CIRJE-F-714

Incentives in Hedge Funds

Hitoshi Matsushima
University of Tokyo

February 2010

Incentives in Hedge Funds[+]

Hitoshi Matsushima[*]

Faculty of Economics, University of Tokyo

February 4, 2010

Abstract

We investigate a game of delegated portfolio management such as hedge funds featuring risk-neutrality, hidden types, and hidden actions. We show that capital gain tax plays the decisive role in solving the incentive problem. We characterize the constrained optimal fee scheme and capital gain tax rate; the fee after taxation must be linear and affected by gains and losses in a low-powered and symmetric manner. We argue that high income tax incentivizes managers to select this scheme voluntarily. The equity stake suppresses the distortion caused by solvency.

Keywords: Hedge Funds, Hidden Types, Hidden Actions, Tax Policies, Permissive Results.

JEL Classification Numbers: C72, D82, D86, G23, G28.

[+] The research of this paper was supported by a Grant-In-Aid for Scientific Research (KAKENHI 21330043) from the Japan Society for the Promotion of Science (JSPS) and the Ministry of Education, Education, Culture, Sports, Science and Technology (MEXT) of the Japanese government. I appreciate all the participants in the seminars at the University of Shiga, the University of Tokyo, and the University of Kyoto for their helpful comments. All errors are mine.

[*] Faculty of Economics, University of Tokyo, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. E-mail: hitoshi at mark e.u-tokyo.ac.jp

1. Introduction

This paper investigates a game-theoretic model of delegated portfolio management represented by the hedge fund, where a risk-neutral investor delegates to a risk-neutral manager the management of a single unit of his fund for a single period. The purpose of this paper is to clarify whether there exists a manager's fee scheme that is well behaved. Such a scheme would satisfy several incentive constraints when there is informational asymmetry between the manager and the investor in terms of hidden types and hidden actions. The main contribution of this paper is to show that appropriate selection of the capital gain and income tax rates is critical to rendering these incentive constraints compatible with Pareto-improvement and to incentivizing the manager and the investor to select the constrained optimal fee scheme voluntarily.

The hedge fund manager is legally exempted from many regulations on activities such as short selling, derivatives, and leverage.[1] Hence, he is expected to generate alpha, i.e., the return to exceed those generated by the regulated investment funds, where he can make complete use of unregulated investment strategies that benefit from opaqueness. In order to generate such excess returns, he needs to have the special investment skills and needs to engage in costly activity.

In such situations, informational asymmetry in terms of hidden types and hidden actions is a real problem; it is difficult for the investor to ascertain whether the manager is a skilled or an unskilled type. It is also difficult for him to monitor whether the manager's activities are sound; since the manager's strategy is unregulated, opaque, and complicated, investors lacking expertise cannot understand it. In this case, the fee scheme must be contingent not on the manager's type and activity choice but on the generated return. Since the fee is contingent on returns, it must hold that the skilled manager is willing to enter the hedge fund industry, while the unskilled manager has no such incentives. Furthermore, the skilled manager requires strong incentives to engage in the costly activity that will generate an excess return.

As shown by the seminal work of Foster and Young (2009), the incentive problem

[1] For a general introduction to hedge funds, see Lo (2008).

addressed above is much harder to solve than we had perceived.[2] Foster and Young investigated the case of the government levying no capital gain tax; they showed the impossibility result that there exists no fee scheme that satisfies these incentive constraints. Whenever the skilled manager is willing to enter the hedge fund industry, then even the unskilled manager is willing to do the same.

The following are the two reasons why it is very difficult to solve the hedge fund incentive problem. First, the manager may manipulate the fund's return by entering into a side contract with a third party in order to increase his reward; as pointed out by Lo (2001), given the widespread practice of a two-part fee scheme involving a 2% management fee plus a 20% performance fee, the unskilled manager can significantly augment his earnings by selling high-risk covered options named capital decimation partners.

Second, contrary to the popular view in general contract theory literature,[3] the introduction of a penalty system to accompany the fee scheme is not necessarily effective in the case of hedge fund incentives; the penalty system disadvantages the skilled manager more so than the unskilled manager. In order to ensure the fund's solvency, the skilled manager has to place in escrow a part of his personal fund equal in value to the maximal units of the possible penalty. In this case, the skilled manager is prohibited from managing the units in a manner that would earn the excess return, which decreases his interest in entering the hedge fund industry.

In spite of these difficulties, the present paper will show that the impossibility result by Foster and Young crucially depends on the assumption that the government does not levy the capital gain tax; with a positive capital gain tax rate, the skilled manager has the incentive to reduce his tax burden by running the investor's fund at the expense of the success of his personal fund. In contrast, an unskilled manager has no such incentives because he has no investment skill to earn alpha that would become taxable. This difference between the skilled and unskilled manager is the driving force that makes the skilled manager's entry compatible with the unskilled manager's exit in the Pareto-improving manner.

We characterize the optimal fee scheme and capital gain tax rate that maximize the

---

[2] See also Foster and Young (2008a, 2008b).

[3] See Laffont and Martimort (2002) and Bolton and Dewatripont (2005), for instance.

social surplus within these incentive constraints. In this case, the fee after taxation must be linear and symmetric across the zero excess return, so that the manager shares not only gains but also losses with the investor. Previous research on the principal-agent models in fund management, such as Starks (1987), Ackermann, MacEnally, and Ravenscraft (1999), Carpenter (2000), and Hodder and Jackwerth (2006), as well as investors such as Warren Buffet, criticized the asymmetric treatment between gains and losses in the two-part fee scheme's current form of "2 and 20", because the manager is encouraged to take excessive risk that does not necessarily increase the expected return. They proposed an alternative model such as the fulcrum fee scheme, which treats gains and losses in a symmetric manner. These arguments are consistent with our characterization.

Our characterization implies that the constrained optimal fee scheme must be lower-powered than that most preferred by the skilled manager within these incentive constraints; he has the incentive to save his capital gain tax as much as possible, which inevitably increases the maximal amount of the possible penalty, i.e., the amount of his personal fund that has to be put in escrow, which is in turn detrimental to the social welfare.

Based on these observations, we argue that the introduction of income tax on the manager's fee, in addition to the capital gain tax, is crucial in incentivizing the manager to select the constrained optimal fee scheme voluntarily; as the income tax rate increases, the upper bound of the manager's fee after taxation decreases, which has the effect of taking away from him the option of an extremely high-powered fee scheme. In this case, the government must institute an income tax rate that is greater than the capital gain tax rate: such a practice is in contrast with current practice internationally, where the income tax on the manager's fee is kept low. Hence, this paper proposes to argue the theoretical ground for criticizing as unfair the existing system of levying a low income tax on the manager's fee.

This paper mostly assumes that the skilled manager can select investment activities for the investor's fund that differ from those for his own; under this assumption, it is inevitable that the manager's underperformance will attract the positive penalty, thus distorting the social welfare. In the final section of this paper, we eliminate this assumption and make an alternative assumption that the manager can be committed to combining his personal fund with the investor's fund as the equity stake and manage them as one. With some specializations of the model, we show that if the manager possesses a sufficiently large personal fund, it is possible to construct the Pareto-improving fee scheme that satisfies all the incentive constraints without imposing any welfare-distorting penalty. Because of this equity stake, the unskilled manager hesitates to mimic the high performance activity since the manager must use not only the investor's fund but also his personal fund as security when entering a third-party side contract. As a result, even the unskilled manager bears a capital gain tax burden, which makes entry into the hedge fund industry unattractive to him.

In response to Foster and Young, the media such as the Financial Times (2008) and The New York Times (2010) opined that the only way to protect the hedge fund industry is through greater transparency, even if it meant that hedge funds could no longer function as originally intended. This, however, is a rather precipitous judgment since we can solve the hedge fund problem by taxation without changing the fund's original function.

This paper warns against the trend that is tolerant of a manager's indiscriminately escaping tax liability. Excessive tax evasion activities by a manager should be seen as a sign that the manager is unskilled. Hence, the skilled manager should cease such activities if he wants to convince the investor that he is truly skilled.

This paper is organized as follows. Section 2 shows the model and defines the incentive constraints that are required. Section 3 shows that with no capital gain tax levied, it is impossible to implement the incentive constraints with the Pareto-improvement. Sections 4 and 5 constitute the main portion of this paper. In Section 4, we confine our attention to fee schemes that are linear and symmetric after taxation; we show several permissive results implying that the incentive problem may be solved through the beneficial effects of the capital gain tax rate. In Section 5, we take all fee schemes into account and show that the basic meanings of Section 4 are unchanged. In Section 6, we discuss some additional points, such as the investor's optimization, the presence of plural shareholders, and the role of entry cost. In Section 7, we investigate the effect of the equity stake on hedge fund incentives and show a permissive result without any penalty imposed.

2. The Model

This paper investigates a game-theoretic model of delegated portfolio management represented by the hedge fund, where a risk-neutral investor delegates to a risk-neutral fund manager the management of a single unit of his fund for a single period. This manager also possesses $M \in (0,\infty)$ units of his personal fund to manage by himself. A *fee scheme* to the manager is defined as a function $y:[-M,\infty) \to R$; when the management of the investor's fund yields the return $x \in [0,\infty)$, the manager refunds it to the investor and in return receives the fee $y(x) \in [-M,\infty)$ from this investor.[4] Here, the investor is regarded as a limited partner to whom the manager cannot refund a negative return. Let us denote by Y the set of all fee schemes.

We allow the manager's fee to be negative; the investor can impose a penalty for the manager's underperformance. Let us define the *maximal penalty* associated with a fee scheme $y \in Y$ by the following:

$$w(y) \equiv \max[0, \max_{x \in [0,\infty)}[-y(x)]],$$

where we assume that the manager's liability is limited to his personal fund, that is,

$$M \geq w(y).$$

This inequality, however, is not enough to guarantee that the manager is sufficiently solvent; he must put $w(y)$ units of his personal fund in escrow and be prohibited from managing these units unless they are invested in a safe asset, i.e., he manages them in the same manner as the regulated investment funds, where the excess return for the safe asset is assumed to be zero.[5]

The manager's type is either *skilled* or *unskilled*. We assume *hidden types* in that the investor cannot identify whether the manager is skilled or unskilled. A skilled manager can earn *alpha*, i.e., a positive excess return; he selects an action $a \in A \equiv [0,\infty)$

---

[4] This paper assumes for simplicity that the fee is paid to the manager in a lump sum at the expiration of the partnership contract. Foster and Young investigated the multi-period model where fee payments to the manager are made by installment; in this case, we need further incentive devices to prevent the manager from taking excessive risk such as postponing bonus payments, clawing back bonuses, and high-water marks.

[5] For incentives in general partnerships with limited liability, see Legros and Matsushima (1991), where the unmanageability needed for solvency was not explicitly investigated.

for each unit of the manageable funds that induces the return $a+1$. It is implicitly assumed that in order to earn the excess return a, the skilled manager carries out a dynamic investment strategy at the expense of his non-pecuniary cost $c(a) \geq 0$. In addition to hidden types, we assume *hidden actions* in that the investor cannot monitor the manager's action choice. Hence, a fee scheme cannot be made contingent on his action choice and type. The cost function per unit, given by $c: A \to R_{+} \cup \{0\}$, is twice-differentiable and convex, where $c''(a) > 0$, $c(0) = 0$, $c'(0) = 0$, and $\lim_{a \uparrow \infty} c'(a) > 1$.

Let us call $a = 0$ the *null* action; the manager invests in the safe asset. In contrast to the skilled manager, an unskilled manager has no investment skill to earn alpha; he cannot select any action other than the null action. Any manager, either skilled or unskilled, is prohibited by law from diverting the investor's fund to personal use.

The government levies a *capital gain tax* on the excess return at a rate $t \in [0,1]$. The government also levies an *income tax* on the manager's fee at a rate $\tau \in [0,1]$.

2.1. Skilled Manager's Entry

Throughout this paper, except for Section 7, we assume that the skilled manager can select actions for the investor's fund that differ from those he selects for his personal fund. When a skilled manager selects the actions $a \in A$ for the investor's fund and $a' \in A$ for his manageable personal fund $M - w(y)$, he has to pay the income tax on the manager's fee given by

$$\tau \max[y(a+1), 0],$$

and also pay the capital gain tax given by

$$t\{M - w(y)\}a'.$$

Hence, the resulting payoff for the skilled manager should be given by

$$\min[(1-\tau)y(a+1), y(a+1)] - c(a) + \{M - w(y)\}\{(1-t)a' - c(a')\}.$$

The skilled manager selects the actions $a \in A$ and $a' \in A$ so as to maximize this value. Hence, his payoff associated with any combination of the fee scheme and tax rates (y, t, τ) should be defined as

$$V(y,t,\tau) \equiv \max_{a \in A}\{\min[(1-\tau)y(a+1), y(a+1)] - c(a)\}$$
$$+\{M - w(y)\}\max_{a' \in A}\{(1-t)a' - c(a')\}.$$

For each $r \in [0,\infty)$, we denote by $\tilde{a}(r) \in A$ the action that maximizes $ra - c(a)$ with respect to $a \in A$, where $r = c'(\tilde{a}(r))$. We assume that $\tilde{a}(r) - c(\tilde{a}(r))$ is *concave* with respect to $r \in [0,\infty)$. We denote by $a^*(y,\tau) \in A$ the action that maximizes $\min[(1-\tau)y(a+1), y(a+1)] - c(a)$ with respect to $a \in A$. It is clear that the skilled manager is willing to select $a = a^*(y,\tau)$ and $a' = \tilde{a}(1-t)$:

$$V(y,t,\tau) = \min[(1-\tau)y(a^*(y,\tau)+1), y(a^*(y,\tau)+1)] - c(a^*(y,\tau))$$
$$+\{M - w(y)\}\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}.$$

By exiting from the hedge fund industry, the skilled manager can manage his entire personal fund M. Hence, his outside opportunity should be given by

$$\bar{V}(t) \equiv M \max_{a \in A}\{(1-t)a - c(a)\} = M\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}.$$

The following condition requires any skilled manager to enter the hedge fund industry.

Condition 1 (Skilled Manager's Entry): The skilled manager's payoff is not less than his outside opportunity, i.e.,

$$V(y,t,\tau) \geq \bar{V}(t),$$

which is equivalent to

(1) $$\min[(1-\tau)y(a^*(y,\tau)+1), y(a^*(y,\tau)+1)] - c(a^*(y,\tau))$$
$$\geq w(y)\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}.$$

2.2. Unskilled Manager's Exit

Any unskilled manager, after entering the hedge fund industry, can enter into a *side contract* with a risk neutral third party, which is defined as a right-continuous cumulative distribution $F : [0,\infty) \to [0,1]$; we assume that

(2) $$E[z \mid F] = 1,$$

and that there exists the upper bound $\bar{z} > 0$ of the support such that $F(z) = 1$ if and only if $z \geq \bar{z}$, where $E[\cdot \mid F]$ denotes the expectation operator. The unskilled manager gives the investors' fund to the third party and in turn receives a monetary payment $z \in [0,\infty)$ that is randomly determined according to F from this third party. We denote by Φ the set of all such side contracts.

The unskilled manager selects $F \in \Phi$ so as to maximize his expected fee; his payoff should be defined as

$$\max_{F \in \Phi} E[\min[(1-\tau)y(z), y(z)] \mid F].$$

Since he has no management skill, his outside opportunity should be equal to zero. Because of hidden types, the following condition is required, which guarantees that any unskilled manager will exit the hedge fund industry.

Condition 2 (Unskilled Manager's Exit): The unskilled manager's pay-off is not greater than his outside opportunity, i.e.,

(3) $$\max_{F \in \Phi} E[\min[(1-\tau)y(z), y(z)] \mid F] \leq 0.$$

For instance, for each fee scheme $y \in Y$ and each income tax rate $\tau \in [0,1]$, let us specify a side contract $F^{y,\tau} \in \Phi$ by

$$F^{y,\tau}(z) = \frac{a^*(y,\tau)}{a^*(y,\tau)+1} \quad \text{for all} \quad z \in [0, a^*(y,\tau)+1),$$

and

$$F^{y,\tau}(z) = 1 \quad \text{for all} \quad z \in [a^*(y,\tau)+1, \infty).$$

According to $F^{y,\tau}$, with the probability of $\dfrac{1}{a^*(y,\tau)+1}$, the unskilled manager receives the same return from the third party as the return $a^*(y,\tau)+1$ that the skilled manager can earn, i.e., he can mimic the skilled manager's performance. Under Condition 2, the unskilled manager has no incentive to mimic the skilled manager's performance in this manner; the unskilled manager's expected pay-off induced by $F^{y,\tau}$ should not be greater than his zero outside opportunity:

$$E[\min[(1-\tau)y(z), y(z)] \mid F^{y,\tau}]$$

$$= \frac{1}{a^*(y,\tau)+1}\min[(1-z)y(a^*(y,\tau)+1), y(a^*(y,\tau)+1)]$$
$$+\frac{a^*(y,\tau)}{a^*(y,\tau)+1}\min[(1-\tau)y(0), y(0)]$$
$$\leq 0,$$

which implies that

(4) $$(1-\tau)y(a^*(y,\tau)+1) \leq w(y)a^*(y,\tau).$$

The unskilled manager can implement any side contract $F \in \Phi$ by selling the following covered option according to the concept of *capital decimation partners* addressed by Lo (2001); we suppose that there exists a random variable $\tilde{\gamma}$ such as the S&P 500 stock index, the realization of which is determined according to a continuous and increasing cumulative distribution G on $[\underline{\gamma},\overline{\gamma}]$, where $\underline{\gamma} < \overline{\gamma}$. Let us denote by $g:[\underline{\gamma},\overline{\gamma}] \to [0,\overline{z}]$ the right-continuous and non-decreasing function defined as

$$\lim_{z \uparrow g(\gamma)} F(z) \leq G(\gamma) \leq F(g(\gamma)) \quad \text{for all} \quad \gamma \in (\underline{\gamma},\overline{\gamma}].$$

Based on G and g, the unskilled manager sells a covered option to the third-party arbitrageur in the following manner: he puts the investor's fund in escrow, and then transfers $\frac{\overline{z}-g(\gamma)}{\overline{z}}$ units from the escrow account to the buyer of this option, provided that $\tilde{\gamma} = \gamma$ is realized. The price of this option equals the expected value of his transfer payment:

$$\frac{\overline{z}-E[g(\gamma)\,|\,G]}{\overline{z}} = \frac{\overline{z}-E[z\,|\,F]}{\overline{z}} = \frac{\overline{z}-1}{\overline{z}}.$$

By putting the earning from this option sale $\frac{\overline{z}-1}{\overline{z}}$ in escrow again, the manager can re-sell the covered option; by repeating this step indefinitely, the unskilled manger can make the total size of sold options equal to

$$1+\frac{\overline{z}-1}{\overline{z}}+\left(\frac{\overline{z}-1}{\overline{z}}\right)^2+\left(\frac{\overline{z}-1}{\overline{z}}\right)^3+\cdots=\overline{z}.$$

In this case, the manager's net earning from his option sale is equal to $\frac{\overline{z}-1}{\overline{z}}\times\overline{z}=\overline{z}-1$, and the total units that he gives to the third party are equal to $\frac{\bar{z}-z}{\bar{z}} \times \bar{z} = \bar{z} - z$. Hence, the return yielded by his entire option sales equals

$$\bar{z} - 1 + 1 - (\bar{z} - z) = z .$$

2.3. Investor's Entry

We assume that the manager's fee is deducted from the investor's taxed capital gain; the investor pays the capital gain tax given by

$$t \max[a^*(y,\tau) - y(a^*(y,\tau)+1), 0] .$$

His payoff should be defined as

$$U(y,t,\tau) \equiv \min[(1-t)\{a^*(y,\tau) - y(a^*(y,\tau)+1)\}, a^*(y,\tau) - y(a^*(y,\tau)+1)] .$$

Since the investor cannot earn any capital gain by himself, his outside opportunity should be equal to zero. The following condition requires the investor to enter the hedge fund industry, provided the manager is, as he expects, skilled.

Condition 3 (Investor's Entry): The investor's payoff is not less than his outside opportunity, i.e.,

$$U(y,t,\tau) \geq 0 ,$$

which is equivalent to

(5) $$a^*(y,\tau) \geq y(a^*(y,\tau)+1) .$$

2.4. Welfare Improvement

Associated with any combination of the fee scheme and tax rates (y,t,τ), we define the *social surplus* $S(y,t,\tau)$ as the summation of the skilled manager's payoff, the investor's payoff, and the government's tax revenue, i.e., the summation of the skilled manager's payoff before taxation and the investor's payoff before taxation:

$$S(y,t,\tau) \equiv a^*(y,\tau) - c(a^*(y,\tau)) + \{M - w(y)\}\{\tilde{a}(1-t) - c(\tilde{a}(1-t))\} .$$

For each capital gain tax rate $t \in [0,1]$, let us define the *status quo* social surplus as

$$\bar{S}(t) \equiv M\{\tilde{a}(1-t) - c(\tilde{a}(1-t))\},$$

where we assume that the hedge fund industry does not exist in this status quo and the manager manages his entire personal fund M. Note that $\bar{S}(t)$ is decreasing with respect to $t \in [0,1]$. The following condition requires the hedge fund industry to be associated with a fee scheme y that would improve the social surplus more than the status quo social surplus associated with the zero rate of capital gain tax.

Condition 4 (Welfare Improvement): The social surplus associated with (y,t,τ) is greater than the status quo social surplus associated with the zero rate of capital gain tax, i.e.,

$$S(y,t,\tau) > \bar{S}(0),$$

which is equivalent to

(6) $$a^*(y,\tau) - c(a^*(y,\tau)) + \{M - w(y)\}\{\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$
$$> M\{\tilde{a}(1) - c(\tilde{a}(1))\}.$$

It is clear that if y satisfies Condition 4, it must hold that $w(y) < 1$.

2.5. Skilled Manager's Non-mimicry

Not only an unskilled manager but also a skilled manager can enter into a side contract with a third party; for every $a \in A$, let us denote by $\Phi(a)$ the set of all possible side contracts defined as a right-continuous cumulative distribution $F:[0,\infty) \to [0,1]$, where we assume that $E[z \mid F] = a+1$ and that there exists the upper-bound of its support. By selecting an action $a \in A$ and signing a side contract $F \in \Phi(a)$, the skilled manager can receive the payoff given by

$$E[\min[(1-\tau)y(z), y(z)] \mid F] - c(a)$$
$$+\{M - w(y)\}\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}.$$

Given that the skilled manager selects an action $a \in A$, he can implement any side contract $F \in \Phi(a)$ by selling covered options to the third party after the return $a+1$

is realized and before this return is refunded to the investor in the same manner as the unskilled manager did in Subsection 2.2. The following condition requires that any skilled manager have no incentive to enter into any such side contract, i.e., no incentive to mimic higher performance than his own.

Condition 5 (Skilled Manager's Non-mimicry): There exist no $\boldsymbol{a} \in \boldsymbol{A}$ and no $\boldsymbol{F} \in \Phi(\boldsymbol{a})$ such that

(7) $$\boldsymbol{V(y,t,\tau) < E[\min[(1-\tau)y(z), y(z)] \mid F] - c(a)}$$
$$\boldsymbol{+\{M - w(y)\}\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}}$$

3. No Capital Gain Tax

Since the maximal penalty must be put in escrow, the skilled manager loses the opportunity to earn alpha by managing the corresponding part of his personal fund. Hence, the presence of positive maximal penalty strengthens the constraint of the skilled manager's entry, while keeping the constraint of the unskilled manager's exit unchanged since the unskilled manager has no investment skill to earn alpha. On the basis of this difference between the skilled and unskilled types, we can show that when no capital gain tax is levied, it is impossible to make the skilled manager's entry compatible with the unskilled manager's exit in the Pareto-improving manner.

Theorem 1: *If* (y, t, τ) *satisfies Conditions 1, 2, and 4, then*

$$t > 0.$$

Proof: Let us suppose that $t = 0$. Suppose that a fee scheme $y \in Y$ and an income tax rate $\tau \in [0,1]$ satisfy Conditions 1, 2, and 4. The minimal penalty must be positive in this case, i.e., $w(y) > 0$. From Condition 4 and $t = 0$, it follows that

(8) $$0 < w(y) \leq 1 \text{ and } a^*(y,\tau) > 0.$$

From Condition 1 and $y(a^*(y,\tau)+1) \geq 0$, it follows that

(9) $$(1-\tau)y(a^*(y,\tau)+1) - c(a^*(y,\tau)) \geq w(y)\{\tilde{a}(1) - c(\tilde{a}(1))\}.$$

Because of Condition 2, an unskilled manager has no incentive to mimic the skilled manager's performance; inequality (4) must hold, i.e.,

$$(1-\tau)y(a^*(y,\tau)+1) \leq w(y)a^*(y,\tau),$$

which along with inequality (9), implies that

$$w(y)a^*(y,\tau) \geq c(a^*(y,\tau)) + w(y)\{\tilde{a}(1) - c(\tilde{a}(1))\}.$$

This inequality, along with inequality (8), implies that

$$a^*(y,\tau) - c(a^*(y,\tau)) > \tilde{a}(1) - c(\tilde{a}(1)),$$

which, however, contradicts the fact that $a = \tilde{a}(1)$ maximizes $a - c(a)$.

Q.E.D.

4. Fee Schemes with Linearity After Taxation

This section considers the case that the capital gain tax is positive. The positive capital gain tax rate plays the decisive role in making the skilled manager's entry compatible with the unskilled manager's exit. As the capital gain tax rate increases, the skilled manager's outside opportunity decreases; any skilled manager has the incentive to reduce his capital gain tax burden by running the investor's fund at the expense of keeping a part of his personal fund unmanageable. On the other hand, the unskilled manager's outside opportunity is kept unchanged; the unskilled manager has no such incentives, because he has no investment skill to earn alpha that becomes taxable.

In this section, we shall confine our attention to the subsets of fee schemes denoted by $Y^*(\tau) \subset Y$, which will be defined afterward. As shown by the next section, this confinement was irrelevant to the results of this paper, i.e., it was made just for the convenience of our argument.

For every $\tau \in [0,1]$, let us denote by $Y^*(\tau) \subset Y$ the set of fee schemes y such that there exists $k \in [0, M]$ which satisfies

$$\min[(1-\tau)y(x), y(x)] = k(x-1) \text{ for all } x \in [0,\infty),$$

that is, the fee after taxation is linear with respect to the return. Note that

$$y(x) = \frac{k}{1-\tau}(x-1) \text{ for all } x \in [1,\infty),$$

and

$$y(x) = k(x-1) \text{ for all } x \in [0,1).$$

Let us denote by $y = y^{k,\tau}$ such a fee scheme. Note that

$$a^*(y^{k,\tau},\tau) = \tilde{a}(k) \text{ and } w(y^{k,\tau}) = k.$$

Any fee scheme $y^{k,\tau} \in Y^*(\tau)$ satisfies the following properties.

(i) Condition 1 is equivalent to

$$k\tilde{a}(k) - c(\tilde{a}(k)) \geq k\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}.$$

(ii) Conditions 2 and 5 *automatically* hold.

(iii) Condition 3 is equivalent to

$$k \leq 1-\tau.$$

The property (ii) implies that any fee scheme that belongs to $Y^*(\tau)$ can prevent a skilled or unskilled manager from mimicking higher performance than what he actually does.

4.1. Existence and Pareto-Improvement

Note that for every $k \in (0,1]$, there *uniquely* exists a capital gain tax rate $t = \hat{t}(k) \in [0,1)$ that satisfies inequality (1) in Condition 1 with an equality, i.e.,

(10) $$k\tilde{a}(k) - c(\tilde{a}(k)) = k\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\},$$

which implies that the skilled manager's payoff equals his outside opportunity, i.e.,

$$V(y^{k,\tau}, t, \tau) = \bar{V}(t).$$

Note that $\hat{t}(k)$ is decreasing, $\hat{t}(1) = 0$, $\lim_{k \downarrow 0} \hat{t}(k) = 1$, and

(11) $$\hat{t}(k) > 1 - k \text{ for all } k \in (0,1).$$

Any fee scheme $y^{k,\tau} \in Y^*(\tau)$ satisfies Conditions 1, 2, 3, and 5 if and only if

$$\hat{t}(1-\tau) \le t \text{ and } k \le 1-\tau.$$

The social surplus $S(y^{k,\tau}, t, \tau)$ decreases as the capital gain tax rate $t \in [0,1]$ increases, because any skilled manager becomes less incentivized to manage his personal fund efficiently. Hence, the government can improve welfare by making $t \in [0,1]$ as small as possible within the constraints of Conditions 1, 2, and 3. Since the capital gain tax rate t is only relevant to Condition 1, the government should select it so as to satisfy the equality (10), i.e., to make Condition 1 binding. The following theorem shows that there exists (y, t, τ), which makes the skilled manager's entry compatible with the unskilled manager's exit in the Pareto-improving manner.

Theorem 2: *There exist* $(t, \tau) \in [0,1]^2$ *and* $y \in Y^*(\tau)$ *that satisfy Conditions 1, 2, 3, 4, and 5.*

Proof: From the equality (10) and the definition of $\hat{t}(k)$, it follows that

(12) $$\left.\frac{d\hat{t}(k)}{dk}\right|_{k=1} = -\frac{c(\tilde{a}(1))}{\tilde{a}(1)}.$$

Let us assume that $\tau = 0$. Suppose that $M \geq 1$. From the definition of $S(y,t,\tau)$ and $\hat{t}(1) = 0$,

$$S(y^{1,0}, \hat{t}(1), 0) = \bar{S}(0).$$

From the equality (12),

$$\left.\frac{dS(y^{k,0}, \hat{t}(k), 0)}{dk}\right|_{k=1} = \left.\frac{\partial S(y^{k,0}, 0, 0)}{\partial k}\right|_{k=1} + (\left.\frac{d\hat{t}(k)}{dk}\right|_{k=1})(\left.\frac{\partial S(y^{1,0}, t, 0)}{\partial t}\right|_{t=0})$$

$$= -\{\tilde{a}(1) - c(\tilde{a}(1))\} < 0.$$

Hence, by selecting k less than but close to unity, we can make the social surplus greater than the status quo surplus associated with the zero rate of capital gain tax, i.e.,

$$S(y^{k,0}, \hat{t}(k), 0) > \bar{S}(0),$$

which implies Condition 4. It is clear that $(y,t,\tau) = (y^{k,0}, \hat{t}(k), 0)$ satisfies Conditions 1, 2, 3, and 5.

Suppose that $M < 1$. Then,

$$S(y^{M,0}, 1, 0) = \tilde{a}(M) - c(\tilde{a}(M)) > M\{\tilde{a}(1) - c(\tilde{a}(1))\} = \bar{S}(0),$$

the inequality of which was derived from the assumption that $\tilde{a}(k) - c(\tilde{a}(k))$ is concave. Hence, $(y,t,\tau) = (y^{M,0}, 1, 0)$ satisfies Condition 4. It is clear that $(y,t,\tau) = (y^{M,0}, 1, 0)$ satisfies Conditions 1, 2, 3, and 5.

Q.E.D.

4.2. Welfare Optimization

Let us define $k^* = k^*(M) \in [0, \min[M,1]]$ as maximizing

(13) $$\tilde{a}(k) - c(\tilde{a}(k)) - (M-k)\{\tilde{a}(1-\hat{t}(k)) - c(\tilde{a}(1-\hat{t}(k)))\}$$

with respect to $k \in [0, \min[M,1]]$. Let us define

$$t^* = t^*(M) \equiv \hat{t}(k^*(M)).$$

Note that the value (13) is equivalent to $S(y^{k,\tau}, \hat{t}(k), \tau)$. Hence, $(y^{k^*,\tau}, t^*, \tau)$

maximizes the social surplus $S(y^{k,\tau},t,\tau)$ with respect to $(k,t,\tau)\in[0,M]\times[0,1]^2$ under the constraints of Conditions 1 and 3, where $k^*\le 1-\tau$ was satisfied. Since $(y^{k^*,\tau},t^*,\tau)$ clearly satisfies Conditions 1, 2, 3, 4, and 5, we have proven the following theorem.

Theorem 3: *For every* $\tau\in[0,1-k^*]$, *the combination of the fee scheme and tax rates* $(y^{k^*,\tau},t^*,\tau)$ *satisfies Conditions 1, 2, 3, 4, and 5, and*

$$V(y^{k^*,\tau},t^*,\tau)=\bar{V}(t^*).$$

Moreover, there exists no $(k,t,\tau')\in[0,M]\times[0,1]^2$ *that satisfies Conditions 1, 2, and 3, and*

$$S(y^{k,\tau'},t,\tau')>S(y^{k^*,\tau},t^*,\tau).$$

4.3. Skilled Manager's Optimization

Any skilled manager prefers the largest possible maximal penalty $\min[M,1-\tau]$; he is willing to select the fee scheme $y^{\min[M,1-\tau],\tau}$ instead of $y^{k^*,\tau}$ so as to reduce his capital gain tax burden as much as possible. The following theorem shows that this skilled manager's selection may make the welfare even worse than status quo.

Theorem 4: For every $(t,\tau)\in[0,1]^2$, *there exists no fee scheme* $y\in Y^*(\tau)$ *that satisfies Conditions 1, 2, 3, and*

$$V(y,t,\tau)>V(y^{\min[M,1-\tau],\tau},t,\tau).$$

Suppose that $\hat{t}(1-\tau)\le t$. *Then, the fee scheme* $y^{\min[M,1-\tau],\tau}$ *satisfies Condition 4 if*

$$M\le 1-\tau.$$

If $M>1$ *and* $\tau=0$, *then it does not satisfy Condition 4 in the strict sense, i.e.,*

$$\bar{S}(0)>S(y^{\min[M,1],0},t,0).$$

Proof: The first part of this theorem holds directly from the fact that $V(y^{k,\tau},t,\tau)$ is increasing with respect to $\boldsymbol{k \in [0, \min[M, 1-\tau]]}$. Suppose that $\boldsymbol{M \le 1-\tau}$. Then, from the concavity of $\boldsymbol{\tilde{a}(r) - c(\tilde{a}(r))}$, it follows that

$$\boldsymbol{S(y^{\min[M,1-\tau],\tau}, t, \tau) = \tilde{a}(1-\tau) - c(\tilde{a}(1-\tau)) \ge (1-\tau)\{\tilde{a}(1) - c(\tilde{a}(1))\}}$$

$$\boldsymbol{\ge M\{\tilde{a}(1) - c(\tilde{a}(1))\} = \bar{S}(0)},$$

which implies Condition 4.

Suppose that $\boldsymbol{M > 1}$, $\boldsymbol{t > 0}$, and $\boldsymbol{\tau = 0}$. Note that

$$\boldsymbol{S(y^{\min[M,1],0}, t, 0) = \tilde{a}(1) - c(\tilde{a}(1)) + (M-1)\{\tilde{a}(1-t) - c(\tilde{a}(1-t))\}}.$$

Since $\boldsymbol{\tilde{a}(r) - c(\tilde{a}(r))}$ is increasing with respect to $\boldsymbol{r \in [0,1]}$, it follows that

$$\boldsymbol{S(y^{\min[M,1],0}, t, 0) < \tilde{a}(1) - c(\tilde{a}(1)) + (M-1)\{\tilde{a}(1) - c(\tilde{a}(1))\} = \bar{S}(0)}.$$

Q.E.D.

4.4. Role of Income Tax

From Theorem 4, it follows that in order to move the skilled manager to select voluntarily the constrained optimal fee scheme, the government should set the income tax rate $\boldsymbol{\tau}$ so as to make $\boldsymbol{y^{\min[M,1-\tau],\tau}}$ equivalent to $\boldsymbol{y^{k^*,\tau}}$; let us denote the minimal income tax rate that is compatible with the investor's entry by the following:

$$\boldsymbol{\tau^* = \tau^*(M) \equiv 1 - k^*(M)}.$$

Clearly, $\boldsymbol{y^{k^*,\tau^*}}$ is the only fee scheme in $\boldsymbol{Y^*(\tau^*)}$ that satisfies Conditions 1, 2, 3, and 5, which implies that

$$\boldsymbol{y^{\min[M,1-\tau^*],\tau^*} = y^{k^*,\tau^*}}.$$

Hence, the government can implement the constrained optimal social surplus by setting $\boldsymbol{\tau = \tau^*}$ while leaving the selection of the fee scheme to the manager.

Theorem 5: $\boldsymbol{y^{k^*,\tau^*}}$ *is the only fee scheme in* $\boldsymbol{Y^*(\tau^*)}$ *that satisfies Conditions 1, 2, 3, and 5 for* $\boldsymbol{(t,\tau) = (t^*,\tau^*)}$.

From inequalities (11) and $\boldsymbol{k^{*} < 1}$, it follows that

$$\boldsymbol{t^{*} < \tau^{*}}.$$

Hence, the government should set the income tax rate higher than the capital gain tax rate.

5. General Fee Schemes

In the previous section, we confined our attention to the subsets of fee schemes $\boldsymbol{Y^*(\tau)}$. This section will show that the basic meaning of the previous section remains unchanged even if we take into account all fee schemes in $\boldsymbol{Y}$.

First, we show that it is impossible for the government to further enhance the social surplus by selecting any fee scheme that does not belong to $\boldsymbol{Y^*(\tau)}$. Condition 5 may be more restrictive in this case; the skilled manager comes to mimic even more the higher performance than his own.

For instance, let us specify a fee scheme $\boldsymbol{y \in Y}$ by

$$\boldsymbol{y(\tilde{a}(1)+1) = \frac{k}{1-\tau}\tilde{a}(1)},$$

and

$$\boldsymbol{y(x) = -k} \quad \text{for all} \quad \boldsymbol{x \neq \tilde{a}(1)+1},$$

where $\boldsymbol{k < 1-\tau}$ and $\boldsymbol{k\tilde{a}(1) - c(\tilde{a}(1)) > -k}$. Note that the specified fee scheme $\boldsymbol{y}$ does not belong to $\boldsymbol{Y^*(\tau)}$. Since $\boldsymbol{a^*(y,\tau) = \tilde{a}(1)}$, it follows that without any third-party side contract, the skilled manager's best choice is the first-best action given by $\boldsymbol{\tilde{a}(1)}$.

However, he does not prefer to select $\boldsymbol{\tilde{a}(1)}$; instead, he is willing to select the less active investment strategy $\boldsymbol{\tilde{a}(k)}$ and to mimic the initial best performance by entering into a third-party side contract $\boldsymbol{F \in \Phi(\tilde{a}(k))}$ that is specified by

$$\boldsymbol{F(z) = \frac{\tilde{a}(1)-\tilde{a}(k)}{\tilde{a}(1)+1}} \quad \text{for all} \quad \boldsymbol{z \in [0, \tilde{a}(1)+1)},$$

and

$$\boldsymbol{F(z) = 1} \quad \text{for all} \quad \boldsymbol{z \in [\tilde{a}(1)+1, \infty)}.$$

With the probability of $\boldsymbol{\frac{\tilde{a}(k)+1}{\tilde{a}(1)+1}}$, the manager receives $\boldsymbol{\tilde{a}(1)+1}$ from the third party; he receives nothing otherwise. Hence, he obtains the expected payoff, which is given by

$$\boldsymbol{\frac{\tilde{a}(k)+1}{\tilde{a}(1)+1}k\tilde{a}(1) - \frac{\tilde{a}(1)-\tilde{a}(k)}{\tilde{a}(1)+1}k - c(\tilde{a}(k))}$$

$$\boldsymbol{+(M-k)\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}}$$

$$= k\tilde{a}(k) - c(\tilde{a}(k)) + (M - k)\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\},$$

and it is greater than

$$k\tilde{a}(1) - c(\tilde{a}(1)) + (M - k)\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$

$$= V(y, t, \tau).$$

This contradicts Condition 5.

Theorem 6: *Suppose that* $(y, t, \tau) \in Y \times [0,1]^2$ *satisfies Conditions 1, 2, 3, 4, and 5. Then, it holds that* $\hat{t}(1-\tau) \le t$*, i.e.,* $(y^{\tilde{k},\tau}, t, \tau)$ *also satisfies these conditions, where* $\tilde{k}$ *was defined by* $t = \hat{t}(\tilde{k})$*, and*

$$S(y^{\tilde{k},\tau}, t, \tau) > S(y, t, \tau),$$

Proof: See the Appendix.

Theorem 6 implied that there exists no $(y, t, \tau) \in Y \times [0,1]^2$ that satisfies Conditions 1, 2, 3, 4, and 5, and

$$S(y, t, \tau) > S(y^{k^*,\tau^*}, t^*, \tau^*).$$

Hence, it is impossible for the government to improve the social surplus further by selecting any fee scheme that does not belong to $Y^*(\tau)$. We have proven also that for every $(t, \tau) \in [0,1]^2$, there exists a fee scheme $y \in Y$ that satisfies Conditions 1, 2, 3, 4, and 5, if and only if

$$\hat{t}(1-\tau) \le t.$$

Next, we show that it is impossible for the skilled manager to increase his payoff by selecting any fee scheme that does not belong to $Y^*(\tau)$. Condition 2 may be more restrictive in this case. Any unskilled manager would therefore mimic the skilled manager's performance even further.

Theorem 7: *For every* $(t, \tau) \in [0,1]^2$*, there exists no* $y \in Y$ *that satisfies Conditions 1, 2 and 3, and*

$$V(y, t, \tau) > V(y^{\min[M, 1-\tau], \tau}, t, \tau).$$

Proof: See the Appendix.

6. Discussions

6.1. Investor's Optimization

Neither the skilled manager nor the investor necessarily prefers $y^{k^*,\tau}$ under the constraints of Conditions 1, 2, 3, and 5; $k^+(\tau) \in [k^*, \min[M, 1-\tau]]$ is defined as maximizing the investor's payoff $U(y^{k,\tau}, t^*, \tau) = (1-t^*)(1-\frac{k}{1-\tau})\tilde{a}(k)$ with respect to $k \in [k^*, \min[M, 1-\tau]]$. Since $(y^{k^+(\tau),\tau}, t^*, \tau)$ satisfies Conditions 1, 2, 3, and 5, the investor clearly prefers $y^{k^+(\tau),\tau}$ to $y^{k^*,\tau}$.

In contrast with the skilled manager, the investor prefers a fee scheme that does not belong to $Y^*(\tau)$ as his best choice; supposing that

$$\tau > \tau^* \text{ and } k^+(\tau) > k^*.$$

Let us denote by $y = y^{k,\tau,w} \in Y$ the fee scheme specified by

$$y^{k,\tau,w}(x) = \max[\frac{kx - w}{1-\tau}, kx - w] \text{ for all } x \in [0, \infty),$$

where $0 < k \leq w \leq \min[M, 1-\tau]$, and $y^{k,\tau,w}$ satisfies Conditions 2 and 5. Specify

$$w^+(\tau) \equiv k^+(\tau) + V(y^{k^+(\tau),\tau}, t^*, \tau) - \bar{V}(t^*).$$

Since $w^+(\tau)$ is greater than $k^+(\tau)$, it follows that the associated fee after taxation is not linear, and therefore, $y^{k,\tau,w}$ does not belong to $Y^*(\tau)$. Note that $y^{k^+(\tau),\tau,w^+(\tau)}$ satisfies Condition 1 with equality. Since $w^+(\tau) > k^+(\tau)$, it follows that

$$U(y^{k^+(\tau),\tau,w^+(\tau)}, t^*, \tau) - U(y^{k^+(\tau),\tau}, t^*, \tau) = w^+(\tau) - k^+(\tau) > 0,$$

which implies that the investor prefers $y^{k^+(\tau),\tau,w^+(\tau)} \notin Y^*(\tau)$ to $y^{k^+(\tau),\tau} \in Y^*(\tau)$.

By transferring the entire tax revenue to the investor, the government can incentivize him to select $y^{k^*,\tau}$; the investor's payoff should be replaced with

$$(15)\qquad U(y^{k,\tau},t^*,\tau)+\{t^*(1-\frac{k}{1-\tau})+\tau\frac{k}{1-\tau}\}\tilde{a}(k)+t^*(M-k)\tilde{a}(1-t^*)$$

$$=S(y^{k,\tau},t^*,\tau)-V(y^{k,\tau},t^*,\tau).$$

Note that $V(y^{k,\tau},t^*,\tau)$ is increasing with respect to $k\in[0,\min[M,1]]$, that $S(y^{k^*,\tau},t^*,\tau)\geq S(y^{k,\tau},t^*,\tau)$ for all $k\in[k^*,\min[M,1]]$, and that $y^{k,\tau}$ satisfies Condition 1 only if $k\geq k^*$. Hence, $k=k^*$ maximizes the value of (15) subject to Condition 1, and therefore, the investor prefers $y^{k^*,\tau}$ the most.

There, however, exists a drawback to this manner of transfer payments; the investor's payoff equals

$$(1-k^*)\tilde{a}(k^*)+t^*(M-k^*)\tilde{a}(1-t^*),$$

while the skilled manager's pay off per unit equals

$$(1-t^*)\tilde{a}(1-t^*)-c(\tilde{a}(1-t^*)).$$

In this case, the skilled manager's payoff per unit is not necessarily greater than the investor's payoff. Thus, it may be advantageous for the skilled manager to close down his hedge fund business and instead delegate to another skilled manager the management of his personal fund.

6.2. Plural Shareholders

We can extend our model to the hedge fund owned by plural shareholders, where no informational asymmetry exists among the manager and the shareholders; the shareholders know whether the manager is skilled or unskilled, monitor the manager's decisions, and the interests of the manager accord with that of the shareholders.

Note that the shareholders are regarded as sophisticated investors who should be distinguished from the unsophisticated investors that this paper has investigated. In this respect, this paper has argued about the possibility of even unsophisticated investors being attracted to a hedge fund that poses no danger.

6.4. Entry Cost

We have assumed that the manager can enter the hedge fund industry with no entry cost. This subsection eliminates this assumption and makes an alternative assumption that any manager has to take a fixed cost $T > 0$ for his entry. In this case, the impact of the entry cost is different depending on whether this cost is pecuniary or nonpecuniary.

Suppose that the entry cost is nonpecuniary; the skilled manager does not lose his personal fund. In this case, the welfare can be improved compared with the no-entry cost case. By replacing any fee scheme $y \in Y$ with $y + T$, i.e., by letting the investor take over the manager's entry cost T, the government can save the manager's unmanageable personal fund from $w(y)$ to $w(y) - T$.

Next, suppose that the entry cost is pecuniary; the manager loses his personal fund for T units, which makes it impossible to improve the welfare because the skilled manager is prevented from earning alpha from these units.

To examine this further, let us regard T as the *lump-sum tax* that the government levies on the manager when he enters. Then, $V(y,t,\tau)$, $U(y,t,\tau)$, and $S(y,t)$ should be replaced with

$$V(y,t,\tau) - T - T\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\},$$

$$U(y,t,\tau) - T,$$

and

$$S(y,t,\tau) - T\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\},$$

respectively. Let us consider any fee scheme $y \in Y$ that satisfies the skilled manager's entry and the unskilled manager's exit. Let us specify another fee scheme $y' \in Y$ by

$$\min[(1-\tau)y'(x), y'(x)] = \min[(1-\tau)y(x), y(x)] - \varepsilon \quad \text{for all} \quad x \in [0,\infty),$$

where we assume ε to be positive but close to zero. Since $w(y') = w(y) - \varepsilon$, by replacing the lump-sum tax T with the lesser amount $T - \varepsilon$ and replacing the fee scheme y with y', the government can keep the social surplus unchanged without any change in the manager's incentive.

7. Personal Fund as Equity Stake

Throughout this paper, we have assumed that the skilled manager can select actions for the investor's fund that differ from those he selects for his personal fund. In this section, we eliminate this assumption and make an alternative assumption that the manager is committed to combining his personal fund with the investor's fund as the equity stake and to managing them as one; the manager must select the same action for his personal fund as he does for the investor's fund. More importantly, he must use not only the investor's fund but also his personal fund as security when entering into a third-party side contract. In this case, even the unskilled manager bears a capital gain tax burden. This section shows that the unskilled manager's resulting capital gain tax burden is the main factor dissuading him from entering the hedge fund industry, where there is no welfare-distorting penalty put in escrow.[6]

For simplicity of the arguments, we assume that $A = \{0, a^*\}$, i.e., that the skilled manager selects either the null action 0 or an action denoted by $a^* > 0$ with a positive cost $c^* = c(a^*) > 0$. We assume also that $\tau = 0$, i.e., that the income tax rate is set equal to zero. Let us consider any fee scheme y such that $y(x) \geq 0$ for all $x \in [0, \infty)$, i.e., no positive penalty is imposed in any case, where we assumed without loss of generality that $y(0) = 0$, and that $(1-t)a^* - c^* \geq 0$, i.e.,

(16) $$t \leq \frac{a^* - c^*}{a^*}.$$

Inequality (16) guarantees that the skilled manager prefers to select a^* rather than the null action 0 when he decides to exit from the hedge fund industry.

The constraint of the skilled manager's entry is given by

$$y(a^* + 1) - c^* + M\{(1-t)a^* - c^*\} \geq M\{(1-t)a^* - c^*\},$$

that is,

(17) $$y(a^* + 1) \geq c^*.$$

The constraint of the unskilled manager's exit is given by

[6] Foster and Young (2009) showed that the equity stake may serve to deter the unskilled manager even when there is no capital gain tax, provided that he is assumed to be, not risk neutral, but risk averse.

$$\max_{F \in \Phi} E[y(z) + M \min[(1-t)(z-1), z-1] \mid F] \leq 0,$$

where the unskilled manager uses not only the investor's fund but also his personal fund as security. Depending on the use of the capital decimation partners specified in Subsection 2.2, the necessary condition for the unskilled manager's exit is given by

$$\frac{1}{a^*+1}[y(a^*+1) + (1-t)Ma^*] - \frac{a^*}{a^*+1}M \leq 0,$$

that is,

(18) $$y(a^*+1) \leq tMa^*.$$

From inequalities (17), and (18), it is necessary to satisfy

(19) $$tMa^* \geq c^*,$$

which implies that

$$t > 0.$$

Hence, we have shown that the capital gain tax rate should be positive. This deduction is related to that of Foster and Young (2009), who showed that with $t = 0$, it is impossible to solve the incentive problem in the hedge fund, even if we utilize the equity stake device. Based on this impossibility result and Theorem 1, we can show a more general result that with no capital gain tax, it is impossible to solve the incentive problem even if we utilize both the equity stake and the positive penalty.

From inequalities (16) and (19), it must hold that

(20) $$0 < t \leq \frac{a^* - c^*}{a^*} \quad \text{and} \quad M \geq \frac{c^*}{a^* - c^*}.$$

Inequalities (20) are not only necessary but also sufficient for solving the incentive problem; with these inequalities, the fee scheme specified by

$$y(x) \equiv \max[x-1, 0] \quad \text{for all} \quad x \in [0, \infty)$$

and the capital gain tax specified by

$$t = \frac{a^* - c^*}{a^*}$$

jointly satisfy all the constraints of the skilled manager's entry, the unskilled manager's exit, the investor's entry, and the skilled manager's non-mimicry.

On the other hand, we can verify in the same manner as done in the previous sections that if the manager can select actions for the investor's fund that differ from those for his personal fund, the maximal penalty that is permitted will be positive, and the inequality given by

(21) $$M \geq \frac{c^*}{a^*}$$

holds; then the fee scheme specified by

$$y(x) = \frac{c^*}{a^*}(x-1) \quad \text{for all} \quad x \in [0,\infty),$$

and the capital gain tax specified by

$$t = \frac{a^* - c^*}{a^*}$$

jointly satisfy all the constraints of Conditions 1, 2, 3, 4, and 5.

The argument of this section indicates that if the skilled manager can commit himself to combining his personal fund with the investor's fund as the equity stake, then it might be possible for the widespread practice of a two-part fee scheme involving a 2% management fee plus a 20% performance fee to solve the incentive problem. However, it must be noted that inequality (21) is less restrictive than the latter part of inequalities (20). Hence, in order to incentivize the manager and the investor without using any welfare-distorting penalty, we must require the skilled manager to possess a much larger personal fund than that needed in the case that the positive penalty is allowed to be put in escrow.

The Appendix

Proof of Theorem 6: Let us consider any $(y,t,\tau)\in Y\times[0,1]^2$ that satisfies Conditions 1, 2, 3, 4, and 5. It must hold that

$$w(y)\le 1 \text{ and } a^*(y)\ge(1-\tau)y(a^*(y)+1).$$

Let us specify

$$w=-y(0) \text{ and } k=\frac{(1-\tau)y(a^*(y)+1)+w}{a^*(y)+1}.$$

Note that

(A-1) $$k\le w\le w(y)\le \min[M,1],$$

and

(A-2) $$V(y,t,\tau)=k\{a^*(y)+1\}-w-c(a^*(y))$$
$$+\{M-w(y)\}\{(1-t)\tilde{a}(1-t)-c(\tilde{a}(1-t))\}.$$

Suppose that $a^*(y,\tau)>\tilde{a}(k)$. Let us specify $F\in\Phi(\tilde{a}(k))$ by

$$F(z)=\frac{a^*(y)-\tilde{a}(k)}{a^*(y)+1} \text{ for all } z\in[0,a^*(y)+1),$$

and

$$F(z)=1 \text{ for all } z\in[a^*(y)+1,\infty),$$

where $E[z\,|\,F]=\tilde{a}(k)$ and

$$E[\min[(1-\tau)y(z),y(z))\,|\,F]=k\{\tilde{a}(k)+1\}-w.$$

The skilled manager's expected payoff induced by the action choice $\tilde{a}(k)$ and the specified side contract $F\in\Phi(\tilde{a}(k))$ is given by

(A-3) $$k(\tilde{a}(k)+1)-w-c(\tilde{a}(k))+\{M-w(y)\}\{(1-t)\tilde{a}(1-t)-c(\tilde{a}(1-t))\}.$$

From Condition 5, the value of $V(y,t,\tau)$, given by (A-2), minus the value of (A-3) must be non-negative:

$$ka^*(y,\tau)-c(a^*(y,\tau))-k\tilde{a}(k)+c(\tilde{a}(k))\ge 0,$$

which implies that $a^*(y)=\tilde{a}(k)$, i.e., contradicts the supposition. Hence, it must hold that

$$a^*(y,\tau)\le\tilde{a}(k).$$

From inequalities (A-1), it follows that

$$V(y,t,\tau) \le k\tilde{a}(k) - c(\tilde{a}(k)) + (M-k)\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$
$$= V(y^{k,\tau},t,\tau),$$

which, along with $V(y,t,\tau) \ge \bar{V}(t)$, implies that $(y^{k,\tau},t,\tau)$ satisfies Condition 1. Hence, from the argument in Subsection 4.1, it follows that $\hat{t}(1-\tau) \le t$, i.e., $(y^{\tilde{k},\tau},t,\tau)$ satisfies Conditions 1, 2, 3, 4, and 5. Since $a^*(y,\tau) \le \tilde{a}(k) \le \tilde{a}(1)$, it follows that

$$S(y^{k,\tau},t,\tau) = \tilde{a}(k) - c(\tilde{a}(k)) + (M-k)\{\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$
$$\ge a^*(y,\tau) - c(a^*(y,\tau)) + \{M - w(y)\}\{\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$
$$= S(y,t),$$

which implies that

$$S(y^{\tilde{k},\tau},t,\tau) \ge S(y^{k,\tau},t,\tau) \ge S(y,t,\tau).$$

Q.E.D.

Proof of Theorem 7: Suppose that $y \in Y$ satisfies Conditions 1, 2, and 3. From inequality (4), which was implied by Condition 2, it follows that the manager's fee after taxation $(1-\tau)y(a^*(y)+1)$ must be less than or equal to $w(y)a^*(y,\tau)$. Since $V(y^{k,\tau},t,\tau)$ is increasing with respect to $k \in [0, \min[M, 1-\tau]]$, it follows that

$$V(y,t,\tau) = (1-\tau)y(a^*(y,\tau)+1) - c(a^*(y,\tau))$$
$$+\{M - w(y)\}\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$
$$\le w(y)a^*(y,\tau) - c(a^*(y,\tau)) + \{M - w(y)\}\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$
$$\le w(y)\tilde{a}(w(y)) - c(\tilde{a}(w(y))) + \{M - w(y)\}\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$
$$= V(y^{w(y),\tau},t,\tau) \le V(y^{\min[M,1-\tau],\tau},t,\tau).$$

Q.E.D.

References

Ackermann, C., R. MacEnally, and D. Ravenscraft (1999): The Performance of Hedge Funds: Risk, Return, and Incentives, *Journal of Finance* 54, 833-874.

Bolton, P. and M. Dewatripont (2005): *Contract Theory*, MIT Press.

Carpenter, J. (2000): Does Option Compensation Increase Managerial Risk Appetite?, *Journal of Finance* 55, 2311-2331.

Financial Times (2008): “Why today’s hedge fund industry may not survive,” by Martin Wolf, March 18.

Foster, D. and P. Young (2008a): The Hedge Fund Game: Incentive, Excess Returns, and Piggy-Backing, mimeo.

Foster, D. and P. Young (2008b): Hedge Fund Wizards, The Economists’ Voice 5 (2), Article 1.

Foster, D. and P. Young (2009): Gaming Performance Fee by Portfolio Managers, mimeo.

Hodder, J. and J. Jackwerth (2006): Incentive Contracts and Hedge Fund Management, mimeo.

Laffont, J.-J. and D. Martimort (2002): *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press.

Legros, P. and H. Matsushima (1991): Efficiency in Partnerships, *Journal of Economic Theory* 55, 296-322.

Lo, A. (2001): Risk Management for Hedge Funds: Introduction and Overview, *Financial Analysts Journal*, 16-33.

Lo, A (2008): *Hedge Funds: An Analytic Perspective*, Princeton University Press.

New York Times (2010), “Hedge Funds: A ‘Lemon’ Market?,” April 3.

Starks, L. (1987): Performance Incentive Fees: An Agency Theoretic Approach, *Journal of Financial and Quantitative Analysis* 22, 17–32.