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Stochastic Macro-equilibrium and A Microfoundation for the Keynesian Economics*

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Abstract

In place of the standard search equilibrium, this paper presents an alternative concept of stochastic macro-equilibrium based on the principle of statistical physics. This concept of equilibrium is motivated by unspecifiable differences of economic agents and the presence of all kinds of micro shocks in the macroeconomy. Our model mimics the empirically observed distribution of labor productivity. The distribution of productivity resulting from the matchings of workers and firms depends crucially on aggregate demand. When aggregate demand rises, more workers are employed by firms with higher productivity while at the same time, the unemployment rate declines. The effect of the reservation wage on unemployment also depends on aggregate demand. The model provides a micro-foundation for Keynes' principle of effective demand.

Key words: Labor Search Theory, Microeconomic Foundations, Stochastic Macro-equilibrium, Unemployment, Productivity, Keynes' Principle of Effective Demand

JRL: D39, E10, J64

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1. Introduction

The purpose of this paper is to present a new concept of stochastic macro-equilibrium which provides a micro-foundation for the Keynesian theory of effective demand. Our problem is *cyclical* changes in effective utilization of labor. The concept of stochastic macro-equilibrium is meant to clarify how the labor market works under the Keynesian problem of aggregate demand deficiency. Unemployment and distribution of labor productivity both depend crucially on aggregate demand. The model is similar to standard search equilibrium in spirit, but it takes a different approach following the method of statistical physics.

The textbook interpretation of Keynesian economics originating with Modigliani (1944) regards it as economics of inflexible prices/wages. If price and wages were flexible enough, the economy would be led to the Pareto optimal neoclassical equilibrium. However, whatever the reason, when prices and wages are inflexible, the economy may fall into equilibrium with unemployment. Keynesian economics is meant to analyze such economy. In this frame of thoughts, micro-foundations for Keynesian economics would be to provide reasonable explanations for inflexible prices and wages. Toward this goal, a number of researches were done, summarized under the heading of “New Keynesian economics” (Mankiw and Romer (1991)). The crux of these studies is to consider optimizing behaviors of the representative household and firm which are compatible with inflexible prices and wages. New Keynesian dynamic stochastic general equilibrium (DSGE) models are built in the same spirit.

A different interpretation of Keynesian economics was advanced by Tobin (1993).

“The central Keynesian proposition is not nominal price rigidity but the principle of effective demand (Keynes, 1936, Ch. 3). In the absence of instantaneous and complete market clearing, output and employment are frequently constrained by aggregate demand. In these excess-supply regimes, agents’ demands are limited by their inability to sell as much as they would like at prevailing prices. Any failure of price adjustments to keep markets cleared opens the door for quantities to determine quantities, for example real national income to determine consumption demand, as described in Keynes’ multiplier calculus. ...

In Keynesian business cycle theory, the shocks generating fluctuations are generally shifts in *real* aggregate demand for goods and services, notably in capital investment.(Tobin, 1993)”

Tobin dubbed his own position an “Old Keynesian view”. Certainly, the main message of Keynes (1936) is that *real* demand rather than factor endowment and technology determines the level of aggregate production in the short-run simply because the rate of utilization of production factors such as labor and capital endogenously changes responding to changes in real demand. Keynes maintained that this proposition

holds true regardless of flexibility of prices and wages; he, in fact, argued that a fall of prices or wages would aggravate, not alleviate the problems facing the economy in deep recession because it may lower aggregate demand.

Following Tobin, let us call this proposition the Old Keynesian view. According to the Old Keynesian view, changes in *real* aggregate demand for goods and services generate fluctuations of output. The challenge is then to clarify the market mechanism by which aggregate demand conditions the allocation of production factors in such a way that total output follows changes in real aggregate demand. A decrease of aggregate output is necessarily accompanied by lower utilization of production factors, and *vice versa*. Since the days of Keynes, economists have taken unemployment as a most important sign of possible under-utilization of labor. However, unemployment is by definition job search, a kind of economic activity of worker, and as such calls for explanation. Besides, unemployment is only a partial indicator of under-utilization of labor in the macroeconomy. The celebrated Okun's law which relates the unemployment rate to the growth rate of real GDP demonstrates the significance of under-utilization of employed labor other than unemployment¹. In this paper, we consider not only unemployment but also on productivity dispersion in the economy.

To consider Keynes' principle of effective demand, we must obviously depart from the Walrasian general equilibrium as represented by Arrow and Debreu (1954). The most successful example of "non-Walraian economics" which analyzes labor market in depth is equilibrium search theory surveyed by its pioneers Rogerson, Shimer, and Wright (2005), Diamond (2011), Mortensen (2011), and Pissarides (2000, 2011). The standard general equilibrium abstracts itself altogether from the search and matching costs which are always present in the actual markets. By explicitly exploring search frictions, search theory has succeeded in shedding much light on the workings of labor market.

While acknowledging the achievement of equilibrium search theory, we find several fundamental problems with the standard theory. In particular, the theory fails to provide a useful framework for explaining *cyclical* changes in effective utilization of labor in the macroeconomy². The reason, in our view, is that though blurred by the Poisson modeling, the standard theory effectively assumes perfect competition in the

¹ Okun (1963) found that a decline of the unemployment rate by one percent raises the growth rate of real GDP by three percent. The Okun coefficient three is much larger than the elasticity of output with respect to labor which is supposed to be equal to the labor share, and roughly one third. This finding demonstrates that there always exists significant under-employment of labor other than unemployment in the macroeconomy: See Okun(1973).

² Shimer (2005) demonstrates that the standard search theory fails to account for stylized empirical facts on cyclical fluctuations of unemployment and vacancy.

product market. No doubt, prices and wages guide economic agents in market economy. However, quantity constraints also play a crucial role, particularly in the presence of frictions and uncertainty which the equilibrium search theory emphasizes. With the concept of stochastic equilibrium of the macroeconomy based on the method of statistical physics, this paper clarifies the meaning of unemployment due to demand deficiency.

Section 2 points out limitations of standard search theory. After brief explanation of the concept of equilibrium based on statistical physics in Section 3, Section 4 presents a model of stochastic macro-equilibrium. The model explains how unemployment is determined together with distribution of productivity. Section 5 then explains that the stochastic macro-equilibrium provides a micro-foundation for Keynes' principle of effective demand. It also presents suggestive evidence supporting the model. The final section offers brief concluding remarks.

2. Limitations of Search Theory

The search theory starts with the presence of various frictions and accompanying matching costs in market transactions. Once we recognize these problems, we are led to heterogeneity of economic agents and multiple outcomes in equilibrium³. In the simplest retail market, for example, with search cost, it would be possible to obtain high and low (more generally multiple) prices for the same good or service in equilibrium. This break with the law of one price is certainly a big step toward reality. Frictions and matching costs are particularly significant in labor market. And the analysis of labor market has direct implications for macroeconomics. In what follows, we will discuss labor search theory.

In search equilibrium, potentially similar workers and firms experience different economic outcomes. For example, some workers are employed while others are unemployed. In this way, search theory well recognizes, even emphasizes heterogeneity of workers and firms. Despite this recognition, when it comes to model behavior of economic agent such as worker and firm, it, in effect, presumes the representative agent in the sense that *stochastic economic environment is common to all the agents*; Workers and firms differ only in terms of the realizations of stochastic variable of interest whose probability distribution is common. Specifically, it is routinely assumed that the job arrival rate, the job separation rate, and probability distributions of wages and

³ Similarly, Tobin (1972) advanced the notion of "stochastic macro-equilibrium" in his explanation of the Phillips curve as a macro equation.

productivity are common to all the workers and firms⁴.

The job separation includes layoffs as well as voluntary quits. It makes no sense that all the firms and workers face the same job separation rate, particularly the probability of layoffs. White collar and blue collar workers face different risks of layoff. Everybody knows the difference between new expanding industry and old declining industry. In any case, the probability of layoffs depends crucially on the state of demand for the firm's product, and as a result, among other things, on industry, region, and ultimately the firm's performance in the product market.

We can always calculate the average job arrival rate and job separation rate, of course. However, the average job arrival rate and the economy wide wage distribution which by definition determine the average duration of an unemployment spell would be relevant only to decisions made by say, the department of labor. When individual private economic agent makes decisions, they are not relevant because the job arrival rate, the job separation rate, and the wage distribution facing individual worker and firm are all different.

On the assumption that the reservation wage R is the same for all the workers, the standard analysis typically goes as follows. In the equilibrium labor market, we must obtain

$$fu = s(1 - u) \tag{1}$$

where u is the unemployment rate, and s and f are the separation rate and the job finding rate, respectively. Equation (1) makes sure the balance between in- and out-flows of the unemployment pool.

If λ is the offer arrival rate and $F(w)$ is the cumulative distribution function of wage offers, the job finding rate f is

$$f = \lambda(1 - F(R)). \tag{2}$$

From equations (1) and (2), we obtain

$$u = \frac{s}{s + \lambda(1 - F(R))}. \tag{3}$$

Equation (2), the standard equation in the literature, presumes that the offer arrival rate λ , the reservation wage R , and the cumulative distribution function of wage offers F are common to all the workers. However, as we argued above, it is obvious that in reality, λ , R and $F(w)$ differ significantly across workers. It is difficult to imagine that workers of different educational attainments face the same probability distribution

⁴ In some models such as Burdett and Mortensen (1998), the arrival rate is assumed to depend on a worker's current state, namely either employed or unemployed. However, within each group, the job arrival rate is common to all the workers.

of wage offers; it is plainly unrealistic to assume that a youngster working at gas station faces the same probability of getting a well paid job offer from bank as a graduate of business school. And yet, in standard search models, the assumption that $F(w)$ is common to all the workers is routinely made, and the common $F(w)$ is put into the Bellman equations describing behaviors of firms and workers. The assumption is simply untenable; $F(w)$ for the i -th worker must be $F_i(w)$.

Besides, although wages are one of the most important elements in any job offer, workers care not only wages but other factors such as job quality, tenure, and location. Preferences for these other factors which define a job offer certainly differ widely across workers, and are constantly changing over time. Rogerson, Shimer and Wright (2005; P.962) say that “although we refer to w as the wage, more generally it could capture some measure of the desirability of the job, depending on benefits, location, prestige, etc.” However, this is an illegitimate proposition. All the complexities they refer to simply strengthens the case that we cannot assume that the probability distribution of w , $F(w)$ is common to all the workers. In fact, wage may be even a lexicographically inferior variable to some workers. For example, pregnant female worker might prefer job closer to her home at the expense of lower wage. Workers and firms all act in their own different universes.

The second problem of standard search model pertains to the behavior of firm. Though blurred by the standard Poisson modeling, it is routinely assumed that the product market is perfectly competitive in the sense that individual demand curve facing the firm is flat. In Burdett and Mortensen (1998), for example, the flow of revenue generated by employed worker, p is constant and a firm’s steady-state profit given the wage offer w is simply $(p-w)l$ where l is the number of workers of this firm. l depends on the firm’s wage setting. This is the standard assumption in the literature where p is often called “productivity “. Given p , the determination of wages plays a crucial role. It is essentially a model of labor shortage in the sense that the firm’s output and profit are determined by labor supply at the level of wages the firm offers to workers. It is curious that the standard search theory makes so much effort to consider the determination of wages within firm taking into account strategic behavior of rival firms while at the same time it leaves the price unexplained under the naïve assumption of perfect competition. Most firms regard the determination of the prices of their own products as important as (possibly more important than) the determination of wages taking into account behavior of their competitors.

Under the assumption of perfect competition in the product market, given firm’s wage offer, whenever a worker comes, the firm is ready to hire him/her though the

worker could turn down the offer or will leave the firm when better offers arrive. The level of employment l is determined only by the number of successful matching.

This analytical framework leads us to some awkward conclusion. Postel-Vinay and Robin (2002), for example, interpret the empirically observed decreasing number of workers at high productivity job sites as a consequence of less recruitment efforts made by high productivity firms than by low productivity firms. However, there is no reasonable reason why high productivity firms make less recruitment efforts. A much more plausible reason is that firms are demand constrained in the product market, and that demand for goods and services produced by high productivity firm is limited.

Shimer (2005) also demonstrates that in the standard search model, an increase in the job separation rate raises both the unemployment and vacancy rates. This analytical result is in stark contradiction to the well-known negative relationship between two rates, the Beveridge curve⁵. The strange result is obtained because the standard model assumes perfect competition in the product market, and considers strategic behavior of firms only in labor market. In the economy constrained by real demand, an increase in the job separation rate is likely to be generated by an increase in layoff which in turn, is caused by a fall of aggregate demand. The unemployment rate rises while firms laying off workers must post less vacancy signs.

From our perspective, the most serious problem with standard search theory is the assumption that the product market is perfectly competitive. This assumption is crucial because the firm's demand for labor depends basically on demand for the firm's products. It actually contradicts to the spirit of search theory, and is particularly ill-suited for studying *cyclical* changes in effective utilization of labor. In this paper, following Negishi (1979), we assume that firms are monopolistically competitive in the sense that they face downward sloping individual demand curve in the product market.

Although the most important factor constraining the firm's demand for labor is demand constraint in the product market, it is absolutely impossible for us to know individual demand curve facing each firm. The standard assumption in theoretical models of imperfectly competitive markets is that the demand system is symmetric, namely that firms face the same demand condition in equilibrium; See, for example, Asplund and Nocke (2006). This assumption might be justified in some cases for the analysis of *an industry* or *a local market*, but is absolutely untenable for the purpose of studying the macroeconomy. Demand is far from symmetric across firms, and there is no knowing how asymmetric it is in the economy as a whole. Our approach is based on

⁵ Most of efforts to reconcile standard matching models with the observed data rest on artificial assumptions : Mortensen and Nagypál (2007).

this fact.

In summary, standard search models are built on several unrealistic assumptions. First, the job arrival rate, the job separation rate, and probability distributions of wages and productivity are common to all the workers and firms. Some models assume that workers and firms are different in terms of their ability, preferences, and productivity (Burdett and Mortensen (1998), Postel-Vinay and Robin (2002)). However, the probability distributions of those characteristics are assumed to be utilized by all the economic agents in common; firms strategically determine wages, and workers do job search using the common wage distribution, and Poisson arrival rates.

We can always find the distribution of any variable of interest for the economy as a whole. The point is that such macro distribution is not relevant to the decisions made by individual economic agent because the “universe” of each economic agent is all different and keeps changing. In this sense, the macroeconomy is fundamentally different from a local retail market or an industry consisting of a small number of dominant firms. In the case of a local retail market, one may reasonably assume that all the consumers in the small region share the same distribution of prices. However, this assumption is untenable for the economy as a whole. It is a cliché that the labor market is segmented in many ways.

In fact, each agent acts in his/her own universe. For example, a worker may suffer from illness. This amounts to shocks to his utility function on one hand and to his resource constraint or “ability” on the other. His preference for job including the reservation wage, the location and working hours, necessarily changes. At the same time, the job arrival rate and the wage distribution relevant for his decision making also changes; the corresponding economy-wide information is not relevant.

Secondly, the standard assumption that firms are perfectly competitive without any demand constraint in the product market is particularly ill-suited for the analysis of *cyclical* changes in under-utilization of labor. We must assume that instead firms face downward-slowing demand curve. Because we analyze the macroeconomy, we cannot assume that demand is symmetric across firms, and moreover, we never know the details of demand constraints facing firms.

The point is *not* that we must *explicitly* introduce all the complexities characterizing labor and product markets into analytical model. It would simply make model intractable. Rather, we must fully recognize that it is absolutely impossible to trace the microeconomic behaviors, namely decision makings of workers and firms in detail. When the problem is *micro* (say, oligopolistic market dominated by a few firms, or sales/purchases of old cars), strategic/optimizing behaviors of economic agents must

be explicitly considered. However, the macroeconomy is fundamentally different. In the labor market, microeconomic shocks are indeed *unspecifiable*. Thus, for the purpose of the analysis of the macroeconomy, sophisticated optimization exercises based on representative agent assumptions do not make much sense (Aoki and Yoshikawa (2007)).

This is actually partly recognized by search theorists themselves. The recognition has led them to introduce the “matching function” into the analysis. The matching function relates the rate of meetings of job seekers and firms to the numbers of the unemployed and job vacancies. The idea behind it is explained by Pissarides (2011) as follows.

“Although there were many attempts to derive an equilibrium wage distribution for markets with search frictions, I took a different approach to labor market equilibrium that could be better described by the term “matching”. The idea is that the job search underlying unemployment in the official definitions is not about looking for a good wage, but about looking for a good job match. Moreover, it is not only the worker who is concerned to find a good match, with the firm passively prepared to hire anyone who accepts its wage offer, but the firm is also as concerned with locating a good match before hiring someone.

The foundation for this idea is that each worker has many distinct features, which make her suitable for different kinds of jobs. Job requirements vary across firms too, and employers are not indifferent about the type of worker that they hire, whatever the wage. The process of matching workers to jobs takes time, irrespective of the wage offered by each job. A process whereby both workers and firms search for each other and jointly either accept or reject the match seemed to be closer to reality.

..... It allowed one to study equilibrium models that could incorporate real-world features like differences across workers and jobs, and differences in the institutional structure of labor markets.

The step from a theory of search based on the acceptance of a wage offer to one based on a good match is small but has far-reaching implications for the modeling of the labor market. The reason is that in the case of searching for a good match we can bring in the matching function as a description of the choices available to the worker. The matching function captures many features of frictions in labor markets that are not made explicit. It is a black box, as Barbara Petrongolo and I called it in our 2001 survey, in the same sense that the production function is a black box of technology. (Pissarides, 2011; pp.1093-1094)”

What job seeker is looking for is not simply a good wage, but a good job offer which cannot be uniquely defined but differs significantly across workers. It is simply *unspecifiable*. Pissarides recognizes such “real-world features” as differences across workers and jobs; the “universe” differs across workers and firms. Then, at the same time, he recognizes that we need a macro black box. The matching function is certainly a black box not explicitly derived from micro optimization exercises, and is, in fact, not a function of any economic variable which directly affects the decisions of individual

workers and firms. Good in spirit, but the matching function is still only a half way in our view.

The matching function is based on a kind of common sense in that the number of job matching would increase when there are a greater number of both job seekers and vacancies. However, it still abstracts itself from an important aspect of reality. As Okun (1973) emphasizes, the problem of unemployment cannot be reduced only to numbers.

“The evidence presented above confirms that a high-pressure economy generates not only more jobs than does a slack economy, but also a different pattern of employment. It suggests that, in a weak labor market, a poor job is often the best job available, superior at least to the alternative of no job. A high-pressure economy provides people with a chance to climb ladders to better jobs.

The industry shifts are only one dimension of ladder climbing. Increased upward movements within firms and within industries, and greater geographical flows from lower-income to higher-income regions, are also likely to be significant. (Okun, 1973; pp.234-235)”

Dynamics of unemployment cannot be separated from qualities of jobs, or more specifically distribution of productivity on which we focus in the present paper.

To explicitly consider these problems, we face greater complexity and, therefore, need a “greater macro black box” than the standard matching function. Our analysis, in fact, demonstrates that the “matching function” is *not* a structurally given function, but depends crucially on the level of aggregate demand. This is the motive for stochastic macro-equilibrium we explain in the next section.

3. Stochastic Macro-equilibrium — The Basic Idea

Our vision of the macroeconomy is basically the same as standard search theory. Workers are always interested in better job opportunities, and occasionally change their jobs. Job opportunities facing workers are stochastic depending on vacancy signs posted by firms. Firms, on the other hand, make their efforts to recruit and retain the best workers for their purposes. We assume that firms are demand-constrained in the product market. Demand determines the level of production and, as a consequence, demand for labor.

Unemployment, a great challenge to any economy, deserves special attention in economic analysis. However, a significant part of workers actually change their jobs without experiencing any spell of unemployment. The importance of on the job search also means that job turnover depends crucially on the distribution of qualities of jobs in the economy. Postel-Vinay and Robin (2002), in fact, analyze on-the-job search explicitly considering workers with different abilities on one hand and firms with

different productivities on the other. Their analysis, however, still depends on restrictive assumptions that all the workers face the same job offer distribution independent of their ability and employment status, and that the product market is perfectly competitive.

While workers search for suitable jobs, firms also search for suitable workers. Firm's job offer is, of course, conditional on its economic performance. The present analysis focuses on the firm's labor productivity. The firm's labor productivity increases thanks to capital accumulation and technical progress or innovations. However, those job sites with high productivity remain only *potential* unless firms face high enough demand for their products; firms may not post job vacancy signs or even discharge the existing workers when demand is low. As noted above, we assume that firms are all monopolistically competitive in the sense that they face downward sloping individual demand curves, and the levels of production are determined by demand rather than increasing marginal costs.

Formally, a most elegant general equilibrium model of monopolistic competition is given by Negishi (1960-61). Negishi (1979) persuasively argues that when the firm is monopolistically competitive, the individual demand curve is not only downward-sloping, but must be kinked at the current level of output and price. The corresponding marginal revenue becomes discontinuous as shown in Figure 1. The response of the firm's sales to a change in price is asymmetric because of the asymmetric reactions of rival firms on one hand, and the asymmetric reactions of customers on the other. Drèze (1979) also shows that for a risk-averse firm, uncertainty about the price elasticity of demand has an effect equivalent to that of kinked demand curve with the kink located at the current price and quantity. Therefore, in the economy with uncertainty and frictions as emphasized by search theory, it is reasonable to expect kinked individual demand curve facing monopolistically competitive firm.

Analytically, firm j ($j = 1, 2, \dots, K$) maximizes its profit

$$p_j(y_j, \bar{p}_j, \bar{y}_j) y_j - wn_j \quad (4)$$

with respect to labor n_j . Here, y_j and p_j are the firm's output and its price, respectively. For simplicity, we assume linear production function.

$$y_j = c_j n_j \quad (c_j > 0) \quad (5)$$

where c_j is the labor productivity. The price p_j depends not only on y_j but also on the current \bar{p}_j and \bar{y}_j . The firm perceives that it is prevented by competition from raising the price above its current level \bar{p}_j , while at the same time, it realizes the

difficulty of selling a larger quantity of its product without reducing the price below \bar{p}_j . The perceived individual demand curve $p_j(y_j, \bar{p}_j, \bar{y}_j)$ satisfies the following conditions :

$$\bar{p}_j = p_j(\bar{y}_j, \bar{p}_j, \bar{y}_j) \quad (6)$$

$$p_j = \bar{p}_j \quad \text{for } y_j \leq \bar{y}_j \quad (7)$$

and

$$\frac{\partial p_j}{\partial y_j} < 0 \quad \text{for } y_j > \bar{y}_j. \quad (8)$$

When the firm's expectation on sales is realized, *i.e.* $y_j = \bar{y}_j$ and $p_j = \bar{p}_j$, the conditions for maximization are

$$p_j(1 - \eta_j) c_j \leq w \quad (9)$$

and

$$p_j c_j \geq w \quad (10)$$

at (\bar{p}_j, \bar{y}_j) where η_j is the right-hand side elasticity of p_j with respect to y_j at \bar{y}_j .

Conditions (9) and (10) mean that the marginal cost of production w/c_j is no larger than the price, and no smaller than the marginal revenue $p_j(1 - \eta_j)$ for any increase in the level of output. It can be shown that any change in the level of sales at the unchanged price, namely a shift in demand is completely absorbed in a change in output leaving the price unchanged because inequality conditions (9) and (10) remain undisturbed. We must note that this result does not ensue only from the assumption that the firm's individual demand curve is downward-sloping. If there is no kink in the perceived demand curve, an increase in sales at the unchanged price may be followed by an increase in price leaving output little changed. With the kinked individual demand curve, the marginal revenue becomes discontinuous, and a shift in demand is all absorbed in a change in output leaving the price unchanged.

We must also note that given inequality conditions (9) and (10), a change in wages which disturbs the marginal cost leaves output and the price unchanged. This result is in stark contrast to the standard search literature. In the equilibrium search theory, wages determined within firm by means of such mechanism as bargaining crucially affect the

job creation rate, and accordingly the level of output. This is because labor employment and output are assumed to depend on firm's strategic behavior of recruitment under the assumption of perfect competition in the product market, and it, in turn, depends on wages by way of profitability. In the standard search framework, therefore, the determination of wages plays a crucial role in accounting for volatility of employment or unemployment (Pissarides (2009)). However, it is hard to believe that wages regardless how they are determined, have anything to do with a sharp increase of the US unemployment rate from 4.6% to 9.8% during the Great Recession (2007 - 2010).

In the present framework the way wages are determined is actually of secondary importance for the determination of employment and output because within a certain limit, wages do not affect inequality conditions (9) and (10). Here, output and labor employment are basically determined by real demand facing individual firm in the product market, not by wages. With the perceived kinked individual demand curve, a shift of demand is basically absorbed by a change in output leaving price unchanged. As Tobin (1993) says, the model opens the door for "quantities determine quantities." In this framework, we can concentrate on quantities (output and labor employment) without explicitly considering price and wages.

Negishi (1979; Chapter 6) shows the existence of general equilibrium of monopolistically competitive firms facing the kinked individual demand curve. Dreze and Herings (2008) establish the existence of general equilibrium under much more general assumptions. Though the equilibrium exists, it is indeterminate because the initial levels of the firms' output and price (the kink point of the individual demand curve, or point P in Figure 1) must be exogenously given. To give such initial condition amounts to determining the level of individual demand curve facing each firm.

The present analysis provides a rule for allocating the aggregate demand D to monopolistically competitive firms based on the principle of statistical physics.

$$D = \sum_{j=1}^K \bar{y}_j \tag{11}$$

Negishi (1979) and Dreze and Herings (2008)s' general equilibrium models of monopolistically competitive firms abstract itself from frictions and uncertainty in the labor market. Unlike capital, however, worker either stays at a job, moves to another, or searches for one as the unemployed. Whenever necessary, firms try to recruit the best workers for their purposes. These are, of course, aspects of labor market which search theory analyses. We follow its spirit, but not its analytical method.

In the present analysis, we simply assume that firms with higher productivity make more attractive job offers to workers. This assumption means that whenever possible, workers move to firms with higher productivity though we never know particular reasons for such moves. For workers to move to firms with higher productivity, it is necessary that those firms must decide to fill the vacant job sites, and post enough number of vacancy signs. They post such vacancy signs only when they face an increase of demand for their products, and decide to raise the level of production. It also goes without saying that high productivity firms keep their existing workers only when they face high enough demand.

The question we ask is what the distribution of employed workers is across firms whose productivities differ. As we argued in the pervious section, because microeconomic shocks to both workers and firms are so complex and unspicifiable, optimization exercises based on representative agent assumptions do not help us much. In particular, we never know how the aggregate demand is distributed across monopolistically competitive firms. To repeat, among other things, the job arrival rate, the job separation rate, and the probability distribution of wages (or more generally measure of the desirability of the job) differ across workers and firms. This recognition is precisely the starting point of the fundamental method of statistical physics⁶. At first, one might think that allowing too large a dispersion of individual characteristics leaves so many degrees of freedom that almost anything can happen. However, it turns out that the methods of statistical physics provide us not only with qualitative results but also with quantitative predictions.

In the present model, the fundamental constraint on the economy as a whole is aggregate demand D . Accordingly, to each firm facing the downward-sloping kinked individual demand curve, the level of demand for its product is the fundamental constraint. The problem is how the aggregate demand D is allocated to these monopolistically competitive firms, namely the determination of \bar{y}_j . Our model provides a solution. The basic idea behind the analysis can be explained with the help of the simplest case.

Suppose that n_k workers belong to firms whose productivity is c_k ($c_k < c_{k'}$ where $k < k'$). There are K levels of productivity in the economy ($k = 1, 2, \dots, K$). The total number of workers N is given.

$$\sum_{k=1}^K n_k = N \tag{12}$$

⁶ Foley (1994), in his seminal application of this approach to general equilibrium theory, advanced the idea of “statistical equilibrium theory of markets”.

A vector $n = (n_1, n_2, \dots, n_K)$ shows a particular allocation of workers across firms with different productivities. The combinatorial number W_n of obtaining this allocation, n , is equal to that of throwing N balls to K different boxes. It is

$$W_n = \frac{N!}{\prod_{k=1}^K n_k!} \quad (13)$$

Because the number of all the possible ways to allocate N different balls to K different boxes is K^N , the probability that a particular allocation $n = (n_1, n_2, \dots, n_K)$ is obtained is

$$P_n = \frac{W_n}{K^N} = \frac{1}{K^N} \frac{N!}{\prod_{k=1}^K n_k!} \quad (14)$$

It is the fundamental postulate of statistical physics that the state or the allocation $n = (n_1, n_2, \dots, n_K)$ which maximizes the probability P_n or (14) *under macro-constraints* is to be actually observed in the economy. The idea is similar to maximum likelihood in statistics/econometrics.

Maximizing P_n is equivalent to maximizing $\ln P_n$. Applying the Stirling formula for large number x

$$\ln x! \cong x \ln x - x, \quad (15)$$

we find that the maximization of $\ln P_n$ is equivalent to that of S .

$$S = -\sum_{k=1}^K p_k \ln p_k \quad (p_k = \frac{n_k}{N}) \quad (16)$$

S is the Shannon entropy, and captures the combinatorial aspect of the problem. It can be easily understood with the help of a simple example. When we throw a pair of dice, the possible sums range from two to twelve. Six is much more likely than twelve simply because the combinatorial number of the former is five whereas that of the latter is only one. Though the combinatorial consideration summarized in the entropy plays a decisive role for the final outcome, that is not the whole story, of course. The qualification “under macro-constraints” is crucial.

The first macro-constraint concerns the labor endowment, (12). The second macro-constraint concerns the effective demand.

$$D = \sum_{k=1}^K y_k = \sum_{k=1}^K c_k n_k \quad (17)$$

Here, aggregate demand D is assumed to be given. In our analysis, we explicitly analyze the allocation of labor (n_1, n_2, \dots, n_K) though the allocation of labor is basically

equivalent to the allocation of the aggregate demand to monopolistically competitive firms.

To maximize entropy S under two macro-constraints (12) and (17), set up the following Lagrangean form L :

$$L = -\sum_{k=1}^K \left(\frac{n_k}{N} \right) \ln \left(\frac{n_k}{N} \right) + \alpha \left[N - \sum_{k=1}^K n_k \right] + \beta \left[D - \sum_{k=1}^K c_k n_k \right] \quad (18)$$

with two Lagrangean multipliers, α and β . Maximization of this Lagrangean form with respect to n_k leads us to the first-order conditions:

$$\ln \left(\frac{n_k}{N} \right) = -1 - \alpha N - \beta N c_k \quad (k = 1, 2, \dots, K) \quad (19)$$

which are equivalent to

$$\frac{n_k}{N} = \exp[-1 - \alpha N - \beta N c_k]. \quad (20)$$

Because n_k/N sums up to one, we obtain

$$\frac{n_k}{N} = \frac{e^{-\beta N c_k}}{\sum_{k=1}^K e^{-\beta N c_k}} \quad (21)$$

Thus, the number of workers working at firms with productivity c_k is exponentially distributed. It is well known as the Boltzmann distribution in physics, but here arises a crucial difference between economics and physics. In physics, c_k corresponds to the level of energy. Whenever possible, particles tend to move toward the *lowest* energy level. To the contrary, in economics, workers always strive for better jobs offered by firms with higher productivity c_k . In fact, if allowed, all the workers would move up to the job sites with the highest productivity, c_K . This situation shown in Figure 2 corresponds to the textbook Pareto optimal equilibrium with no frictions and uncertainty. However, this state is actually impossible unless the level of aggregate demand D is so high as equal to the maximum level $D_{\max} = c_K N$. Only when D is equal to D_{\max} , demand-constrained firms with the highest productivity face high enough individual demand curves so that they have incentives to employ N workers. On the part of workers, they would be able to find job sites with the highest productivity because firms with c_K try hard to keep their existing workers, and at the same time post enough vacancy signs to aggressively recruit. With frictions, D must be actually higher than D_{\max} for the state shown in Figure 2 to be realized. The maximum level of aggregate demand D_{\max} is only imaginary, however. When D is lower than D_{\max} , the story is quite different. Some workers — a majority of workers, in fact, must work

at job sites with productivity lower than c_K .

How are workers distributed over job sites with different productivity? Obviously, it depends on the level of aggregate demand. When D reaches its lowest level, D_{\min} , workers are distributed evenly across all the sectors with different levels of productivity, c_1, c_2, \dots, c_K (Figure 3). Here, D_{\min} is defined as $D_{\min} = N(c_1 + c_2 + \dots + c_k) / K$. It is easy to see that the lower the level of D is, the greater the combinatorial number of distribution (n_1, n_2, \dots, n_K) which satisfies aggregate demand constraint (17) becomes. Note that this combinatorial number basically corresponds to the distribution of demand for goods and services across job sites at monopolistically competitive firms with different levels of productivity.

As explained above, the combinatorial number W_n of a particular allocation $n = (n_1, n_2, \dots, n_K)$ is basically equivalent to the Shannon entropy, S defined by (16). S increases when D decreases. In the extreme case where D is equal to (or more realistically greater than) the maximum level D_{\max} , all the workers work at job sites with the highest productivity because whenever possible, workers move to better job sites with higher productivity (Figure 2). In this case, the entropy S becomes zero, its lowest level, because $n_K / N = 1$ and $n_k / N = 0$ ($k \neq K$). In the other extreme where aggregate demand is equal to the minimum level D_{\min} , we have $n_k = N / K$ (Figure 3), and the entropy S defined by (16) becomes $\ln K$, its maximum level. The relation between the entropy S and the level of aggregate demand D , therefore, looks like the one shown in Figure 4. In the present model, the entropy basically corresponds to dispersion of job quality in the labor market which in turn, comes from dispersion of demand facing firms in the product market. Workers face greater uncertainty and friction when the level of aggregate demand is low because dispersion of job quality becomes larger in that case. Low productivity firms can attract workers as high productivity firms facing more stringent demand constraints make less job openings under lower aggregate demand.

At this stage, we can recall that the Lagrangean multiplier β in (18) for aggregate demand constraint is equal to

$$\beta = \frac{\partial L}{\partial D} = \frac{\partial S}{\partial D}. \quad (22)$$

β is the slope of the tangent of the curve as shown in Figure 4, and, therefore, is negative⁷. Table 1 shows how negative β relates to the level of aggregate demand D .

⁷ In physics, β is positive in most cases. This difference arises because workers strive for job sites with higher productivity, not the other way round (Iyetomi (2012)). In physics, β is equal to the inverse of temperature, or more precisely, temperature is defined as the inverse of $\partial S / \partial D$ when

With negative β , the exponential distribution (21) looks like the one shown in Figure 5. Whenever possible, workers always move toward job sites with higher productivity. As a result, the distribution is upward-sloping. However, unless the aggregate demand is equal to (or greater than) the maximum level, D_{\max} , workers' efforts to reach job sites with the highest productivity c_K must be frustrated because firms with the highest productivity do not need to employ a large number of workers and are less aggressive in recruitment, and accordingly it becomes harder for workers to find such jobs. As a consequence, workers are distributed over all the job-sites with different levels of productivity.

The maximization of entropy under the aggregate demand constraint (18), in fact, balances two forces. On one hand, whenever possible, workers move to better jobs identified with job sites with higher productivity here. It is the outcome of successful job matching resulting from the worker's search and the firm's recruitment. When the level of aggregate demand is high, this force dominates. However, when D is lower than D_{\max} , there are in general a number of different allocations (n_1, n_2, \dots, n_K) which are consistent with D .

As we explained in the previous section, micro shocks facing both workers and firms are truly unspecifiable. We simply do not know, on one hand, which firms with what productivity face how much demand constraint and need to employ how many workers with what qualifications, and on the other, which workers are seeking what kind of jobs with how much productivity. Here comes the maximization of entropy. It gives us the distribution (n_1, n_2, \dots, n_K) which corresponds to the maximum combinatorial number consistent with given D .

The entropy maximization under aggregate demand constraint plays, therefore, the role similar to the matching function in the standard search theory. Note that unlike the standard matching function which focuses only on the number of jobs, the matching of job quality characterized by productivity plays a central role in the present analysis. The matching function presumes that the number of job matching increases when the numbers of the unemployed and vacancy increase. In the present analysis, the matching of high productivity jobs is ultimately conditioned by the level of aggregate demand because high aggregate demand loosens demand constraints facing monopolistically competitive high productivity firms, and *vice versa*. Uncertainty and frictions emphasized by the standard search theory are *not* exogenously given, but depend

S is the entropy and D energy. Thus, negative β means the negative temperature. It may sound odd, but the notion of negative temperature is perfectly legitimate in such systems as the one in the present analysis; see Ramsey (1965) and Appendix E of Kittel and Kroemer (1980).

crucially on aggregate demand; In a booming gold-rush town, one does not waste a minute to find a good job!

The distribution of workers over job-sites with different productivity levels which results from this analysis is an upward-sloping exponential distribution as shown in Figure 5. In the figure, the bold line ($\beta = -1/5000$) corresponds to high aggregate demand, D and the dotted line ($\beta = -1/7500$) to low aggregate demand. As one should expect, the higher the level of aggregate demand D is, the steeper the distribution becomes meaning that more workers are mobilized to jobs with higher productivity (Okun (1973)).

It is essential to understand that the present approach does *not* regard economic agents' behaviors as random. Certainly, firms and workers maximize their profits and utilities. The present analysis, in fact, presumes that workers always strive for better jobs characterized by higher productivity. However, firms are demand-constrained facing downward-sloping kinked demand curves. As a result of profit-maximization, the optimal level of production is constrained by demand. Unless the level of aggregate demand is extremely high and equal to (with frictions, greater than) its maximum level $C_K N$, micro demand constraints become effective. In this case, the entropy matters because we can never know details of micro behaviors. Randomness underneath the entropy maximization comes from the fact that both the objective functions of and constraints facing a large number of economic agents are constantly subject to *unspecifiable* micro shocks. We must recall that the number of households is of order 10^7 , and the number of firms, 10^6 . Therefore, there is nothing for outside observers, namely economists analyzing the macroeconomy but to regard a particular allocation under macro-constraints as equi-probable. Then, it is most likely that the allocation of the aggregate demand and workers which maximizes the probability P_n or (14) *under macro-constraints* is realized⁸.

⁸ This method has been time and again successful in natural sciences when we analyze object comprising many micro elements. Economists might be still skeptical of the validity of the method in economics saying that inorganic atoms and molecules comprising gas are essentially different from optimizing economic agents. Every student of economics knows that behavior of dynamically optimizing economic agent such as the Ramey consumer is described by the Euler equation for a problem of calculus of variation. On the surface, such a sophisticated economic behavior must look remote from "mechanical" movements of an inorganic particle which only satisfy the law of motion. However, every student of physics knows that the Newtonian law of motion is actually nothing but the Euler equation for a certain variational problem; particles minimize the energy or the Hamiltonian!. It is called the principle of least action: see Chapter 19 of Feynmann (1964)'s *Lectures on Physics*, Vol. II. Therefore, behavior of dynamically optimizing economic agent and motions of inorganic particle are on a par to the extent that they both satisfy the Euler equations for respective variational problems. The method of statistical physics can be usefully applied not because motions of micro units are "mechanical," but because object under investigation comprises many micro units

4. The Model

The above analysis shows that the distribution of workers at firms with different productivities depends crucially on the level of aggregate demand. Though the simple model is useful to explain the basic idea, it is too simple to apply to the empirically observed distribution of labor productivity.

Empirical Distribution of Productivity

Figures 6 (a), (b) and (c) show the distributions of workers at different productivity levels for the Japanese economy ((a) total, (b) manufacturing and (c) non-manufacturing industries). The data used are the Nikkei Economic Electric Database (NEEDS, <http://www.crd-office.net/CRD/english/index.html>) and the Credit Risk Database (CRD, <http://www.crd-office.net/CRD/english/index.html>) which cover more than a million large and medium/small firms for 2007.

The “productivity” here is simply value added of the firm divided by the number of employed workers, that is, the average labor productivity. Theoretically, we should be interested in unobserved *marginal* productivity, not the average productivity. Besides, proper “labor input” must be in terms of work hour, or for that matter even in terms of work efficiency units rather than the number of workers. For these reasons, the average labor productivity shown in the figure is a crude measure of theoretically meaningful unobserved marginal productivity. However, Aoyama *et. al.* (2010; p.38-41) demonstrates that when the average productivity and measurement errors are independent, the distribution of true marginal productivity obeys the power law with the same exponent as that for the measured average productivity. In other words, distribution is robust with respect to measurement errors in the present case. Incidentally, we also note that the Pareto efficiency of the economy pertains to marginal products of production factors, *not* to total factor productivity (TFP) some economists focus on.

Figure 6 drawn on the double logarithm plane broadly shows that (1) the distribution of labor productivity is single-peaked, (2) in the low productivity (left) region, it is upward-sloping exponential whereas (3) in the high productivity (right) region, it obeys downward-sloping power-law (Aoyama *et. al.*, (2010)). Ikeda and Souma (2009) find a similar distribution of productivity for the U.S. while DelliGatti *et. al.* (2008) find power-law tails of productivity distribution for France and Italy. In what follows, we present an extended model based on the principle of statistical physics for explaining the broad shape of this empirically observed distribution.

individual movements of which we are unable to know. It leads us to a specific exponential distribution (21).

We explained in the previous section that the entropy maximization under macro constraints leads us to exponential distribution. This distribution with negative β can explain the broad pattern of the left-hand side of the distribution shown in Figures 6 (a) (b) and (c), namely an upward-sloping exponential distribution (Iyetomi (2012)). However, we cannot explain the downward-sloping power distribution for high productivity firms. To explain it, we need to make an additional assumption that the number of potentially available high-productivity jobs is limited, and that it decreases as the level of productivity rises.

Dynamics of Potential Job Creation/Destruction

Potential jobs are created by firms by accumulating capital and/or introducing new technologies, particularly new products. On the other hand, they are destroyed by firms' losing demand for their products permanently. Schumpeterian innovations by way of creative destruction raise the levels of some potential jobs, but at the same time lower the levels of others. In this way, the number of potential jobs with a particular level of productivity keeps changing. They remain *potential* because firms do not attempt to fill all the job sites with workers. To fill them, firms either keep the existing workers on the job or post job vacancy signs, but it is an economic decision, and depends crucially on the economic conditions facing the firms. The most important constraint on the firm's employment is the level of demand in the product market. The number of potential job sites, therefore, is not exactly equal to, but rather imposes a ceiling on the sum of the number of filled job sites, namely employment and the number of job openings or vacancy signs.

The statistical theory we will discuss later explains how employment is determined. In this sub-section, we first consider dynamics of *potential* job creation and destruction. Causes of creation/destruction of potential job sites are micro-shocks, and as explained in the previous section, unspecifiable. The best way to describe them is Markov model. Good examples in economics are Champernowne (1950) on income distribution, and Ijiri and Simon (1979) on size distribution of firms. Here, we adapt the model of Marsili and Zhang (1998) to our own purpose. The goal of the analysis is to derive a power-law distribution such as the one for the tail of the empirically observed distribution of labor productivity.

Suppose that there are f_j "potential" jobs with productivity, c_j ($c_j < c_{j+1}$),

when $j < j'$). In small time interval dt , the level of productivity c of a potential job site increases with probability $w_+(c) dt$ by a small amount which we can assume is unity without loss of generality. We denote this probability as $w_+(c)$ because w_+ depends on the level of c . Likewise, it decreases by a unit with productivity $w_-(c) dt$. Thus, $w_+(c)$ and $w_-(c)$ are transition rates for the processes $c \rightarrow c+1$ and $c \rightarrow c-1$, respectively. As noted above, the productivity of job site changes for many reasons. It may reflect technical progress or innovations. Given significant costs of adjusting labor, productivity also changes when demand for firm's products change. For example, when demand for firm's products falls, labor productivity necessarily declines. The decline of productivity in this case reflects labor hoarding (Fay and Medoff (1985)). These changes are captured by transition rates, $w_+(c)$ and $w_-(c)$ in the model.

We also assume that a new job site is born with a unit of productivity with probability $p dt$. On the other hand, a job site with productivity $c=1$ will cease to exist if c falls to zero. Thus the probability of exit is $w_-(c=1) dt$. A set of the transition rates and the entry probability specifies a jump Markov process.

Given this Markov model, the evolution of the average number of job sites of productivity c at time t , $f(c, t)$, obeys the following master equation:

$$\begin{aligned} \frac{\partial f(c, t)}{\partial t} = & w_+(c-1)f(c-1, t) + w_-(c+1)f(c+1, t) \\ & - w_+(c)f(c, t) - w_-(c)f(c, t) + p\delta_{c,1}, \end{aligned} \quad (23)$$

Here, $\delta_{c,1}$ is 1 if $c=1$, and 0 if otherwise. This equation shows that change in $f(c, t)$ over time is nothing but the *net* inflow to the state c .

We consider the steady state or the stationary solution of equation (23) such that $\partial f(c, t)/\partial t = 0$. The solution $f(c)$ can be readily obtained by using the boundary condition that $w_-(1) f(1) = p$:

$$f(c) = f(1) \prod_{k=1}^{c-1} \frac{w_+(c-k)}{w_-(c-k+1)}. \quad (24)$$

Here, we make an important assumption on the transition rates, w_+ and w_- . Namely, we assume that the probabilities of an increase and a decrease of productivity depend on the current level of productivity of job site. Specifically, the higher the current level of productivity, the larger a chance of unit productivity change. This

assumption means that the transition rates can be written as $w_+(c) = a_+ c^\alpha$ and $w_-(c) = a_- c^\alpha$, respectively. Here, a_+ and a_- are positive constants, and α is greater than 1. Under this assumption, the stationary solution (16) becomes

$$f(c) = \frac{f(1)}{1 - f(1)/C_{(\alpha)}} \frac{(1 - f(1)/C_{(\alpha)})^c}{c^\alpha} \cong c^{-\alpha} e^{-c/c^*}. \quad (25)$$

where

$$c^* \equiv (f(1)/C_{(\alpha)})^{-1} \quad \text{and} \quad C_{(\alpha)} \equiv \sum_{c=1}^{\infty} c^\alpha f(c).$$

We use the relation $a_+/a_- = 1 - f(1)/C_{(\alpha)}$. The approximation in equation (25) follows from $n(1)/C_{(\alpha)} \ll 1$, and the exponential cut-off works as c approaches to c^* . However, the value of c^* is practically large, and therefore, we observe the power law distribution $n(c) \propto c^{-\alpha}$ for a wide range of c in spite of the cut-off. Therefore, under the reasonable assumption, we obtain power law distribution for job sites, f_j .

The above model can be understood easily with the help of an analogy with the formation of cities. Imagine that $f(c, t)$ is the number of cities with population c at time t . $w_+(c)$ corresponds to a birth in a city with population c , or an inflow into the city from another city. Similarly, $w_-(c)$ represents a death or an exit of a person moving to another city. These rates are the instantaneous probabilities that the population of a city with the current population c either increases or decreases by 1. They are, therefore, the entry and exit rates of *one person* times the population c of the city, respectively. In addition, a drifter forms his own one-person city with the instantaneous probability p . In this model, the dynamics of $f(c, t)$ or the average number of cities with population c , is given by equation (23). In the case of population dynamics, one might assume that the entry (or birth) and exit (or death) rates of a person, a_+ and a_- , are independent of the size of population of the city in which the person lives. In that case $w_+(c)$ and $w_-(c)$ become linear functions of c , namely $a_+ c$ and $a_- c$. However, even in population dynamics, one might assume that the entry rate of a person into a large city is higher than its counterpart in a smaller one because of better job opportunities or the social attractiveness of such places, as encapsulated in the words of the song, “bright lights, big city”. The same may hold true for exit and death rates because of congestion or epidemics.

It turns out that in the dynamics of productivity of job site, both the “entry” and

“exit” rates of an existing “productivity job site” are increasing functions of c , namely the level of productivity in which that particular job site happens to be located; to be concrete, a_+c and a_-c . Thus, $w_+(c)$ becomes a_+c times c which is equal to a_+c^2 . Likewise, we obtain $w_-(c) = a_-c^2$. This is the case of the so-called Zipf law (Ijiri and Simon (1975)). Thus, under the reasonable assumption that the probability of a unit change in productivity is an increasing function of its current level c , we obtain power law distribution for job sites f_j .

Economists often presume that changes in productivity are caused by supply-side factors such as technical progress and entry/exit of firm alone. However, an important source of productivity change is actually a sectoral shift of demand. Indeed, Fay and Medoff (1985) documented such changes in firm’s labor productivity by way of changes in the rate of labor hoarding. Stochastic productivity changes as described in our Markov model certainly include technical progress, particularly in the case of an increase, but at the same time they embrace *allocative demand disturbances*. By their careful study of industry-level productivity and worker flows, Davis et al. (1996) found that while job creation is higher in industries with high total factor productivity growth, job destruction is not systematically related to productivity growth; job destruction is actually highest in the industries in the top productivity growth quintile (Table 3.7, Davis *et. al.* (1996, P.52)). This finding suggests the importance of negative *demand* shocks for job destruction.

Distribution of Productivity

We showed that under reasonable assumptions, distribution of potential job sites with high productivity becomes downward-sloping power law. However, the determination of employment by firms with various levels of productivity is another matter. To fill potential job sites with workers is the firm’s economic decision. The most important constraining factor is the level of demand facing the firm in the product market. Whatever the level, to fill potential job sites, the firm must either keep the existing workers on the job or post vacancy signs toward successful job matching. Such actions of the firms and job search of workers are purposeful. However, micro shocks affecting firms and workers are just unspecifiable. Then, how are workers actually employed at firms with various levels of productivity? This is the problem we considered in the previous section. In what follows, we will consider it in a more

general framework.

The number of workers working at the firms with productivity, c_j , namely n_j is

$$n_j \in \{0, 1, \dots, f_j\} \quad (j = 1, 2, \dots, K). \quad (26)$$

Here, f_j is the number of potential jobs with productivity c_j , and puts a ceiling on n_j ⁹. We assume that in the low productivity region, f_j is large enough meaning that n_j is virtually unconstrained by f_j . In contrast, in the high productivity region, f_j constrains n_j , and its distribution is power distribution as we have analyzed above.

Like many others, Postel-Vinay and Robin (2002) assume that there is no ceiling for job-sites with high productivity. On this assumption as well as the assumption that the product market is perfectly competitive, they regard a decreasing number of workers with high productivity jobs as a consequence of less recruitment efforts made by high productivity firms than by low productivity firms. This is a strange interpretation. There is no reasonable reason why high productivity firms make less recruitment efforts than low productivity firms. A more plausible assumption is that firms are monopolistically competitive facing the downward-sloping individual demand curve rather than the price takers in the product market, and that jobs with high productivity are limited in number by such demand constraints. Suppose, for example, that automobile industry is a high productivity industry. It would be unreasonable to argue that the level of employment in the industry is only the outcome of job matching, and that a limited size of employment is due to a lack of the firms' recruitment efforts. Plainly, the size of car producers is given by their capacity which is, in turn, determined by the level of demand for cars.

The total number of employed workers is simply the sum of n_j :

$$N = \sum_{j=1}^K n_j. \quad (27)$$

In the basic model in section 3, the total number of employed workers, N is exogenously given (equation (12)). In the extended model, N is assumed to be variable. N is smaller than the exogenously given total number of workers or labor force, L ($N < L$). The difference between L and N is the number of the unemployed, U :

$$U = L - N. \quad (28)$$

⁹ When the number of potential jobs with high productivity is limited, behavior of economic agents necessarily becomes correlated; If good jobs are taken by some workers, it becomes more difficult for others to find such jobs. Garibaldi and Scalas (2010) suggest that we study the problem by Markov model with such constraints.

As in the basic model, firms are monopolistically competitive facing the downward-sloping kinked individual demand curve. The firm's output is constrained by demand, which is conditioned by the level of aggregate demand, D (Equation (17)). In the basic model, D is literally given. Accordingly, total output is also given by equation (17), and is constant. In the present model, we more realistically assume that in accordance with fluctuations of aggregate demand, total output Y also fluctuates. Specifically, Y defined by

$$Y = \sum_{k=1}^K y_k = \sum_{k=1}^K c_k n_k \quad (29)$$

is now stochastic, and its expected value $\langle Y \rangle$ is equal to constant D . That is, we have

$$\langle Y \rangle = D. \quad (30)$$

Aggregate demand constrains total output in the sense of its expected value.

Under this assumption, the probability of total output Y turns out to be exponential; The density function $g(Y)$ is

$$g(Y) = \frac{e^{-\beta Y}}{\sum_i e^{-\beta Y_i}} \quad (31)$$

This result is obtained by the method of Gibbs' canonical ensemble. Gibbs established the statistical mechanics by introducing the concept of "canonical ensemble" which is a collection of macro states, Y in our present case. Suppose that there are K possible levels of Y denoted by Y_1, \dots, Y_K . For the moment, we reinterpret n_k as the number of cases where Y takes the value Y_k ($k = 1, \dots, K$). The sum of n_k , N is given. Then, n_k satisfies equation (12). We assume that the average of Y is equal to constant D .

$$\sum_{k=1}^K Y_k \left(\frac{n_k}{N} \right) = D \quad (32)$$

Replacing c_k by Y_k / N , we observe that equation (32) is equivalent to equation (17). Thus, we can apply the exactly same entropy maximization as we did in the basic model in section 3. It leads us to equation (31). In (31), β is the Lagrange multiplier for constraint (32) in the maximization of entropy (16).

Obviously, Y constrained by aggregate demand D affects the distribution of workers, n_k (equation (29)). In the present model, the number of employed workers N is not constant, but changes causing changes in unemployment. Besides, the number

of potential job sites with high productivity, f_j constrains n_j . Under these assumptions, we seek the state which maximizes the probability P_n or equation (14).

Before we proceed, it is necessary to explain *partition function* because it is rarely used in economics, but we will use it intensively in the subsequent analysis. When a stochastic variable Y is exponentially distributed, that is, its density function $g(Y)$ is given by equation (31), partition function Z is defined as

$$Z = \sum_i e^{-\beta Y_i} . \quad (33)$$

This function is extremely useful as moment generating function. For example, the first moment or the average of Y can be simply found by differentiating $\log Z$ with respect to β .

$$\begin{aligned} -\frac{d \log Z}{d\beta} &= -\frac{d}{d\beta} \log\left(\sum_i e^{-\beta Y_i}\right) = -\frac{\sum_i (-Y_i) e^{-\beta Y_i}}{\sum_i e^{-\beta Y_i}} \\ &= \sum_i Y_i \left(\frac{e^{-\beta Y_i}}{\sum_i e^{-\beta Y_i}}\right) = \sum_i Y_i g(Y_i) = E(Y_i) \end{aligned} \quad (34)$$

As in the basic model, we want to find the state which maximizes the probability, P_n or equation (14). We have two macro-constraints, equations (27) and (29). The total number of workers employed N is, however, not constant but variable. The aggregate output Y is also not constant but obeys the exponential distribution, namely equation (31).

Because the level of total output depends on the total number of employed workers N , we denote Y_i as $Y_i(N)$. Then, the canonical partition function Z_N can be written as

$$Z_N = \sum_i e^{-\beta Y_i(N)} . \quad (35)$$

Using equation (29), we can rewrite this partition function as follow:

$$Z_N = \sum_{\{n_i\}} \exp\left(-\beta \sum_{i=1}^K c_i n_i\right) . \quad (36)$$

It is generally difficult to carry out the summation with respect to $\{n_i\}$ under constraint (27), namely $N = \sum n_i$. Rather than taking N as given, we better allow N

to be variable as we do here, and consider the following extended partition function Φ ¹⁰. With the Lagrangean multiplier for N , μ , this partition function is defined as

$$\Phi = \sum_{N=0}^{\infty} z^N Z_N \quad (37)$$

where

$$z = e^{\beta\mu}. \quad (38)$$

As the Lagrangean multiplier for N , the parameter μ is the marginal condition of an addition employment to the entropy of the macroeconomy¹¹. It measures the marginal product of a worker who newly acquired job out of the pool of unemployment. Thus, μ plays a role similar to the reservation wage in standard models. When μ is high, the unemployed worker is “choosy”, and *vice versa*. In this sense, μ is a barrier between unemployment and employment. However, there is a fundamental difference between the reservation wage and μ . In standard models, the reservation wage R is absolute in the sense that no worker is willing to work with the wages below R . This concept is useful only if R can be well defined and known. As we pointed out in Section 2, the equilibrium search theory presumes the representative worker so that the reservation wage R can be defined. In contrast, μ in the present analysis plays only a relative role as a barrier between unemployment and employment. Each worker has his/her own “reservation job attractiveness” though we do not know what it is. μ represents the average reservation job attractiveness of a heterogeneous group of unemployed workers. We will explain the meaning of μ later, but in general, workers accept jobs with productivity below μ depending on the level of aggregate demand.

Substituting equation (36) into equation (37), the grand canonical partition function Φ becomes as follows:

$$\Phi = \sum_{N=0}^{\infty} z^N \sum_{n_j} \exp\{-\beta \sum_j n_j c_j\} \quad \text{where } z = e^{\beta\mu} \quad (39)$$

Using the definitions of z , (38), and also N , (27), we have

$$\Phi = \sum_{N=0}^{\infty} e^{\beta\mu(n_1+\dots+n_K)} \sum_{n_j} \exp\{-\beta \sum_j n_j c_j\} = \prod_{j=1}^K \sum_{n_j} \exp[\beta(\mu - c_j)n_j]. \quad (40)$$

¹⁰ It is called the grand canonical partition function in physics.

¹¹ μ is called the chemical potential in physics, and measures the marginal contribution in terms of energy of an additional particle to the system under investigation.

Because there is ceiling f_j for n_j (constraint (26)), (40) can be rewritten as follows:

$$\Phi = \prod_{j=1}^K [1 + e^{\beta(\mu-c)} + \dots + e^{f_j \beta(\mu-c)}] = \prod_{j=1}^K \left[\frac{1 - e^{(f_j+1)\beta(\mu-c)}}{1 - e^{\beta(\mu-c)}} \right] \quad (41)$$

With this grand canonical partition function Φ , we can easily obtain the expected value of the total number of employed workers N , $\langle N \rangle$ by differentiating $\log \Phi$ with respect to μ which corresponds to the reservation wage of the unemployed worker. This can be seen by differentiating (37) and noting the definition of z , (38).

$$\frac{1}{\beta} \left[\frac{\partial}{\partial \mu} \log \Phi \right] = \frac{1}{\beta} \left[\frac{\partial}{\partial \mu} \log \left(\sum_{N=0}^{\infty} e^{\beta \mu N} Z_N \right) \right] = \frac{1}{\beta} \left[\frac{\beta \sum_{N=0}^{\infty} N e^{\beta \mu N} Z_N}{\sum_{N=0}^{\infty} e^{\beta \mu N} Z_N} \right] = \langle N \rangle. \quad (42)$$

In the present case, Φ is actually given by equation (41). Therefore, we can find $\langle N \rangle$ as follows.

$$\begin{aligned} \langle N \rangle &= \frac{1}{\beta} \left[\frac{\partial}{\partial \mu} \log \Phi \right] \\ &= \frac{1}{\beta} \sum_{j=1}^K \frac{\partial}{\partial \mu} \{ \log(1 - e^{(f_j+1)\beta(\mu-c_j)}) - \log(1 - e^{\beta(\mu-c_j)}) \} \\ &= \sum_{j=1}^K \left[\frac{(f_j+1)e^{(f_j+1)\beta(\mu-c_j)}}{e^{(f_j+1)\beta(\mu-c_j)} - 1} - \frac{e^{\beta(\mu-c_j)}}{e^{\beta(\mu-c_j)} - 1} \right] \end{aligned} \quad (43)$$

The expected value of the number of workers employed on the job sites with productivity c_j , $\langle n_j \rangle$ is simply the corresponding term in the summation of $\langle N \rangle$ or equation (43).

$$\langle n_j \rangle = \frac{(f_j+1)e^{(f_j+1)\beta(\mu-c_j)}}{e^{(f_j+1)\beta(\mu-c_j)} - 1} - \frac{e^{\beta(\mu-c_j)}}{e^{\beta(\mu-c_j)} - 1} \quad (44)$$

Equation (44) determines the distribution of workers across job-sites with different levels of productivity in our stochastic macro-equilibrium.

Figure 7 shows how this model works. On one hand, there is dynamics of creation and destruction of potential job sites with various levels of productivity (Figure 7 (a)). We presented a Markov model which leads us to power-law tail of productivity distribution in the steady state. At the same time, there is another dynamics of job matching which corresponds to the standard search theory (Figure 7(b)). Convolution of two dynamics determines the distribution of workers at job sites with various productivities. The result is equation (44).

This distribution is fundamentally conditioned by aggregate demand. When the level of aggregate demand is high, it is more likely that high productivity firms make more job openings. They attract not only the unemployed but also workers currently on the inferior jobs. As Okun (1973) vividly illustrates, “a high pressure economy provides people with a chance to climb ladders to better jobs.” And people actually climb ladders in such circumstances.

It is useful to consider $\langle n_j \rangle$ in (44) under the extremely high and low levels of aggregate demand respectively. This exercise clarifies the meaning of a “barrier” between unemployment and employment, μ .

First is the case of the extremely high aggregate demand. In this case, β is $-\infty$ (Table 1). Given (44), for $c_j > \mu$, $\langle n_j \rangle$ becomes f_j , whereas for $c_j < \mu$, $\langle n_j \rangle$ is zero. That is, when aggregate demand is extremely high, no worker takes jobs whose productivity is lower than μ whereas employment at job sites with productivity greater than μ is constrained only by capacity, f_j . When $c = \mu$, $\langle n_j \rangle$ is equal to $f_j/2$.

In contrast, when aggregate demand is extremely low ($\beta = -0$), $\langle n_j \rangle$ becomes equal to $f_j/2$ irrespective of c_j and μ . Note that $\langle n_j \rangle$ is not zero for c_j below μ . When aggregate demand is low, a number of workers take “odd jobs” whose productivity is lower than μ . The more realistic case with the finite absolute value of β ($-\infty < \beta < 0$) will be discussed shortly

Unemployment and the Reservation Job Attractiveness

The distribution of employed workers across job-sites with different productivity, equation (44) simultaneously determines the level of unemployment because the number of unemployed U is simply the difference between exogenously given labor force, L and the number of employed workers, N which is nothing but the sum of $\langle n_j \rangle$. The expected value of N , $\langle N \rangle$ is given by (43).

When aggregate demand D rises, more workers work at job sites with higher productivity while at the same time, unemployment decreases. On the other hand, an increase in μ *ceteris paribus* raises unemployment. This can be most clearly seen in the case of the extremely high aggregate demand. As we noted above, in such a case ($\beta = -\infty$), μ plays a role similar to the reservation wage in the standard theory in the sense that no worker takes a job whose productivity is lower than μ . Thus, it is trivial to see that an increase in μ lowers employment and raises unemployment. More generally, we must recognize that the reservation job attractiveness differs across workers, and that it cannot be reduced to a single number of a particular variable such as

wages. μ in this model is a “barrier” between unemployment and employment, but unlike the reservation wage, it is not an absolute barrier. Given μ , when aggregate demand is low, workers actually work at job-sites whose productivity is lower than μ . As we will see shortly, the effect of a change in μ on unemployment depends on the level of aggregate demand.

Unemployment which arises because of frictions and uncertainty in the labor market is by definition job search. Each worker has his/her own reservation job attractiveness. However, frictions and uncertainty depend on the level of aggregate demand. When aggregate demand is high, high productivity firms make many job openings so that workers can find attractive jobs easily. When aggregate demand is low, just the opposite holds true. In this sense, some of the “frictional” unemployment is actually caused by aggregate demand deficiency.

A Numerical Example

With the help of a simple numerical example, we can better understand how equation (44) looks like, and also how unemployment is determined in the present model. In this example, the level of productivity are assumed to be $c_1 = 1, \dots, c_{200} = 200$. μ is set equal to 25. The labor force L is assumed to be 630. The number of potential jobs or the ceiling at each productivity level, f_j , is assumed to be constant at 10 for c_1, \dots, c_{50} , while it declines for c_j ($j = 50, \dots, 200$) as c_j increases. Specifically, for c_j ($j = 50, \dots, 200$), f_j obeys a power distribution: $f_j \sim 1/c_j^2$. This assumption means that low productivity jobs are potentially abundant whereas high productivity jobs are limited. In the following figures, the number of potential jobs or “capacity” is shown by a dotted line.

Figure 8 shows labor productivity distributions (equation (44)) under (A) the extremely high and (B) low levels of aggregate demand, respectively. As explained previously, when aggregate demand is extremely high ($\beta = -\infty$), high quality jobs are fully occupied at the ceiling level while no worker takes jobs whose productivity is lower than $\mu = 25$. In contrast, when aggregate demand is extremely low ($\beta = 0$), employment is equal to $f_j/2$ regardless of the level of productivity c_j . A number of workers take jobs with productivity lower than μ , while at the same time high quality job sites are left unfilled.

Figure 9 shows two realistic cases; Case A corresponds to high aggregate demand whereas Case B to low aggregate demand. Specifically, β is assumed to be (A) -0.05, and (B) -0.02. As explained in Table 1, Case (A) $\beta = -0.05$ corresponds to high demand D whereas Case (B) $\beta = -0.02$ to low demand. In Figure 9, we observe that

n_j increases up to $j = 50$, and then declines from $j = 50$ to 200 in both cases. Whenever possible, workers strive to get better jobs offered by firms with higher productivity. That is why the number of workers n_j increases as the level of productivity rises in the relatively low productivity region. Note that the number of potential jobs or the ceiling is constant in this region, and yet that n_j increases as the level of productivity c_j rises. The number of workers n_j turns to be a decreasing function of productivity c_j in the high productivity region simply because the number of potentially available jobs f_j declines as c_j rises. Note, however, that n_j is not equal to f_j ; n_j is strictly smaller than f_j .

Single-peaked distributions shown in Figure 9 are broadly consistent with the observed pattern of productivity dispersion (Figure 6).

When aggregate demand D goes up, the distribution shifts in such a way that more workers work at high productivity job sites and less workers at low productivity job sites. Figure 10 shows how employment at job sites with different productivity depends on the level of aggregate demand. When aggregate demand is high, the percentage of filled jobs with high productivity is high whereas the percentage of filled jobs with low productivity (to be precise, lower than μ) is low, and *vice versa*.

When aggregate demand increases, the number of employed workers N which corresponds to the area below the distribution curve, increases. Specifically, N is 618 in case (A) while it is 582 in case (B). It means that given labor force $L = 630$, the unemployed rate $U/L = (L - N)/L$ declines when aggregate demand D rises. In this example, the unemployment rate is 1.8% in case (A) while it is 7.6% in case (B).

Table 2 shows (a) the unemployment rate, (b) the share of workers on job sites with productivity higher than μ , and (c) the corresponding share of workers on job sites with productivity lower than μ for various levels of aggregate demand or β . When aggregate demand rises, the unemployment rate declines while at the same time, the share of high quality job goes up.

Using this numerical example, we can also consider the effects of a change in the reservation job attractiveness μ on employment / unemployment. In the base model, μ is set equal to 25. Table 3 shows how employment and unemployment change when μ increases from 25 to 28. μ corresponds to the reservation wage in standard models so that an increase in μ raises the unemployment rate. However, Table 3 demonstrates that the extent to which the unemployment rate rises depends crucially on the level of aggregate demand. The higher the level of aggregate demand is, the more significant the effect of an increase μ on the unemployment rate is. It is important to recognize that the effects of a change in μ on unemployment gets smaller when the level of

aggregate demand is low.

Summary

Let us summarize economics of the present model. Potential job sites with various levels of productivity are determined not only by capital accumulation and technical progress but also by allocative disturbances to demand. The existing stock of capital and technology only slowly change, but allocative demand disturbances can swiftly change the economic values of the job sites associated with those capital and technology. Creative destructions due to Schumpeterian innovations raise the levels of productivity of some job sites, but at the same time, lower the levels of productivity of others. We consider a Markov model to describe dynamics of creations and destructions of potential job sites, and derive the conditions for the stationary state. The number of job sites with high productivity in the stationary state turns out to be power distribution. The important point is that job sites with relatively low productivities are abundant whereas those with high productivities are limited following power distribution¹².

Now, let us visualize a potential job site as a “box” or a “desk” for a worker. Each job site or box is associated with a particular level of productivity. It is either empty or occupied by a worker. We can consider that a firm is nothing but a cluster of many job sites or boxes. It is conceivable that productivity differs across boxes within a single firm.

To fill job sites or boxes with workers is a firm’s economic decision. To do so, the firm can either keep the existing workers on the job or post vacancy signs for workers searching better jobs. The most important factor constraining the firm’s labor employment is the level of the product demand which depends ultimately on the level of aggregate demand.

Because micro shocks and constraints facing firms and workers are so complex and unspecifiable, we cannot usefully pursue the exact micro behaviors. The single matching function for the economy as a whole will not do. Wages are obviously important, but attractiveness of job is not fully determined by wages. It depends on many factors; workers are interested in tenure, location, fringe benefits and other work conditions. The relative importance of these factors differs across workers. Likewise, the type of workers the firm is looking for cannot be simply defined, but depends on many factors. Again, the relative importance of these factors differs across firms.

¹² Houthakker (1955) shows that the aggregate production function becomes Cobb-Douglas on the assumption that the distribution of productivity (labor and capital coefficients in his model) is the power distribution: See also Sato (1974).

Besides, economic conditions facing workers and firms keep changing in unspecifiable ways. Given such complexity, optimization exercises based on the assumption that all the agents face the same probability distributions do not help us find the outcome of random matching of workers and firms. We cannot even define the unique reservation wage in a meaningful way. Here comes the method of statistical physics.

The basic assumption of the model is that firms with higher productivity can afford to make more attractive job offers to workers. However, firms are monopolistically competitive in that they face the downward-sloping kinked individual demand curve for their products. Then, their production and employment decisions depend ultimately on the level of aggregate demand; see Negishi (1979), Solow (1986), and Drèze and Herings (2008). We never know how aggregate demand is distributed across firms (or job sites) with different levels of productivity. However, when the level of aggregate demand is high, it is more likely that high productivity firms face high enough demand, and as a consequence, they keep more workers on the job and post more vacancy signs to attract good workers. Workers are certainly aware of it; we know that quit rates go up in boom and down in recession despite of the fact that the conditions of employed workers deteriorate in recession. As a result of such actions of both firms and workers, the distribution of productivity tilts to the direction of high productivity (Figure 9). As Okun (1973) argues, when aggregate demand rises, workers on the job “climb ladders to better jobs” without experiencing any spell of unemployment. At the same time, more workers currently in the unemployment pool find acceptable jobs. Employment N increases, and the unemployment rate declines.

5. The Principle of Effective Demand

Keynes (1936) argued that the aggregate demand determines the level of output in the economy as a whole. Factor endowment and technology may set a ceiling on aggregate output, but the actual level of output is effectively determined by aggregate demand. Factor endowment and technology change only slowly, but aggregate demand can change swiftly.

Unemployment is obviously a problem of the labor market, but Keynes (1936) argued that it is basically caused by demand deficiency in the markets for goods and services; Changes in aggregate demand determine cyclical changes in unemployment. Our economy really experiences occasional aggregate demand shocks. Figure 11 shows indices of exports and industrial production of Japan during the post-Lehman “great recession”. It is most reasonable to regard a sudden fall of exports as *exogenous real demand shocks* to the Japanese economy; plainly, we cannot regard it as a “technology”

or “productivity” shock in the ordinary sense of words. Figure 11 demonstrates that the principle of effective demand is alive and well! (See Iyetomi *et. al.* (2011))

The post-Lehman “great recession” provides us with an excellent example of the negative aggregate demand shock. The unemployment rate certainly rose. The relatively low and cyclically insensitive Japan’s unemployment rate was 3.6% as of July 2007, but after the global financial crisis, rose to 5.5 percent by July 2009. During the same period, the unemployment rate rose from 4.6% to 9.8% in the US. Unemployment is not the whole story, however, because distribution of productivity also changes.

Figures 12 (a), (b), (c) compare the distributions of productivity *before* and *after* the Lehman crisis, namely 2007 and 2009; (a) total, (b) manufacturing sector, and (c) non-manufacturing sector. As our theory indicates, the distribution as a whole, in fact, tilts toward lower productivity in severe recession. Figure 12 shows that the tilt of the distribution toward low productivity is more conspicuous for the manufacturing industry than for the non-manufacturing industry. It is due to the fact that in Japan, the 2009 recession after the bankruptcy of the Lehman Brothers was basically caused by a fall of exports (Figure 11), and that exports consist mainly of manufactured products such as cars. We can observe, however, that the distribution tilts toward low productivity for the non-manufacturing industry as well, particularly in the high productivity region. This is, of course, due to the fact that a fall of demand in the manufacturing sector spills over to the non-manufacturing sector.

The existing literature focuses on “productivity shocks,” and takes them as exogenous. Our analysis shows, however, that cyclical changes in “productivity” can be nothing but the *result* of aggregate demand shocks.

When output changes responding to changes in aggregate demand, the level of utilization of production factors must necessarily change. An example is cyclical changes of capacity utilization of capital. Unemployment of labor is another. However, as a measure of underutilization of labor, unemployment is not sufficient because distribution of productivity of employed workers significantly changes. Our model shows that aggregate demand determines the degree of uncertainty and frictions with respect to dispersion of job quality in the labor market, and as a consequence, changes both unemployment and distribution of labor productivity across firms and job sites. This is the market mechanism beneath Keynes’ principle of efficient demand.

6. Concluding Remarks

It is a cliché that the Keynesian problem of unemployment and under-utilization of production factors arises because prices and wages are inflexible. Tobin (1993), in

fact, Keynes (1936) himself argued that the principle of effective demand holds true regardless of flexibility of prices and wages because adjustment in terms of quantity is faster.

The natural micro picture underneath the Keynesian economics is monopolistic competition of firms facing the downward sloping individual demand curve, not perfect competition in the product market. Negishi (1979)'s model of general equilibrium of monopolistically competitive firms *with the kinked individual demand curve* provides a neat microfoundation for what Tobin (1993) called the Old Keynesian view in which "quantities determine quantities." In this framework, we focus on the determination of quantities such as output and labor employment. This model, however, abstracts itself from frictions and uncertainty present in the labor market.

The standard equilibrium search theory has filled a gap by explicitly considering frictions and matching costs in the labor market. While acknowledging the achievement of standard search theory, we pointed out two fundamental problems with the theory. First is the assumption that the job arrival rate, the job separation rate, and the probability distribution of wages (more generally, some measure of the desirability of the jobs) are common to all the workers and firms. This assumption, though it is taken for granted in most models, is simply untenable. There is always the economy-wide distribution of economic variables of interest such as the job arrival rate, the job separation rate, and wages, of course. However, it is not relevant distribution for individual worker and firm. Each economic agent faces different job arrival rate, job separation rate and probability distribution of wages. In short, each economic agent acts in its own "universe". For this reason, the unique reservation wage as a determinant of unemployment cannot be defined. It is, in fact, frictions and uncertainty emphasized by the equilibrium search theory that makes the economy-wide distribution or the average irrelevant to economic decisions made by individual economic agent.

Secondly, we maintain that the standard assumption that the product market is perfectly competitive in the sense that the firm's individual demand curve is flat is ill-suited for studying *cyclical* changes in effective utilization rate of labor. It is indeed curious that the standard search theory puts so much emphasis on frictions and uncertainty in the labor market while at the same time it is content with the naïve assumption of perfect competition in the goods market. Prices and wages certainly guide economic agents in market economy. However, they do so only partially with frictions and uncertainty; quantity constraints also play an equally important role in the resource allocation.

Under the assumption of perfect competition, cyclical driving forces are

identified with changes in the average productivity (See, for example, Barlevy (2002), Shimer (2005, 2010), Mortensen and Nagypál (2007) and Hagedorn and Manovskii (2008)). This exercise is essentially the real business cycle (RBC) theory (Kydland and Prescott (1982)). Shimer (2005)'s comparative static analysis however, in effect, demonstrates that a change in labor productivity *a la* RBC cannot reasonably explain the empirically observed magnitude of fluctuations of the unemployment and vacancy rates¹³. There is, in fact, a strong case that cyclical changes are caused by demand shocks rather than productivity shocks (Mankiw (1989), Summers (1986), and Iyotomi *et. al.* (2011)).

We assume that instead of perfect competition, firms face the downward sloping individual demand curve with a kink (Negishi (1979) and Drèze and Herings (2008)). This theoretical framework provides a neat microeconomic foundation for the old Keynesian view in which quantities determine quantities (Tobin (1993)). However, the equilibrium is indeterminate in the sense that we never know how the aggregate demand is distributed across firms or job sites with different levels of productivity. We are unable to know micro behaviors of workers and firms, either. A model of stochastic macro-equilibrium based on the principle of statistical physics provides a solution.

The concept of stochastic macro-equilibrium is motivated by the presence of all kinds of unspecifiable micro shocks. At first, one might think that allowing all kinds of unspecifiable micro shocks leaves so many degrees of freedom that almost anything can happen. However, the methods of statistical physics — the maximization of entropy under macro-constraints — actually provide us with the quantitative prediction about the equilibrium distribution of productivity, namely equation (44).

It is extremely important to recognize that the present approach does not regard behaviors of workers and firms as random. They certainly maximize their objective functions perhaps dynamically in their respective stochastic environments. The maximization of entropy under the aggregate demand constraint (18), in fact, balances two forces. On one hand, whenever possible, workers are assumed to move to better jobs which are identified with job sites with higher productivity. It is the outcome of successful job matching resulting from the worker's search and the firm's recruitment. When the level of aggregate demand is high, this force dominates because demand for labor of high productivity firms is high. However, as the aggregate demand gets lower, the number of possible allocations consistent with the level of aggregate demand

¹³ The present paper shows that a change in the labor productivity and a change in aggregate demand are different things. The Keynesian model with demand deficiency can easily reconcile the magnitudes of fluctuations of labor productivity on one hand and unemployment and vacancy on the other.

increases. More workers are forced to be satisfied with or look for low productivity jobs. Randomness which plays a crucial role in our analysis basically comes from the fact that the distribution of demand constraints in the product market across firms with different productivity, and optimizing behaviors of workers and firms under such constraints are so complex and unspecifiable that those of us who analyze the macroeconomy must take micro behaviors as random. The method is straight-forward, and does not require any arbitrary assumptions on the behavior of economic agents.

When the level of aggregate demand is high, it is most likely that high productivity firms keep more workers on the job¹⁴, and put more vacancy signs than in the period of low demand. Workers are certainly aware of such a change. It is demonstrated by the fact that quit rates are higher in high-demand periods despite of the fact that the employed workers are treated better in such periods. Whether or not better jobs are really offered and workers move to those jobs depends ultimately on the level of aggregate demand. Our analysis demonstrates that the most probable outcome of random matching of firms and workers is given by equation (44) which depends on aggregate demand. It broadly coincides with the empirically observed distribution of productivity.

We emphasize that *frictions and uncertainty in the labor market are not exogenously given, but depend crucially on the aggregate demand*. The aggregate demand, therefore, fundamentally conditions the rate of successful matching. The entropy maximization plays the role of matching function in standard search theory. We must note that the role of the reservation job attractiveness, μ in the model also depends crucially on the aggregate demand.

Keynes' theory has been long debated in terms of unemployment or "involuntary" unemployment. Though unemployment is one of the most important economic problems in any society, to focus only on unemployment is inadequate for the purpose of providing micro-foundations for the Keynesian economics. The real issue is whether or not there is any room for mobilizing labor to high productivity jobs, firms, or sectors. The famous Okun's law demonstrates that there is always such a room in the economy (Okun (1963)); See Syverson (2011) on more recent research on productivity dispersion.

Based on the methods of statistical physics, the present paper quantitatively shows how labor is mobilized when the aggregate demand rises. The level of aggregate

¹⁴ To be precise, more workers are on the "job sites" with higher productivity because productivity may differ across job sites within a single firm. The method I used in this paper should apply to "job sites" rather than firms. A firm is a cluster of job sites with different productivity levels.

demand is the ultimate factor conditioning the outcome of random matching of workers and monopolistically competitive firms. By so doing, it changes not only unemployment but also distribution of productivity, and as a consequence the level of aggregate output. This is the market mechanism beneath Keynes' principle of effective demand.

Table 1: Negative β and Aggregate Demand

β	0- $-\infty$
Aggregate Demand D	Low High

Table 2: Aggregate Demand, Job Quality, and Unemployment

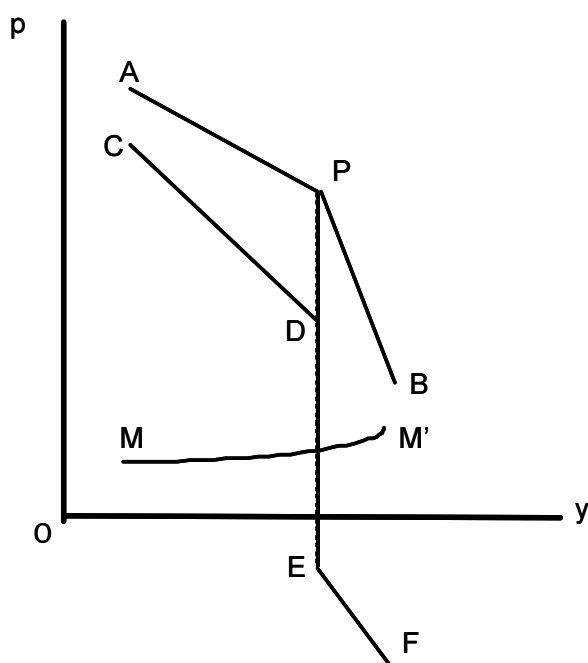
Aggregate Demand β		← High			Low →	
	-0.1	-0.05	-0.03	-0.02	-0.01	-0.001
Unemployment Rate (U / L) %	0.9	1.8	4.1	7.6	15.9	29.2
Share of Employment with Productivity $c_i > \mu$ (%)	97	94	91	88	83	74
Share of Employment with Productivity $c_i < \mu$ (%)	3	6	9	12	17	26

Note : μ = The Critical “Reservation Job Attractiveness.”
See the main text for details.

Table 3: The Effects of a Change in the Reservation Job Attractiveness μ on Unemployment Depends on the Level of Aggregate Demand

Aggregate Demand β	-0.001	← Low -0.005	-0.01	High → -0.02	-0.05
(1) Employment when $\mu = 25$	446	487	530	582	618
(2) Employment when $\mu = 28$	444	477	513	557	590
(3) Change in Employment when μ increased from 25 to 28 ((2) - (1))	-2	-10	-17	-25	-28
(4) Change in Unemployment Rate (3) / $L = 630$	0.3	1.6	2.7	4.0	4.5

**Figure 1: The Kinked Individual Demand Curve
Facing Monopolistically Competitive Firm: Negishi (1979)**



Note : APB = Individual Demand Curve
CDEF = Marginal Revenue
MM' = Marginal Cost

Figure 2: All the Workers Work at Job sites with the Highest Level of Productivity under Extremely High Aggregate Demand, D_{\max}

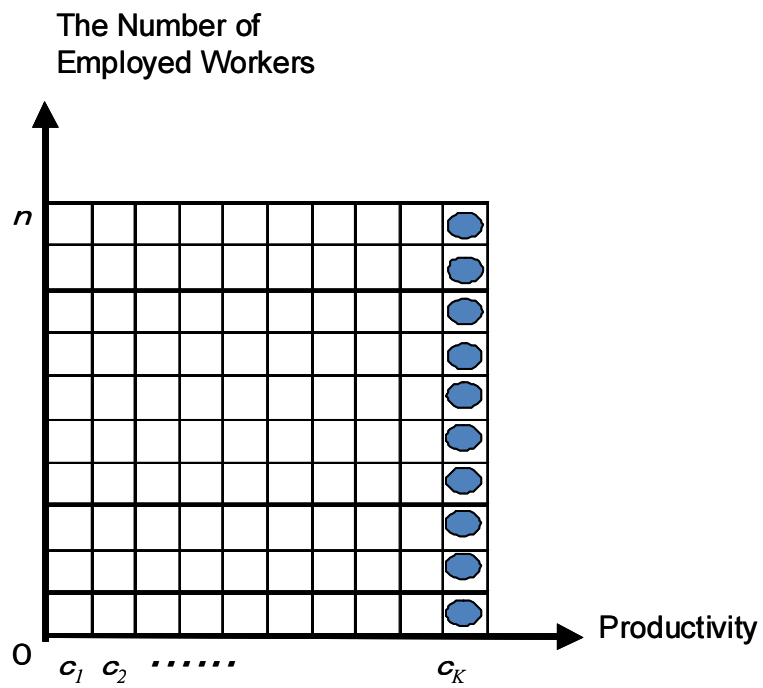


Figure 3: Workers are Distributed Evenly across All the Sectors with Different Levels of Productivity under Extremely Low Aggregate Demand, D_{\min}

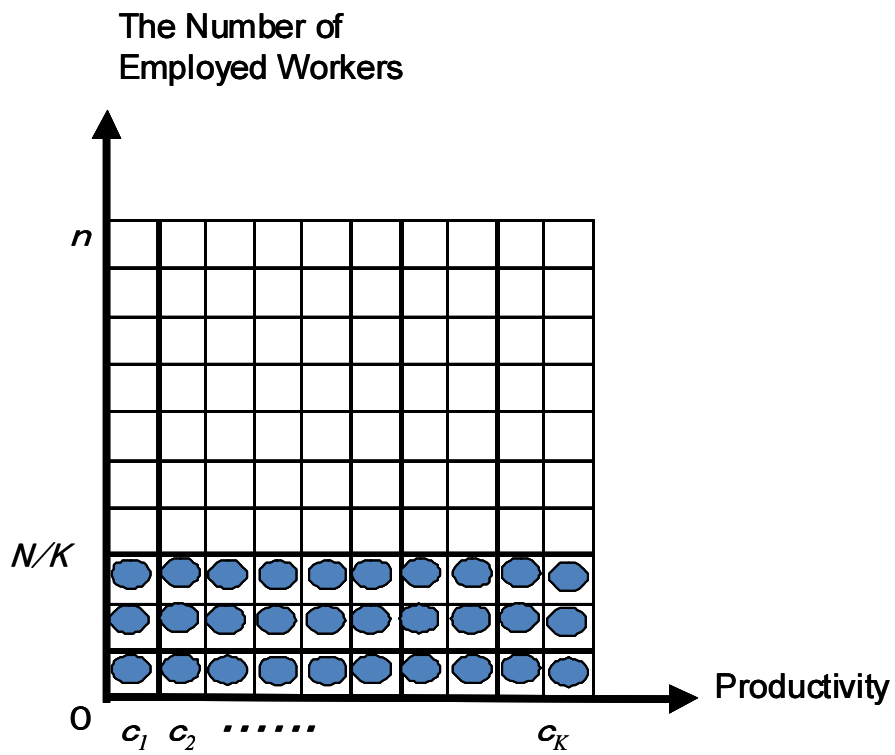


Figure 4: Entropy S and Aggregate Demand D

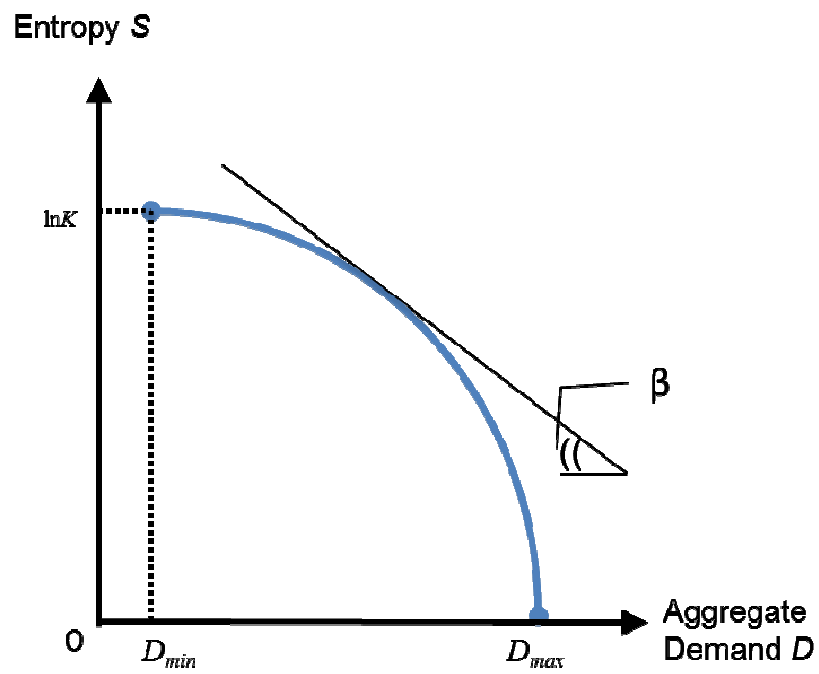


Figure 5: Distribution of Labor across Sectors with Different Productivity

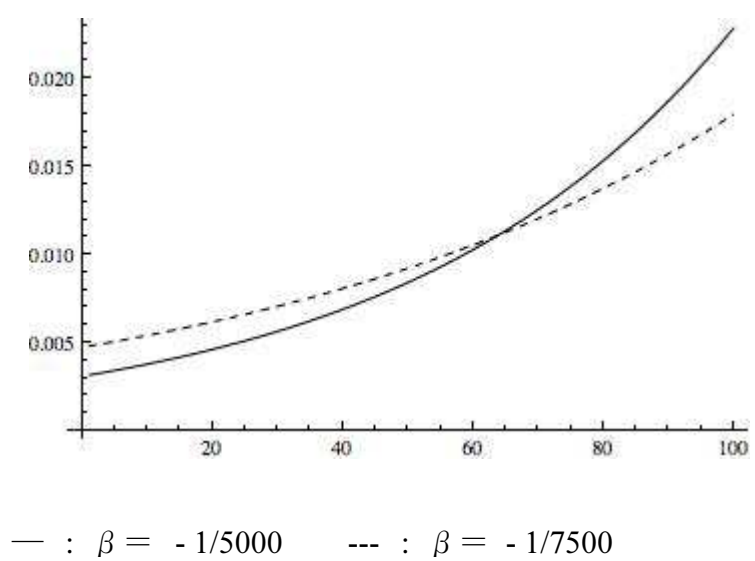
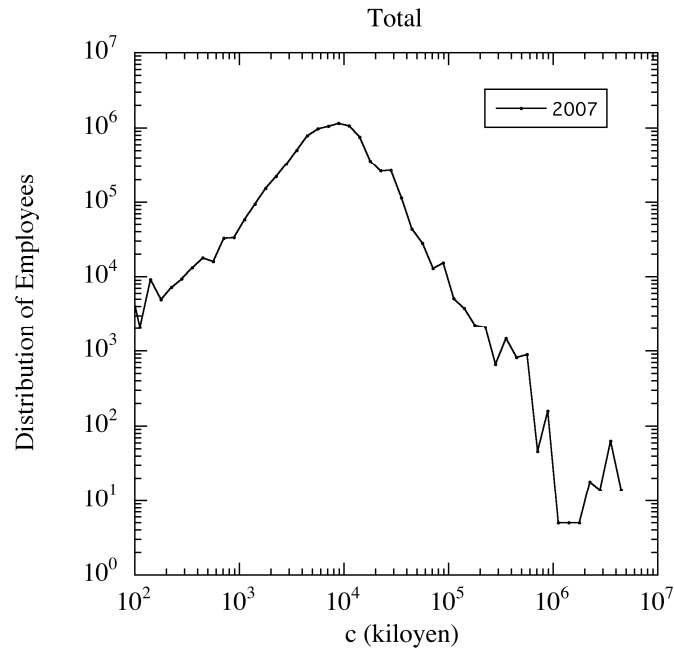


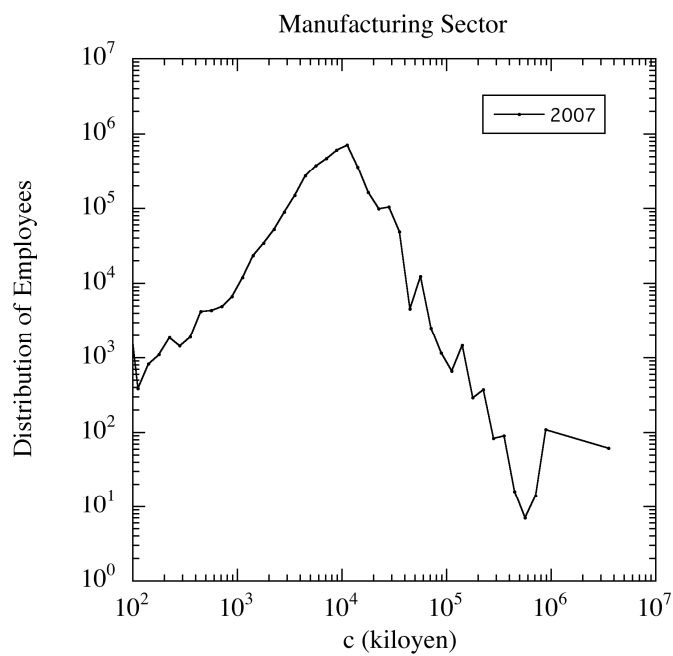
Figure 6: Distribution of Labor Productivity in Japan (2007)

(a)



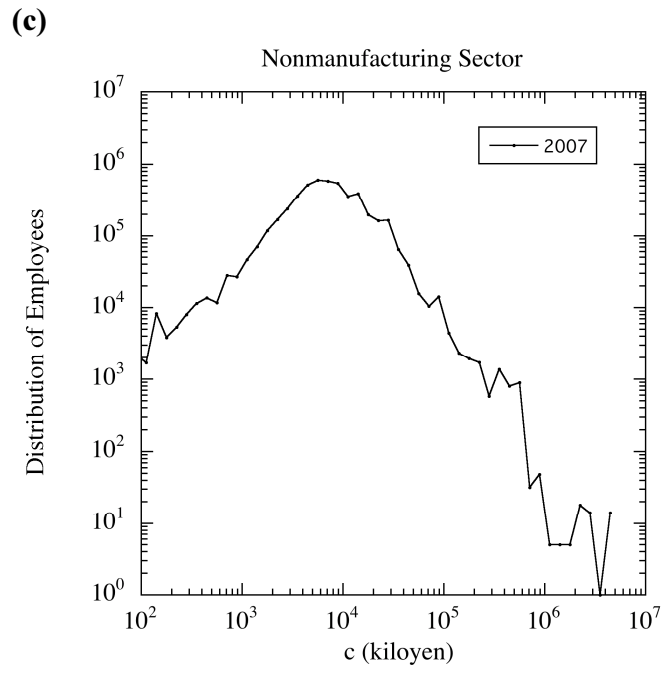
Mean= 1.29×10^4 , Standard Deviation= 2.37×10^4

(b)



Mean= 1.30×10^4 , Standard Deviation= 2.21×10^4

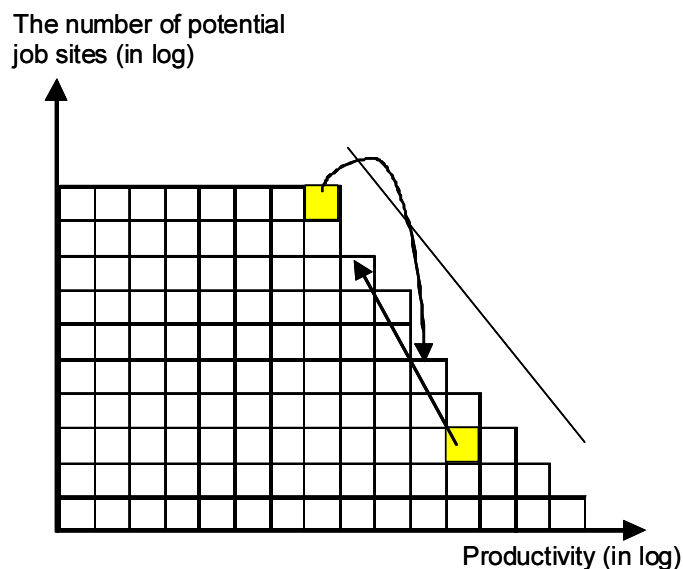
Figure 6: Distribution of Labor Productivity in Japan (2007)



Mean= 1.28×10^4 , Standard Deviation= 2.48×10^4

Figure 7: Model of Stochastic Macro-equilibrium

(a) Dynamics of Creation and destruction of Potential Jobs



Note: Both productivity and the number of potential job sites are in the natural logarithm. The straight line as drawn in the figure means that the distribution of productivity is power-law.

(b) Stochastic Macro-equilibrium

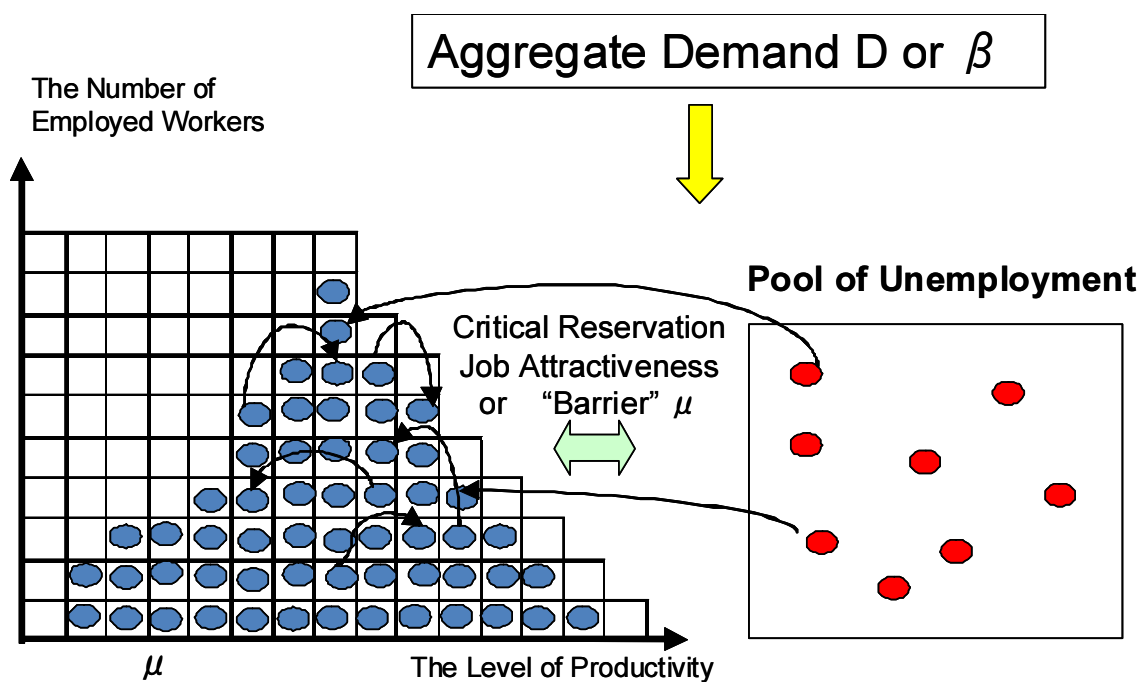
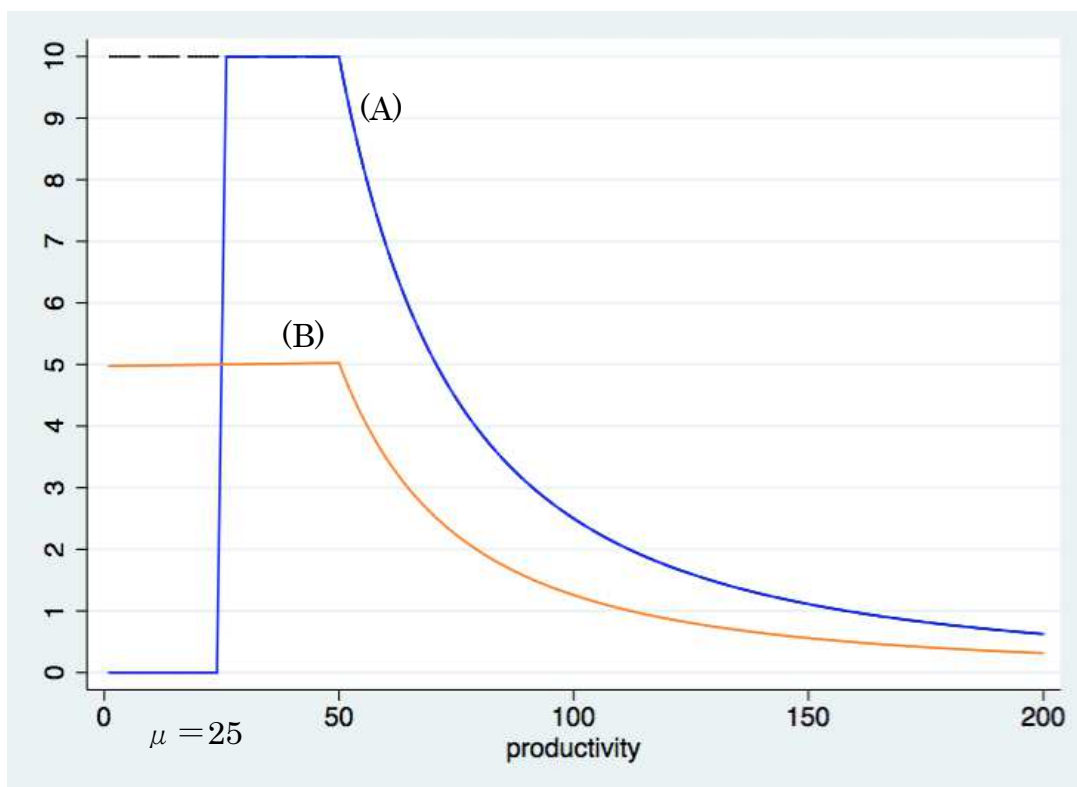


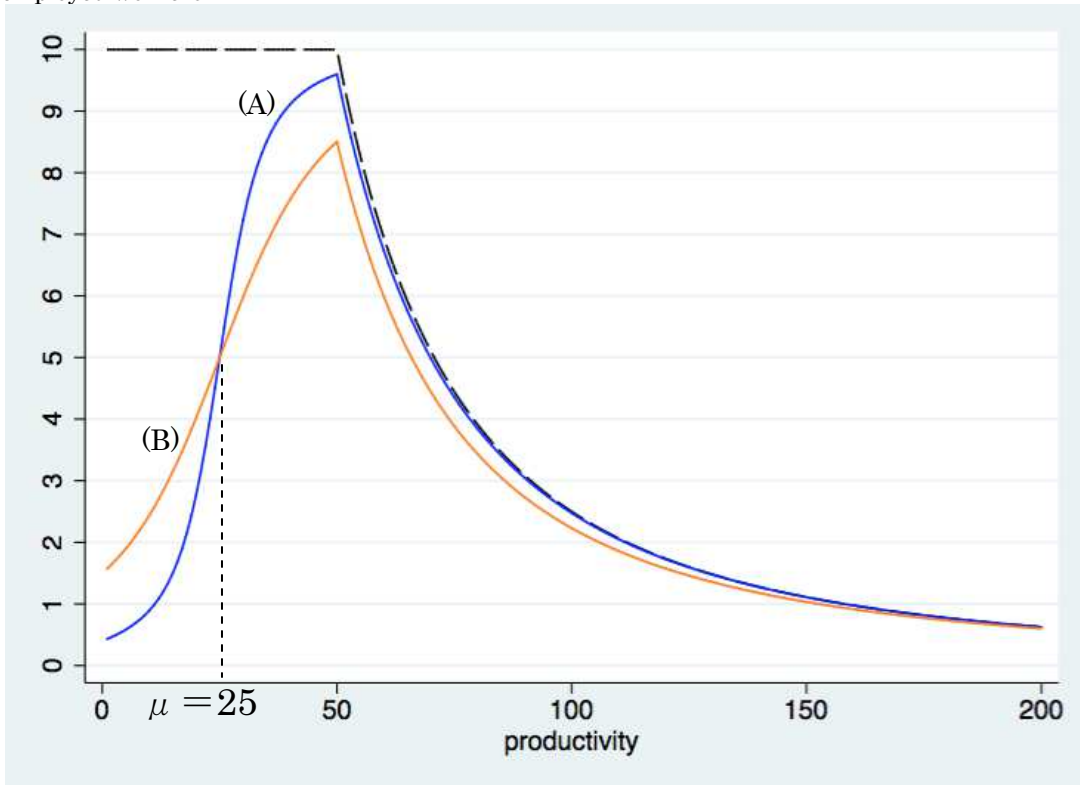
Figure 8: Job Quality Distribution under the Extremely High and Low Levels of Aggregate Demand



Note: (A) Extremely High Aggregate Demand ($\beta \cong -\infty$),
(B) Extremely Low Aggregate Demand ($\beta \cong 0$).

Figure 9: Distribution of Productivity and Aggregate Demand

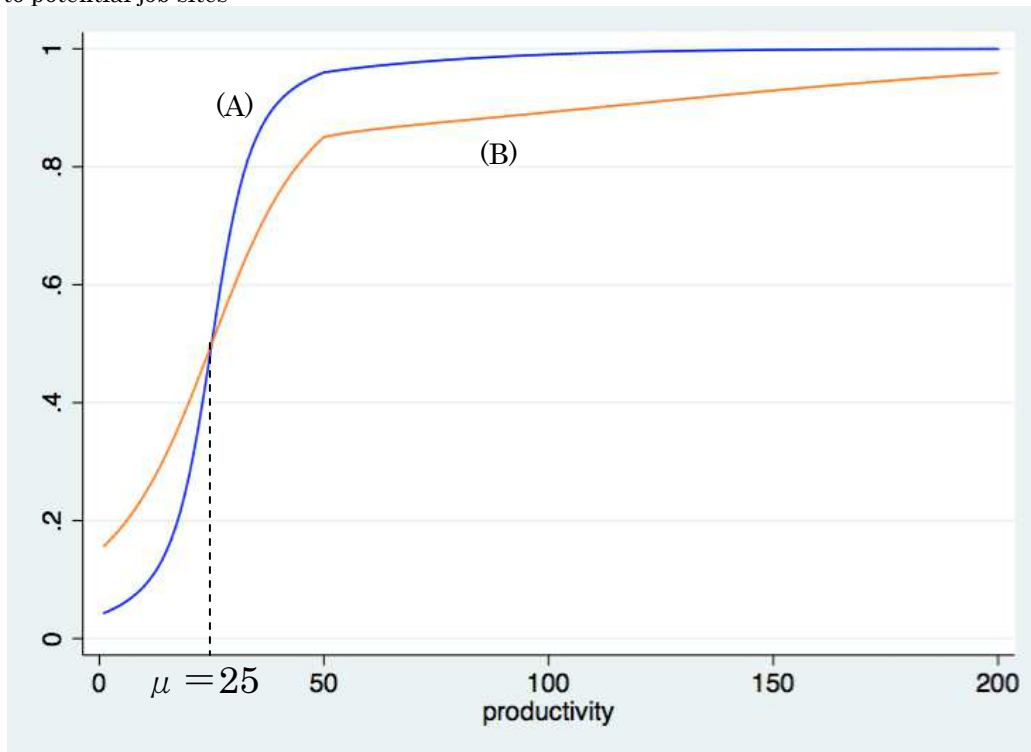
The number of employed workers



Note: (A) High Aggregate Demand ($\beta = -0.05$),
(B) Low Aggregate Demand ($\beta = -0.02$).
See the main text for details.

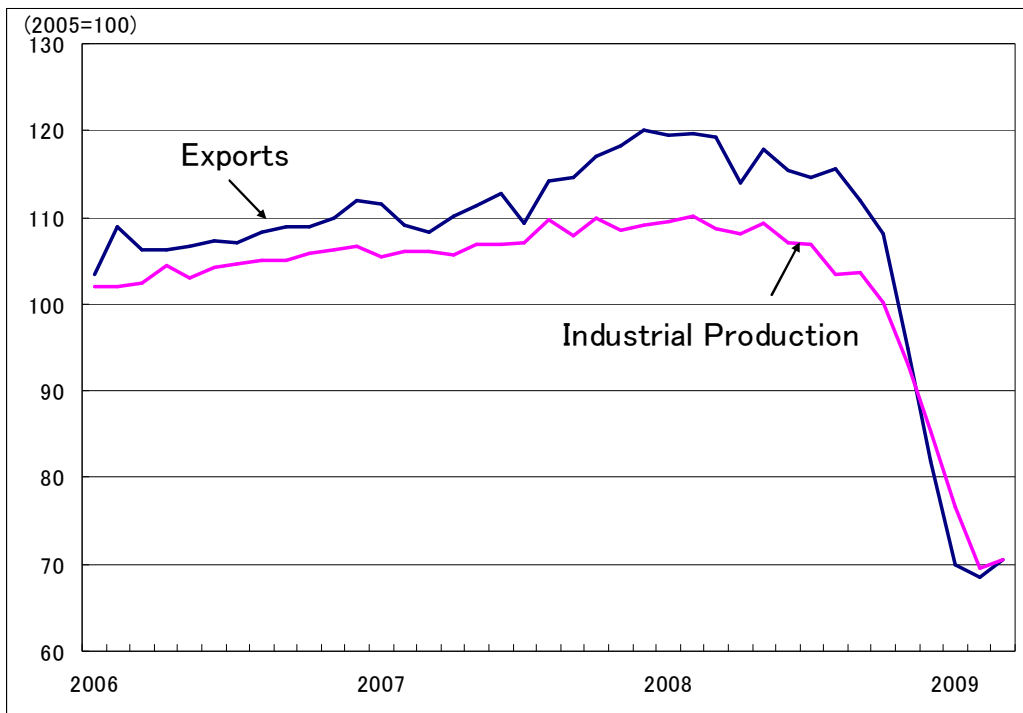
Figure 10: Percentage of Potential Job Sites Occupied by Employed Workers

Ratio of employed workers
to potential job sites



Note: (A) High Aggregate Demand ($\beta = -0.05$),
(B) Low Aggregate Demand ($\beta = -0.02$).
See the main text for details.

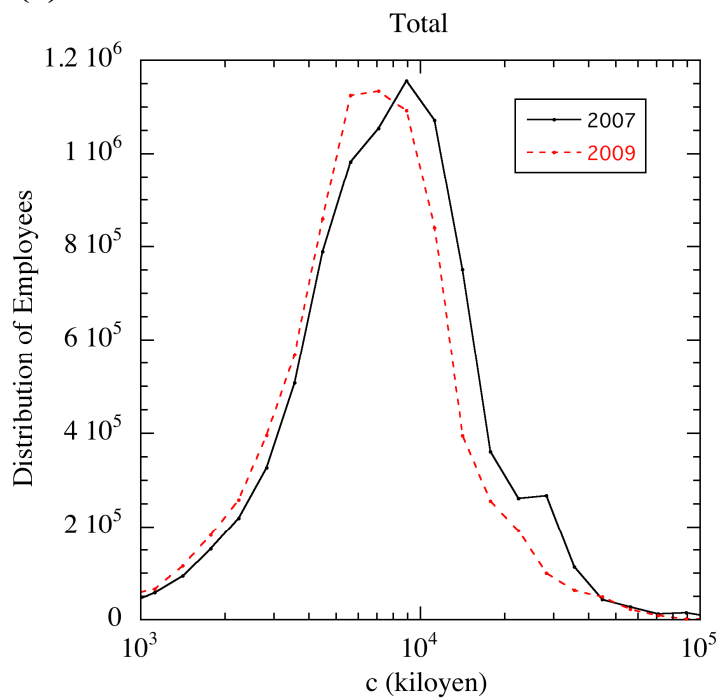
Figure 11: Indices of exports and the Industrial Production, normalized to 100 for 2005



Source: Cabinet Office, Ministry of Economy, Trade and Industry

Figure 12:

(a)



(b)

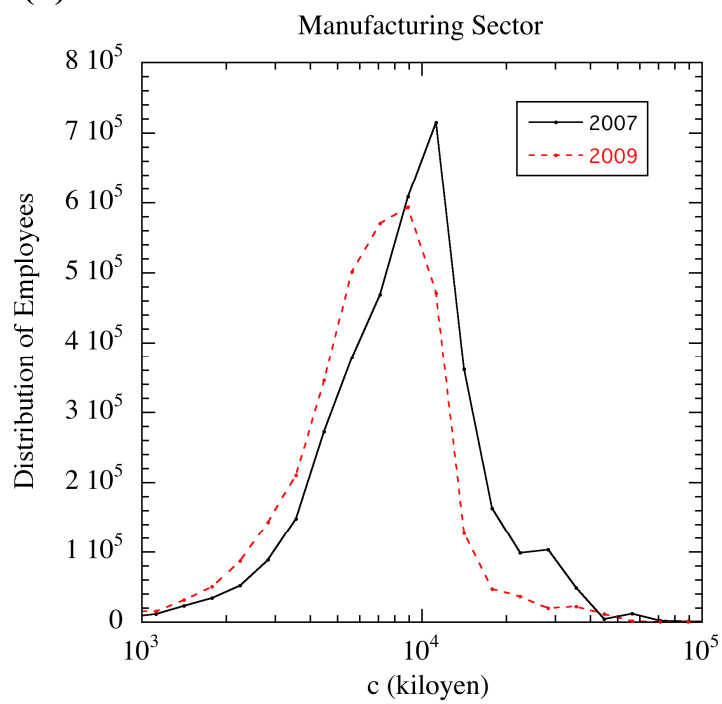
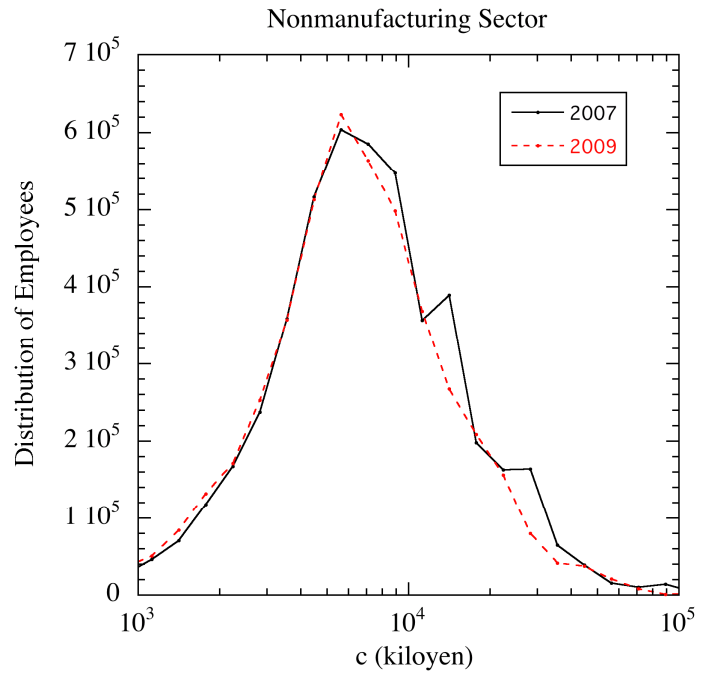


Figure 12:

(c)



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