Overlapping Tax Revenue, Soft Budget, and Rent Seeking

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Abstract

This paper investigates how the soft-budget constraint with grants from the central government to local governments tends to internalize the vertical externality of local public investment by stimulating local expenditure when both the central and local governments impose taxes on the same economic activities financed by public investment. The model incorporates the local governments' rent-seeking activities in a multi-government setting. The soft-budget constraint is welfare deteriorating because it stimulates rent-seeking activities, although a soft-budget game may attain the first-best level of public investment.

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1. Introduction

This paper investigates how intergovernmental financing may stimulate both insufficient local public expenditures and inefficient rent seeking by highlighting the soft-budget constraint of grants from the central (or federal) government to subnational governments (hereafter, local governments) with overlapping tax bases. This paper theoretically analyzes the welfare implications of the soft-budget constraint of intergovernmental financing by developing a simple game between two governments. By incorporating both the vertical externality of public investment and the rent-seeking activities of local politicians or governments, we explore both positive and negative welfare effects of soft-budget outcomes with ex post grants from the central government.

The soft-budget constraint means that the local government (LG) may, after making a decision, receive an additional grant from the central government (CG), while the hard-budget constraint means the LG may not receive such a grant. It is well recognized that if local governments face soft-budget constraints, they will have an incentive to overspend, overborrow, and/or pay insufficient attention to the quality of investments that their borrowing finances. Such welfare-deteriorating overspending/overborrowing can occur through a common pool mechanism. See, for example, Wildasin (1997, 2004), Goodspeed (2002), Akai and Sato (2005), and Boadway and Tremblay (2005), among others. That is, the standard result is that if the CG imposes soft-budget constraints, inefficiently high levels of investment will occur. On the other hand, Besfamille and Lockwood (2004) show that hard-budget constraints can be too hard and discourage socially efficient investment. They point out the possibility that the hard-budget constraint would create an incentive to penalize investment too much for project failure, ultimately leading to the possibility that socially efficient projects may not be undertaken. Thus, the welfare implications of the soft-budget constraint seem ambiguous, and depend on their effects on public investment. In other words, the conventional thinking is that a soft budget is welfare deteriorating if public investment is too high.

We pay attention to the vertical externality of shared tax bases of the central and local governments in a real economy. Multilevel government normally means some commonality of tax base between the central and local governments. As a result the tax base may overlap, and shared tax bases may create the common pool problem[1]. In this paper, we do not consider such vertical/horizontal tax competition between the central and multi-local governments, but simply assume that tax rates are exogenously given for both the central and multi-local governments. Instead, we would like to focus on another externality of local expenditures due to the overlapping tax bases.

When the tax share is exogenously given, useful local public investment may have a positive externality effect. That is, if an increase in local expenditure on

infrastructure stimulates macroeconomic activities, this response may enlarge the overlapping tax base, which would then increase taxes for the central government at the given shares of the tax base of the two governments. This creates a positive spillover of the vertical externality. The non cooperative Nash equilibrium level of local public investment in the hard-budget game may well be too low. Thus, the soft-budget game could be welfare improving if it stimulates insufficient public investment.

In this paper, however, we also incorporate the rent-seeking activities of local politicians who are trying to obtain as much revenue for themselves subject to a utility constraint. We consider wasteful spending by the local government. In the tradition of the "Leviathan" models of government (see Brennan and Buchanan [1980] among others), wasteful projects are too high because of subsidies from the central government due to local governments' political demands. Local politicians prefer "wasteful" public spending, which provides them with rent-seeking opportunities. In reality, this feature is very relevant. Some of the local expenditures are wasteful and associated with rent-seeking activities.

We assume that government officials are trying to obtain as much revenue for themselves subject to a utility constraint. Suppose there are many types of politicians. The type of politician may be expressed as the degree of rent seeking. Voters may choose a type of politician by undertaking political efforts. If they spend more on political efforts, they may find or select a less rent-seeking politician. For simplicity, we assume that the optimizing behavior of voters is exogenously given in the model, so that both their political efforts and the resulting reservation utility are fixed. The opportunity of receiving an additional grant in the soft-budget game would stimulate wasteful rent-seeking activities, and hence reduce social welfare. We shall show that although the soft-budget game leads to the first-best level of public investment, it deteriorates equilibrium welfare by stimulating rent-seeking activities.

In the standard-type model, the soft-budget outcome normally arises from information asymmetry or cost sharing. On the contrary, we consider a very simple two-period model without uncertainty or information asymmetry[2]. When the hard-budget game is over, the rent seeking is also over. Then, the benevolent central government may have an incentive to stimulate useful local expenditures in period 2 by means of additional grants ex post. Such additional grants could produce the soft-budget problem. If local governments anticipate this, the soft-budget outcome is caused by two channels: public investment and rent seeking. First of all, the soft-budget constraint would stimulate insufficient local public investment, which is beneficial. Second, it would also stimulate rent-seeking activities, which is not beneficial. We show that so long as the survival constraint on local governments is effectively fixed, the second deteriorating effect is more

powerful than the first beneficial effect, even if the beneficial effect helps in attaining the first-best level of public investment. We may conclude as follows. Since the utility from local public goods is fixed by the survival constraint of local politicians, an increase in public investment would not directly raise social welfare but will increase rent. Hence, the second deteriorating effect becomes more powerful than the first beneficial effect.

This paper consists of six sections. In section 2, we develop a theoretical model of the central and local governments, and then consider the outcome of the first best as the reference point. In section 3, we investigate the hard-budget game between the benevolent central government and the rent-seeking local government. In section 4, we consider the soft-budget game and compare these two games. In section 5, we present some remarks. Finally, we conclude the paper with section 6.

2. Analytical Model of Central and Local Governments
2.1 Analytical Framework

We develop a two-period intergovernmental financing model of the two governments, the central government, and the lower-level local government, in a small open economy in order to explore how local public investment and wasteful spending may be stimulated under the soft-budget constraint. For simplicity, we consider a representative LG, and do not examine the free-riding and/or spillover effects within multi-local governments. Several studies have explored the horizontal and vertical externalities due to non-cooperative competition among multi-local governments (see Wilson [1999] among others). However, in reality, many LGs do cooperate. Furthermore, our model is a good approximation to Japan, where most LGs cooperate with the central government; their behavior may be suggestive of our model local government[3].

The central government would not like to commit itself to the initial decision of making no grants to the local government when the hard-budget game is over and period 2 starts. One contribution of this paper is to show that the soft-budget outcome could occur with a representative LG even if the central government intends to attain an optimal allocation of central and local public goods. This is a new result since the conventional literature on soft budgets normally assumes multi-local governments where the CG intends to transfer public goods to local governments in order to attain an optimal allocation of local public goods. Moreover, in this paper the soft-budget outcome may occur even if we assume away information asymmetry or cost sharing. Another important contribution is that the soft budget is shown to be welfare deteriorating even if the soft-budget game may attain the first-best level of public investment. In our framework, the inclusion of rent seeking is crucial to obtain the analytical result.

The representative LG provides useful local public goods g_t and the CG provides useful nationwide public goods G_t in each period. Each public good is beneficial, and its utility is given by a twice-continuously differentiable and strictly quasi-concave function. Moreover, we assume that all goods are normal ones. The relative price of each good is set to unity for simplicity. Thus, social welfare W, which reflects the representative agent's preferences over public goods, is given by

$$W = v(g_1) + \delta\{u(G_2) + v(g_2)\} \tag{1}$$

where $0 < \delta < 1$ is a discount factor. For simplicity, we do not explicitly consider the first-period spending by the CG, G_1, in the social welfare function since it is exogenously fixed. Private consumption is also assumed to be fixed, and hence we only consider utility from public goods. This formulation may be justified because we assume that the tax rate on income is fixed and labor supply is exogenously given, so that the private optimizing behavior is not incorporated. If we explicitly consider private consumption for social welfare, the analytical results would be almost the same, qualitatively. See section 5.3.

The local government undertakes public investment k in period 1, which has the productive effect of raising tax revenue in period 2. Let Y_t represent the total tax revenue of the two governments in period t ($t = 1, 2$). We assume that Y_1 is exogenously given but Y_2 increases as a result of public works implemented by the LG in period 1, k. $Y_2 = Y_1 + f(k)$. The investment product function $f(\ )$ satisfies the standard Inada condition $f'(\ ) > 0, f''(\ ) < 0$. For simplicity, we do not consider public investment by the central government. In a multi-local government setting, local public investment may have spillover effects across regions. However, since we consider a representative local government, we do not incorporate such horizontal spillovers. See section 5.2.

Both central and local governments levy taxes on overlapping economic activities in period 2. The tax revenue is shared by the two governments. We set β as the LG's portion of the total tax revenue, $0 < \beta < 1$. Thus, the share of the central government in the total tax revenue is $1 - \beta$. The share parameter β is assumed to be exogenously given. Public investment has a vertical externality effect on the CG's tax revenue.

We consider pork barrel spending by the local government. As shown in Del Rossi and Inman (1999), wasteful pork barrel projects could be too high due to subsidies from the central government in response to local governments' political demands. In the tradition of the Leviathan models of government (see Brennan and Buchanan [1980] among others), local politicians prefer "wasteful" public spending (S_1, S_2), which provides them with rent-seeking opportunities but does not benefit voters or consumers. In this sense, useful local spending is divided among g_1, g_2, and k, and wasteful local spending is divided between S_1 and S_2. In a real

economy, some of the public works are wasteful. Such wasteful public investment corresponds to S_i, and not k, in this model. In section 5.1, we discuss how the analytical result would be altered if the LG is benevolent.

Next, we specify each government's budget constraint. The budget constraints of the CG are given as follows:

$$G_1 + A_1 = B + (1 - \beta)Y_1 \tag{2-1}$$

$$G_2 + (1 + r)B = (1 - \beta)Y_2 - A_2 \tag{2-2}$$

where A_i is a lump sum subsidy to the LG in period $i = 1, 2$ ($A_i \geq 0$), B is CG debt, and $r > 0$ is the exogenously given world interest rate. Note that in period 1 $G_1 = (1 - \beta)Y_1$ is exogenously given, and hence we do not explicitly consider G_1 or $(1 - \beta)Y_1$ in the analytical framework. In the hard-budget game, the CG may choose A_1. At the same time, the CG commits itself to the initial value $A_2 = 0$. In this game the CG is the leader, while the LG is the follower. If the CG changes the value of A_2 in period 2 when the hard-budget game is over, we call this state the soft-budget game. In this game, the LG becomes the leader, while the CG is the follower.

The period-by-period budget constraints of the LG are given as follows:

$$D = g_1 + k - \beta Y_1 + S_1 + A_1 \tag{3-1}$$

$$g_2 + S_2 + (1 + r)D = \beta Y_2 + A_2 \tag{3-2}$$

where D is the LG debt, which is controlled by the CG. Alternatively, we may simply assume that the LG cannot issue any local debt: $D = 0$.

From (3-1) and (3-2) we can write the objective function of the local government as follows:

$$S \equiv S_1 + \frac{S_2}{1 + r} = \beta Y_1 + \frac{\beta Y_2}{1 + r} + A_1 + \frac{A_2}{1 + r} - g_1 - \frac{g_2}{1 + r} - k \tag{3-3}$$

2.2 Pareto-Efficient Solution

First of all, we investigate the Pareto-efficient first-best allocation in this model as a benchmark. A unitary benevolent government, consolidating the CG and the LG, could attain the first best by optimally allocating the total tax revenue among nationwide public goods and local public goods in each period. That is, the unitary government, which implements an optimal allocation of $\{G_t, g_t, k\}$, maximizes the social welfare (1) subject to the following overall feasibility constraint:

$$\beta Y_1 + \frac{Y_2}{1 + r} = \frac{G_2}{1 + r} + g_1 + \frac{g_2}{1 + r} + k + S_1 + \frac{S_2}{1 + r} \tag{4}$$

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This equation is obtained by eliminating A_1 and A_2 from (2-1)(2-2) and (3-1)(3-2), respectively.

The first-order conditions of this optimization problem are as follows:

$$\delta u_{G2} - \frac{\mu}{1+r} = 0 \qquad \text{where} \quad u_{G2} \equiv \frac{\partial u(G_2)}{\partial G_2}$$

$$v_{g1} - \mu = 0$$

$$\delta v_{g2} - \frac{\mu}{1+r} = 0 \qquad \text{where} \quad v_{gt} \equiv \frac{\partial v(g_t)}{\partial g_t}$$

$$\mu \left\{ \frac{f'(k\ )}{1+r} - 1 \right\} = 0$$

$$S_1 = S_2 = 0$$

Further, μ is the Lagrangian multiplier of equation (4). From these conditions, we have

$$u_{G2} = v_{g2} \tag{5-1}$$

$$\frac{v_{g1}}{v_{g2}} = (1+r)\delta \tag{5-2}$$

$$f'(k) = 1+r \tag{5-3}$$

$$S_1 = S_2 = 0 \tag{5-4}$$

The above optimality conditions (5-1), (5-2), (5-3), and (5-4) and the overall feasibility condition (4) determine the Pareto-efficient allocation as the benchmark case. Condition (5-1) means that the marginal benefit of pubic goods is equalized between the CG and the LG. Condition (5-2) governs the standard (intertemporal) optimal allocation of local public spending between the two periods. Condition (5-3) is the standard first-best criterion of public investment. Finally, condition (5-4) is obviously the efficiency condition. We do not have any rent-seeking activities with the first-best solution.

3. Hard-Budget Game

We now investigate the outcomes in a decentralized multi-government non-cooperative system, where the benevolent central and rent-seeking local governments decide their policy variables non-cooperatively. First of all, we consider the hard-budget game in this section. Here, we investigate the fully (or isolated) decentralized Nash equilibrium at the exogenously given $\beta > 0$. In this game the CG is the leader, and the LG is the follower. In the hard-budget game the CG may set $A_2 = 0$ throughout the game.

At the first stage, the CG determines the grants in period 1, A_1 (and hence its debt issuance B and public goods, G_2). Then, at the second stage, the LG determines its expenditures, g_1, g_2, k, and rent-seeking activities S_1 and S_2. At the start of period 2, the CG maintains the committed value of $A_2 = 0$ [4].

The CG maximizes (1) subject to (2) by choosing the size of transfer to LG in period 1, A_1. On the other hand, the LG, which represents the interests of rent-seeking local politicians, maximizes the present value of wasteful public spending or rent, $S = S_1 + S_2/(1 + r)$ by choosing local public goods and investment subject to the following survival constraint:

$$v(g_1) + \delta v(g_2) = \bar{U} \qquad (6)$$

where $\bar{U}$ means the reservation utility, which depends on the preferences of voters. If condition (6) is not satisfied, voters do not re-elect the current government, and the ruling local politicians cannot stay in office. In this sense, we implicitly assume that there are many politicians available in each region. It is plausible to assume that

$$\bar{U} < U^F \equiv v(g_1^F) + \delta v(g_2^F)$$

where g_1^F, g_2^F are the first-best levels of g_1, g_2, respectively.

Suppose there are many types of politicians. The type of politician may be expressed in terms of the degree of rent seeking. If the reservation utility, $\bar{U}$, is lower, the associated politician is a greater rent seeker. Voters may choose the desired type of politician by undertaking political efforts. If they spend more on political efforts, they may find or select a less rent-seeking politician. For simplicity, we assume that the optimizing behavior of voters is given in the present model, so that both their political efforts and the resulting reservation utility are fixed. Note that nationwide public goods are not controlled by local politicians, so their survival does not depend on them [5].

3.1 Second Stage

Let us investigate the outcome of this hard-budget game. The LG's problem at the second stage is to maximize

$$S = \beta(Y_1 + \frac{Y_2}{1+r}) - (g_1 + \frac{g_2}{1+r} + k) + A_1 \quad \text{subject to (6).}$$

Then, the first-order conditions with respect to g_1, g_2, and k are as follows:

$$-1 + \psi v_{g1} = 0$$

$$-\frac{1}{1+r} + \psi \delta v_{g2} = 0$$

$$\frac{f'(k)\beta}{1+r} - 1 = 0 \qquad (7)$$

where ψ (>0) is the Lagrangian multiplier of (6). Thus, we have

$$\frac{v_{g1}}{v_{g2}} = (1+r)\delta \qquad (5\text{-}2)$$

From these conditions, (6), (7), and (5-2), the optimal levels of g_1^*, g_2^*, k^*, and S are determined. Condition (5-2) means that the total expenditure on local public goods, $g_1 + \dfrac{1}{1+r} g_2$, is minimized under the survival condition (6). Both conditions (6) and (5-2) determine the equilibrium values of g_1, g_2 in this game, g_1^*, g_2^*. Condition (7) determines k in this game, k^*. Note that the optimal levels of g_1^*, g_2^*, k^* are not dependent on the CG's choice variable, A_1. Note also that S increases with A_1. S_1 and S_2 are uniquely determined at a given level of D to meet the budget constraints (3-1) and (3-2) [6].

3.2 First Stage

The CG maximizes (1) subject to (2-1) and (2-2) by choosing a transfer to the LG, A_1, considering the optimizing behavior of the LG, described in section 3.1. That is, the CG anticipates that the LG determines its choice variables under constraint (6) at the second stage. Hence, the resulting social welfare reduces to $W = \delta u(G_2) + \bar{U}$. The equilibrium social welfare increases with G_2, and hence decreases with A_1 at a given level of k.

Thus, it is always desirable for the CG to reduce A_1 (and hence S) by raising G_2 as much as possible. That is, a decrease in A_1 raises social welfare by reducing S_1 and S_2. Considering the non-negativity constraint, $A_1 \geq 0$, the optimal level of A_1 for the CG is given by $A_1 = 0$. The social welfare, which increases with G_2, is maximized at $A_1 = 0$ at the given level of local public expenditures, g_1, g_2, S_1, S_2, associated with k^*. Although S is minimized at $A_1 = 0$, we still have positive values of S_1, S_2.

3.3 Outcome

The subgame perfect outcome of this hard-budget game is given by

$$f'(k) = \frac{1+r}{\beta} > 1 + r \qquad (7)$$

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$$\frac{v_{g1}}{v_{g2}} = (1+r)\delta \qquad\qquad (5\text{-}2)$$

$$A_1 = A_2 = 0 \qquad\qquad (8)$$

Condition (5-2) implies that the relative (intertemporal) allocation between g_1 and g_2 is efficient in this game. However, these public goods and local investments are not necessarily provided optimally. In other words, condition (5-1) does not necessarily hold since the total level of public goods, G_2 and $g_1 + \dfrac{g_2}{1+r}$, is arbitrarily set, depending on the exogenous parameter β, the rent-seeking behavior of the LG, and the survival condition (6).

      Condition (5-3) does not hold, either. Considering $\beta < 1$, (7) means that k is under-provided due to the vertical externality of the overlapping tax base, $k* < k^F$, where $k*$ is the level of k in this game and k^F is the level of k at the first best. Since the local government does not take into account the positive spillover effect of the increasing overlapping tax base of the central government, local public investment is not sufficient. Further, the total tax revenue shared by both governments in period 2 is too little.

      To sum up, there are three sources of inefficiency in the decentralized system. First, since β is not necessarily set at the optimal level, the allocation of public spending between the CG and LG is not determined optimally. Second, there is the vertical externality of public investment due to the overlapping tax base, and hence k is too little ($k^* < k^F$). Finally, because of the rent-seeking activities of the LG ($\bar{U} < U^F$), local public goods, g_1, g_2, are too little, and wasteful public expenditures S_1 and S_2 become positive.

4 Soft-Budget Game
4.1 CG's Ex Post Transfer: Second Stage

      As explained before, at the first stage of the hard-budget game, the CG determines A_1 anticipating that the LG will choose g_1, g_2, k, S_1, S_2 to meet the survival condition (6) at the second stage, and maintains the committed value of $A_2 = 0$. Then, it is optimal to set $A_1 = 0$ since a reducing A_1 depresses rent seeking to some extent.

      However, when the hard-budget game is over, the LG's optimizing behavior also becomes fixed, so that the survival condition is not binding ex post. The CG may now have an incentive to change the committed value of $A_2 = 0$. That is, the CG may raise A_2 by creating grants to the LG ex post in order to increase social

welfare. Thus, the LG would face the soft-budget constraint. This is a time inconsistency problem. In this soft-budget game the LG becomes the leader, and the CG the follower.

We first investigate the optimizing behavior of the CG at the beginning of the second period. After the LG determines local expenditures, g_1, $\widehat{g}_2$, S_1, S_2, and k, at the first stage of this soft-budget game, the CG may choose its public spending, G_2 (and effectively g_2), subject to the budget conditions (2-2) and (3-2) by creating an additional grant, A_2, appropriately, in period 2. Here, $\widehat{g}_2$ is the level of second-period local public spending chosen by the LG at the first stage of the soft-budget game, and g_2 is the final outcome of second-period local public spending effectively chosen by the CG. Note that $g_2 = \widehat{g}_2 + A_2$. At a given level of $\widehat{g}_2$, an increase in A_2 means an increase in g_2 by the same amount because k and S_2 are chosen at the first stage of this game.

Eliminating A_2 from (2-2) and (3-2) gives the overall budget constraint in period 2 as

$$G_2 + g_2 + S_2 + (1+r)D = Y_2 \qquad (9)$$

Note that S_2 and Y_2 are determined by the LG at the first stage. By choosing A_2 in period 2, the CG may in fact choose the allocation of G_2 and g_2 under the overall budget constraint (9) to maximize social welfare in period 2, $u(G_2) + v(g_2)$. Since the rent-seeking activity was already performed at the first stage of this game, the survival condition (6) is no longer binding at this stage. When the rent-seeking activity is over, an additional grant to the LG may increase g_2. This is why we have the soft-budget outcome without information asymmetry or cost sharing in this paper.

Thus, the first-order condition at the second stage of this game is given by

$$u_{G2} = v_{g2} \qquad (5\text{-}1)$$

From the optimality condition (5-1) and the second-period budget constraints (2-2) and (3-2), at given levels of local expenditures, S_2 and k, which are chosen by the LG at the first stage, we may derive the optimal responses A_2 and g_2 (and hence G_2) of the CG as functions of S_2 and k, respectively. Although A_2 and g_2 change in the same direction by the same amount at a given level of $\widehat{g}_2$, they may change in a different way when S_2 and k (and hence $\widehat{g}_2$) change:

$$A_2 = A(S_2, k) \qquad (10\text{-}1)$$

$$g_2 = g(S_2, k) \qquad (10\text{-}2)$$

By totally differentiating the budget conditions (2-2) and (9) and the optimality condition (5-1), we have

$$dG_2 = (1-\beta)f'(k)dk - dA_2$$

$$dG_2 + dg_2 + dS_2 = f'(k)dk$$

$$(1-\eta)dG_2 = \eta dg_2$$

where $\eta \equiv |v_{gg2}|/[|u_{GG2}| + |v_{gg2}|]$ represents the relative evaluation of G_2 compared with g_2. It is assumed for simplicity that $0 < \eta < 1$ is constant. Then, we have the property of the response functions as follows:

$$A_S = \frac{\partial A_2}{\partial S_2} = -\frac{\partial G_2}{\partial S_2} = \eta > 0 \tag{11-1}$$

$$A_k = \frac{\partial A_2}{\partial k} = -\frac{\partial G_2}{\partial k} + (1-\beta)f'(k) = (1-\beta)f'(k) - \eta f'(k) \tag{11-2}$$

$$g_{S_2} = \frac{\partial g_2}{\partial S_2} = -(1-\eta) < 0 \tag{11-3}$$

$$g_k = \frac{\partial g_2}{\partial k} = (1-\eta)f'(k) \tag{11-4}$$

Equation (11-1) shows the plausible outcome of the soft-budget constraint due to rent-seeking activities. That is, an increase in S_2 (or a decrease in $\hat{g}_2$) at the given levels of k and A_2 results in a decrease in g_2, leading to more grants, A_2, from the central government. Hence, we have $A_S > 0$. We may conclude as follows. When the LG conducts more rent-seeking activities, S_2, at the first stage of this soft-budget game, $\hat{g}_2$ (and hence g_2) falls relative to (3-2). This outcome is not good for the CG since it would like to realize the optimality condition (5-2) to increase social welfare ex post. Thus, the CG has an incentive to provide additional subsidies to the LG in period 2 in order to raise the ex post level of g_2 and reduce the ex post level of G_2. This positive effect of S_2 on A_2 is an important outcome of the soft-budget game.

Moreover, we have another outcome of the soft-budget result due to the effect of public investment on A_2, A_k, which is another new channel caused by the vertical externality. As shown in (11-2), the sign of A_k is generally ambiguous. If $1 - \beta > \eta$, then $A_k > 0$ (and vice versa). That is, if the marginal valuation of G_2 is relatively small and $1 - \beta$ is too high, then g_2 would be too low compared with G_2. In such a case, when k increases, the CG would react by increasing A_2 in order to maximize the ex post social welfare. We may conclude as follows. An increase in k

raises the CG's tax revenue by $(1-\beta)f'$, resulting in an increase in the utility from G_2 by $(1-\beta)f'u_{G2}$ at a given level of A_2. This increase in G_2 is optimal if the utility from g_2 increases by $(1-\beta)(1-\eta)f'v_{g2}/\eta$. On the other hand, it actually raises the marginal utility of g_2 by $\beta f'v_{g2}$ at a given level of A_2. Hence, if $1-\beta > \eta$, it is desirable that g_2 is increased by a reduction of G_2, so the CG would react by giving more grants A_2 to the LG to stimulate g_2.

A key part of the model is the interaction between the CG and LG. The CG intends to allocate revenues to equalize the marginal gains of public goods between the central and local governments so long as the rent-seeking activity is fixed. The CG's benevolent incentives result in a soft-budget constraint by creating additional grants in period 2 when the LG conducts more rent seeking and undertakes more investment. The soft-budget outcome is caused by two channels: rent seeking and public investment. First, more rent seeking means a decline in local public goods in period 2 and hence upsets the central government's optimal allocation strategy. Second, more investment means an increase in the tax revenue of the CG and hence may raise the amount of central public goods too much. Then, the CG has an incentive to make additional grants in period 2 by stimulating g_2 in order to raise the ex post social welfare.

4.2 LG's Behavior: First Stage

We now investigate the optimizing behavior of the LG at the first stage of the soft-budget game and the resulting subgame perfect outcome. The local government's survival constraint (6) is effectively binding here with the LG anticipating that the CG would change A_2 in response to local expenditures, as summarized by equation (10-1) and (10-2). Therefore, the survival condition and the objective function for the LG are rewritten as

$$v(g_1) + \delta v(g(S_2, k)) = \overline{U} \tag{12}$$

$$S = \beta Y_1 + \frac{\beta[Y_1 + f(k)]}{1+r} + \frac{A(S_2, k)}{1+r} - g_1 - \frac{g(S_2, k)}{1+r} - k \tag{13}$$

The LG maximizes the objective S, (13), subject to the survival condition (12) at given levels of the tax share parameter β and the reservation utility $\overline{U}$.

The first-order conditions with respect to its choice variables, g_1, S_2, and k, are respectively given as follows:

$$-1 + \omega v_{g1} = 0 \tag{14-1}$$

$$-\frac{g_S - A_S}{1+r} + \omega \delta v_{g2} g_S = 0 \tag{14-2}$$

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$$-(1+\frac{g_k}{1+r}-\frac{\beta}{1+r}f'(k)-\frac{A_k}{1+r})+\omega\delta v_{g2}g_k=0 \qquad (14\text{-}3)$$

where ω (>0) is the Lagrange multiplier of the survival constraint (12). Equations (14-1) and (14-2) govern the allocation of g_1 and g_2 at a given level of the tax share parameters β and $\bar{U}$.

Substituting (11-1), (11-2), and (11-3) into (14-1) and (14-2), we have

$$v_{g1}=\delta v_{g2}(1-\eta)(1+r) \qquad (15\text{-}1)$$

Thus, the (first-best) optimality condition between g_1 and g_2 given by (5-2) is not realized here.

$$\frac{v_{g1}}{v_{g2}}=(1+r)\delta \qquad (5\text{-}2)$$

This is an important difference between the hard-budget and soft-budget outcomes. If the CG did not provide additional grants A_2, as in the hard-budget case ($A_2=0$), the optimizing behavior of the LG could have attained this optimality condition (5-2) with respect to the relative allocation of g_1 and g_2.

On the contrary, the response functions of the CG, (10-1) and (10-2), would effectively reduce the marginal cost of raising g_1, stimulating g_1 in period 1. Hence, the LG would reduce $\hat{g}_2$ (and hence g_2) but increase g_1 to satisfy the survival condition (6). By doing so, the LG may increase the present value of rent seeking, S, by receiving more grants from the CG. Equation (15-1) means that g_1 is too high compared with g_2 and G_2. The soft-budget constraint leads to an increase in A_2, which has a positive effect on g_1 as well as S_2. It should be noted that the soft-budget game actually reduces the equilibrium levels of g_2 and G_2 compared with the hard-budget game. Anticipating more A_2, the LG has an incentive to reduce $\hat{g}_2$, which results in a decrease in g_2 although the CG raises A_2. This is because the LG may reduce g_2 by increasing g_1 in order to meet the survival condition (6).

In the hard-budget case we have shown that (5-2) is attained but (5-4) is not. On the contrary, in the soft-budget case, substituting (11-2), (11-3), and (11-4) into (14-3), we have

$$v_{g1}=\delta(1-\eta)f'v_{g2} \qquad (15\text{-}2)$$

Considering (15-1) and (15-2), we finally get

$$1+r=f' \qquad (5\text{-}4)$$

The first-best level of k is attained here. We may conclude as follows. When k rises,

the LG may expect additional grants, A_2, from the CG due to an increase in $(1-\beta)Y_2$, in addition to an increase in its own tax revenue, βY_2, so that the effective marginal benefit of an increase in k becomes f', instead of $\beta f'$. As shown in (11-2), the direct effect of an increase in k on A_2 is $(1-\beta)f' - \eta f'$. In addition, as shown in (11-4), an increase in k would raise g_2, and indirectly benefit the LG by alleviating the survival constraint. This indirect benefit may be expressed as

$$\delta \omega v_{g2} g_k (1+r) - g_k = \eta f' + (1+r) - f'.$$

Thus, the overall beneficial effect through the CG's response in the soft-budget game is $(1-\beta)f'$ at $1+r = f'$, and hence it may internalize the vertical externality. It follows that, at the subgame perfect solution, k is given by the first-best level, k^F, which is larger than k^* in the hard-budget case of $A_1 = A_2 = 0$. This is an interesting result of the soft-budget constraint. The CG does not intend to internalize the vertical externality by raising A_2 to the LG. Nevertheless, the CG's additional grants in response to k may effectively internalize the vertical externality.

4.3 Welfare Comparison

We have shown that the soft-budget constraint stimulates public investment and the first-best level of k is attained. However, at the same time, A_2 becomes positive, which hurts social welfare. We now investigate the overall welfare effect of the soft-budget constraint. At the subgame perfect equilibrium of the soft-budget game, we have

$$W = \delta u(G_2) + \bar{U}.$$

The ex post welfare increases with G_2. Note that even in the soft-budget case the subgame perfect equilibrium actually satisfies the survival condition because the LG behaves under condition (12) at the first stage.

An increase in k may raise A_2 both directly and indirectly as shown in (11-1) and (11-2). On the other hand, when k rises, it can increase the tax revenue of the CG. The overall impact of an increase in k on G_2 (or CG's net tax revenue, excluding A_2) is expressed by a marginal change in the right-hand side of (2). This term may be written as

$$R \equiv (1-\beta)f' - (A_k + A_S \frac{dS_2}{dk}) \tag{16}$$

The sign of this term seems ambiguous. However, we can show that this is actually negative.

Since (5-1) holds in the soft-budget game, an increase in G_2 corresponds to an increase in g_2. On the other hand, (15-1) means that g_2 actually declines due to rent-seeking activities. It follows that the equilibrium level of G_2 is also lower in the soft-budget game than in the hard-budget game. Hence, the resulting social welfare is lower in the soft-budget game as well. This undesirable outcome occurs because the soft budget stimulates rent seeking. A rise in k does not lead to an increase in either the overall tax revenues available to the CG or to the provision of G_2. Thus, because of the survival condition, an increase in k will not improve social welfare, but the amount of rent will rise.

Under the vertical externality due to overlapping tax bases, local public investment is too little in the hard-budget game. From this viewpoint the soft budget may improve welfare by stimulating inefficient local investment. Actually, we have shown that the first-best level of public investment is attained. Thus, it naturally follows that the soft budget could be welfare improving since public investment is too little in the hard-budget game.

However, once we incorporate rent-seeking activities, the soft-budget game is actually welfare deteriorating by depressing useful public goods provided by both the central and local governments. Since the utility from local public goods is fixed by the survival constraint of local politicians, an increase in public investment would not directly improve social welfare but would increase rent. Hence, the deteriorating effect becomes more powerful than the beneficial effect. We have shown that the welfare implications of the soft budget are dependent on not only the efficiency of local public investment but also the degree of rent seeking[6].

5. Comments
5.1 Benevolent LG

Suppose that the LG is also benevolent. Suppose also that the LG may choose its debt issuance D as well. Otherwise, the LG's choice of g_1, g_2 becomes meaningless when $S_1 = S_2 = 0$ and the tax rate is exogenously given. The following budget constraint, rather than the survival constraint (6), is now binding on the LG:

$$g_1 + \frac{g_2}{1+r} + k = \beta Y_1 + \frac{\beta Y_2}{1+r} + A_1 \qquad (17)$$

Then, the LG maximizes the social welfare (1) subject to the budget constraint (17) by choosing local public goods, local debt issuance, and investment, while assuming that nationwide public goods and grants from the CG are fixed. The first-order conditions of the LG at the second stage of the hard-budget game are as follows:

$$\frac{f'(k)\beta}{1+r} - 1 = 0 \tag{7}$$

$$\frac{v_{g1}}{v_{g2}} = (1+r)\delta \tag{5-2}$$

Since the LG does not internalize the spillover effect of public investment on the CG's tax revenue, k is still too little. This result is the same as in section 3.1.

The welfare implications of a rent-seeking LG are different from those of a benevolent LG. The latter does not conduct any rent-seeking activity, and the equilibrium levels of g_1, g_2 are higher than in the rent-seeking case. The utility from local public goods, $v(g_1) + \delta v(g_2)$, is also higher than the reservation utility in the rent-seeking case, $\overline{U}$. Also, note that an increase in A_1 has a positive income effect on g_1, g_2; that is, g_1, g_2 are increasing functions of A_1:

$$g_1 = g_1(A_1) \tag{18-1}$$

$$g_2 = g_2(A_1) \tag{18-2}$$

At the first stage of the hard-budget game, the CG maximizes social welfare by choosing A_1 (and hence G_2). In contrast to the rent-seeking case, the utility from local public goods, $v(g_1) + \delta v(g_2)$, may be affected here by the CG's choice of A_1. In other words, the CG now maximizes

$$W = \delta u((1-\beta)Y_2 - (1+r)A_1) + v(g_1(A_1)) + \delta v(g_2(A_1))$$

by choosing A_1. It is easy to show that the first-order condition reduces to

$$u_{G2} = v_{g2} \tag{5-1}$$

The optimal level of A_1 is determined to satisfy (5-1), which is generally positive if β is low.

Since the optimality condition (5-1) is attained as the outcome of the hard-budget game, this is time consistent. It is optimal for the CG to commit itself to the level of $A_2 = 0$ after the hard-budget game is over. The CG does not have an incentive to raise A_2 ex post. The soft-budget game is not relevant. The first-best level of public investment cannot be attained in the case of a benevolent LG since the CG does not have an incentive to make an additional grant to the LG.

5.2 Spillover Effect of Local Public Investment

The vertical externality is a key factor to obtain the result that local public investment is too low in the hard-budget constraint. If we incorporate the spillover effect of local public investment in a framework of multi-local governments, we could get a similar result regarding the level of public investment. That is, local

public investment becomes too low in a system of multi-local governments where each local government determines its public investment non-cooperatively. Generally, the main result in section 4 would not hold in such a case.

However, we might obtain a similar result if the CG internalizes the spillover effect. For example, suppose the CG now intends to redistribute resources among LGs. The spillover effect could be internalized by the CG's redistribution policy. Then, we could obtain a similar analytical result even if we incorporate the spillover effect of local public investment in a system of multi-local governments.

5.3 Inclusion of Private Consumption

Suppose we explicitly incorporate private consumption into the model. Now, c_1, c_2 represent private consumption in periods 1 and 2, respectively, t is the tax rate, and y_i is the income. We may define tax revenue as $ty_i = Y_i$. (i = 1, 2). For simplicity, we assume that c_1 is exogenously fixed, similar to G_1. We do not consider private savings, either. The budget constraint of the household in period 2 is given as

$$c_2 = (1-t)y_2 \tag{19}$$

In the hard-budget game, the second-stage outcome is the same as in section 3 since the rent-seeking LG behaves in the same way. Suppose now that the CG may choose the tax rate t optimally. At the first stage, the CG intends to maximize

$$\delta U(c_2) + \delta u(G_2)$$

by choosing t and A_1 subject to (19), $A_2 = 0$, and

$$G_2 = (1-\beta)ty_2 + A_1(1+r) \tag{2'}$$

The optimality conditions are

$$U_{c2} = (1-\beta)u_{G2} \tag{20}$$

and A_1 = 0. The tax rate t is set so as to equate $(1-\beta)$ times the marginal utility of public goods $\{(1-\beta)u_{G2}\}$ and the marginal cost of public goods $\{U_{c2}\}$. With the soft-budget outcome, the analytical result is qualitatively the same as in section 4. The CG has an incentive to raise A_2 to attain (5-1). However, it does not have an incentive to change the tax rate because (20) is also the ex post optimality condition in the soft-budget game.

The welfare implications will change to some extent. Since Y_2 is an increasing function of k, y_2 also increases with k at a given tax rate. Thus, the utility from private consumption increases with k as well. In section 4 we have shown that since G_2 decreases because of the soft-budget constraint, its overall

welfare effect is negative in the model without private consumption. On the contrary, here, if an increase in k raises private consumption, this increase is welfare improving. This is an additional positive effect. Hence, the welfare implication becomes ambiguous. That is to say, the soft-budget game is welfare deteriorating so long as the welfare-improving effect due to an increase in private consumption is not strong enough. In other words, the main analytical result would almost hold when private consumption is explicitly incorporated.

So far, we have assumed that the survival utility, $\bar{U}$, is fixed. When k rises, it increases the disposal income of voters. This might well stimulate their political effort to select a less rent-seeking politician. Then, $\bar{U}$ may increase with k, which is beneficial for the voters. If we allow for this possibility, the soft-budget outcome will not necessarily be deteriorating as in section 4.

6. Concluding Remarks

In this paper, we have theoretically investigated the hard-budget and soft-budget outcomes with grants from the central government to the local government by clarifying the vertical externality of local expenditures due to the overlapping tax bases between the two governments, using a simple two-period model. We have explicitly incorporated rent-seeking activities by local politicians or local governments to explore both the benefits and costs of the soft-budget constraint.

We called the committed version of $A_2 = 0$ the hard-budget game and the uncommitted version of $A_2 > 0$ the soft-budget game, respectively. When the central government, as the leader of the intergovernmental game, commits itself to a predetermined value of $A_2 = 0$, the local government is subject to the hard-budget constraint. In this game the CG chooses $A_1 = 0$, although the outcome is not the first best, because of several factors such as the predetermined value of β, the rent-seeking activity of the LG, and the overlapping taxes. Public investment is too low because of the vertical externality. In the hard-budget game, the CG may depress the LG's rent-seeking behavior to some extent by setting $A_1 = 0$. This is the merit of the hard-budget game, although the game is not time consistent.

In the standard-type model, the soft-budget constraint comes from information asymmetry or cost sharing. On the contrary, we have considered a very simple two-period model without uncertainty or information asymmetry. It has been shown that rent seeking is crucial to the soft-budget problem considered here. The CG could raise the ex post social welfare by increasing A_2 after the LG's rent-seeking behavior is already over.

In the soft-budget game, the local government has an incentive to raise local public investment and rent-seeking activities. First, more rent seeking results in a decline in local public goods in period 2 and hence upsets the central government's optimal allocation strategy. Second, more public investment means a higher revenue for the CG, which may therefore subsidize the local government more. These are new lines of thought explored in this paper.

The central government intends to provide more local public goods by raising grants ex post. We have shown that the soft budget works to internalize the vertical externality, so the first-best level of public investment is attained. However, the equilibrium level of local public goods in period 2 is actually lower than it is in the hard-budget game, harming social welfare. So long as the reservation utility is fixed by the survival constraint of local politicians, the local politician may obtain more rent by raising g_1 and reducing g_2. An increase in public investment will not improve social welfare directly, but it will increase rent. Hence, the deteriorating effect becomes more powerful than the beneficial effect.

With the vertical externality due to overlapping tax bases, local public investment is too little in the hard-budget game. One could argue that the soft budget may be welfare improving because the first-best level of public investment is attained. Nevertheless, the soft-budget game has been shown to be actually welfare deteriorating by depressing useful public goods provided by both the central and local governments.

We make no claim that the soft-budget outcome is always deteriorating. If the survival utility increases with a larger *k*, social welfare could be higher in the soft-budget game despite more rent seeking. We have shown that the welfare implications of the soft budget are dependent on the degree of rent seeking. Our model is admittedly highly stylized and abstracts from several possible directions of generalization such as asymmetric information between the two governments. Nevertheless, this paper has explored the possibility that the normative implications of the soft budget are dependent not only on whether public investment is too little or too much but also on the degree of rent seeking.

Appendix 1: Multiple Local Governments

Suppose there are n (≥ 2) local governments. If we define the total amount of local public goods as g_1, g_2 and each local government's supply of public goods as g_1^i, g_2^i, then we have

$$g_1 = \sum_{i=1}^{n} g_1^i, \quad g_2 = \sum_{i=1}^{n} g_2^i \tag{A1}$$

We may define other variables of local governments as in (A1). Then the budget constraints of the CG and the LG are the same as in the text. For simplicity, suppose all local governments are identical. The social welfare (1) can now be rewritten as

$$W^i = v(g_1^i) + \delta\{u(G_2) + v(g_2^i)\} \tag{A2}$$

where W^i is the social welfare in the representative agent in region i. Then, the social welfare is given as

$$W = \sum_{i=1}^{n} W^i$$

It follows that as in section 2.2 the first-best conditions are given by (5-2), (5-3), (5-4), and

$$nu_{G2} = v_{g2} \tag{A3}$$

Equation (A3) corresponds to the well-known Samuelson condition of the pure public good, G. Analytically, we have the same results as in sections 3 and 4.

For example, suppose in the soft-budget game the CG chooses A_2^i at a given level of A_2^j ($i \neq j$), then the ex post optimality condition would be

$$u_{G2} = v_{g2}^i$$

which is the same as in the text but differs from the first-best condition (A3). Then, regarding the game between the CG and the LG, we may assume that each LG behaves non-cooperatively and regards other LGs' choice variables as given. Equation (7) may be rewritten as

$$G_2 + g_2^i + \sum_{j \neq i} g_2^j + S_2^i + \sum_{j \neq i} S_2^j + (1+r)D = Y_2^i + \sum_{j \neq i} Y_2^j \tag{A4}$$

Further, the central government's response functions in place of (10-1) and (10-2) are given as

$$A_2^i = A^i(S_2^i, k^i) \tag{A5-1}$$

$$g_2^i = g^i(S_2^i, k^i) \tag{A5-2}$$

Similarly, we have

$$dG_2 = (1-\beta)f'(k^i)dk^i - dA_2^i$$

$$dG_2 + dg_2^i + dS_2^i = f'(k^i)dk^i$$

$$(1-\eta)dG_2 = \eta dg_2^i$$

Hence, we have (11-1), (11-2), (11-3), and (11-4) as in the text.

Intuitively, this is because an increase in k^i has a spillover effect on the CG only. Since each LG's behavior does not have any spillover effects on other LGs,

the number of LGs would not make much of a difference as far as the game between each LG and the CG is concerned. Thus, the analytical result would qualitatively be the same as in the main text. That is, the first-best level of k is attained in the soft-budget game, but the resulting social welfare is lower due to rent-seeking activities.

Notes

1. It is now well recognized in the tax competition literature that such vertical externalities are likely to leave local taxes too high. This is because each local government unduly discounts the pressure on the central government's spending it creates by raising its own tax rate. See Keen and Kotsogiannis (2002), Keen (1998), and Wilson (1999) among others.

2. It is shown that rent seeking is crucial to the soft-budget problem considered here because the setting of benevolent local governments does not create time inconsistency.

3. As shown in Appendix 1, the analytical results would be qualitatively almost the same even if we consider non-cooperative behavior by multi-local governments.

4. Here, we assume the restricted scheme of local debt issuance, and the LG may not choose the optimal amount of D. Note that this assumption would not restrict the opportunity set of the LG since it may transfer resources effectively by choosing S_1 and S_2 appropriately.

5. A more general specification would assume that the probability the officials can remain employed is a function of the utility of local residents compared with the utility available elsewhere. See Wilson and Gordon (2003).

6. If we assume that D is optimally chosen by the LG but either S_1 or S_2 is exogenously given, we still obtain the same conditions, (6), (7), and (5-2).

7. These analytical results are relevant to Japan. In Japan, local expenditures and grants from the CG to the LG actually increased in the 1990s. The Japanese central government mainly increased borrowing at the Special Account of Allocation and Transfer Taxes ex post facto, in response to excessive local expenditures by local governments. Doi and Ihori (2009) point out that the soft-budget constraint produced wasteful public spending in the form of local public works.

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