b_i) \geq E[\tilde{w}_i(\omega) \,|\, b_i', b_{-i}] - c_i(b_i') \quad \textit{for all } b_i' \in B_i,$$

if and only if $E_{\omega_{-i}}[w_i(\omega_i, \omega_{-i}) - \tilde{w}_i(\omega_i, \omega_{-i}) \,|\, b_{-i}]$ is independent of $\omega_i \in \Omega_i$.

Proof: The proof of this lemma parallels that of Lemma 2. See the Appendix.

Proposition 7: *Consider an arbitrary combination of an action profile and a mechanism $(b,(g,x))$. Suppose that b is privately rich and (g,x) induces b. For every payment rule $\tilde{x}$, the associated mechanism $(g,\tilde{x})$ induces b if and only if $E_{\omega_{-i}}[x_i(\omega_i, \omega_{-i}) - \tilde{x}_i(\omega_i, \omega_{-i}) \,|\, b_{-i}]$ is independent of $\omega_i \in \Omega_i$.*

Proof: The proof of this proposition parallels that of Theorem 1. See the Appendix.

Fix an arbitrary combination of an action profile and an allocation rule (b,g). Consider two arbitrary payment rules x and $\tilde{x}$ such that both (g,x) and $(g,\tilde{x})$ induce b. Let $U_i \in R$ and $\tilde{U}_i \in R$ denote the respective ex-ante expected payoff for each agent $i \in N$:

$$U_i \equiv E[v_i(g(\omega),\omega) - x_i(\omega) \,|\, b] - c_i(b_i),$$

and

$$\tilde{U}_i \equiv E[v_i(g(\omega),\omega) - \tilde{x}_i(\omega) \mid b] - c_i(b_i) .$$

Proposition 7 implies that *the interim* (rather than ex-post) *payment for each agent* i *is unique up to constants*, in that

$$E_{\omega_{-i}}[\tilde{x}_i(\omega_i,\omega_{-i}) \mid b_{-i}] = E_{\omega_{-i}}[x_i(\omega_i,\omega_{-i}) \mid b_{-i}] - U_i + \tilde{U}_i \quad \text{for all} \quad \omega_i \in \Omega_i .$$

Proposition 8: *Consider an arbitrary combination of an allocation rule and an action profile* (b,g)*, where* b *is privately rich. If there exists a payment rule* x *such that* $(b,(g,x))$ *satisfies BI, then, for every payment rule* $\tilde{x}$*, whenever* $(g,\tilde{x})$ *induces* b*,* $(b,(g,\tilde{x}))$ *satisfies BI.*

Proof: The proof of this proposition parallels that of Theorem 2. See the Appendix.

Let us consider an arbitrary combination of action profile and allocation rule (b,g) that are efficient. A payment rule x is said to be *expectation-Groves* if for each $i \in N$, there exist $r_i : \Omega \to R$ such that for every $i \in N$,

$$x_i(\omega) = -\sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(g(\omega),\omega) + r_i(\omega) \quad \text{for all} \quad \omega \in \Omega ,$$

and $E_{\omega_{-i}}[r_i(\omega_i,\omega_{-i}) \mid b_{-i}]$ is independent of $\omega_i \in \Omega_i$. Note that, with efficiency, any expectation-Groves payment rule guarantees inducibility. Note that any Groves payment rule is expectation-Groves. Whenever x is expectation-Groves, then any payment rule $\tilde{x}$ is expectation-Groves if and only if $E_{\omega_{-i}}[x_i(\omega_i,\omega_{-i}) - \tilde{x}_i(\omega_i,\omega_{-i}) \mid b_{-i}]$ is independent of $\omega_i \in \Omega_i$.

Proposition 9: *Suppose that* (b,g) *is efficient. With independence,* (g,x) *induces* b *if* x *is expectation-Groves. Suppose that* b *is privately rich and* (b,g) *is efficient. With independence,* (g,x) *induces* b *if and only if* x *is expectation-Groves.*

Proof: See the Appendix.

The replacement of richness with private richness dramatically reduces the difficulty of achieving efficiency without deficits. In fact, the VCG payment rule is not pure Groves, but is expectation-Groves and satisfies EPIC. With private values, it generally guarantees the non-negativity of the ex-post payment from each agent to the central planner.

With private values, the mechanism (g,x) explored by Arrow (1979) and D'Aspremont and Gerard-Varet (1979) (i.e., *the AGV mechanism*) is expectation-Groves, where $r_i(\omega)$ $(\equiv x_i(\omega)+\sum_{j\in N\cup\{0\}\backslash\{i\}} v_j(g(\omega),\omega))$ is specified as

(17) $$r_i(\omega)=\sum_{j\in N\cup\{0\}\backslash\{i\}} v_j(g(\omega),\omega_j)-E_{\tilde{\omega}_{-i}}[\sum_{j\in N\cup\{0\}\backslash\{i\}} v_j(g(\omega_i,\tilde{\omega}_{-i}),\tilde{\omega}_j)\,|\,b_{-i}]$$
$$+\frac{1}{n-1}\sum_{j\in N\backslash\{i\}} E_{\tilde{\omega}_{-j}}[\sum_{h\in N\cup\{0\}\backslash\{j\}} v_h(g(\omega_j,\tilde{\omega}_{-j}),\tilde{\omega}_h)\,|\,b_{-j}].$$

This mechanism clearly satisfies *budget-balancing*:

$$\sum_{i\in N} x_i(\omega)=0 \quad \text{for all} \quad \omega\in\Omega.$$

Accordingly, with independence, the central planner can achieve efficiency under the constraints of BI and budget-balancing.

Hatfield, Kojima, and Kominers (2015) studied a relevant problem by focusing on mechanisms that are detail-free (i.e., independent of specifications such as action spaces and cost functions). They showed that efficient and inducible mechanisms must be Groves. In contrast, we permit mechanisms to not be detail-free, and then show that expectation-Groves mechanisms, which includes Groves as a proper subclass, are necessary and sufficient.[20]

8. Alternative Definition of Richness: Full Dimensionality

[20] Note that the construction of an expectation-Groves mechanism does not depend much on fine details of the specification of (f,B,c). In fact, the only knowledge needed for this construction is the shape of $f(\cdot\,|\,b)$ at the efficient action profile b.

So far, we have defined richness as a condition implying that each agent can change the distribution in *all* directions by selecting *pure* actions. This section introduces an alternative definition of richness, in which each agent can change the distribution in *finitely many* directions by selecting pure actions.

Denote $K \equiv \dim(\Delta(\Omega)) = |\Omega| - 1$. Taking advantage of the finiteness of the state space, we assume the following condition as an alternative to richness: For each $i \in N$, there exist $\bar{\alpha} > 0$, $\{\delta^k\}_{k=1}^K$, and $\{\beta_i^k : [-\bar{\alpha}, \bar{\alpha}] \to B_i\}_{k=1}^K$ such that the K vectors $\{\delta^k(\cdot) - f(\cdot \mid b)\}_{k=1}^K$ are linearly independent, and for every $k \in \{1, ..., K\}$,

$$\beta_i^k(0) = b_i,$$

$$\lim_{\alpha \to 0} \frac{f(\cdot \mid \beta_i^k(\alpha), b_{-i}) - f(\cdot \mid b)}{\alpha} = \delta_i^k(\cdot) - f(\cdot \mid b),$$

and $c_i(\beta_i^k(\alpha))$ is differentiable in α at $\alpha = 0$. In contrast to the original definition of richness, we only assume that each agent can change the distribution in finitely many directions, i.e., $\{\delta^k(\cdot) - f(\cdot \mid b)\}_{k=1}^K$, by selecting pure actions. However, according to the linear independence of $\{\delta^k(\cdot) - f(\cdot \mid b)\}_{k=1}^K$, it follows that for every $\delta \in \Delta(\Omega)$, there exists a *mixed* action for agent $i \in N$, by selecting which this agent can change the distribution in the direction of $\delta(\cdot) - f(\cdot \mid b)$ at a differentiable cost. Hence, with the alternative richness, each agent can change the distribution in all directions by selecting not pure but *mixed* actions.

This finding has important implications. Even if we replace the original definition of richness with the above-mentioned alternative, we can prove all results of this study due to richness. For the satisfaction of the alternative richness, we do not require each agent to have a huge breadth of action space. All we have to do for the satisfaction of the alternative richness is to clarify whether each agent can change the distribution by selecting pure actions in *finitely many* directions.

In the same manner as the above argument, we can also replace the definition of private richness with an alternative concerning only finitely many directions.

9. Conclusion

We have studied the general collective decision problem with quasi-linearity and risk-neutrality that includes aspects of both hidden action and hidden information. We have assumed that each agent has a wide availability of activities before the state occurs, and that such activities have a significant externality effect on other agents' valuations. We have shown that the class of mechanisms that successfully induce the desired action profile in hidden action is restrictive in the following manner.

First, the payment rule that satisfies inducibility, if such a rule exists, is unique up to constants. We, therefore, obtained the equivalence properties in the ex-post term with respect to payoff, payment, and revenue. We have also shown that, if there is a mechanism that induces the desired action profile in hidden action but fails to satisfy incentive compatibility in hidden information, then we cannot discover a mechanism that satisfies both inducibility and incentive compatibility.

Second, focusing on the achievement of efficiency, we have shown that the mechanisms that satisfy both inducibility and incentive compatibility must be pure Groves (which is the simplest form of the Groves mechanism). Accordingly, it is difficult to satisfy both inducibility and incentive compatibility in the interdependent value case, while it is generally possible in the private value case. Even in the private value case, though, the central planner has to struggle to avoid deficits.

The above-mentioned difficulties are caused by the central planner's perception that each agent has, potentially, a wide availability of activities with externality effects. Accordingly, to calm these difficulties, the central planner should collect information in advance about which aspects of activities are actually relevant to the problem posed.

This paper has assumed richness as describing an extreme range of externalities. The substances of this paper's results, however, are unchanged, even if we weaken the richness assumption.

In future research, it would be important to investigate the collective decision problem by replacing richness with a much milder condition. We have shown that, with richness (or its alternative), the incentive constraint in hidden action alone determines the shape of well-behaved mechanisms, without mentioning hidden information. Without richness, we can expect incentive constraints in hidden information to supplement the indeterminacy of the shape of the mechanism. However, careful research of the unification of hidden action and hidden information in this line are beyond the scope of this paper.

References

Arozamena, L. and E. Cantillon (2004): "Investment Incentives in Procurement Auctions," *Review of Economic Studies*, 71, 1–18.

Arrow, K. (1979): "The Property Rights Doctrine and Demand Revelation under Incomplete Information," in *Economics and Human Welfare*, ed. by M. Boskin, New York: Academic Press, 24–39.

Athey, S. and I. Segal (2013): "An Efficient Dynamic Mechanism," *Econometrica*, 81, 2463–2485.

Bergemann, D., X. Shi, and J. Välimäki (2009): "Information Acquisition in Interdependent Value Auctions," *Journal of the European Economic Association*, 7, 61–89.

Bergemann, D. and J. Välimäki (2002): "Information Acquisition and Efficient Mechanism Design," *Econometrica*, 70, 1007–1033.

Carroll, G. (2015): "Robustness and Linear Contracts," *The American Economic Review*, 105, 536–563

Clarke, E. H. (1971): "Multipart Pricing of Public Goods," *Public choice*, 11, 17–33.

Crémer, J. and R. P. McLean (1985): "Optimal Selling Strategies under Uncertainty for a Discriminating Monopolist when Demands are Interdependent," *Econometrica*, 53, 345–361.

——— (1988): "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica*, 56, 1247–1257.

Dasgupta, P. and E. Maskin (2000): "Efficient Auctions," *The Quarterly Journal of Economics*, CXV, 341–388.

D'Aspremont, C. and L.-A. Gerard-Varet (1979): "Incentives and Incomplete Information," *Journal of Public Economics*, 11, 25–45.

Green, J. R. and J.-J. Laffont (1977): "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods," *Econometrica*, 45, 427–438.

——— (1979): Incentives in Public Decision Making, Amsterdam: North-Holland.

Groves, T. (1973): "Incentives in Teams," *Econometrica*, 41, 617–631.

Harris, M. and A. Raviv (1979): "Optimal Incentive Contracts with Imperfect Information," *Journal of Economic Theory*, 259, 231–259.

Hatfield, J. W., F. Kojima, and S. D. Kominers (2015): "Strategy-Proofness, Investment Efficiency, and Marginal Returns: An Equivalence," Working paper.

Hausch, D. B. and L. Li (1991): "Private Values Auctions with Endogenous Information: Revenue Equivalence and Non-Equivalence," Working paper.

——— (1993): "A Common Value Auction Model with Endogenous Entry and Information Acquisition," *Economic Theory*, 3, 315–334.

Holmström, B. (1979): "Groves' Scheme on Restricted Domains," *Econometrica*, 47, 1137–1144.

Holmström, B. and P. Milgrom (1987): "Aggregation and Linearity in the Provision of Intertemporal Incentives," *Econometrica*, 55, 303–328.

Klemperer, P. (2004): *Auctions: Theory and Practice*, Princeton: Princeton University Press.

Legros, P. and H. Matsushima (1991): "Efficiency in Partnerships," *Journal of Economic Theory*, 55, 296–322.

Maskin, E. S. (1992): "Auctions and Privatization," in *Privatization*, ed. by H. Siebert, Kiel: Institut fur Weltwirschaften der Universitat Kiel, 115–136.

Matsushima, H. (1989): "Efficiency in Repeated Games with Imperfect Monitoring," *Journal of Economic Theory*, 48, 428–442.

Matthews, S. (1984): "Information Acquisition in Discriminatory Auctions," in *Bayesian Models in Economic Theory*, ed. by M. Boyer, and R. Kihlstrom, 49, 1477–1500.

Mezzetti, C. (2004): "Mechanism Design with Interdependent Valuations: Efficiency," *Econometrica*, 72, 1617–1626.

Milgrom, P. R. (2004): *Putting Auction Theory to Work*, Cambridge: Cambridge University Press.

Noda, S. (2016): "Full Surplus Extraction and Within-Period Ex Post Implementation in Dynamic Environments," Working Paper.

Obara, I. (2008): "The Full Surplus Extraction Theorem with Hidden Actions," *The B.E. Journal of Theoretical Economics*, 8.

Persico, N. (2000): "Information Acquisition in Auctions," *Econometrica*, 68, 135–148.

Stegeman, M. (1996): "Participation Costs and Efficient Auctions," *Journal of Economic Theory*, 71, 228–259.

Tan, G. (1992): "Entry and R & D in Procurement Contracting," *Journal of Economic Theory*, 60, 41–60.

Vickrey, W. (1961): "Counterspeculation, Auctions, and Competitive Sealed Tenders," *The Journal of Finance*, 16, 8–37.

Williams, S. R. and R. Radner (1995): "Efficiency in Partnerships when the Joint Output is Uncertain," in *The Economics of Informational Decentralization*, ed. by J. O. Ledyard, Boston: Kluwer, 79–99.

Appendix

Proof of Proposition 3: From Theorem 3, we can focus on pure Groves payment rules, where the constraint for EPIR is equivalent to

(A1) $$\min_{\omega\in\Omega} \sum_{j\in N\cup\{0\}} v_j(g(\omega),\omega_0,\omega_j) \geq z_i \quad \text{for all } \; i\in N.$$

We can maximize the expected revenue by letting z_i satisfy (A1) with equality for each $i \in N$. Accordingly, the central planner can receive from each agent i the expected value given by

$$\min_{\omega \in \Omega} \sum_{j \in N \cup \{0\}} v_j(g(\omega), \omega_0, \omega_j) - E\left[\sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(g(\omega), \omega_0, \omega_j) \middle| b \right],$$

which implies (11), where we add $E[v_0(g(\omega), \omega_0) \mid b]$.

Similarly, the constraint for IIR is equivalent to

(A2) $$\min_{\omega_i \in \Omega_i} E_{\omega_{-i}}\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega_i, \omega_{-i}), \omega_0, \omega_j) \middle| \omega_i, b \right] \geq z_i \text{ for all } i \in N \text{ and } \omega_i \in \Omega_i.$$

We can maximize the expected revenue by letting z_i satisfy (A2) with equality for each $i \in N$. Accordingly, the central planner can receive from each agent i the expected value given by

$$\min_{\omega_i \in \Omega_i} E_{\omega_{-i}}\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega_i, \omega_{-i}), \omega_0, \omega_j) \middle| \omega_i, b \right] - E\left[\sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(g(\omega), \omega_0, \omega_j) \middle| b \right],$$

which implies (12), where we add $E[v_0(g(\omega), \omega_0) \mid b]$.

The constraint for EAIR is equivalent to

(A3) $$E\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega), \omega_0, \omega_j) \middle| b \right] - c_i(b_i) \geq z_i \text{ for all } i \in N.$$

We can maximize the expected revenue by letting z_i satisfy (A3) with equality for each $i \in N$. Accordingly, the central planner can receive from each agent i the expected value given by

$$E\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega), \omega_0, \omega_j) \middle| b \right] - c_i(b_i) - E\left[\sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(g(\omega), \omega_0, \omega_j) \middle| b \right]$$

$$= E\left[v_i(g(\omega), \omega_0, \omega_i) \middle| b \right] - c_i(b_i),$$

which implies (13), where we add $E[v_0(g(\omega), \omega_0) \mid b]$.

The constraint for IIR and EAIR is equivalent to

(A4)
$$\min\left\{E\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega),\omega_0,\omega_j)\middle| b\right]-c_i(b_i),\right.$$
$$\left.\min_{\omega_i\in\Omega_i} E_{\omega_{-i}}\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega_i,\omega_{-i}),\omega_0,\omega_j)\middle| b,\omega_i\right]\right\}\geq z_i .$$

We can maximize the expected revenue by letting z_i satisfy (A4) with equality for each $i\in N$. Accordingly, the central planner can receive from each agent i the expected value given by

$$\min\left\{E\left[v_i(g(\omega),\omega_0,\omega_i)\middle| b\right]-c_i(b_i),\right.$$
$$\left.\min_{\omega_i\in\Omega_i} E_{\omega_{-i}}\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega_i,\omega_{-i}),\omega_0,\omega_j)\middle| b,\omega_i\right]-E\left[\sum_{j\in N\cup\{0\}\setminus\{i\}} v_j(g(\omega),\omega_0,\omega_j)\middle| b\right]\right\},$$

which implies (14), where we add $E[v_0(g(\omega),\omega_0)\,|\,b]$.

Q.E.D.

Proof of Proposition 4: If $R^{EAIR}<0$, the conclusion would be immediate from the fact that $R^{EPIR}\leq R^{EAIR}$. Suppose $R^{EAIR}\geq 0$. Because for $(\underline{\omega}_0,\underline{\omega}_1,\cdots,\underline{\omega}_n)$, $\sum_{i\in N\cup\{0\}} v_i(a,\underline{\omega}_0,\underline{\omega}_i)=0$ holds for all $a\in A$, we have

$$\min_{\omega\in\Omega}\sum_{i\in N\cup\{0\}} v_i(g(\omega),\omega_0,\omega_i)\leq 0 .$$

It follows from $R^{EAIR}\geq 0$ and $\sum_{i\in N} c_i(b_i)>0$ that we have

$$E\left[\sum_{i\in N\cup\{0\}} v_i(g(\omega),\omega_0,\omega_i)\middle| b\right]\geq\sum_{i\in N} c_i(b_i)>0 .$$

From these observations,

$$R^{EPIR}=n\min_{\omega\in\Omega}\sum_{j\in N\cup\{0\}} v_j(g(\omega),\omega_j)-(n-1)E\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega),\omega_0,\omega_j)\middle| b\right]<0 .$$

Q.E.D.

Proof of Proposition 5: We have shown that there is a pure Groves mechanism that achieves $R^{IIR,EAIR}$. With efficiency, conditional independence, and private values, let us consider an arbitrary Groves mechanism (g,x), where for each $i \in N$, there exists $y_i : \Omega_{-i} \to R$ such that

$$x_i(\omega) = -\sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(g(\omega), \omega_0, \omega_j) + y_i(\omega_{-i}) \quad \text{for all} \quad \omega \in \Omega .$$

EAIR implies

$$E\left[y_i(\omega_{-i}) \middle| b \right] \le E\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega), \omega_0, \omega_j) \middle| b \right] - c_i(b_i),$$

while IIR requires

$$E\left[y_i(\omega_{-i}) \middle| b, \omega_i \right] \le E_{\omega_{-i}}\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega_i, \omega_{-i}), \omega_0, \omega_j) \middle| b, \omega_i \right]$$

$$\text{for all} \quad \omega_i \in \Omega_i ,$$

or, equivalently,

$$E\left[y_i(\omega_{-i}) \middle| b \right] \le \min_{\omega_i \in \Omega_i} E_{\omega_{-i}}\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega_i, \omega_{-i}), \omega_0, \omega_j) \middle| b, \omega_i \right].$$

Here, we utilized $E\left[y_i(\omega_{-i}) \middle| b \right] = E\left[y_i(\omega_{-i}) \middle| b, \omega_i \right]$ because of conditional independence. Accordingly, we have

$$E\left[y_i(\omega_{-i}) \middle| b \right] \le \min \left\{ E\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega), \omega_0, \omega_j) \middle| b \right] - c_i(b_i), \right.$$

$$\left. \min_{\omega_i \in \Omega_i} E_{\omega_{-i}}\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega_i, \omega_{-i}), \omega_0, \omega_j) \middle| b, \omega_i \right] \right\},$$

which implies

$$E[v_0(g(\omega), \omega_0) + \sum_{i \in N} x_i(\omega) \mid b] \le R^{IIR,EAIR} .$$

Q.E.D.

Proof of Proposition 6: From conditional independence, non-negative valuations, and null state, it follows that for every $i \in N$ and $\omega_i \in \Omega_i$,

$$E_{\omega_{-i}}\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega_i, \omega_{-i}), \omega_0, \omega_j) \middle| b\right] = E_{\omega_{-i}}\left[\max_{a \in N} \sum_{j \in N \cup \{0\}} v_j(a, \omega_0, \omega_j) \middle| b\right]$$

$$\geq E_{\omega_{-i}}\left[\max_{a \in N} \sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(a, \omega_0, \omega_j) \middle| b\right]$$

$$= E_{\omega_{-i}}\left[v_i(g(\underline{\omega}_i, \omega_{-i}), \omega_0, \underline{\omega}_i) + \sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(g(\underline{\omega}_i, \omega_{-i}), \omega_0, \omega_j) \middle| b\right],$$

which implies

$$\min_{\omega_i \in \Omega_i} E_{\omega_{-i}}\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega_i, \omega_{-i}), \omega_0, \omega_j) \middle| b\right] = E_{\omega_{-i}}\left[\max_{a \in A} \sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(a, \omega_0, \omega_j) \middle| b\right],$$

that is,

$$\min_{\omega_i \in \Omega_i} E_{\omega_{-i}}\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega_i, \omega_{-i}), \omega_0, \omega_j) \middle| b\right] - E[\sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(g(\omega), \omega_0, \omega_j) \mid b] \geq 0.$$

From the assumption of non-negative expected payoffs, we have

$$E\left[v_i(g(\omega), \omega_0, \omega_i) \middle| b\right] - c_i(b_i) \geq 0.$$

From these observations, for every $i \in N$,

$$\min\Big\{E\left[v_i(g(\omega), \omega_0, \omega_i) \middle| b\right] - c_i(b_i),$$

$$\min_{\omega_i \in \Omega_i} E_{\omega_{-i}}\left[\sum_{j \in N \cup \{0\}} v_j(g(\omega_i, \omega_{-i}), \omega_0, \omega_j) \middle| b\right] - E[\sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(g(\omega), \omega_0, \omega_j) \mid b]\Big\}$$

$$\geq 0.$$

which, along with non-negative valuations, implies $R^{IIR,EAIR} \geq 0$.

Q.E.D.

Proof of Lemma 3: The proof of the sufficiency is straightforward. We present the proof of the necessity as follows. Take an arbitrary w which satisfy (3) and consider an arbitrary agent $i \in N$. Take an arbitrary $\tilde{w}_i$ such that

$\xi_i(\omega_i) \equiv E_{\omega_{-i}}[\tilde{w}_i(\omega_i, \omega_{-i}) - w_i(\omega_i, \omega_{-i}) \,|\, b_{-i}]$ is a non-constant function. Then, there exists $\bar{\omega}_i \in \Omega_i$ such that $\xi_i(\bar{\omega}_i) > E[\xi_i(\omega_i) \,|\, b_i]$. Let $\delta_i^{\bar{\omega}_i} \in \Delta(\Omega_i)$ denote the degenerate distribution where $\delta_i^{\bar{\omega}_i}(\bar{\omega}_i) = 1$. Due to private richness, there exist $\bar{\alpha} > 0$ and $\beta_i(\delta_i^{\bar{\omega}_i}, \cdot) : [-\bar{\alpha}, \bar{\alpha}] \to B_i$ such that

$$\lim_{\alpha \to 0} \frac{f_i(\cdot \,|\, \beta_i(\delta_i^{\bar{\omega}_i}, \alpha)) - f_i(\cdot \,|\, b_i)}{\alpha} = \delta_i^{\bar{\omega}_i}(\cdot) - f_i(\cdot \,|\, b_i) \,.$$

Since w satisfies (3), the first order condition along $\beta_i(\delta_i^{\bar{\omega}_i}, \cdot)$ must hold, i.e.,

$$\frac{\partial}{\partial \alpha} \left\{ E_{\omega_i} \left[E_{\omega_{-i}}[w_i(\omega) \,|\, b_{-i}] \middle| \beta_i(\delta_i^{\bar{\omega}_i}, \alpha) \right] - c_i(\beta_i(\delta_i^{\bar{\omega}_i}, \alpha)) \right\} \Bigg|_{\alpha=0} = 0 \,.$$

On the other hand,

$$\frac{\partial}{\partial \alpha} \left\{ E_{\omega_i} \left[E_{\omega_{-i}}[\tilde{w}_i(\omega) \,|\, b_{-i}] \middle| \beta_i(\delta_i^{\bar{\omega}_i}, \alpha) \right] - c_i(\beta_i(\delta_i^{\bar{\omega}_i}, \alpha)) \right\} \Bigg|_{\alpha=0}$$

$$= \frac{\partial}{\partial \alpha} \left\{ E_{\omega_i} \left[\xi_i(\omega_i) + E_{\omega_{-i}}[w_i(\omega) \,|\, b_{-i}] \middle| \beta_i(\delta_i^{\bar{\omega}_i}, \alpha) \right] - c_i(\beta_i(\delta_i^{\bar{\omega}_i}, \alpha)) \right\} \Bigg|_{\alpha=0}$$

$$= \frac{\partial}{\partial \alpha} \left\{ E[\xi_i(\omega_i) \,|\, \beta_i(\delta_i^{\bar{\omega}_i}, \alpha)] \right\} \Big|_{\alpha=0} = \xi_i(\bar{\omega}_i) - E[\xi_i(\omega_i) \,|\, b_i] > 0 \,.$$

Hence, if $\xi_i(\omega_i) \equiv E_{\omega_{-i}}[\tilde{w}_i(\omega_i, \omega_{-i}) - w_i(\omega_i, \omega_{-i}) \,|\, b_{-i}]$ is non-constant, agent i has an incentive to increase α along $\beta_i(\delta^{\bar{\omega}}, \cdot)$ from $\alpha = 0$. Accordingly, whenever w and $\tilde{w}$ satisfy (3), then $E_{\omega_{-i}}[w_i(\omega_i, \omega_{-i}) - \tilde{w}_i(\omega_i, \omega_{-i}) \,|\, b_{-i}]$ is independent of ω_i.

Q.E.D.

Proof of Proposition 7: The proof of sufficiency is straightforward. Let us show the proof of necessity as follows. Suppose that (g, x) induces b. According to the proof of Lemma 1, we specify w by

$$w_i(\omega) = v_i(g(\omega), \omega) - x_i(\omega) \text{ for all } i \in N \text{ and } \omega \in \Omega \,.$$

Suppose also that $(g, \tilde{x})$ induces b. Similarly, we specify $\tilde{w}$ by

$$\tilde{w}_i(\omega) = v_i(g(\omega),\omega) - \tilde{x}_i(\omega) \text{ for all } i \in N \text{ and } \omega \in \Omega.$$

Lemma 3 implies that $E_{\omega_{-i}}[w_i(\omega_i,\omega_{-i}) - \tilde{w}_i(\omega_i,\omega_{-i}) \mid b_{-i}]$ is independent of $\omega_i \in \Omega_i$ (i.e., $E_{\omega_{-i}}[x_i(\omega_i,\omega_{-i}) - \tilde{x}_i(\omega_i,\omega_{-i}) \mid b_{-i}]$ is independent of $\omega_i \in \Omega_i$).

Q.E.D.

Proof of Proposition 8: Suppose that (g,x) and $(g,\tilde{x})$ induce b. From Proposition 7, $E_{\omega_{-i}}[x_i(\omega_i,\omega_{-i}) - \tilde{x}_i(\omega_i,\omega_{-i}) \mid b_{-i}]$ is independent of $\omega_i \in \Omega_i$. This automatically implies that $(b,(g,x))$ satisfies BI if and only if $(b,(g,\tilde{x}))$ satisfies BI.

Q.E.D.

Proof of Proposition 9: Sufficiency is straightforward from the fact that $E_{\omega_{-i}}[r_i(\omega_i,\omega_{-i}) \mid b_{-i}]$ is independent of ω_i and b_i. For necessity, note that whenever (b,g) is efficient and x is pure Groves, then (g,x) induces b. Because pure Groves is expectation-Groves, it follows from Proposition 7 that $(g,\tilde{x})$ induces b if and only if $\tilde{x}$ is expectation-Groves.

Q.E.D.