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Impacts of Trade and Capital Movement

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Firm Heterogeneity under Financial Imperfection:
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Abstract

The paper examines the impacts of trade and capital movement between North and South, which differ in the quality of financial institution, on the productivity distribution and other characteristics of a financially-dependent industry. We find that financial imperfection causes firm heterogeneity and that trade and capital movement are complements in the sense that trade in goods affects the productivity distribution only when accompanied by international capital movement (trade induces capital outflow from South when capital has been internationally mobile). We also find that an international difference in financial development induces reciprocal foreign direct investment.

JEL Classification: F12, F21, G20, G30
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1 Introduction

Recent financial turmoil reminded us of the importance of the high-quality credit market on the economy. The subprime loan problem in the United States seriously hurt the financial systems in the United States and other countries in the world, which led to the global economic downturn as banks and other financial intermediaries became cautious and reluctant to lend money that is necessary for firms to smoothly operate. The financial crisis had spread very quickly and its impact was large worldwide because the financial market had been globalized in the last decades.

The impact of globalization of the financial market on the economy has been extensively analyzed. Henry (2007), for example, finds evidence that financial globalization contributes to economic growth. Klein (2005) shows that countries with better (but not the best) institutions exhibit positive effects of capital account liberalization on economic growth. Kose, Prasad, Rogoff, and Wei (2006) argue that countries that meet threshold conditions (about institutional quality and trade openness, for example) are better able to reap the growth and stability benefits of financial globalization. Financial development itself depends on general institutional quality and political and economic environment. Chinn and Ito (2006) find evidence that capital account liberalization leads to equity market development only if a threshold level of legal development has been attained and that trade openness is a prerequisite for capital account liberalization. Rajan and Zingales (2003) and Do and Levchenko (2007) find that trade and international capital movement induce financial development. Indeed, the quality of financial institution has long been recognized to be critical to the economic prosperity. McKinnon (1973, 1993), for example, emphasizes that less-developed countries and countries in transition from socialism to democracy should develop reliable financial institution in order to achieve economic growth. He argues that countries should first improve their internal financial institutions before opening to trade in goods. Rajan and Zingales (1998) find evidence that financial development contributes positively to the

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1Kose, Prasad, Rogoff, and Wei (2006, 2009) provide excellent surveys of the studies on the economic impacts of financial globalization.
economic growth.

In this paper, we examine the impacts of international trade in goods as well as the impact of international capital movement (i.e., capital account liberalization) on the economy when two countries with different qualities of financial institutions exchange goods and capital. We are especially interested in the impacts at an industry level and in economic interaction between trade in goods and capital movement. More specifically, we examine how financial imperfection affects firm heterogeneity within an industry and how international trade and capital movement affect the industry in individual countries with different qualities of financial institutions. We also allow countries to differ from each other in their wealth distributions (which can be considered as capital endowments for now) in order to distinguish the effects caused by the difference in countries’ financial development from those caused by a traditional difference in factor endowments. We find that international flows of goods and capital when countries are different in their financial development are quite different from those when they are different in their wealth distributions. More importantly, relationships between trade and capital movement are quite different between these two cases: trade and capital movement are substitute when countries are different in their factor endowments (Mundell 1957 and Krugman 1979) while they are complements when countries are different in their financial development. We also investigate foreign direct investment (FDI) flows between two countries with different financial development, and find that reciprocal FDI may arise in such situations. On one hand, FDI from a (relatively) financially-developed country (which we call North) to a financially-less-developed country (which we call South) arises since Northern firms, which locally finance part of their FDI projects, attempt to exploit interest rate differential. On the other hand, there also exists FDI from South to North aiming to overcome financial constraints.

In order to show the impacts of trade and capital movement between two countries with different qualities of financial institutions on the industry in the individual countries, we build a two-country model in which individuals with different wealth become either entrepreneurs who produce a differentiated good or lenders. Entrepreneurs would be faced with credit
constraints if financial institution is imperfect and if they do not have sufficient wealth to cover the setup costs for the firm. The differentiated-good industry is under monopolistic competition such that each firm produces a commodity that is differentiated from other commodities produced by other firms. If international trade in goods is allowed, firms will be faced with absolutely no trade costs, and they compete monopolistically with all other firms in both domestic and foreign markets. If capital is mobile between the two countries, there will be no barrier when capital moves across the countries so that some entrepreneurs may finance the set-up cost through international borrowing.

In this framework, we first show an important proposition that firms become heterogeneous in their productivities only if the country’s financial institution is imperfect. This paper, therefore, has a significant contribution to the recent literature on firm heterogeneity pioneered by Bernard, et al. (2003) and Melitz (2003). Ederington and McCalman (2008, 2009) show that heterogeneous timing of technology adoption causes firm heterogeneity. Yeaple (2005) considers worker heterogeneity as a source of firm heterogeneity.\(^2\) We propose another source of firm heterogeneity, which is the financial imperfection. This source is particularly important because international capital movement, which is critically affected by international heterogeneity in financial development, is large in recent decades and capital movement significantly affects firm heterogeneity (as we show later). Indeed, Kumar, Rajan, and Zingales (1999) find evidence that the average size of firms in industries that depend on external finance is larger in countries with better financial markets. They also find that institutional development is correlated with lower dispersion in firm size within an industry. The analysis of this paper predicts that the average productivity is higher and the degree of firm heterogeneity is lower in financially developed countries; our analysis provides a theoretical background for these important empirical findings about the relationship between institutional quality and firm size.

\(^2\)Bustos (2005), Atkeson and Burnstein (2010), and Constantini and Melitz (2007) allow firms to upgrade their production technologies with a fixed amount of investment, and show that inherently productive firms have more incentives than others to upgrade their technology and engage in the export. Furusawa and Sato (2008) demonstrate that firms with inherently different productivities choose different production technologies with different factor intensities.
Chaney (2005), Manova (2008a), and Suwantaradon (2008) also develop their models in which heterogeneous firms are faced with credit-constraints when they finance trade costs. Their models predict that more-productive and wealthier firms engage in export, while other sell their products only domestically.\(^3\) Our model is quite different from theirs in that financial imperfection leads to firm heterogeneity in not just their attitudes toward exporting but their productivities themselves (which of course affect export activities). We also investigate the impact of international trade and capital movement and their interactions under financial imperfection. Foellmi and Oechslin (2009) also theoretically investigate the effect of international trade on exogenously-heterogeneous firms within an industry and show that rich entrepreneurs wins while poor ones lose from opening to trade.

There also exists the literature on the effect of financial imperfection on the trade structure and trade policies. Kletzer and Bardhan (1987), Beck (2002), Matsuyama (2005), Wynne (2005), Ju and Wei (2008), and Antrás and Caballero (2009) argue that the cross-country differences in the quality of financial institutions significantly affect the structure of countries’ comparative advantage and trade patterns. Antrás and Caballero (2009) also theoretically examine the complementarity between international trade in goods and capital movement under financial imperfection. They show among others that trade in goods and capital movement are complements in the sense that trade in goods induces capital to flow into a financially-less-developed country (i.e., South). This result is in a stark contrast to a typical result in the traditional literature that trade in goods and international capital movement are substitutes (Mundell 1957).

We also find that trade in goods and capital movement are complement under imperfect financial institution. But we find that trade in goods induces capital to flow out of South. Antrás and Caballero (2009) find the complementarity between trade and capital inflow from the perspective of South in a general equilibrium model with financial friction. The key to this complementarity is that trade induces South to specialize to the financially-unconstrained industry so that labor employed disproportionately in that sector depresses

\(^3\) Manova (2008b) and Chor and Manova (2009) find evidence that credit constraints are an important determinant of international trade flows.
(compared to North) the wage rate in South and raises the rental rate of capital that is mobile across countries. This traditional, general-equilibrium mechanism of specialization causes the complementarity between trade and capital inflow (to South). Trade and capital outflow (from South) are complements in our model, on the other hand, because trade benefits Northern firms and harms Southern firms (the positive market expansion effect outweighs the negative competition enhancement effect for Northern firms but not for Southern firms) so that trade pushes up the Northern interest rate (or rental rate) but pushes down the Southern interest rate. That is, the key to our complementarity is the competition effect in the industry. Our competition effect is relevant and important especially for trade between similar countries because the proportion of intra-industry trade to inter-industry trade tends to be higher in trade between similar countries. In addition, our analysis applies better to trade among developed countries and emerging-market economies since they tend to trade manufactures whose production operation depends on external finance.

After setting out the model in the next section, we briefly examine trade equilibrium when financial institution is perfect, and confirm that trade and capital movement are (almost) perfect substitutes. Section 4, which is the main section of the paper, examines the economy under imperfect financial institution. We first show that firms are heterogeneous in their productivities only if the country’s financial institution is imperfect. Then we show that trade in goods alone will not affect the productivity distribution of the industry in either country, but capital movement makes a significant impact on the industry, i.e., capital flight from South to North, which is known as Lucas Paradox (Lucas 1990), arises if the difference in the financial institution is significant, for example. Finally in this section, we show that trade induces capital movement from South to North, affecting the productivity distribution of the industry in both countries; trade affects the productivity distribution only when it is accompanied by international capital movement. Section 5 considers the possibility of FDI. We find that reciprocal FDI (FDI from South to North as well as FDI from North to South) arises under financial imperfection. To our knowledge, this paper is the first to present a

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4Our result can be considered to be in contrast to Krugman’s (1979) substitutability result, just as Antràs and Caballero’s (2009) complementarity is in contrast to Mundell’s (1957) substitutability result.
theoretical framework in which financial imperfection induces reciprocal FDI.

2 Model

There are two countries, which we call North (N) and South (S). In country $k \in \{N, S\}$, there is a mass $m_k$ of individuals, each owning one unit of labor and a wealth of $\omega$ that is uniformly distributed on $[0, \bar{\omega}_k]$; thus the density of individuals whose wealth is $\omega \in [0, \bar{\omega}_k]$ equals $m_k/\bar{\omega}_k$. All individuals share the same utility function over the two goods, a differentiated good $X$ and a numeraire good $Y$, which is characterized by

$$u = \log u_x + y,$$

where

$$u_x = \left[ \int_{\Omega_k} x(i) \frac{\sigma}{\sigma - 1} di \right]^{\frac{\sigma}{\sigma - 1}} ; \quad \sigma > 1,$$

(1)

denotes the subutility derived from the consumption of continuum varieties of good $X$, $\{x(i)\}_{i \in \Omega_k}$ (where $\Omega_k$ denotes the set of all varieties available in country $k$), and $y$ denotes the consumption of good $Y$. The numeraire good is competitively produced such that one unit of labor produces one unit of the good, so the wage rate equals one.

Each individual chooses a consumption profile of good $X$ to maximize $u_x$ subject to $\int_{\Omega_k} p(i)x(i)di \leq E$, where $p(i)$ and $E$ denote the price for variety $i$ and the total expenditure on all varieties of good $X$, respectively. It is immediate to obtain $x(i) = p(i)^{-\sigma}E/P_k^{1-\sigma}$, where $P_k \equiv \left[ \int_{\Omega_k} p(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$ denotes the price index of good $X$. We substitute this result into (1) to obtain $u_x = E/P_k$. Therefore, an individual’s utility function can be written as $u = \log E - \log P_k + y$. Maximizing the utility with the constraint $E + y \leq I$, where $I$ denote the individual’s income (which is the sum of her labor income and the investment return from her wealth), we obtain $E = 1$. That is, each individual spends $E = 1$ on good $X$, so the country $k$’s aggregate expenditure on good $X$ is $m_k$.

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5We assume this particular probability distribution of the wealth only for concreteness. We can easily extend our model to one with a general wealth distribution with a continuous cumulative distribution function.
The differentiated-good industry is characterized by the monopolistic competition with free-entry and free-exit. When a firm enters, however, it incurs an R&D (or setup) investment. R&D and production costs are the cost of labor for those operations. We assume that there are two types of production technology (or facility). The higher the investment, the lower is the marginal cost of production. More specifically, if a firm invests $g_h \ (g_l)$ units of the numeraire good, its marginal cost becomes $1/\varphi_h \ (1/\varphi_l)$. We assume that $g_l < g_h \leq \bar{\omega}_k$, $\varphi_l \equiv \varphi$, and $\varphi_l < \varphi_h \equiv \beta \varphi$, where $\beta > 1$ represents the productivity gap.\footnote{We can relax the assumption that $\bar{\omega}_k \geq g_h$ at the cost of complicating the exposition of the analysis.} To obtain the profits for firm $i$ in country $k$ (in autarky), we define the competition index by

$$\tilde{\varphi}_k \equiv \left[ \int_{i \in \Omega_k} \varphi(i)^{\sigma-1} di \right]^{\frac{1}{\sigma-1}}. \quad (2)$$

Since there is a continuum of varieties, each firm naturally ignores the impact of its pricing on the price index, so that firms select prices that are $\sigma/(\sigma - 1)$ times their individual marginal costs. It is easy to see that the profits for firm $i$ in country $k$ equal

$$\pi_k(\varphi(i), \tilde{\varphi}_k) = \frac{m_k}{\sigma} \left( \frac{\varphi(i)}{\tilde{\varphi}_k} \right)^{\sigma-1}. \quad (3)$$

Individuals in country $k$ decide whether or not they become entrepreneurs who can borrow money at a gross interest rate of $R_k$ to finance their investments if necessary. If an individual decides to become an entrepreneur, she will choose either the high-productivity technology or the low-productivity technology with which her firm operates. If she decides not to be an entrepreneur or if part of her wealth is left after the investment for her firm, she will lend out her (remaining) wealth.

The critical feature of the model is that entrepreneurs are faced with a financial constraint. We assume that entrepreneurs in country $k$ can only pledge themselves to repay only a fraction $\theta_k \in (0, 1]$ of the profits that they will earn, and hence entrepreneur $i$ in country $k$ can borrow only up to the amount such that the repayment does not exceed $\theta_k \pi_k(\varphi(i), \tilde{\varphi}_k)$. The fraction $\theta_k$ represents the quality of the financial institution of the country. (Matsuyama 2000 adopts this formulation of financial imperfection.\footnote{Matsuyama (2007) describes various economic implications of the credit market imperfection of this type.}) A financial institution is perfect
if $\theta_k = 1$; any entrepreneur with any amount of wealth can finance the investment for either high-productivity technology or low-productivity technology, effectively without any constraint. A financial institution is imperfect if $\theta_k < 1$; individuals with small amounts of wealth may not be able to finance the investment in this case.

We can list several reasons why $\theta$ (from which we drop the subscript $k$ for the following general argument) can be smaller than one. A natural cause of financial imperfection is the imperfection of legal enforcement. If the legal enforcement is perfect, as assumed in the traditional literature, a court can enforce a borrowing contract as long as the repayment under the contract does not exceed the profit from the project, denoted by $\pi$. Empirical evidences show, however, the enforcement power is not perfect (La Porta, et al., 1998). Thus, in reality, a court may be able to force a borrower to pay only up to a fraction of the profits, i.e., $\theta\pi$ where $\theta < 1$, even though the realized profit is $\pi$. Hence, unless the non-pecuniary penalty on the default is large enough, the borrower is likely to refuse to pay more than $\theta\pi$ even if the promised payment exceeds this amount. This behavior is called the “strategic default.” A contract cannot be a perfect commitment device if the legal enforcement is imperfect; it is difficult for a lender to expect that a borrower will sincerely make the promised payment. Given that, lenders will not lend more than the amount such that the return from the lending equals $\theta\pi$. Another cause of financial imperfection is the agency problem of the lender-borrower relationship, which is explained briefly in a simple model in the Appendix.

In the economy that we consider, there are two types of the constraints that must be satisfied: the profitability constraints and borrowing constraints. The profitability constraints

$$
\pi_k(\varphi_h, \tilde{\varphi}_k) - R_k g_h \geq 0,
$$  

(4)

$$
\pi_k(\varphi_l, \tilde{\varphi}_k) - R_k g_l \geq 0,
$$  

(5)

for the high-productivity firm and the low-productivity firm, respectively, simply mean that the net profits must be non-negative if firms of the respective type operate at all. The

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8See for example Hart(1995).
borrowing constraints, on the other hand, can be written as
\[
\theta_k \pi_k(\phi_h, \tilde{\phi}_k) \geq R_k (g_h - \omega),
\]
\[
\theta_k \pi_k(\phi_l, \tilde{\phi}_k) \geq R_k (g_l - \omega),
\]
which mean that in country \( k \), an entrepreneur with the wealth of \( \omega \) can borrow money only up to the amount such that the repayment does not exceed the fraction \( \theta_k \) of the profits. It is easy to see that for each type of the firm, the profitability constraint is tighter than the borrowing constraint if \( \theta_k \) is large, whereas the borrowing constraint is tighter if \( \theta_k \) is small. The borrowing constraint tends to be tighter for entrepreneurs with a small amount of wealth.

We investigate the effects of trade and international capital movement under an imperfect financial institution on the economy (mainly on the productivity distribution of the differentiated-good industry). But before that, we briefly analyze the benchmark case in which the financial institution is perfect. For the rest of the analysis, we assume that North is (weakly) capital-abundant (i.e., \( \bar{\omega}_N \geq \bar{\omega}_S \)) and North has a (weakly) better financial institution than South (i.e., \( \theta_N \geq \theta_S \)).

We also emphasize here that the numeraire good is always tradable in all cases that we consider, so “opening to trade in goods” here means opening to trade in the differentiated good as well as the numeraire good. We need this assumption in order to meaningfully analyze the effect of capital movement. Due to the static nature of our model, some goods must be traded for capital to flow from one country to the other from the balance-of-payment requirement.

### 3 Equilibrium under a perfect financial institution

This section shows that if there is no financial constraint, all entrepreneurs choose the same production technology and hence all firms in the differentiated-good sector become homogeneous. Moreover, trade in goods and international capital movement are shown to be substitutes in a sense that is made clear shortly.
Consider a decision made by an individual with the wealth $\omega$. If she invests $g_h$ on the high-productivity technology, she would obtain $\pi_k(\varphi_h, \tilde{\varphi}_k) - R_k(g_h - \omega)$. If $\omega < g_h$, she borrows $g_h - \omega$ to earn $\pi_k(\varphi_h, \tilde{\varphi}_k)$ and pay $R_k(g_h - \omega)$ back to the lenders. If $\omega \geq g_h$, on the other hand, she obtains $\pi_k(\varphi_h, \tilde{\varphi}_k)$ from the production of good $X$ (from the investment of $g_h$) and $-R_k(g_h - \omega)$ from lending out.\(^9\) Similarly, if she invests $g_l$, she would obtain $\pi_k(\varphi_l) - R_k(g_l - \omega)$. Finally, if she lends out the entire wealth of hers, she would get $R_k\omega$.

An entrepreneur chooses the high-productivity technology rather than the low-productivity technology if

$$\pi_k(\varphi_h, \tilde{\varphi}_k) - R_k(g_h - \omega) > \pi_k(\varphi_l, \tilde{\varphi}_k) - R_k(g_l - \omega),$$

which can be written as

$$(1 - \beta^{1-\sigma})\pi_k(\varphi_h, \tilde{\varphi}_k) > R_k(g_h - g_l).$$

(8)

Note that this inequality does not depend on $\omega$, so all entrepreneurs choose the same technology.

Whether or not the inequality (8) holds depends on the productivity and investment-cost parameters. In this paper, we assume that (8) holds so that all entrepreneurs choose the high-productivity technology if they are not financially constrained.\(^{10}\) In equilibrium, some individuals become entrepreneurs while some others must be lending money to them, and hence the net benefit of being an entrepreneur and that of lending money must be the same. That is,

$$\pi_k(\varphi_h, \tilde{\varphi}_k) - R_k(g_h - \omega) = R_k\omega,$$

which is reduced to

$$\pi_k(\varphi_h, \tilde{\varphi}_k) = R_kg_h.$$  

(9)

Note that this equality simply shows that the net profits for high-tech firms are zero: running a business does not yield extra benefits to individuals.

Now, substituting this equality into (8) and rearranging terms, we obtain $\beta^{\sigma-1} > g_h/g_l$, which we assume for the rest of our analysis.

\(^9\)We assume that every individual runs at most one firm.

\(^{10}\)The results of the paper would not be affected qualitatively by assuming otherwise.
Assumption 1

\[ \beta^{\sigma - 1} > g_h / g_l. \]

This assumption indicates that the productivity gap is so large that the more-costly high-productivity technology is effectively more economical than the low-productivity technology. Consequently, all entrepreneurs choose the high-productivity technology while some individuals lend their wealth to those entrepreneurs. Moreover, it is easy to check that under this assumption, there does not exist equilibrium in which entrepreneurs choose the low-productivity technology.

**Proposition 1**  Under a perfect financial institution, all entrepreneurs in the differentiated-good sector choose the same production technology upon entry, and hence firms are homogeneous within the sector.

### 3.1 Autarkic Equilibrium

In this subsection, we derive the autarkic equilibrium in some details. To this end, we first investigate the credit market. Let \( n_k \) denote the mass of firms (or equivalently the mass of entrepreneurs) in country \( k \) in equilibrium. Then, the total credit demands equal \( n_k g_h \), while the total credit supply equals

\[
\frac{m_k}{\bar{\omega}_k} \int_0^{\bar{\omega}_k} \omega d\omega = \frac{m_k \bar{\omega}_k}{2}.
\]

By equating the credit demands and supplies, we find that the mass of firms is given by

\[ n_k = \frac{m_k \bar{\omega}_k}{2g_h}. \tag{10} \]

We need the following assumption to ensure that \( n_k < m_k \).

**Assumption 2**

\[ \bar{\omega}_k < 2g_h. \]
Recall that the decision as to whether or not an individual becomes an entrepreneur does not depend on her wealth. This means that despite that the number of entrepreneurs is unambiguously determined, who become entrepreneurs is indeterminate. But if we suppose that only the wealthiest individuals become entrepreneurs, the wealth level of the poorest entrepreneur $\omega_{h,k}^*$ must satisfy

$$\frac{m_k}{\bar{\omega}_k} (\bar{\omega}_k - \omega_{h,k}^*) = \frac{m_k \bar{\omega}_k}{2g_h},$$

which gives us

$$\omega_{h,k}^* = \bar{\omega}_k - \frac{\bar{\omega}_k^2}{2g_h}.$$  \hspace{1cm} (11)

In this case, individuals become entrepreneurs if and only if their wealth levels lie in the interval $[\omega_{h,k}^*, \bar{\omega}_k]$.

To obtain the equilibrium profits and interest rate, we calculate the autarkic competition index $\tilde{\phi}_A$ from (2) using (10) to obtain

$$\tilde{\phi}_A = \beta \varphi \frac{n_k}{\sigma} = \beta \varphi \left( \frac{m_k \bar{\omega}_k}{2g_h} \right)^{\frac{1}{\sigma-1}}.$$  \hspace{1cm} (12)

Substituting this into (3), we have

$$\pi_k(\varphi_h, \tilde{\phi}_A) = \frac{\varphi}{\sigma n_k} = \frac{2g_h}{\sigma \bar{\omega}_k}.$$  \hspace{1cm} (13)

In autarky, we have $\pi_N(\varphi_h, \tilde{\phi}_A) \leq \pi_S(\varphi_h, \tilde{\phi}_A)$ due to $\bar{\omega}_N \geq \bar{\omega}_S$; profits in capital-abundant North are (weakly) smaller than those in South since there are more firms per capita in North than in South. As for the interest rate, it follows from (9) that

$$R_k = \frac{2}{\sigma \bar{\omega}_k}.$$  

In autarky, we have $R_N \leq R_S$ because $\bar{\omega}_N \geq \bar{\omega}_S$; the interest rate in capital-abundant North is smaller than that in South.

3.2 Free Trade in Goods and Capital Movement

Under a perfect financial institution, the profitability constraint for high-tech firms is binding in equilibrium as (9) shows. This binding profitability constraint is a key to deriving the substitutability between trade in goods and capital movement.
In free trade, all firms compete in a level field regardless of their nationality, so the left-hand side of (9) equals \( \pi_W(\varphi_h, \tilde{\varphi}^T) \) for every firm, where \( \tilde{\varphi}^T = \beta \varphi(n_N + n_S)^{\frac{1}{\sigma - 1}} \) denotes the competition index in each country, and
\[
\pi_W(\varphi_h, \tilde{\varphi}^T) = \frac{m_N + m_S}{\sigma} \left( \frac{\varphi_h}{\tilde{\varphi}^T} \right)^{\sigma - 1}
\]
represents a firm’s profits. Then, it follows directly from (9) that \( R_N = R_S \). Free trade in goods equalizes the interest rates between the two countries as the interest rate reflects firms’ profits which are equalized as a result of free trade. Trade in goods is a substitute of capital movement as it eliminates an interest rate differential.

If capital is mobile between the two countries, on the other hand, the interest rates are equalized (i.e., \( R_N = R_S \)) and hence \( \pi_N(\varphi_h, \tilde{\varphi}_N) = \pi_S(\varphi_h, \tilde{\varphi}_S) \) as (9) indicates. Moreover, we show in the Appendix that \( \pi_N(\varphi_h, \tilde{\varphi}_N) = \pi_S(\varphi_h, \tilde{\varphi}_S) = \pi_W(\varphi_h, \tilde{\varphi}^T) \). That is, capital movement is a substitute of trade for the firms as they earn the same profits in these two regimes.

It is interesting that the equilibrium profits under capital movement are the same as those under free trade in goods. Trade in goods effectively expands the market for Northern firms and shrinks the market for Southern firms, which increases Northern firms’ profits and decreases Southern firms’. Capital movement (without trade), on the other hand, decreases the number of Northern firms while increases the number of Southern firms, and thereby changes their profits accordingly. Trade induces a net flow of the good from North to South, while capital movement induces a flow of firms themselves from North to South. Northern firms’ profits increase while Southern firms’ profits decrease as a result.

**Proposition 2** Under a perfect financial institution, trade in goods and international capital movement are perfect substitutes in the sense that (i) trade will equalize the two countries’ interest rates, eliminating an incentive for capital to move internationally, (ii) capital movement will also equalize the interest rate at the same level as the equilibrium interest rate in free trade, and (iii) the profits are equalized between the two countries at a common level in either of the two regimes.
Although some important economic variables, such as the profits and interest rates, are the same between the two regimes, consumers can enjoy more varieties in the case of free trade in goods than in the case of free capital mobility, as trade allows consumers in either country to consume varieties produced in the foreign country as well as those produced in the home country. Trade in goods and capital movement are perfect substitutes from the perspective of the production side of the economy. But trade in goods is a better alternative than capital movement when the consumption side is taken into account.

4 Financial Imperfection and Firm Heterogeneity

We have shown that under a perfect financial institution, an individual’s wealth is irrelevant in her decision as to whether or not she becomes an entrepreneur. As expected, individual’s wealth will be an important factor under financial imperfection. Due to a financial constraint, only wealthy individuals can borrow money to finance a project. Moreover, since individuals are heterogeneous in their wealth, their choice of technology may also be heterogeneous leading to the firm heterogeneity in productivity.

For the rest of the analysis, we focus on the case in which $\theta$ is small such that the borrowing constraints, (6) and (7), hold with equality while profitability constraints, (4) and (5), hold with strict inequalities. Thus, the relevant constraints are the borrowing constraints, (6) and (7). If $\theta_k$ is small enough that the borrowing constraint for either type is binding, wealthiest individuals become entrepreneurs with the high-productivity technology, those who own intermediate levels of wealth become entrepreneurs with the low-productivity technology, and the poorest individuals lend out their wealth.

We define critical levels of wealth, $\omega_{h,k}$ and $\omega_{l,k}$, such that all individuals with $\omega \in [\omega_{h,k}, \bar{\omega}_k]$ become entrepreneurs choosing the high-productivity technology while all individuals with $\omega \in [\omega_{l,k}, \omega_{h,k})$ become entrepreneurs choosing the low-productivity technology.

A condition that $\omega_{h,k}$ and $\omega_{l,k}$ must satisfy is the credit-market clearing condition. In autarky, it is written as

$$m_k \frac{\bar{\omega}_k - \omega_{h,k}}{\bar{\omega}_k} g_h + m_k \frac{\omega_{h,k} - \omega_{l,k}}{\bar{\omega}_k} g_l = \frac{m_k \bar{\omega}_k}{2}, \quad (14)$$
which can be solved for \( \omega_{l,k} \) to define the function \( \hat{\omega}_{l,k} \):

\[
\hat{\omega}_{l,k}(\omega_{h,k}) = \frac{2gh\bar{\omega}_k - \bar{\omega}_k^2}{2gl} - \frac{gh - gl}{gl} \omega_{h,k}.
\]  

(15)

This function represents the relationship between \( \omega_{l,k} \) and \( \omega_{h,k} \) under the credit-market clearing condition. We can easily see that \( \hat{\omega}_{l,k} \) is a decreasing function and that \( \omega_{h,k} - \hat{\omega}_{l,k}(\omega_{h,k}) \) increases with \( \omega_{h,k} \). An increase in \( \omega_{h,k} \) releases part of capital used for the high-tech firms, which is absorbed by the low-tech entrants whose mass exceeds that of the exiting high-tech firms.

### 4.1 Autarkic Equilibrium

We use (15) to write profits for firms as functions of \( \omega_{h,k} \). In this case of autarky, the competition index defined by (2) can be written as

\[
\tilde{\varphi}_k(\omega_{h,k}) = \left\{ \left( \beta \varphi \right)^{\sigma-1} \frac{m_k}{\omega_k} (\bar{\omega}_k - \omega_{h,k}) + \varphi^{\sigma-1} \frac{m_k}{\omega_k} [\omega_{h,k} - \hat{\omega}_{l,k}(\omega_{h,k})] \right\}^{\frac{1}{\sigma-1}}
\]

\[
= \varphi \frac{m_k}{\sigma} \phi_k(\omega_{h,k})^{\frac{1}{\sigma-1}},
\]

where

\[
\phi_k(\omega_{h,k}) = \beta^{\sigma-1} \frac{\bar{\omega}_k - \omega_{h,k}}{\bar{\omega}_k} + \frac{\omega_{h,k} - \hat{\omega}_{l,k}(\omega_{h,k})}{\bar{\omega}_k}.
\]  

(16)

The competition index \( \tilde{\varphi}_k(\omega_{h,k}) \) is decreasing in \( \omega_{h,k} \) as the derivative of the normalized average productivity \( \phi_k(\omega_{h,k}) \) with respect to \( \omega_h \) equals \( [(g_h/g_l) - \beta^{\sigma-1}] \bar{\omega}_k \), which is negative under Assumption 1; the effect of the contraction of the high-tech group (i.e., an increase in \( \omega_{h,k} \)) outweighs the effect of the expansion of the entire mass of firms. The profits for the firms can be written as

\[
\pi_k(\varphi_h, \tilde{\varphi}_k(\omega_{h,k})) = \frac{m_k}{\sigma} \left( \frac{\beta \varphi}{\tilde{\varphi}_k(\omega_{h,k})} \right)^{\sigma-1} = \frac{\beta^{\sigma-1}}{\sigma \phi_k(\omega_{h,k})},
\]  

(17)

\[
\pi_k(\varphi_l, \tilde{\varphi}_k(\omega_{h,k})) = \frac{1}{\sigma \phi_k(\omega_{h,k})},
\]  

(18)

for the high-tech and low-tech firms, respectively. Since \( \phi_k(\omega_{h,k}) \) decreases with \( \omega_{h,k} \), both \( \pi_k(\varphi_h, \tilde{\varphi}_k(\omega_{h,k})) \) and \( \pi_k(\varphi_l, \tilde{\varphi}_k(\omega_{h,k})) \) increase with \( \omega_{h,k} \).

\[11\]Note that the population \( m_k \) is irrelevant for firms’ profits. A rise in population is good news for a firm as its market expands, but is also bad news as the number of firms increases. These two effects are completely offset against each other in this environment.
Now, we are ready to determine equilibrium levels of $\omega_{h,k}$, $\omega_{l,k}$, and $R_k$. The binding borrowing constraint for the high-tech firms, $\theta_k \pi_k(\varphi_h, \tilde{\varphi}_k(\omega_{h,k})) = R_k(g_h - \omega_{h,k})$, can be written as

$$R_k = \frac{\theta_k \beta \sigma - 1}{\sigma \phi_k(\omega_{h,k})(g_h - \omega_{h,k})} = \omega_{h,k} - \tilde{\omega}_{l,k}(\omega_{h,k}), \quad (19)$$

while the one for the low-tech firms, $\theta_k \pi_k(\varphi_l, \tilde{\varphi}_k(\omega_{h,k})) = R_k(g_l - \tilde{\omega}_{l,k}(\omega_{h,k}))$, can be written as

$$R_k = \frac{\theta_k}{\sigma \phi_k(\omega_{h,k})(g_l - \tilde{\omega}_{l,k}(\omega_{h,k}))}. \quad (20)$$

It immediately follows from (19) and (20) that $\omega_{A,h,k}$ is given by

$$\beta \sigma - 1 = \frac{g_h - \omega_{A,h,k}}{g_l - \tilde{\omega}_{l,k}(\omega_{A,h,k})}. \quad (21)$$

This condition reveals that the ratio of the maximum amount of borrowing by high-tech firms and that of low-tech firms must be the same as the profit gap between the firms of the two types.

The Appendix (Proof of Proposition 3) shows that the following assumption ensures that (21) determines $\omega_{A,h,k}$ such that $\omega_{A,h,k} - \tilde{\omega}_{l,k}(\omega_{A,h,k}) > 0$ so low-tech firms exist in equilibrium.

**Assumption 3**

$$\frac{\beta \sigma - 1 g_l - g_h}{\beta \sigma - 1 - 1} < \tilde{\omega}_k - \frac{\omega_k^2}{2g_h}.$$  

Figure 1 depicts the feasible set of $\tilde{\omega}_k$ as the interval $[g_h, \tilde{\omega}_k]$. As the figure suggests, to guarantee that the feasible set of $\tilde{\omega}_k$ to be nonempty, we need to have $g_h/2 > (\beta \sigma - 1 g_l - g_h)/(\beta \sigma - 1 - 1)$, or equivalently $g_h/g_l > 2\beta \sigma - 1(1 + \beta \sigma - 1)$.

**Assumption 4**

$$\frac{g_h}{g_l} > \frac{2\beta \sigma - 1}{1 + \beta \sigma - 1}.$$  

With Assumption 4, we can find $\tilde{\omega}_k$ that satisfies Assumption 3. For such a $\tilde{\omega}_k$, we have $\tilde{\omega}_{l,k}(\omega_{A,h,k}) < \omega_{A,h,k}$ and hence the following proposition.
Proposition 3 Firm heterogeneity within the differentiated-good sector arises under a poor financial institution.

Under a perfect financial institution, entrepreneurs can freely borrow money (if necessary) to finance the most-efficient production technology. Under an imperfect financial institution, however, their initial wealth levels become important. Due to the borrowing constraint, only wealthiest individuals can afford the most-efficient and most-expensive production technology; other entrepreneurs must be content with the less-efficient but less-expensive production technology. Firm heterogeneity arises only if the financial institution is imperfect.

Note that equation (21) does not involve $\theta_k$. As long as $\theta_k$ is small so that the borrowing constraints are binding for both high-tech and low-tech firms, the productivity distribution of the industry is not affected by a change in the quality of the financial institution; it is the credit-market equilibrium that determines the productivity distribution. With the productivity distribution given by (21), the interest rate $R_k^A$ is determined by the borrowing constraint (for high-tech firms, for example):

$$R_k^A = \frac{\theta_k \beta^{\sigma-1}}{\sigma \phi_k (\omega_{h,k}^A)(gh - \omega_{h,k}^A)}.$$  

(22)

As (22) indicates, any change in $\theta_k$ will induce offsetting change in $R_k$. In partial equilibrium analyses, financial development generally induces firms to enter the market or to upgrade their production technologies, because it becomes easier for entrepreneurs to finance the investment costs. But this seemingly obvious causality breaks down in this general equilibrium model. The productivity distribution of the industry hinges critically on the total credit supply that is fixed in the autarkic economy. That is why financial development would increase the interest rate to offset an induced increase in credit demands.

Lemma 1 Under financial imperfection (such that both borrowing constraints, (6) and (7), are binding), financial development would only raise the interest rate, leaving the size and the productivity distribution of the industry unchanged.

An increase in the wealth level of the wealthiest individuals, on the other hand, will change the productivity distribution of the industry such that it increases the normalized
average productivity. It follows directly from (15) and (21) that a rise in \( \bar{\omega}_k \) will decrease both \( \omega_{h,k} \) and \( \hat{\omega}_{l,k}(\omega_{h,k}) \), which unambiguously lower \( \omega_{h,k}/\bar{\omega}_k \) and \( \hat{\omega}_{l,k}(\omega_{h,k})/\bar{\omega}_k \). Then, it is readily seen from (16) that \( \phi_k(\omega_{h,k}) \) increases as a consequence.

The impact of a rise in \( \bar{\omega}_k \) on the mass of firms of each type can also be seen readily. It is easy to see that a rise in \( \bar{\omega}_k \) increases the total mass of firms, 
\[
\left( m_k/\bar{\omega}_k \right) [\bar{\omega}_k - \hat{\omega}_{l,k}(\omega_{h,k})] = m_k[1 - (\hat{\omega}_{l,k}(\omega_{h,k})/\bar{\omega}_k)].
\]
The mass of high-tech firms increases as 
\[
(m_k/\bar{\omega}_k)(\bar{\omega}_k - \omega_{h,k}) = m_k[1 - (\omega_{h,k}/\bar{\omega}_k)]
\]
rises when \( \omega_{h,k}/\bar{\omega}_k \) falls. As we can see from (21), on the other hand, \( \hat{\omega}_{l,k}(\omega_{h,k}) \) decreases when \( \omega_{h,k} \) falls at a smaller rate. Thus, \( \omega_{h,k} - \hat{\omega}_{l,k}(\omega_{h,k}) \) decreases when \( \omega_{h,k} \) falls in response to an increase in \( \bar{\omega}_k \). As a consequence, the mass of low-tech firms, 
\[
(m_k/\bar{\omega}_k)[\omega_{h,k} - \hat{\omega}_{l,k}(\omega_{h,k})]
\]
falls if \( \bar{\omega}_k \) increases.

Applying this argument to the case of the two countries when they have different wealth distributions such that \( \bar{\omega}_N > \bar{\omega}_S \), we obtain the following lemma.

**Lemma 2** Suppose \( \bar{\omega}_N > \bar{\omega}_S \). In autarky, there are more firms in North than in South. There are more high-tech firms in North than in South, while there are less low-tech firms in North than in South. Northern market is more competitive than Southern as indicated by the fact that the normalized average productivity is higher in North.

It follows from Lemmas 1 and 2 that (i) \( R^A_N < R^A_S \) if \( \theta_N = \theta_S \) and \( \bar{\omega}_N > \bar{\omega}_S \), and (ii) \( R^A_N > R^A_S \) if \( \theta_N > \theta_S \) and \( \bar{\omega}_N = \bar{\omega}_S \). In a general case where \( \theta_N \geq \theta_S \) and \( \omega_N \geq \omega_S \), which of \( R^A_N \) and \( R^A_S \) is greater than the other depends on whether or not the difference in the quality of the financial institutions between the two countries is more significant than the difference in the wealth levels.

**Proposition 4** In autarky, the interest rate is higher in North than in South if the difference in the quality of the financial institution between North and South is more significant than the difference in the wealth levels.

### 4.2 Equilibrium with Free Trade in Goods

In this subsection, we show that opening to trade induces intra-industry trade so that consumers enjoy an increase in variety of the good, but it does not affect the productivity
distribution of the industry in either country. Trade in goods, however, generally narrows the gap between the countries’ interest rates.

To see how the profits for firms change with trade, we calculate the competition index, defined by (2), as

$$
\tilde{\phi}_W = \left\{ (\beta \varphi)^{\sigma - 1} \sum_{k=N,S} m_k \left( \bar{\omega}_k - \omega_{h,k} \right) + \varphi^{\sigma - 1} \sum_{k=N,S} m_k \left[ \omega_{h,k} - \bar{\omega}_{l,k}(\omega_{h,k}) \right] \right\}^{\frac{1}{\sigma - 1}}
$$

= $\varphi (m_N + m_S)^{\frac{1}{\sigma - 1}} \phi_W (\omega_{h,N}, \omega_{h,S})^{\frac{1}{\sigma - 1}},$ (23)

where

$$
\phi_W (\omega_{h,N}, \omega_{h,S}) \equiv \beta^{\sigma - 1} \sum_{k=N,S} m_k \left( \bar{\omega}_k - \omega_{h,k} \right) + \sum_{k=N,S} m_k \left( \omega_{h,k} - \bar{\omega}_{l,k}(\omega_{h,k}) \right) \varphi \left( m_N + m_S \right)^{\frac{1}{\sigma - 1}} \phi_W (\omega_{h,N}, \omega_{h,S})^{\frac{1}{\sigma - 1}},
$$

(24)

Then the profits for high-tech firms are given by

$$
\pi_W (\varphi_h, \tilde{\varphi}_T) = \frac{m_N + m_S}{\sigma} \left( \frac{\beta \varphi}{\varphi (m_N + m_S)^{\frac{1}{\sigma - 1}} \phi_W (\omega_{h,N}, \omega_{h,S})^{\frac{1}{\sigma - 1}}} \right)^{\sigma - 1}
$$

= $\frac{\beta^{\sigma - 1}}{\sigma \phi_W (\omega_{h,N}, \omega_{h,S})},$ (25)

while the profits for low-tech firms are given by

$$
\pi_W (\varphi_l, \tilde{\varphi}_T) = \frac{1}{\sigma \phi_W (\omega_{h,N}, \omega_{h,S})}.$$(26)

The borrowing constraints for high-tech and low-tech firms can be written respectively as

$$
R_k = \frac{\theta_k}{\beta^{\sigma - 1} \varphi_W (\omega_{h,N}, \omega_{h,S})} \left( g_h - \bar{\omega}_{h,k} \right),
$$

(27)

$$
R_k = \frac{\theta_k}{\beta^{\sigma - 1} \varphi_W (\omega_{h,N}, \omega_{h,S})} \left( g_l - \bar{\omega}_{l,k}(\omega_{h,k}) \right),
$$

(28)

The equilibrium values of $\omega_{h,k}$ and $R_k$, which we call as $\omega_{h,k}^T$ and $R_k^T$, satisfy the two equations for each $k = N, S$:

$$
\beta^{\sigma - 1} = \frac{g_h - \omega_{h,k}^T}{g_l - \bar{\omega}_{l,k}(\omega_{h,k}^T)},
$$

(29)

$$
R_k^T = \frac{\theta_k \beta^{\sigma - 1}}{\varphi_W (\omega_{h,N}^T, \omega_{h,S}^T)} \left( g_h - \omega_{h,k}^T \right),
$$

(30)
which are directly derived from (27) and (28).

Since equation (29) is identical to the one in (21), we find that the critical levels of wealth are the same between the two cases, i.e., $\omega^T_{h,N} = \omega^A_{h,N}$ and $\omega^T_{h,S} = \omega^A_{h,S}$. That is, opening to trade will not change the productivity distribution. The borrowing constraints require the ratio of the maximum borrowing of the high-tech and low-tech firms to remain the same before and after opening to trade, as indicated by (29). In addition, capital supply is inelastic, so the productivity distribution of the industry will not change with trade.

**Proposition 5** Under financial imperfection, international trade in goods between two countries will not affect the productivity distribution of the industry in either country.

International trade in goods, however, affects the interest rates in general through its effects on firms’ profits. We investigate the impacts of trade on profits and interest rates separately in the case where the countries are different in their wealth levels ($\theta_N = \theta_S$ and $\bar{\omega}_N > \bar{\omega}_S$) and in the case where they are different in financial development ($\theta_N > \theta_S$ and $\bar{\omega}_N = \bar{\omega}_S$).

### 4.2.1 Interest rates when $\theta_N = \theta_S$ and $\bar{\omega}_N > \bar{\omega}_S$

In this case, Northern market is more competitive than Southern market in autarky, so opening to trade will increase Northern firms’ profits and decrease Southern firms’ as they earn the same profits in free trade. Consequently, the interest rate increases in North and decreases in South.

To see this effect of trade on the interest rates more precisely, we note that $\omega^T_{h,N} = \omega^A_{h,N}$, $\omega^T_{h,S} = \omega^A_{h,S}$ and find from (24) that

$$\phi_N(\omega^A_{h,N}) > \phi_W(\omega^T_{h,N}, \omega^T_{h,S}) > \phi_S(\omega^A_{h,S}),$$

(31)

and hence $\pi_N(\varphi_h, \tilde{\varphi}^A_N) < \pi_W(\varphi_h, \tilde{\varphi}^T_W) < \pi_S(\varphi_h, \tilde{\varphi}^A_S)$ as (17), (18), (25), and (26) indicate. That is, trade will increase Northern firms’ profits while decrease Southern firms’ profits. These effects on profits imply that the interest rate rises in North and drops in South, i.e., $R^T_N > R^A_N$ and $R^T_S < R^A_S$ as the comparison between (22) and (30) also reveals. We also find
that the interest rate is still lower in North than in South, i.e., $R^T_N < R^T_S$. It follows from $\omega^A_{h,N} < \omega^A_{h,S}$ when $\bar{\omega}_N > \bar{\omega}_S$ and $\omega^T_{h,k} = \omega^A_{h,k}$ for $k = N, S$ that $g_h - \omega^T_{h,N} > g_h - \omega^T_{h,S}$. Then, since $\theta_N = \theta_S$, (30) implies that $R^T_N < R^T_S$.

The interest rate is lower in North than in South in autarky, reflecting the difference in their wealth levels. The gap between them narrows, although not completely, as a result of trade.

### 4.2.2 Interest rates when $\theta_N > \theta_S$ and $\bar{\omega}_N = \bar{\omega}_S$

In this case, the normalized average productivities are the same between the two countries in autarky, and so are the profits for the firms of each type. The autarkic interest rate is higher in North than in South, reflecting the difference in the quality of the financial institutions. As seen from (24), the firms’ profits are the same between the two countries in autarky, and so are the profits in free trade; the market expansion effect of trade liberalization completely offsets the competition enhancement effect. Consequently, the firms’ profits and also the individual interest rates of the two countries will not change as a result of trade. To be more precise, we compare the normalized average productivities before and after the trade liberalization to find that $\phi_N(\omega^A_{h,N}) = \phi_W(\omega^T_{h,N}, \omega^T_{h,S}) = \phi_S(\omega^A_{h,S})$ and that $\pi_N(\varphi_h, \hat{\varphi}_N) = \pi_W(\varphi_h, \hat{\varphi}_W) = \pi_S(\varphi_h, \hat{\varphi}_S)$. Then, it follows from (22), (30), and $\theta_N > \theta_S$ that $R^T_N = R^A_N > R^A_S = R^T_S$.

### 4.3 Equilibrium with International Capital Movement

As Proposition 4 indicates, whether or not North has a higher interest rate than South depends on whether or not the difference in the quality of the financial institution between the two countries is more significant than the difference in the wealth levels. If capital is allowed to move internationally, it moves from the country with a lower interest rate to the country with a higher interest rate, shrinking the industry in the former country and expanding the industry in the latter.

To see the impacts of capital movement more closely, we first derive the equilibrium competition index for each country. Let us define the amount of capital that moves from North to South by $K$ (which takes a negative value when capital moves from South to North).
Then, the credit-market clearing condition in North can be written as

\[
\frac{m_N}{\bar{\omega}_N} (\bar{\omega}_N - \omega_{h,N}) g_h + \frac{m_N}{\bar{\omega}_N} (\omega_{h,N} - \omega_{l,N}) g_l = \frac{m_N \bar{\omega}_N}{2} - K,
\]

which is solved for \(\omega_{l,N}\) to obtain the threshold wealth level for the low-tech firms in North as

\[
\hat{\omega}_{l,N}(\omega_{h,N}, K) = \frac{2(g_h + K)\bar{\omega}_N - \omega_{h,N}^2}{2g_l} - \frac{g_h - g_l}{g_l} \omega_{h,N},
\]

with slight abuse of notation. Similarly, the threshold wealth level for the low-tech firms in South can be written as

\[
\hat{\omega}_{l,S}(\omega_{h,S}, K) = \frac{2(g_h - K)\bar{\omega}_S - \omega_{h,S}^2}{2g_l} - \frac{g_h - g_l}{g_l} \omega_{h,S}.
\]

Now, we can write the competition index for \(k = N, S\) as

\[
\tilde{\varphi}_k(\omega_{h,k}, K) = \left\{ (\beta \varphi)^{\sigma^{-1}} \left( \frac{m_k}{\bar{\omega}_k} (\bar{\omega}_k - \omega_{h,k}) + \varphi^{\sigma^{-1}} \left( \frac{m_k}{\bar{\omega}_k} [\omega_{h,k} - \hat{\omega}_{l,k}(\omega_{h,k}, K)] \right) \right) \right\}^{\frac{1}{\sigma - 1}}
\]

\[
= \varphi m_k^{\frac{1}{\sigma - 1}} \phi_k(\omega_{h,k}, K)^{\frac{1}{\sigma - 1}}, \tag{32}
\]

where

\[
\phi_k(\omega_{h,k}, K) \equiv \beta^{\sigma - 1} \frac{\bar{\omega}_k - \omega_{h,k}}{\bar{\omega}_k} + \frac{\omega_{h,k} - \hat{\omega}_{l,k}(\omega_{h,k}, K)}{\bar{\omega}_k}.
\]

It follows from (32) that profits for high-tech and low-tech firms in country \(k\) are written as

\[
\pi_k(\varphi_h, \tilde{\varphi}_k(\omega_{h,k}, K)) = \frac{m_k}{\sigma} \left( \frac{\beta \varphi}{\varphi m_k^{\frac{1}{\sigma - 1}} \phi_k(\omega_{h,k}, K)^{\frac{1}{\sigma - 1}}} \right)^{\sigma - 1} = \frac{\beta}{\sigma \phi_k(\omega_{h,k}, K)},
\]

\[
\pi_k(\varphi_l, \tilde{\varphi}_k(\omega_{h,k}, K)) = \frac{1}{\sigma \phi_k(\omega_{h,k}, K)},
\]

respectively. The borrowing constraints for high-tech and low-tech firms in country \(k\) can be written as

\[
R_k = \frac{\theta_k \beta^{\sigma - 1}}{\sigma \phi_k(\omega_{h,k}, K)(g_h - \omega_{h,k})},
\]

\[
R_k = \frac{\theta_k}{\sigma \phi_k(\omega_{h,k}, K)[g_l - \hat{\omega}_{l,k}(\omega_{h,k}, K)]},
\]

respectively.
As a consequence of international capital movement, interest rates $R_N$ and $R_S$ are equalized at a level, which we call $R_W$. The equilibrium values of $K$, $R_W$, and $\omega_{h,k}$ for $k = N, S$, which we call $K^K$, $R^K_W$, and $\omega^K_{h,k}$, satisfy the following three equations:

\begin{align}
\beta^\sigma - 1 &= \frac{g_h - \omega^K_{h,N}}{g_I - \hat{\omega}_{l,N}(\omega^K_{h,N}, K^K)}, \quad (33) \\
\beta^\sigma - 1 &= \frac{g_h - \omega^K_{h,S}}{g_I - \hat{\omega}_{l,S}(\omega^K_{h,S}, K^K)}, \quad (34) \\
\frac{\theta_N \beta^\sigma - 1}{\sigma \phi_N(\omega^K_{h,N}, K^K)(g_h - \omega^K_{h,N})} &= \frac{\theta_S \beta^\sigma - 1}{\sigma \phi_S(\omega^K_{h,S}, K^K)(g_h - \omega^K_{h,S})}. \quad (35)
\end{align}

The effects of capital movement on the industry are very different from those of trade in goods. Capital movement, induced by the difference in financial development and the difference in wealth levels, will change the productivity distribution and other characteristics of the industry. To see these effects, we examine the two cases separately again.

4.3.1 Effects of capital movement when $\theta_N = \theta_S$ and $\bar{\omega}_N > \bar{\omega}_S$

As we have seen in the above, the normalized average productivity is higher in North than in South in autarky, i.e., $\phi_N(\omega^A_{h,N}) > \phi_S(\omega^A_{h,S})$, so firms’ profits are smaller in North. Consequently, the interest rate is lower in North than in South. If capital is allowed to move internationally, therefore, capital flows out of North to South, which shrinks Northern industry (i.e., both $\omega_{h,N}$ and $\omega_{l,N}$ increase) and expand Southern industry (i.e., both $\omega_{h,S}$ and $\omega_{l,S}$ decrease). As a result, the normalized average productivity decreases in North and increases in South to their individual equilibrium values.

We know from (35) that

$$
\phi_N(\omega^K_{h,N}, K^K)(g_h - \omega^K_{h,N}) = \phi_S(\omega^K_{h,S}, K^K)(g_h - \omega^K_{h,S}) \quad (36)
$$

in equilibrium. Here, we show that (36) implies that $\phi_N(\omega^K_{h,N}, K^K) > \phi_S(\omega^K_{h,S}, K^K)$ and $g_h - \omega^K_{h,N} < g_h - \omega^K_{h,S}$. Suppose on the contrary that $g_h - \omega^K_{h,N} \geq g_h - \omega^K_{h,S}$, or equivalently $\omega^K_{h,N} \leq \omega^K_{h,S}$. Then, it follows from (33) and (34) that $\hat{\omega}_{l,N}(\omega^K_{h,N}, K^K) < \hat{\omega}_{l,S}(\omega^K_{h,S}, K^K)$ and hence $\omega^K_{h,N}/\bar{\omega}_N < \omega^K_{h,S}/\bar{\omega}_S$ and $\hat{\omega}_{l,N}(\omega^K_{h,N}, K^K)/\bar{\omega}_N < \hat{\omega}_{l,S}(\omega^K_{h,S}, K^K)/\bar{\omega}_S$, which in turn imply

$$
\phi_N(\omega^K_{h,N}, K^K) > \phi_S(\omega^K_{h,S}, K^K) \quad \text{as we have seen in the case of autarky.}
$$

But then (36) would
be violated, so we must have \( g_h - \omega^K_{h,N} < g_h - \omega^K_{h,S} \) and hence \( \phi_N(\omega^K_{h,N}, K^K) > \phi_S(\omega^K_{h,S}, K^K) \), again from (36).

Capital movement from capital-abundant North shrinks Northern industry and expands Southern industry. This change can be considered as large because Northern thresholds of wealth, \( \omega_{h,N} \) and \( \omega_{l,N} \), are smaller than Southern counterparts, \( \omega_{h,S} \) and \( \omega_{l,S} \), in autarky, but are greater now in the case where capital is allowed to move internationally; the poorest entrepreneurs who adopt the high-productivity technology and low-productivity, respectively, must be richer now in North than in South. Note, however, that \( \phi_N(\omega^K_{h,N}, K^K) > \phi_S(\omega^K_{h,S}, K^K) \) so that Northern firms still earn less profits than Southern firms in equilibrium as there are more rich entrepreneurs in North than in South. As Northern firms’ profits increase while Southern firms’ profits decrease, the interest rate increases in North and decreases in South to the common rate \( R^K_W \).

**Proposition 6** When the two countries differ in their wealth levels, capital moves from capital-abundant North to capital-scarce South, shrinking the industry in North and expanding the industry in South. Northern market is still more competitive than Southern in equilibrium with capital movement, so Northern firms earn smaller profits than Southern firms.

4.3.2 Effects of capital movement when \( \theta_N > \theta_S \) and \( \bar{\omega}_N = \bar{\omega}_S \)

The normalized average productivities are the same between the two countries in autarky due to \( \bar{\omega}_N = \bar{\omega}_S \), so the fact that \( \theta_N > \theta_S \) leads to \( R^A_N > R^A_S \). Consequently, capital flight from South occurs if capital is allowed to move internationally.

It follows from (35) that

\[
\frac{\theta_N}{\theta_S} = \frac{\phi_N(\omega^K_{h,N}, K^K) g_h - \omega^K_{h,N}}{\phi_S(\omega^K_{h,S}, K^K) g_h - \omega^K_{h,S}}.
\]

(37)

Since capital moves from South to North, this condition implies that \( \omega^K_{h,N} < \omega^K_{h,S} \) and \( \phi_N(\omega^K_{h,N}, K^K) > \phi_S(\omega^K_{h,S}, K^K) \): the normalized average productivity is higher in North than in South also in this case.

**Proposition 7** When the two countries differ in the quality of the financial institution,
capital moves from South with the relatively poor financial institution to North, shrinking the industry in South and expanding the industry in North.

4.4 Equilibrium with Trade in Goods and Capital Movement

We have seen that the normalized average productivity is higher in North than in South even in the equilibrium with international capital movement. Thus, if trade is allowed (in addition to capital movement) so that all firms compete in a level field, Northern firms’ profits rise while Southern firms’ profits fall, which will induce further capital movement from South to North.

To see this more formally, we first note that the formula for the competition index is the same as in the case where only trade is allowed, i.e., the formula given in (23), since capital movement simply relocate the firms from one country to the other without affecting the total number of firms of each type for given \( \omega_{h,N} \) and \( \omega_{h,S} \). Thus, the borrowing constraints for high-tech and low-tech firms in country \( k \) can be similarly written as

\[
R_k = \frac{\theta_k \beta^{\sigma-1}}{\sigma \phi_W(\omega_{h,N}, \omega_{h,S}, K)(g_h - \omega_{h,k})},
\]

\[
R_k = \frac{\theta_k}{\sigma \phi_W(\omega_{h,N}, \omega_{h,S}, K)[g_l - \hat{\omega}_{l,k}(\omega_{h,k}, K)]},
\]

where

\[
\phi_W(\omega_{h,N}, \omega_{h,S}, K) \equiv \sum_{k=N,S} \frac{m_k}{m_N + m_S} \phi_k(\omega_{h,k}, K).
\]

When trade in goods is allowed in addition to capital movement, the normalized average productivity that faces Northern firms decrease from \( \phi_N(\omega_{h,N}, K^K) \) and the one that faces Southern firms increase from \( \phi_S(\omega_{h,S}, K^K) \) to \( \phi_W(\omega_{h,N}, \omega_{h,S}, K^{KT}) \), where \( K^{KT} \) represents the equilibrium capital flow from North to South. Induced changes in profits will tend to increase the interest rate in North and decrease the interest rate in South. Capital moves from South to North to counter these movements in order to keep the interest rate parity between the two countries.

**Proposition 8** Trade in goods and capital movement are complement such that trade in goods induces further capital movement when capital has been mobile internationally. Trade
induces capital flight from South, expanding Northern industry and shrinking Southern.

International capital movement alone is not sufficient to equalize firms’ profits between North and South; the market in the capital-abundant or financially-developed North is more competitive than the one in the South. Opening to trade allows Northern firms to penetrate less-competitive Southern market and induces Southern firms to penetrate more-competitive Northern market, raising Northern firms’ profits while lowering Southern firms’. That is why opening to trade when capital has been internationally mobile induces additional capital movement from South to North. Proposition 8, which expresses this complementarity of trade and capital movement, is a core result of the paper.

4.4.1 Effects of trade and capital movement when $\theta_N = \theta_S$ and $\bar{\omega}_N > \bar{\omega}_S$

It follows from (38) and $R_N = R_S$ that $\omega_{h,N}^{KT} = \omega_{h,S}^{KT}$ and consequently $\omega_{l,N}^{KT} = \omega_{l,S}^{KT}$. Note that although the threshold wealth levels are the same between the two countries, the ratio of the mass of high-tech firms to the mass of low-tech firms is higher in North than in South due to $\bar{\omega}_N > \bar{\omega}_S$. Moreover, despite that trade induces a capital flight from South when capital has been mobile between the countries, capital is relocated from capital-abundant North to South relative to the autarky. Figure 2(a) shows the equilibrium thresholds between high-tech and low-tech entrepreneurs, i.e., $\omega_{h,k}$, in the four scenarios that we have considered. The arrows there indicate the movement of the threshold when the regime changes from autarky (or free trade) to free capital movement, and from free capital movement to trade and capital movement. Note again that trade will change the threshold only when accompanied by capital movement. The change in the threshold for being an entrepreneur, i.e., $\omega_{l,k}$, is similar, so it is not depicted in the figure for clarity.

4.4.2 Effects of trade and capital movement when $\theta_N > \theta_S$ and $\bar{\omega}_N = \bar{\omega}_S$

It follows from (38) and $R_N = R_S$ that the equilibrium $\omega_{h,N}$ and $\omega_{h,S}$ satisfy

$$\frac{\theta_N}{\theta_S} = \frac{g_h - \omega_{h,N}^{KT}}{g_h - \omega_{h,S}^{KT}}.$$
Comparing this condition to the one in the case where only capital is allowed to move, i.e., the condition shown in (37), we immediately find (also from $\phi_N(\omega_{h,N}, K) > \phi_S(\omega_{h,S}, K)$) that $\omega_{h,N}^{KT} < \omega_{h,N}^K$ and $\omega_{h,S}^{KT} > \omega_{h,S}^K$. This, of course, is consistent with our observation that trade (in addition to capital movement) induces further capital movement from South to North. Figure 2(b) shows the thresholds for the choice of technology. Again, it shows that trade changes the productivity distribution of the industry only when accompanied by capital movement.

5 Foreign Direct Investment under Financial Imperfection

We have shown how financial imperfection affects international capital movement and the resulting adjustment of the industry. The type of capital movement that we have considered is portfolio investment such that capital moves to the country in which borrowers utilize the capital to establish their firms. In this section, we consider foreign direct investment (FDI) such that it is entrepreneurs that move from one country to the other where they borrow money and invest to produce the good.\(^{12}\)

FDI naturally arises if trade is prohibited. In such cases, entrepreneurs in the country with a competitive market have an incentive to locate their firms in the foreign country with a less-competitive market. Here, we consider a situation in which such incentives do not exist. We consider a free-trade situation in which profits are the same for all firms regardless of their locations. To derive a sharp result, we also assume that $\tilde{\omega}_N = \tilde{\omega}_S$. We show that even in such situations, some firms engage in FDI. Indeed, we find that there co-exist two types of FDI in equilibrium: one that firms engage in FDI to exploit interest rate differential between the two countries, and the other that firms invest in the foreign country to overcome their borrowing constraints. Under financial imperfection, reciprocal FDI arises: Northern firms invest in South to exploit interest rate differential whereas Southern firms invest in

\(^{12}\)Since we assume that goods are freely traded in this section, whether entrepreneurs themselves move from the FDI source country to the host country or only their operations are relocated does not qualitatively affect the results.
North to overcome borrowing constraints.

Recall that in free trade, the interest rate is higher in North than in South, i.e., \( R_N > R_S \), reflecting the difference in financial development between the two countries, i.e., \( \theta_N > \theta_S \). If FDI is allowed, therefore, some firms in North invest in South, financing the investment at smaller costs. Consequently, the interest rate tends to decrease in North and rise in South, such that they become equal at the world interest rate, \( R_W \) in equilibrium with trade and FDI. Faced with the same interest rate, some Southern individuals are credit-constrained in the South, but not in North, since the borrowing constraint is tighter in South; we have \( \omega_{h,N} < \omega_{h,S} \) and \( \omega_{l,N} < \omega_{l,S} \), and hence Southern individuals with \( \omega \in [\omega_{l,N}, \omega_{l,S}] \cup [\omega_{h,N}, \omega_{h,S}] \) will invest in North to overcome their borrowing constraint for either becoming entrepreneurs or upgrading production technology.

To derive the equilibrium with trade and FDI, let \( K(\omega) \) denote the amount of capital that moves from North to South as FDI by entrepreneurs with the wealth \( \omega \); the entire FDI flows are characterized by \( \{K(\omega)\}_{\omega \in [0,\bar{\omega}_N]} \), which will be written henceforth simply as \( \{K(\omega)\} \). (Note that we have used the supposition that \( \bar{\omega}_S = \bar{\omega}_N \) in this definition of \( K \).) In equilibrium, the borrowing constraints must be binding for both high-tech and low-tech firms in each country \( k \):

\[
R_k = \frac{\theta_k \beta^{-1}}{\sigma \phi_W(\omega_{h,N}, \omega_{h,S}, \{K(\omega)\})(g_h - \omega_{h,k})},
\]

(39)

\[
R_k = \frac{\theta_k}{\sigma \phi_W(\omega_{h,N}, \omega_{h,S}, \{K(\omega)\})[g_l - \hat{\omega}_{l,k}(\omega_{h,k}, \{K(\omega)\})]},
\]

(40)

where

\[
\phi_W(\omega_{h,N}, \omega_{h,S}, \{K(\omega)\}) = \sum_{h=N,S} \frac{m_k}{m_N + m_S} \phi_k(\omega_{h,k}, \{K(\omega)\}).
\]

As in the case of trade and capital movement in the previous section, we have from (39) for \( k = N, S \) that

\[
\frac{\theta_N}{\theta_S} = \frac{g_h - \omega_{SD,N}}{g_h - \omega_{SD,S}},
\]

where \( \omega_{SD,k} \) represents the threshold for high-tech firms in equilibrium with trade and FDI. This condition immediately gives us \( \omega_{SD,N} < \omega_{SD,S} \) as Figure 3 depicts. Figure 3 also shows
that $\omega_{l,N}^{TD} < \omega_{l,S}^{TD}$, which is given by $\omega_{h,N}^{TD} < \omega_{h,S}^{TD}$ and

$$\beta^{\sigma-1} = \frac{g_h - \omega_{h,k}^{TD}}{g_l - \hat{\omega}_{l,k}^{TD}(\omega_{h,k}^{TD}, \{K(\omega)\})}, \quad (41)$$

which in turn is derived from (39) and (40).

As indicated in Figure 3, some (and not necessarily all) Northern entrepreneurs with $\omega \in [\omega_{l,S}^{TD}, g_l) \cup [\omega_{h,S}^{TD}, g_h)$ move to South, contributing to the interest rate parity between the two countries. At the same time, all Southern entrepreneurs with $\omega \in [\omega_{l,N}^{TD}, \omega_{l,S}^{TD}) \cup [\omega_{h,N}^{TD}, \omega_{h,S}^{TD})$ move to North; those with $\omega \in [\omega_{l,N}^{TD}, \omega_{l,S}^{TD})$ can only run a business in North, while those with $\omega \in [\omega_{h,N}^{TD}, \omega_{h,S}^{TD})$ can adopt the high-productivity technology only in North.

Although the two threshold wealth levels, derived from the borrowing constraints, are higher in South than in North, Southern entrepreneurs effectively gain access to Northern credit market through FDI. Using $\bar{\omega}_S = \bar{\omega}_N$, we can therefore express the effective worldwide credit-market clearing condition as

$$\frac{m_N + m_S}{\bar{\omega}_N(\omega_{h,N}^{TD} - \omega_{h,N})g_h + m_N + m_S(\omega_{l,N}^{TD} - \omega_{l,N})g_l} = \frac{(m_N + m_S)\bar{\omega}_N}{2}.$$  

It is easy to see that this credit-market clearing condition is effectively equivalent to (14) for $k = N$. Together with (41), it implies that $\omega_{h,N}^{TD} = \omega_{h,N}^{A}$ and hence $\omega_{h,N}^{TD} = \omega_{h,N}^{T}$ as Figure 3 shows since $\omega_{h,N}^{T} = \omega_{h,N}^{A}$. The productivity distribution in North would not be affected by allowing FDI in addition to trade, nor would be in South, if we identify a firm by the nationality of its owner. That is, we have

$$\phi_N(\omega_{h,N}^{TD}, \{K^{TD}(\omega)\}) = \phi_N(\omega_{h,N}^{T}) = \phi_S(\omega_{h,N}^{T}) = \phi_S(\omega_{h,S}^{TD}, \{K^{TD}(\omega)\}),$$

which gives us

$$\phi_W(\omega_{h,N}^{TD}, \omega_{h,S}^{TD}, \{K^{TD}(\omega)\}) = \phi_N(\omega_{h,N}^{T})(\omega_{h,S}^{T}).$$

Together with $\omega_{h,N}^{TD} = \omega_{h,N}^{T}$ and $\omega_{l,N}^{TD} = \omega_{l,N}^{T}$, we find that $R_{W}^{TD} = R_{N}^{T}$. Allowing FDI in addition to trade will increase Southern interest rate, but will not change Northern interest rate at all. Note that Northern firms have no (strong) incentive to move to South in equilibrium because the interest rates are the same between the two countries.

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If international portfolio investment is also allowed in addition to FDI (and trade), capital may move from South to North in the form of portfolio investment, to simply supply capital to Northern credit-market in which credit demands have increased due to Southern firms’ FDI in North. That is, portfolio investment from South to North may replace Northern firms’ FDI in South. FDI may exacerbate the capital flight from South.

We record these findings as the final proposition.

**Proposition 9** Reciprocal FDI will arise even when the countries engage in free trade. If international portfolio investment is allowed in addition to trade and FDI, capital may move from South to North in the form of portfolio investment, replacing Northern firms’ FDI in South.

6 Concluding Remarks

In the model where entrepreneurs with different wealth levels choose technology levels when they enter a differentiated-good sector, we have shown that firm heterogeneity in productivity arises only if there exists financial imperfection. We have also examined the impact of international trade in goods and capital movement between two countries. We have found among others that (i) trade in goods alone will not affect the productivity distribution of the industry, (ii) capital tends to move from a wealthy country to the other and from a country with a poorer financial institution to the other, shrinking the industry in the source country and expanding the industry in the host country, (iii) trade in goods affects the productivity distribution in each country only when it is accompanied by capital movement, (iv) when capital is also allowed to move, trade in goods itself induces capital movement from South (with less wealth and/or with less-developed financial institution) to North, and (v) reciprocal FDI arises even in free trade.

These findings regarding the impacts of financial imperfection on the differentiated-good sector (which can be thought of as a manufacture industry) are in general quite different from the conventional wisdom in international trade theory without any consideration of financial imperfection. Since no country has a perfect financial institution in practice, it
is important to know how the traditional theories should be modified when we incorporate financial imperfection into the models.

This paper is one of the first attempts to investigate interactions between financial development and international trade, so there are many related topics to be explored. It would be interesting, for example, to endogenize financial development by incorporating political and legal systems explicitly into the model. It would also be interesting to extend the model to a dynamic one so that the wealth distribution, which has been shown to play an important role in the analysis of this paper, is endogenously determined.
Appendix

A A Cause of Financial Imperfection

Here, we present a simple model to justify an imperfect financial institution. This model setting is a simplified version of Tirole’s (2006).

Let us consider the situation in which an agent tries to borrow $g$ from a lender to finance a profitable project. This project potentially generates profits of $\pi(> Rg)$ where $R$ is the exogenous (gross) interest rate. In order to complete the project successfully with a high probability, however, the agent must exert effort, which is unobservable to the lender. If the agent exerts effort, the project generates $\pi$ with the probability 1. If the agent shirks, on the other hand, the project generates $\pi$ with the probability $p^L(< 1)$ and 0 with the probability $1 - p^L$. By shirking, however, the agent can get non-pecuniary benefits $b\pi$, where $0 < b < 1$.

The agent unambiguously shirks if the entire $\pi$ goes to the lender. In order to induce the agent to exert effort, therefore, the lender must abandon some of $\pi$, giving a contingent reward $w$ to the agent; the reward is given to the agent if and only if the project has successfully generated $\pi$. The reward $w$ should satisfy the incentive condition, $w \geq p^Lw + b\pi$, where the left-hand side is the agent’s payoff when she exerts effort, while the right-hand side is her expected payoff when she shirks. We assume that negative rewards (i.e., penalties) are not allowed perhaps because the asset held by the agent is limited. This incentive condition can be written as

$$w \geq \frac{b}{1 - p^L}\pi.$$  

The lender expects to obtain at most $[1 - (b/(1 - p^L))]\pi$ if he induces the agent to exert effort. Alternatively, he may set $w = 0$ so that he obtains the expected payoff of $p^L\pi$. Consequently, the lender obtains the returns at most $\theta\pi$, where

$$\theta \equiv \max \left\{1 - \frac{b}{1 - p^L}, p^L \right\}.$$  

Obviously, the lender will not lend $g$ if $Rg$ exceeds $\theta\pi$. Note that if $p^L$ is small enough, $\theta$ is
equal to $1 - (b/(1 - p^L))$. Under a developed financial institution with a solid legal system, non-pecuniary benefits tend to be small. The parameter $\theta$ can be considered to represent the quality of a financial institution because $\theta$ increases as $b$ diminishes.

**B Free Trade in Goods and Capital Movement under Perfect Financial Institution**

First, we derive the equilibrium profits and gross interest rate in free trade. Since trade liberalization itself does not change the credit-market clearing condition in each country, the number of firms in country $k$ is still given by $n_k = m_k \bar{\omega}_k / (2g_h)$ as shown in (10). Consequently, the competition index in country $k$ can be written as

$$\tilde{\phi}_T^k = \beta \varphi \left( n_N + n_S \right) \frac{1}{\sigma - 1} = \beta \varphi \left( \frac{m_N \bar{\omega}_N + m_S \bar{\omega}_S}{2g_h} \right)^{\frac{1}{\sigma - 1}}.$$ (42)

Despite the fact that the competition index increases as a result of trade as the comparison between (12) and (42) reveals, firms’ profits do not necessarily decrease because they are now able to sell their products in both countries. The worldwide profits for any firm of any country are derived as

$$\pi_W(\varphi_h, \tilde{\phi}_T) = \frac{m_N + m_S}{\sigma} \left( \frac{\varphi_h}{\tilde{\phi}_T} \right)^{\sigma - 1} = \frac{2g_h(m_N + m_S)}{\sigma(m_N \bar{\omega}_N + m_S \bar{\omega}_S)}.$$ (43)

Then it follows from (9) that

$$R_N = R_S = \frac{2(m_N + m_S)}{\sigma(m_N \bar{\omega}_N + m_S \bar{\omega}_S)}.$$ (44)

Both profits and interest rate in North rise and those in South fall as a result of trade, equating the interest rates between the two countries and hence eliminating individuals’ incentive to invest abroad even when capital is allowed to move internationally.

If capital is internationally mobile while the good is not, capital moves from North to South if $\bar{\omega}_N > \bar{\omega}_S$. The mass of firms decreases in North and increases in South until the profits are equalized between the two countries. That is, it follows from $\pi_k(\varphi_h, \tilde{\phi}_k) =$
m_k/(σn_k) (as (13) shows) that m_N/(σn_N) = m_S/(σn_S). Together with the worldwide credit-market clearing condition, described by

$$(n_N + n_S)g_h = \frac{m_N \bar{\omega}_N}{2} + \frac{m_S \bar{\omega}_S}{2},$$

we obtain

$$n_N = \frac{m_N (m_N \bar{\omega}_N + m_S \bar{\omega}_S)}{2g_h (m_N + m_S)},$$

and

$$n_S = \frac{m_S (m_N \bar{\omega}_N + m_S \bar{\omega}_S)}{2g_h (m_N + m_S)}.$$

Now, it is readily verified that $\pi_N(\varphi_h, \bar{\varphi}_N)$ and $\pi_S(\varphi_h, \bar{\varphi}_S)$ are both equal to $\pi_W(\varphi_h, \bar{\varphi}_T)$ as derived in (43), and as a consequence the interest rate $R_W$ in the world capital market will be the same as $R_N$ and $R_S$ as derived in (44).

C Proof of Proposition 3

As $\bar{\omega}_k$ increases from $g_h$, both $\omega_{h,k}$ and $\hat{\omega}_{l,k}(\omega_{h,k})$ decline to clear the credit market. This change also reduces $\omega_{h,k} - \hat{\omega}_{l,k}(\omega_{h,k})$ as we can see from (21), and eventually makes $\omega_{h,k} - \hat{\omega}_{l,k}(\omega_{h,k})$ equal zero at a certain $\omega_{h,k}$, which we call $\omega^*_A(\omega_{h,k})$, i.e., $\omega^*_A(\omega_{h,k})$ exceeds $\omega_{h,k}$. We substitute $\hat{\omega}_{l,k}(\omega_{h,k}) = \omega^*_A(\omega_{h,k})$ into (21) to obtain

$$\omega^*_A(\omega_{h,k}) = \frac{\beta^{\sigma-1} g_l - g_h}{\beta^{\sigma-1} - 1}.$$ 

If $\omega^*_A(\omega_{h,k}) \geq \omega^*_h(\omega_{h,k})$, there exists nonnegative excess credit supply when $\omega_{h,k} = \omega^*_A(\omega_{h,k})$, in which case $\omega^*_A(\omega_{h,k}) \leq \omega^*_h(\omega_{h,k})$ so that the inequality $\omega_{h,k} > \hat{\omega}_{l,k}(\omega_{h,k})$ does not hold. Therefore, we must have $\omega^*_A(\omega_{h,k}) < \omega^*_h(\omega_{h,k})$, or equivalently,

$$\frac{\beta^{\sigma-1} g_l - g_h}{\beta^{\sigma-1} - 1} < \omega_{h,k} - \frac{\bar{\omega}_k^2}{2g_h},$$

for low-tech firms to exist in equilibrium. In other words, firm heterogeneity arises under Assumption 3.
References


Figure 1. Feasible set of $\bar{\omega}_k$
Figure 2. Thresholds for the choice of high-productivity technology
Figure 3. Reciprocal FDI Flows