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Uninsured CounterCyclical Risk: An Aggregation Result and Application to Optimal Monetary Policy

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Uninsured countercyclical risk: an aggregation result and application to optimal monetary policy*

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Abstract

We consider an incomplete markets economy with capital accumulation and endogenous labor supply. Individuals face countercyclical idiosyncratic labor and asset risk. We derive conditions under which the aggregate allocations and price system can be found by solving a representative agent problem. This result is applied to analyze the properties of an optimal monetary policy in a New Keynesian economy with uninsured countercyclical individual risk. The optimal monetary policy that emerges from our incomplete markets economy is the same as the optimal monetary policy in a representative agent model with preference shocks. When price rigidity is the only friction the optimal monetary policy calls for stabilizing the inflation rate at zero.

Keywords: uninsured idiosyncratic risk; sticky prices; optimal monetary policy.

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1 Introduction

This paper establishes an aggregation theorem for a class of incomplete market economies and uses it to analyze the properties of optimal monetary policy when markets are incomplete.

Our aggregation result is interesting because it applies to a model that captures some of the most significant features of the business cycle. For instance, in the data over 60 percent of the total variation in output over the business cycle is due to variation in labor input. Labor supply is endogenous in our model and monetary policy can affect labor market conditions.

Another property of the business cycle is that capital accumulation makes it possible for the entire economy to insure against variations in economic activity. The significance of this mechanism can be easily discerned in aggregate variability statistics. Aggregate consumption is much less volatile than output while aggregate investment is much more volatile than output. This mechanism is operating in our economy. Capital formation is endogenous and monetary policy can influence the level of investment.

A final aspect of the business cycle that we want to model is a positive correlation between uninsured unemployment and asset risk. For most households their single most important investment is their home.[1] In many localities labor market outcomes are related either directly or indirectly to the economic performance of large employers. When these employers downsize their labor force this implies both a higher probability of unemployment and also a higher probability of lower house prices.[2] We assume that idiosyncratic labor income risk is correlated with asset return risk in our model.

Our paper makes contributions to the literature on incomplete markets models of the business cycle. Producing a tractable real model of the private sector with endogenous labor supply and endogenous capital formation is a challenge. In the current literature there are two approaches to modeling the business cycle with incomplete markets. One approach uses strictly numerical methods. The advantage of this approach is that one can model both labor supply and capital formation. Krusell, Mukoyama, Sahin and Smith (2009), consider the welfare cost of business cycles in a real economy with idiosyncratic, countercyclical labor risk and capital formation and exogenous labor supply. Storesletten, Telmer and Yaron (2001) model countercyclical risk in a real overlapping generations model with capital formation and exogenous labor. Chang and Kim (2007) consider labor supply decisions in an infinite horizon model with capital formation but idiosyncratic risk is acyclical.

The principal disadvantage of this approach is the curse of dimensionality. As the dimension of either the shock space or the list of endogenous state variables is increased one quickly hits the limits of computational feasibility. For this reason the above papers only have a single aggregate shock and a single endogenous aggregate state variable. Deriving optimal state-contingent government policies creates an additional layer of computational difficulty, because

[1] Wolff (2010), for instance, reports that in 2007 over 61 percent of wealth was invested in the primary residence for the bottom 90 percent of the U.S. wealth distribution.

[2] See Foote, Gerardi, Goette and Willen (2010) for empirical evidence on the positive correlation between unemployment and mortgage default risk.

these policies are, in principle, indexed by each possible history.

An alternative strategy is to make assumptions that allow one to derive closed form or nearly closed form results. Most of this research builds on ideas first developed by Constantinides and Duffie (1996). Heathcote, Storesletten and Violante (2008) consider the effects of an increase in labor risk in an incomplete markets economy that admits a closed form solution. However, that model abstracts from capital formation. Krebs (2003) computes the welfare cost of business cycles in a model with countercyclical idiosyncratic risk and capital formation. However, his model abstracts from labor supply. Kruger and Lustig (2010) derive conditions under which incomplete markets are irrelevant for the price of aggregate risk. But, their result requires that idiosyncratic risk be acyclical and they derive their result in an exchange economy.

The real side of the economy we consider extends this previous research by modeling both labor supply and capital formation jointly. Our specification of the risk environment assumes that the labor productivity of each individual follows a geometric random walk, and there are no insurance markets for that risk. We assume that the return to savings of each individual is also subject to idiosyncratic risk. Under these assumptions we establish that the no-trade theorem of Constantinides and Duffie (1996) extends to our production economy with endogenous labor supply. This is accomplished by producing an aggregation result that establishes the existence of a representative-agent economy with preference shocks that yields the same aggregate quantities and prices in equilibrium as the original heterogeneous-agents economy with incomplete markets.[3] Our model has the property that an increase in the variance of idiosyncratic income shocks acts to increase (resp. decrease) the discount factor in the corresponding representative-agent economy if the elasticity of intertemporal substitution of consumption is less (resp. greater) than unity.

Motivated by recent empirical evidence documented in Storesletten, Telmer and Yaron (2004), and Meghir and Pistaferri (2004), we model countercyclical variation in idiosyncratic risk. Modeling countercyclical idiosyncratic risk can produce large welfare costs of business cycles.[4] Our model shares this property. Lucas' (1987) measure exceeds 12 percent of consumption when the coefficient of relative risk aversion is two.

We apply our aggregation result to analyze optimal monetary policy when individuals face uninsured idiosyncratic risk in a New Keynesian model. One challenge to analyzing optimal monetary policy in such an environment arises from the fact that Calvo price setting makes profit maximization of each firm an intertemporal problem. When financial markets are incomplete, shareholders, in general, do not agree on how to value future dividends.[5] In the context of the Calvo model, this implies that when a firm obtains an opportunity to adjust the price of its product, its shareholders do not agree about the price it should charge. In our setup there is no disagreement problem. All shareholders value future dividends in the same way.

[3] For a general discussion on the correspondence between incomplete-markets economies and representative-agent economies, see Nakajima (2005).

[4] See, for instance, Storesletten, Telmer and Yaron (2001), Krebs (2003) and De Santis (2007).

[5] For an overview on the theory of incomplete markets, see, for instance, Magill and Quinzii (1996).

In this paper we focus on sticky prices, and abstract from sticky wages.[6] As is well known, complete price stabilization achieves the first best in the representative-agent New Keynesian model with sticky prices, provided that the average distortion due to monopolistic competition among intermediate goods producers is corrected by a subsidy.[7] Using our aggregation result we establish that price stabilization is also optimal in our incomplete markets model. This is true in spite of the fact that the welfare cost of business cycles is far larger than in the standard representative-agent New Keynesian model.

The intuition for this result is as follows. In our model there are two ways that government policy can improve on laissez faire allocations. One way is to enhance productive efficiency by correcting the dynamic markup distortion. The other way is to provide insurance either via direct redistribution or indirect redistribution. Direct redistribution falls within the domain of fiscal policy, and thus we rule out this possibility. Indirect redistribution involves manipulating the price system in a way that benefits households who experience negative idiosyncratic shocks and thus provides them with implicit insurance. An implication of our aggregation result is that the objective function of a benevolent monetary authority factors in such a way that there is no opportunity for it to influence the conditional distribution of wealth and provide this type of implicit insurance. It follows that the optimal monetary policy in our incomplete markets economy is to stabilize the price level when there is a subsidy to intermediate goods producers that corrects the steady state distortion.

Schmitt-Grohé and Uribe (2007) show that the optimal monetary policy obtained in the New Keynesian model with complete markets continues to call for (nearly) complete price stabilization when there is no such subsidy. In our model the fact that the objective function factors continues to imply that there is no opportunity for the monetary authority to provide implicit insurance by affecting relative prices. Still, the representative agent representation of our incomplete markets model is different from the complete markets economy considered by Schmitt-Grohé and Uribe (2007). We have preference discount shocks that are correlated with the aggregate state of technology and they don't. The welfare costs of business cycles are also large in our model but small in theirs. It turns out that these differences are innocuous. Complete stabilization of the price level is a good policy in our incomplete markets model too.

In the Appendix we consider a more elaborate model costly price and wage adjustment. This second nominal rigidity creates a tradeoff between price stabilization, on the one hand, and wage stabilization on the other hand. Our main finding applies in this setting too. The optimal monetary policy that emerges from our incomplete markets economy is the same that would apply in the analogue representative agent model.[8]

More generally, our results suggest that conclusions about optimal monetary policy in representative agent models are robust to the market structure in the following sense. If one posits shocks to the preference discount rate and allows them to be correlated with aggregate shocks

[6]The appendix considers an extension with both types of price adjustment.

[7]See, for instance, Woodford (2003) and Galí (2008).

[8]As pointed out in e.g. Galí (2008) the optimal monetary policy no longer calls for stabilizing the price level.

then the optimal policies can also be construed as being the optimal policies that emerge in a particular model of incomplete markets.

Finally, our model provides structural foundations for preference discount shifters. Preference discount shifters have been found to be important shocks in the New Keynesian models of Smets and Wouters (2003), Levin, Onatski, Williams and Williams (2005), and Burriel, Fernández-Villaverde and Rubio-Ramírez (2009) among others. Our model explicitly links the persistence and variability of movements in the preference discount rate to the law of motion of the variance of idiosyncratic risk.

The rest of the paper is organized as follows. In Section 2, we describe our heterogeneous-agents economy, and then construct a corresponding representative-agent economy which yields the same equilibrium as the original economy. In Section 3, we present our numerical results. In Section 4, we conclude.

2 The model economy

In this section we describe our model. It is a cashless New Keynesian economy (see Woodford (2003) or Galí (2008)) with nominal price rigidities as in Calvo (1983) and uninsurable idiosyncratic individual risk.

2.1 Individuals

The economy is populated by a continuum of individuals of unit measure, indexed by $i \in [0, 1]$. They are subject to both idiosyncratic and aggregate shocks. We assume that idiosyncratic shocks are independent across individuals, and a law of large numbers applies.

Individuals consume and invest a composite good, which is produced by a continuum of differentiated products, indexed by $j \in [0, 1]$. If the supply of each variety is given by $Y_{j,t}$, for $j \in [0, 1]$, the aggregate amount of the composite good, Y_t, is given by

$$Y_t = \left(\int_0^1 Y_{j,t}^{1-\frac{1}{\zeta}} \, dj \right)^{\frac{1}{1-\frac{1}{\zeta}}} \tag{1}$$

where $\zeta > 1$ denotes the elasticity of substitution across different varieties. This composite good is used for consumption and investment:

$$Y_t = C_t + I_t$$

where C_t and I_t denote the aggregate amounts of consumption and investment in period t, respectively. Let $P_{j,t}$ denote the price of variety j in period t. It then follows from cost minimization that the demand for each variety is given by

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\zeta} Y_t \tag{2}$$

where P_t is the price index defined by

$$P_t = \left(\int_0^1 P_{j,t}^{1-\zeta} \, dj \right)^{\frac{1}{1-\zeta}} \tag{3}$$

Preferences of each individual are described by the utility function defined over stochastic processes of consumption and leisure:

$$u_{i,0} = E_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left[c_{i,t}^{\theta} (1 - l_{i,t})^{1-\theta} \right]^{1-\gamma} \tag{4}$$

where β is a subjective discount factor, $c_{i,t}$ is individual i's consumption of the composite good in period t, and $l_{i,t}$ is her labor supply in period t. We use E_t^i to denote the expectation operator conditional on the history of idiosyncratic shocks to individual i up to and including period t as well as the history of aggregate shocks over the same time period. The expectation operator conditional on the history of aggregate shocks up to and including period t is denoted by E_t. It will prove convenient to define γ_c as

$$\gamma_c \equiv 1 - \theta(1 - \gamma) \tag{5}$$

Then, $1/\gamma_c$ is the intertemporal elasticity of substitution of consumption with a constant level of leisure.

The idiosyncratic risk faced by individual i is represented by a geometric random walk $\{\eta_{i,t}\}$:

$$\ln \eta_{i,t} = \ln \eta_{i,t-1} + \sigma_{\eta,t} \epsilon_{\eta,i,t} - \frac{\sigma_{\eta,t}^2}{2} \tag{6}$$

where $\epsilon_{\eta,i,t}$ is $N(0,1)$ and i.i.d. across individuals and over time. The standard deviation, $\sigma_{\eta,t}$, is allowed to fluctuate over time, in a way that will be specified below. The process $\{\eta_{i,t}\}$ affects individual i's income in two ways. First, $\eta_{i,t}$ affects the productivity of individual i's labor (her efficiency units of labor). Thus, if w_t is the real wage rate per efficiency unit of labor, the labor income of individual i in period t is given by $w_t \eta_{i,t} l_{i,t}$. Second, $\eta_{i,t}$ affects the return on savings.

We will abstract from government bonds. Suppose that claims to the ownership of physical capital and the ownership of firms are traded separately. Let $q_{j,t}$ be the period-t price of a share in firm $j \in [0,1]$, and $e_{i,j,t}$ be the share in firm j held by individual i at the end of period t. Below we conjecture an equilibrium in which all individuals choose the same portfolio weights, and hence they hold equal shares of all firms, that is, $e_{i,j,t} = e_{i,t}$ for all $j \in [0,1]$. We then verify that such an equilibrium exists. Let $s_{i,t}$ be the value of stocks held by individual i: $s_{i,t} \equiv \int_0^1 q_{j,t} e_{i,j,t} \, dj = e_{i,t} \int_0^1 q_{j,t} \, dj$, and let $R_{s,t}$ be the gross rate of return on equities: $R_{s,t} \equiv \int_0^1 (q_{j,t} + d_{j,t}) \, dj / \int_0^1 q_{j,t-1} \, dj$.

Under our assumption that the return to savings is also subject to idiosyncratic risk, the flow budget constraint becomes

$$c_{i,t} + k_{i,t} + s_{i,t} = \frac{\eta_{i,t}}{\eta_{i,t-1}} \left(R_{k,t} k_{i,t-1} + R_{s,t} s_{i,t-1} \right) + \eta_{i,t} w_t l_{i,t} \tag{7}$$

Here $k_{i,t}$ is the amount of physical capital obtained by individual i in period t, and $R_{k,t}$ is the gross rate of return on physical capital, that is,

$$R_{k,t} = 1 - \delta + r_{k,t} \tag{8}$$

where $r_{k,t}$ is the rental rate of capital and δ is its depreciation rate. To rule out Ponzi schemes, we impose $k_{i,t} \geq 0$ and $s_{i,t} \geq 0$. These last two constraints will not bind in equilibrium.[9]

In equation (7), $\eta_{i,t}/\eta_{i,t-1}$ is an idiosyncratic shock to the return on savings. Under this assumption "permanent income" of individual i, which is defined as the sum of human and financial wealth, is proportional to $\eta_{i,t}$.

The assumption that the idiosyncratic risk to labor and capital income is perfectly correlated is strong but it buys us a lot. Under this assumption we are able to derive a tractable solution to what, is in general, a challenging model to solve and analyze. This assumption also strikes us as empirically relevant. As we noted in the introduction home ownership creates a positive correlation between labor risk and financial risk. Labor and financial risks are also likely to be positively correlated for privately held firms as in Angeletos (2007).

At date 0, each individual chooses a contingent plan $\{c_{i,t}, l_{i,t}, k_{i,t}, s_{i,t}\}$ so as to maximize her utility (4) given $\{k_{i,-1}, s_{i,-1}, \eta_{i,-1}\}$ and subject to the sequence of flow budget constraints (7) and the short-selling constraint on $\{k_{i,t}, s_{i,t}\}$.[10] The Lagrangian for the household's problem is

$$\begin{aligned}\mathcal{L} = E_0^i \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \frac{1}{1-\gamma} \left[c_{i,t}^{\theta}(1-l_{i,t})^{1-\theta}\right]^{1-\gamma} \\ & + \lambda_{i,t}\left[\frac{\eta_{i,t}}{\eta_{i,t-1}}(R_{k,t}k_{i,t-1} + R_{s,t}s_{i,t-1}) + \eta_{i,t}w_t l_{i,t} - c_{i,t} - k_{i,t} - s_{i,t}\right]\Bigg\}\end{aligned}$$

Then the first-order conditions are

$$\theta c_{i,t}^{-\gamma_c}(1-l_{i,t})^{(1-\theta)(1-\gamma)} = \lambda_{i,t} \tag{9}$$

$$\frac{1-\theta}{\theta}\frac{c_{i,t}}{1-l_{i,t}} = w_t \eta_{i,t} \tag{10}$$

$$\lambda_{i,t} = \beta E_t^i \lambda_{i,t+1}\frac{\eta_{i,t+1}}{\eta_{i,t}} R_{k,t+1} \tag{11}$$

$$\lambda_{i,t} = \beta E_t^i \lambda_{i,t+1}\frac{\eta_{i,t+1}}{\eta_{i,t}} R_{s,t+1} \tag{12}$$

and the flow budget constraint (7). The transversality conditions for $k_{i,t}$ and $s_{i,t}$ are given respectively as

$$\lim_{t\to\infty} E_0^i \beta^t \lambda_{i,t} k_{i,t} = 0 \tag{13}$$

$$\lim_{t\to\infty} E_0^i \beta^t \lambda_{i,t} s_{i,t} = 0 \tag{14}$$

[9] Constantinides and Duffie (1996) show that in equilibrium agents never choose to borrow. Our economy has this same property.

[10] Note that we are allowing for ex ante heterogeneity.

Given a vector stochastic process $\{R_{k,t}, R_{s,t}, w_t\}$, a solution to the utility maximization problem of each individual is a state-contingent plan $\{c_{i,t}, l_{i,t}, k_{i,t}, s_{i,t}, \lambda_{i,t}\}$ that satisfies the first-order conditions (7)-(12), as well as the transversality conditions (13)-(14) and the initial conditions.

2.2 Aggregation

Here we show that the utility maximization problem of the heterogeneous agents under incomplete markets described in the previous subsection can be aggregated into a utility maximization problem of a representative agent. The key insight in our aggregation result is to recognize that the presence of uninsured idiosyncratic risk induces stochastic shocks to the utility function of the representative agent as in Nakajima (2005).

Consider a representative agent with preferences defined by the utility function:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \nu_t \left[C_t^{\theta} (1-L_t)^{1-\theta} \right]^{1-\gamma} \tag{15}$$

where C_t is the amount of consumption of the composite good defined in (1) in period t, and L is the amount of labor supply in period t. Here, ν_t is the preference shock to the representative agent's utility in period t defined by

$$\nu_t \equiv \exp\left[\frac{1}{2} \gamma_c (\gamma_c - 1) \sum_{s=0}^{t} \sigma_{\eta,s}^2 \right] \tag{16}$$

where γ_c is defined in (5), and $\sigma_{\eta,t}$ is the standard deviation of the idiosyncratic shock in period t, as in (6). Note that ν_t is the cross-sectional average of $\eta_{i,t}^{1-\gamma_c}$:

$$\nu_t = \frac{E_t[\eta_{i,t}^{1-\gamma_c}]}{\eta_{i,-1}}$$

where E_t denotes the expectation operator conditional on the history of aggregate shocks up to and including period t.

Suppose that the representative agent faces the following flow budget constraint:

$$C_t + K_t + S_t = R_{k,t} K_{t-1} + R_{s,t} S_{t-1} + w_t L_t \tag{17}$$

and initial conditions $K_{-1}, S_{-1} > 0$. Here K_t and S_t are the amount of physical capital and the value of stocks held by the representative agent in period t. We assume the short-selling constraints: $K_t, S_t \geq 0$. These two constraints do not bind in equilibrium. Given prices and the initial condition, the representative agent chooses a contingent plan $\{C_t, L_t, K_t, S_t\}$ so as to maximize lifetime utility U_0 in (15) subject to the sequence of flow budget constraints (17) and short-selling constraints.

The Lagrangian for this problem is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \nu_t \Big\{ \frac{1}{1-\gamma} \left[C_t^{\theta} (1-L_t)^{1-\theta} \right]^{1-\gamma} + \lambda_t \left[R_{k,t} K_{t-1} + R_{s,t} S_{t-1} + w_t L_t - C_t - K_t - S_t \right] \Big\}$$

and the first-order conditions are given by

$$\theta C_t^{-\gamma_c}(1-L_t)^{(1-\theta)(1-\gamma)} = \lambda_t \tag{18}$$

$$\frac{1-\theta}{\theta}\frac{C_t}{1-L_t} = w_t \tag{19}$$

$$\lambda_t = E_t \beta \frac{\nu_{t+1}}{\nu_t}\lambda_{t+1}R_{k,t+1} \tag{20}$$

$$\lambda_t = E_t \beta \frac{\nu_{t+1}}{\nu_t}\lambda_{t+1}R_{s,t+1} \tag{21}$$

along with the flow budget constraint (17). The transversality condition for K_t and S_t are, respectively,

$$E_0\beta^t\nu_t\lambda_t K_t = 0 \tag{22}$$

$$E_0\beta^t\nu_t\lambda_t S_t = 0 \tag{23}$$

Given the initial conditions K_{-1} and S_{-1}, a solution to the utility maximization problem of the representative agent is given by $\{C_t, L_t, K_t, S_t, \lambda_t\}$ that satisfies the first-order conditions (17)-(21), as well as the transversality conditions (22)-(23).

The next proposition establishes that the solution to the utility maximization problem of the representative agent, and the solution to the utility maximization problem of each individual described in the previous subsection are the same.

Proposition 1. *Given stochastic processes $\{R_{k,t}, R_{s,t}, w_t, \sigma_{\eta,t}\}$ and initial conditions $\{K_{-1}, S_{-1}\}$, consider the utility maximization problem of individual i described in the previous subsection and the utility maximization problem of the representative agent described in this subsection. Suppose that $\{C_t^*, L_t^*, K_t^*, S_t^*, \lambda_t^*\}_{t=0}^{\infty}$ is a solution to the representative agent's problem. For each $i \in [0,1]$, suppose that the initial conditions have the following form: $\int_0^1 \eta_{i,-1} = 1$, $k_{i,-1} = \eta_{i,-1}K_{-1}$ and $s_{i,-1} = \eta_{i,-1}S_{-1}$. Let $c_{i,t}^* = \eta_{i,t}C_t^*$, $l_{i,t}^* = L_t^*$, $k_{i,t}^* = \eta_{i,t}K_t^*$, $s_{i,t}^* = \eta_{i,t}S_t^*$, and $\lambda_{i,t}^* = \eta_{i,t}^{-\gamma_c}\lambda_t^*$. Then $\{c_{i,t}^*, l_{i,t}^*, k_{i,t}^*, s_{i,t}^*, \lambda_{i,t}^*\}_{t=0}^{\infty}$ is a solution to the problem of individual i.*

Proof. Take stochastic processes $\{R_{k,t}, R_{s,t}, w_t, \sigma_{\eta,t}\}$ and initial conditions $\{K_{-1}, S_{-1}\}$ as given. Suppose that $\{C_t^*, L_t^*, K_t^*, S_t^*, \lambda_t^*\}_{t=0}^{\infty}$ is a solution to the representative agent's problem. Then it satisfies the first-order conditions, (17)-(21), as well as the transversality conditions, (22)-(23). For each $i \in [0,1]$, let $c_{i,t}^* = \eta_{i,t}C_t^*$, $l_{i,t}^* = L_t^*$, $k_{i,t}^* = \eta_{i,t}K_t^*$, $s_{i,t}^* = \eta_{i,t}S_t^*$, and $\lambda_{i,t}^* = \eta_{i,t}^{-\gamma_c}\lambda_t^*$. Then it is straightforward to see that these satisfy the first-order conditions, (7), (9)-(12), and the transversality conditions, (13)-(14), for the problem of individual i. This completes the proof. □

Proposition 1 applies in a setting where agents are ex ante homogeneous $\eta_{i,-1} = \eta_{-1}$. But it also applies in situations where there are ex ante differences among individuals. This second setting will be of interest when we consider the optimal monetary policy problem below.

Proposition 1 also has a number of important implications. First, individual labor allocations are identical across all agents. Note also that $\frac{c_{i,t}}{\eta_{i,-1}}$ is i.i.d. across agents in all periods as in e.g. Constantinides and Duffie (1996), Krebs (2003), and Heathcote, Storesletten and Violante

(2008). It follows from these two properties that, in equilibrium, the utility function of the representative agent (15) is proportionate to the cross-sectional average of individual utility given in equation (4):

$$\begin{aligned}\int_i u_{i,0} di &= \int_i \left[E_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} c_{i,t}^{1-\gamma_c} (1-l_{i,t})^{(1-\theta)(1-\gamma)} \right] di \\ &= \int_i \left[E_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \eta_{i,t}^{1-\gamma_c} C_t^{1-\gamma_c} (1-L_t)^{(1-\theta)(1-\gamma)} \right] di \\ &= \left(\int_i \eta_{i,-1}^{1-\gamma_c} di \right) E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \nu_t C_t^{1-\gamma_c} (1-L_t)^{(1-\theta)(1-\gamma)} \\ &= \left(\int_i \eta_{i,-1}^{1-\gamma_c} di \right) U_0 \end{aligned} \tag{24}$$

Second, by appealing to Proposition 1, it is possible to see in a very transparent way how the size of idiosyncratic shocks, $\sigma_{\eta,t}$, affect the aggregate dynamics of the economy. Let us define the "effective discount factor" between periods t and $t+1$, $\tilde{\beta}_{t,t+1}$, as

$$\tilde{\beta}_{t,t+1} \equiv \beta \frac{\nu_{t+1}}{\nu_t} = \beta \exp \left[\frac{1}{2} \gamma_c (\gamma_c - 1) \sigma_{\eta,t+1}^2 \right] \tag{25}$$

where the second equality follows from (16). This expression illustrates that the presence of idiosyncratic shocks ($\sigma_{\eta,t} > 0$) makes the effective discount factor higher if $\gamma_c > 1$ and lower if $\gamma_c < 1$. These results are associated with relative prudence, which is $1 + \gamma_c$ here. As is well known, if relative prudence is greater (less) than 2 the demand for a risky asset will increase (decrease) with the risk of the asset (see e.g. Gollier (2001)). This effect is reflected here in the relationship between the effective discount factor $\tilde{\beta}_{t,t+1}$ and the size of the idiosyncratic risk $\sigma_{\eta,t+1}^2$ in (25). Note also that cyclical fluctuations in the variance of idiosyncratic shocks, $\sigma_{\eta,t}^2$, induce cyclical variations in the effective discount factor $\tilde{\beta}_{t,t+1}$.

Generally speaking, in incomplete markets economies agents have different consumptions and thus price future cash flows in different ways.[11] A third implication of Proposition 1 though is that in our economy individuals agree on the present value of future dividends of each firm. This is due to the fact that the intertemporal marginal rate of substitution for each individual is independent of the history of idiosyncratic shocks. To see this, note that the stochastic discount factor used by individual i is

$$\begin{aligned}\beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} &= \beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\eta_{i,t+1}}{\eta_{i,t}} \right)^{-\gamma_c} \\ &= \beta \frac{\lambda_{t+1}}{\lambda_t} \exp \left(-\gamma_c \sigma_{\eta,t+1} \epsilon_{\eta,i,t+1} + \frac{\gamma_c}{2} \sigma_{\eta,t+1}^2 \right)\end{aligned}$$

Since $\epsilon_{\eta,i,t+1}$ is i.i.d. across individuals and independent of the stochastic shocks faced by each firm, all individuals value a given future payoff in the same way. In particular, we can use the stochastic discount factor of the representative agent, $\beta \lambda_{t+1} \nu_{t+1} / (\lambda_t \nu_t)$, to value future dividend streams of firms.

[11]See e.g. Magill and Quinzii (1996) for a discussion on this point.

Finally, note that the fact that agents agree about the value of each firm under the allocations described in Proposition 1 also implies that our initial assumption that individuals hold equal shares of all firms, $e_{i,j,t} = e_{i,t}$ for all $j \in [0,1]$, is indeed consistent with utility maximization of each individual.[12]

2.3 Firms

The production side of our economy is standard in the New Keynesian literature and similar to the one considered by Schmitt-Grohé and Uribe (2007). Each differentiated product is produced by a single firm in a monopolistically competitive environment. Firm $j \in [0,1]$ has the production technology:

$$Y_{j,t} = z_t^{1-\alpha} K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} - \Phi_t \tag{26}$$

where z_t is the aggregate productivity shock, $K_{j,t}$ is the physical capital used by firm j in period t, $L_{j,t}$ is its labor input, and Φ_t is the fixed cost of production. The market clearing conditions for capital and labor are

$$\int_0^1 K_{j,t}\, dj = K_{t-1}, \qquad \text{and} \qquad \int_0^1 L_{j,t}\, dj = L_t$$

Here, note that the stock of capital available for production in period t is K_{t-1}. The processes for z_t and Φ_t are specified in the next subsection.

Consider the cost minimization problem of firm j:

$$\min_{K_{j,t}, L_{j,t}} \; w_t L_{j,t} + r_t K_{j,t}, \qquad \text{s.t.} \quad z_t^{1-\alpha} K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} - \Phi_t = Y_{j,t}$$

Since, all firms choose the same capital labor ratio, the first-order conditions of their cost-minimization problems are identical

$$w_t = \mathrm{mc}_t (1-\alpha) z_t^{1-\alpha} K_{t-1}^{\alpha} L_t^{-\alpha} \tag{27}$$

$$r_t = \mathrm{mc}_t\, \alpha z_t^{1-\alpha} K_{t-1}^{\alpha-1} L_t^{1-\alpha} \tag{28}$$

where mc_t is marginal cost which is given by:

$$\mathrm{mc}_t = \alpha^{-\alpha} (1-\alpha)^{-1+\alpha} z_t^{\alpha-1} w_t^{1-\alpha} r_t^{\alpha}$$

The price of each variety is adjusted in a sluggish way as in Calvo (1983) and Yun (1996). For each firm, the opportunity to change the price of its product arrives with probability $1-\xi$ in each period. This random event occurs independently across firms (it is also independent of all other stochastic shocks in our economy). Without such an opportunity, a firm must charge the same price as in the previous period. Suppose that firm j obtains an opportunity to change its price in period t. It chooses $P_{j,t}$ to maximize the present discounted value of profits:

$$\max_{P_{j,t}} E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}\nu_{t+s}}{\lambda_t \nu_t} \xi^s \left[\left(\frac{P_{j,t}}{P_{t+s}} \right)^{1-\zeta} Y_{t+s} - \mathrm{mc}_{t+s} \left\{ \left(\frac{P_{j,t}}{P_{t+s}} \right)^{-\zeta} Y_{t+s} + \Phi_{t+s} \right\} \right]$$

[12] We do not pursue this here but in principle there could be other equilibria in which portfolios differ across individuals.

where $\beta^s \lambda_{t+s}\nu_{t+s}/(\lambda_t \nu_t)$ is the stochastic discount factor used to evaluate (real) payoffs in period $t+s$ in units of consumption in period t.

All firms with the opportunity to change their prices will choose the same price, so denote it by $\tilde{P}_t$. Then the first-order condition for the above profit-maximization problem is given by

$$E_t \sum_{s=0}^{\infty} (\xi\beta)^s \frac{\lambda_{t+s}\nu_{t+s}}{\lambda_t \nu_t} \left\{ (1-\zeta)\tilde{P}_t^{-\zeta} P_{t+s}^{\zeta-1} Y_{t+s} + \zeta \, \mathrm{mc}_{t+s} \, \tilde{P}_t^{-\zeta-1} P_{t+s}^{\zeta} Y_{t+s} \right\} = 0$$

Define $\tilde{\nu}_{t+s}$ as

$$\tilde{\nu}_{t+s} \equiv \frac{\nu_{t+s}}{\nu_t} = \exp\left\{ \frac{1}{2}\gamma_c(\gamma_c - 1) \sum_{u=t+1}^{t+s} \sigma_{\eta,u}^2 \right\}$$

Then, after some algebra, we can rewrite the first-order condition for $\tilde{P}_t$ as

$$x_t^1 = \frac{\zeta - 1}{\zeta} \tilde{p}_t x_t^2 \tag{29}$$

where

$$\begin{aligned}
\tilde{p}_t &\equiv \frac{\tilde{P}_t}{P_t} \\
x_t^1 &\equiv E_t \sum_{s=0}^{\infty} (\xi\beta)^s \lambda_{t+s}\tilde{\nu}_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\zeta} Y_{t+s} \, \mathrm{mc}_{t+s} \\
x_t^2 &\equiv E_t \sum_{s=0}^{\infty} (\xi\beta)^s \lambda_{t+s}\tilde{\nu}_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\zeta-1} Y_{t+s}
\end{aligned}$$

It is convenient to express x_t^1 and x_t^2 in a recursive fashion:

$$x_t^1 = \lambda_t Y_t \, \mathrm{mc}_t + \xi\beta E_t \tilde{\nu}_{t+1} \pi_{t+1}^{\zeta} x_{t+1}^1 \tag{30}$$

$$x_t^2 = \lambda_t Y_t + \xi\beta E_t \tilde{\nu}_{t+1} \pi_{t+1}^{\zeta-1} x_{t+1}^2 \tag{31}$$

where π_{t+1} is the gross inflation rate between periods t and $t+1$:

$$\pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$$

Since all firms that adjust their prices in a given period choose the same new price, $\tilde{P}_t$, equation (3) implies that the price index, P_t, evolves as

$$P_t^{1-\zeta} = \xi P_{t-1}^{1-\zeta} + (1-\xi)\tilde{P}_t^{1-\zeta}$$

which can be rewritten as

$$1 = \xi \pi_t^{-1+\zeta} + (1-\xi)\tilde{p}_t^{1-\zeta} \tag{32}$$

To derive the aggregate production function, rewrite the production function of individual firms (26) as

$$z_t^{1-\alpha} K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} - \Phi_t = \left(\frac{P_{j,t}}{P_t}\right)^{-\zeta} Y_t$$

Using the fact that $K_{j,t}/L_{j,t}$ is the same for all j, and integrating both sides of this equation yields

$$\varsigma_t Y_t = z_t^{1-\alpha} K_{t-1}^{\alpha} L_t^{1-\alpha} - \Phi_t \tag{33}$$

where $\varsigma_t \leq 1$ measures the inefficiency due to price dispersion:

$$\varsigma_t = \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\zeta} dj$$

The evolution of ς_t can be written as

$$\varsigma_t = (1-\xi)\tilde{p}_t^{-\zeta} + \xi \pi_t^{\zeta} \varsigma_{t-1} \tag{34}$$

The aggregate consumption, investment and capital stock satisfy

$$Y_t = C_t + I_t \tag{35}$$

$$K_t = I_t + (1-\delta)K_{t-1} \tag{36}$$

2.4 Aggregate shocks

We consider two specifications of the aggregate productivity shock. One specification we consider is a permanent productivity shock. In particular, we assume that z_t follows a geometric random walk:

$$\ln z_t = \ln z_{t-1} + \mu + \sigma_z \epsilon_{z,t} - \frac{\sigma_z^2}{2} \tag{37}$$

and the fixed cost of production, Φ_t, grows at the rate μ:

$$\Phi_t = \Phi \exp(\mu t) \tag{38}$$

where μ and σ_z are constant parameters, and $\epsilon_{z,t}$ is $N(0,1)$ and i.i.d. across periods. The other specification we consider is a temporary but persistent productivity shock. We assume that z_t follows an AR(1) process:

$$\ln z_t = \rho_z \ln z_{t-1} + \sigma_z \epsilon_{z,t} - \frac{\sigma_z^2}{2(1+\rho_z)} \tag{39}$$

and that the fixed cost is constant:

$$\Phi_t = \Phi \tag{40}$$

For both specifications, the constant Φ is calibrated so that the aggregate profit is zero in the non-stochastic steady state (balanced growth path) with zero inflation.

The standard deviation of innovations to individual labor productivity, $\sigma_{\eta,t}$, is also an aggregate shock. It acts like a preference discount rate shock to the representative agent. Evidence provided by Storesletten, Telmer and Yaron (2004) and Meghir and Pistaferri (2004) suggests that idiosyncratic risk is countercyclical. Krebs (2003) and De Santis (2007) have found that the welfare cost of business cycles can be sizable with countercyclical idiosyncratic risk. The only other aggregate shock in our economy is a shock to the aggregate state of technology. If we allow for a negative correlation between $\sigma_{\eta,t}$ and the aggregate technology shock, idiosyncratic risk will be countercyclical.[13] Specifically, when the evolution of the aggregate productivity is given by (37), we assume that the variance of idiosyncratic shocks evolves as

$$\sigma_{\eta,t}^2 = \bar{\sigma}_\eta^2 + b\sigma_z\epsilon_{z,t} \tag{41}$$

and when z_t follows the temporary process given by (39), we assume that

$$\sigma_{\eta,t}^2 = \bar{\sigma}_\eta^2 + b\ln z_t \tag{42}$$

An important difference between the two specifications of technology shocks is that $\sigma_{\eta,t}^2$ is serially correlated in (42) but not in (41). By combining equation (41) or alternatively (42) with equation (25) one can show that the effective preference discount factor inherits these properties. Under the specification with permanent shocks it is given by:

$$\ln\tilde{\beta}_{t,t+1} = \ln\beta + \frac{1}{2}\gamma_c(\gamma_c - 1)(\bar{\sigma}_\eta^2 + b\sigma_z\epsilon_{z,t+1}) \tag{43}$$

And under the assumption of temporary but persistent shocks it is

$$\ln\tilde{\beta}_{t,t+1} = \ln\beta + \frac{1}{2}\gamma_c(\gamma_c - 1)(\bar{\sigma}_\eta^2 + b\ln z_{t+1}) \tag{44}$$

From these two equations we can see that the law of motion of the effective discount factor for the representative agent has an explicit link to the law of motion of the variance of idiosyncratic shocks. In this sense our model provides explicit micro-foundations for shocks to the subjective discount factor that have been found to be an important source of business cycle variation in the New Keynesian models of Smets and Wouters (2003), Levin, Onatski, Williams and Williams (2005), and Burriel, Fernández-Villaverde and Rubio-Ramírez (2009) among others. Although we don't pursue this here one could investigate the extent to which the estimates that are based on macro data in these papers are consistent with micro observations. One could also investigate how imposing the cross-equation restrictions that emerge from our model affect the properties of the estimated parameters in these other representative agents economies and the significance of this type of shock in accounting for business cycle fluctuations.

2.5 Monetary policy

Government policy is very simple in our economy. First, we abstract from fiscal policy: the government does not consume, and there are no government bonds or taxes. Second, we assume that the monetary authority can directly control the inflation rate. Thus, monetary policy is specified as a state contingent path of the inflation rate, $\{\pi_t\}_{t=0}^{\infty}$.

[13] We are not asserting anything here about the direction of causality. We are following the literature we cited above and abstracting from a formal model that links idiosyncratic risk to the level of aggregate technology. But we can imagine situations in which the causality goes in either direction.

2.6 Definition of equilibrium

The definition of equilibrium for the economy proceeds in two steps. First we define an equilibrium for the representative agent economy. That equilibrium determines aggregate allocations and prices. Then in a second step we show how to derive the individual allocations.

Definition 1. *A representative agent equilibrium consists of a set of stochastic processes for* $\{C_t, L_t, K_t, I_t, Y_t, S_t, \lambda_t, \mathrm{mc}_t, w_t, r_t, R_{s,t}, R_{k,t}, x_t^1, x_t^2, \tilde{p}_t, \varsigma_t\}$ *that satisfy equations (8), (17), (18), (19), (20), (21), (27), (28), (29), (30), (31), (32), (33), (34), (35), and (36) and the transversality conditions (22) and (23) for given* $\{K_{-1}, \varsigma_{-1}\}$*, laws of motion for the exogenous shocks and monetary policy,* $\{\pi_t\}_{t=0}^{\infty}$*.*

The aggregate allocations from the representative agent equilibrium can be used to derive the individual allocations in the following way. Under the assumption of Proposition 1, the initial wealth distribution is given by: $k_{i,-1} = \eta_{i,-1}K_{-1}$ and $s_{i,-1} = \eta_{i,-1}S_{-1}$. Then Proposition 1 implies that the individual allocations for $t = 0, 1, 2, ...$ are given by $c_{i,t} = \eta_{i,t}C_t$, $l_{i,t} = L_t$, $k_{i,t} = \eta_{i,t}K_t$, $s_{i,t} = \eta_{i,t}S_t$, and $\lambda_{i,t} = \eta_{i,t}^{-\gamma_c}\lambda_t$.

2.7 Optimal monetary policy

We consider optimal "Ramsey" monetary policies, where a benevolent monetary authority precommits to a state-contingent path of the inflation rate so as to maximize a weighted average of utility of individuals subject to the restriction that the resulting allocation can be supported as a competitive equilibrium. In the discussion that follows we will explicitly rule out policies such as agent-specific lump-sum transfers or labor or capital taxation. All of these policies fall in the realm of fiscal policy.

Let χ_i denote the Pareto weights which are assumed to be positive $\forall i$ and satisfy $\int_i \chi_i di = 1$. Then the objective function for a benevolent monetary authority can be expressed as:

$$\int_i \chi_i u_{i,0} di = \int_i \chi_i \left[E_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} c_{i,t}^{1-\gamma_c} (1 - l_{i,t})^{(1-\theta)(1-\gamma)} \right] di$$

Generally speaking, the Ramsey planner chooses individual allocations and it is necessary to impose all of the competitive equilibrium restrictions simultaneously. However, the competitive allocations in our economy have two properties that allow us to also factor the objective function into two parts. First, in equilibrium all individuals make identical labor supply decisions, $l_{i,t} = L_t$. Second, $\frac{c_{i,t}}{\eta_{i,-1}}$ is i.i.d. across individuals in all periods. Both of these properties follow from Proposition 1.

Proposition 2. *For all choices of* χ_i *that satisfy* $\chi_i > 0, \forall i$ *and* $\int_i \chi_i di = 1$ *the objective function for the Ramsey planner's problem is* U_0 *in* (15)*.*

Proof. Given that $c_{i,t} = \eta_{i,t} C_t$ and $l_{i,t} = L_t$ for all i in equilibrium, we obtain

$$\begin{aligned}
\int_i \chi_i u_{i,0} di &= \int_i \chi_i \left[E_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \eta_{i,t}^{1-\gamma_c} C_t^{1-\gamma_c} (1-L_t)^{(1-\theta)(1-\gamma)} \right] di \qquad (45) \\
&= \left(\int_i \chi_i \eta_{i,-1}^{1-\gamma_c} di \right) E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \nu_t C_t^{1-\gamma_c} (1-L_t)^{(1-\theta)(1-\gamma)} \\
&= \left(\int_i \chi_i \eta_{i,-1}^{1-\gamma_c} di \right) U_0
\end{aligned}$$

Observe that the term in parenthesis in the final line is a constant that is independent of policy. □

From the proof we can see that individuals are both ex ante and ex post different. For our Ramsey planner, who must honor the restrictions of an incomplete markets equilibrium, these differences get reflected in the constant term. All agents face the same distribution of future consumption growth at all points of time and are proportional to each other. Thus, any manipulation of the price system will affect all agents in the same way. It follows that there is no opportunity for the monetary authority to affect equity in this incomplete markets economy.

Proposition 2 makes it possible to solve for the optimal monetary policy using the same two step procedure that we used to solve the competitive equilibrium. First, we solve a representative agent Ramsey problem. Then in a second step we derive the individual allocations.

Definition 2. *The representative agent Ramsey problem is to maximize U_0 in (15) by choice of the inflation rate $\{\pi_t\}$ subject to (8), (17), (18), (19), (20), (21), (27), (28), (29), (30), (31), (32), (33), (34), (35), and (36) and the transversality conditions (22) and (23) for given $\{K_{-1}, \varsigma_{-1}\}$, and laws of motion for the exogenous shocks.*

Note that conditional on a choice of $\{\pi_t\}$, the remaining equilibrium prices and aggregate quantities are indirectly determined via the constraints. Then the individual allocations can be derived using the same strategy described in the definition of equilibrium above. There is a well known time consistency issue in this class of problem. In the numerical analysis that follows we consider the optimal policy from the timeless perspective as proposed by Woodford (2003).

3 Results

In this section we analyze how the presence of idiosyncratic shocks affects the properties of the optimal monetary policy. We are particularly interested in the case where the idiosyncratic risk, $\sigma_{\eta,t}$, fluctuates countercyclically. We show that even though countercyclical idiosyncratic risk makes the welfare cost of business cycles sizable, properties of the optimal monetary policy are little affected by the presence of idiosyncratic shocks. Namely, the optimal monetary policy is roughly characterized as the zero-inflation policy.

3.1 Analytic results

Let us first consider the case where fiscal policy eliminates the monopoly distortion at the zero-inflation steady state as in Woodford (2003) and Galí (2008). Specifically, suppose that each monopolist's revenue is subsidized at a rate τ, that the subsidies are financed by lump-sum taxes, T_t, on monopolists, and that there are no fixed costs, $\Phi = 0$. Then, net of the tax and subsidy, each monopolist's profit is

$$(1+\tau)\frac{P_{j,t}}{P_t}Y_{j,t} - w_t L_{j,t} - r_t K_{j,t} - T_t$$

where $T_t = \int \tau \frac{P_{j,t}}{P_t} Y_{j,t}\, dj$ to balance the government's budget. If we assume that

$$\tau = \frac{1}{\zeta - 1} \tag{46}$$

then the monopoly distortion is eliminated at the zero-inflation steady state. Let the stochastic processes for $\{z_t\}$ and $\{\sigma^2_{\eta,t}\}$ be given either by (37) and (41), or by (39) and (42), respectively.

Now consider our model with heterogeneous agents under incomplete markets. Market incompleteness introduces a new distortion and thus, in principle, the possibility that there might be a trade off for monetary policy between correcting this distortion and the distortions that arise from costly price adjustment and imperfect competition. However, from Proposition 1 we know that our incomplete markets economy has a representative agent representation in which there are shocks to technology and preferences. It then follows using exactly the same reasoning as Woodford (2003) and Galí (2008) that price stabilization is the optimal monetary policy.

Proposition 3. *Assume that subsidies to the monopolists are given at the rate $\tau = \frac{1}{\zeta-1}$, which are financed by lump-sum taxes on the monopolists. Suppose also that the economy is initially at the zero-inflation steady state. Then the solution to the Ramsey problem is given by*

$$\pi_t = 1,$$

at all dates, under all contingencies and for all Pareto weights.

3.2 Quantitative results

Now let us now consider the case with no subsidy: $\tau = T_t = 0$. With the monopoly distortion, setting the inflation rate to zero at all dates is no longer optimal. The main question asked in this subsection is how different the optimal monetary policy is from the zero-inflation policy. The answer to this question is not immediately obvious. On the one hand, the results we have describe above show that there are no opportunities for an optimal monetary policy to affect equity. However, the same opportunities to enhance efficiency that arise in representative agent models are also present here. Moreover, there is an important difference between our representative agent specification and that considered by e.g. Schmitt-Grohé and Uribe (2007). The effective preference discount factor is correlated with the technology shock and the nature of this dependence varies with the value of γ_c and the law of motion of idiosyncratic risk. It turns out that this distinction can have a first order impact on the welfare cost of business cycles when the variance of idiosyncratic risk is countercyclical. This result occurs when we use an individual's utility function to assess the welfare. From equation (24) we can see that this result also applies when we use the utility function of the representative agent to evaluate welfare.

The parameter values of our model are calibrated as follows. One period in the model corresponds to a quarter. The share of capital is $\alpha = 0.36$, and the depreciation rate is $\delta = 0.02$. These are taken from Boldrin, Christiano and Fisher (2001). The probability of price adjustment is set to 0.2, i.e., $\xi = 0.8$ and the elasticity of substitution across different varieties of products is $\zeta = 5$, following Schmitt-Grohé and Uribe (2007). The fixed cost of production, $\bar{\Phi}$, is set so that the profit of each firm at the non-stochastic steady state under optimal monetary policy is zero. The discount factor β is chosen so that the real interest rate at the non-stochastic steady state is four percent a year. For the preference parameter, we consider two values for γ_c, 0.7 and 2. For each value of γ_c, another preference parameter θ is set so that the labor supply at the stochastic steady state is one third (then, γ is determined as $\gamma = 1 - (1 - \gamma_c)/\theta$). For the case of permanent productivity shock (37), we follow Boldrin, Christiano and Fisher (2001) and set $\mu = 0.004$, and $\sigma_z = 0.018$. For the case of a temporary productivity shock (39), we follow Schmitt-Grohé and Uribe (2007)[14] and set $\rho_z = 0.8556$ and $\sigma_z = 0.0064/(1 - \alpha)$. For the idiosyncratic shock process, we follow De Santis (2007) and set $\bar{\sigma}_\eta = 0.1/2$ and $b = 0$ or $b = -0.8$. It turns out that as long as we adjust β so as to keep the steady state interest rate fixed (i.e., four percent a year), the value of σ_η does not matter. When $b = 0$, the idiosyncratic risk is acyclical; when $b = -0.8$, it is countercyclical. De Santis (2007) chooses $b = -0.8$ based on the evidence provided by Storesletten, Telmer and Yaron (2004).

In what follows, we compare dynamics of different versions of our model economy, which differ in terms of the risk aversion parameter, $\gamma_c \in \{0.7, 2\}$; the cyclicality of the idiosyncratic risk, $b \in \{0, -0.8\}$; the persistence of the aggregate productivity shock, (37) and (39); or the monetary policy: Ramsey and zero inflation-targeting. In addition, for each value of γ_c and b, and for each process for z_t, we compute two normative measures of welfare costs.

The first one is the welfare cost of business cycles as originally estimated by Lucas (1987). Specifically, we consider the real-business-cycle version of our model, in which there are no nominal rigidities, and compare the economy with positive aggregate shocks, $\sigma_z > 0$, and the economy without aggregate shocks, $\sigma_z = 0$. In both cases we assume that there are idiosyncratic shocks, $\bar{\sigma}_\eta > 0$. We also assume that both economies are at the non-stochastic steady state prior to date 0 and compare the welfare conditional on the state vector at $t = -1$.[15] Let X_t denote the vector of the state variables, and let $\bar{X}$ denote its value at the non-stochastic steady state. Further, let $\{C_t^{\text{rbc}}, L_t^{\text{rbc}}\}$ denote the equilibrium process of aggregate consumption and labor supply in the RBC version of our economy, and let $\{\bar{C}, \bar{L}\}$ denote their values in the steady

[14]Note that the productivity level z_t in Schmitt-Grohé and Uribe (2007) corresponds to our $z_t^{1-\alpha}$, so that their standard deviation must be adjusted by $1/(1 - \alpha)$.

[15]In this sense, we are measuring conditional welfare costs. Schmitt-Grohé and Uribe (2007) discuss a related issue.

state. Then, define lifetime utility evaluated at period $t = -1$ by

$$V(\bar{X}, \sigma_z; \mathrm{rbc}) \equiv E_{-1} \sum_{t=0}^{\infty} \beta^t \nu_t \frac{1}{1-\gamma} \left[(C_t^{\mathrm{rbc}})^{\theta} (1 - L_t^{\mathrm{rbc}})^{1-\theta} \right]^{1-\gamma}$$

where ν_t is given by (16). The corresponding value for the non-stochastic economy is given by

$$V(\bar{X}, 0; \mathrm{rbc}) = \sum_{t=0}^{\infty} \beta^t \bar{\nu}_t \frac{1}{1-\gamma} \left[(\bar{C})^{\theta} (1 - \bar{L})^{1-\theta} \right]^{1-\gamma}$$

where $\bar{\nu}_t$ is defined by

$$\bar{\nu}_t \equiv \exp \left[\frac{1}{2} \gamma_c (\gamma_c - 1) \bar{\sigma}_\eta^2 t \right]$$

The welfare cost of business cycles is defined by Δ_{bc} that solves

$$\sum_{t=0}^{\infty} \beta^t \bar{\nu}_t \frac{1}{1-\gamma} \left[((1-\Delta_{\mathrm{bc}})\bar{C})^{\theta} (1 - \bar{L})^{1-\theta} \right]^{1-\gamma} = V(\bar{X}, \sigma_z; \mathrm{rbc})$$

that is,

$$\Delta_{\mathrm{bc}} = 1 - \left\{ \frac{V(\bar{X}, \sigma_z; \mathrm{rbc})}{V(\bar{X}, 0; \mathrm{rbc})} \right\}^{\frac{1}{1-\gamma_c}}$$

The second welfare cost measure is the cost of adopting a non-optimal policy (the zero inflation-targeting policy) as opposed to the optimal monetary policy (the Ramsey policy). Somewhat abusing notation, we again use $\bar{X}$ to denote the non-stochastic steady state under the Ramsey policy. It turns out that the steady-state inflation rate under the Ramsey policy is zero. Therefore, $\bar{X}$ is also the non-stochastic steady state associated with the inflation-targeting policy. Suppose that the economy is at the steady state $\bar{X}$ prior to date 0. Then the welfare cost of the inflation-targeting policy, Δ_{inf}, is given as

$$\Delta_{\mathrm{inf}} = 1 - \left\{ \frac{V(\bar{X}, \sigma_z; \mathrm{inf})}{V(\bar{X}, \sigma_z; \mathrm{ram})} \right\}^{\frac{1}{1-\gamma_c}}$$

where $V(\bar{X}, \sigma_z; \mathrm{inf})$ and $V(\bar{X}, \sigma_z; \mathrm{ram})$ are the lifetime utility associated with the inflation-targeting and Ramsey monetary policies, respectively.

3.2.1 The specification with permanent productivity shocks

Table 1 reports the welfare cost of business cycles, Δ_{bc}, for $\gamma_c = 0.7, 2$ and for $b = 0, -0.8$. When risk aversion is relatively low, $\gamma_c = 0.7$, the welfare cost of business cycles is negative. That is, expected utility is higher when $\sigma_z > 0$ than when $\sigma_z = 0$. Furthermore, in this case, making the idiosyncratic risk countercyclical decreases the welfare cost of business cycles. That is, it increases the welfare gain of business cycles.

These results are similar in nature to a previous finding by Cho and Cooley (2005). They show that a mean-preserving increase of the variance of technology shocks can improve welfare.

To see why, remember that the indirect utility function of a consumer is quasi-convex in prices, and note that technology shocks create fluctuations in the wage rate, i.e., the price of leisure. Of course, the quasi-convexity of the indirect utility function is a partial-equilibrium property. But if this effect is strong enough, increasing the variance of the technology shock makes it possible for agents to concentrate their work effort in periods where their labor productivity is highest, which increases welfare.

On the other hand, when the relative risk aversion is higher, $\gamma_c = 2$, the welfare cost of business cycles is positive. Cyclical fluctuations in $\sigma_{\eta,t}$ act to increase the welfare costs of business cycles. When $\gamma_c = 2$ and $b = -0.8$, the welfare cost of business cycles is about 7.3 percent of consumption, which is a sizable amount.

Table 1 also reports the welfare cost of adopting a strict zero inflation-targeting policy. Observe that the welfare cost of adopting the inflation-targeting policy is negligible for all values of γ_c and b. Even when $\gamma_c = 2$ and $b = -0.8$, it is only 0.0006 percent. For purposes of comparison the welfare cost of business cycles is 7.3 percent for that case. In this sense, under permanent productivity shocks, cyclical fluctuations in the idiosyncratic risk do not change the nature of the optimal monetary policy even when the welfare cost of business cycles is large.

3.2.2 The specification with temporary productivity shocks

Now consider the case where productivity shocks are temporary but persistent. Then the process for z_t is given by (39), and the variance of idiosyncratic shocks follows the process given by (42). This specification differs from the specification in the previous subsection in two important ways. First, the productivity process (39) is stationary. Second, since $\ln z_t$ is autocorrelated, so is $\sigma_{\eta,t}$. This introduces predictable variability in idiosyncratic risk, and thus, to the effective discount factor, which was i.i.d. in the previous subsection.

Specifically, the effective discount factor is now given by

$$\ln \tilde{\beta}_{t,t+1} = \ln \bar{\beta} + \frac{1}{2}\gamma_c(\gamma_c - 1) b \ln z_{t+1}$$

Its conditional expectation then becomes

$$E_t[\ln \tilde{\beta}_{t,t+1}] = \ln \bar{\beta} + \frac{1}{2}\gamma_c(\gamma_c - 1) b \left(\rho_z \ln z_t - \frac{\sigma_z^2}{2(1+\rho_z)} \right)$$

which fluctuates over time. Indeed, when $\gamma_c < 1$ and $b < 0$, the productivity shock today increases z_t as well as the expected value of the effective discount factor, $E_t[\ln \tilde{\beta}_{t,t+1}]$. On the other hand, when $\gamma_c > 1$ and $b < 0$, the shock increasing z_t decreases $E_t[\ln \tilde{\beta}_{t,t+1}]$.

Table 2 shows the welfare costs of business cycles, Δ_{bc}, for $\gamma_c = 0.7, 2$ and for $b = 0, -0.8$. As opposed to the case of permanent shocks in the previous subsection, when $b = 0$, Δ_{bc} is negative for the both values of γ_c. In addition, its absolute value is much smaller. As in the permanent-shock case, countercyclical idiosyncratic risk increases the welfare gain of business cycles for $\gamma_c = 0.7$, and increases the welfare cost of business cycles when $\gamma_c = 2$.

When $\gamma_c = 2$ and $b = -0.8$, the welfare cost of business cycles is sizable (12.2 percent), even though the productivity process is stationary. Note that the welfare costs of business cycles for this specification are about 5 percent larger than the case with permanent technology shocks. To see why the welfare costs are larger here consider a temporary negative shock to technology. If we abstract from variations in the preference discount factor the welfare costs of business cycles would be smaller in the presence of persistent but stationary technology shocks as compared to the case of permanent technology shocks. However, when technology shocks are stationary and persistent a negative technology shock has a second effect. It also increases the effective preference discount factor in a persistent way (see (44)). This second effect increases individuals' saving motives in a bad state and this acts to exacerbate consumption variations.

The welfare costs of price stabilization continue to be small when technology shocks are stationary. Comparing Table 2 with Table 1 we see that the welfare costs are larger when risk aversion is 2 and idiosyncratic risk is countercyclical. This is due to the persistent response of the effective discount factor. However, the size of the welfare cost of price stabilization is still quite small (0.0024 percent).

To summarize, with countercyclical idiosyncratic shocks, the welfare cost of business cycles can be sizable. However, this does not affect how monetary policy should be conducted. The optimal monetary policy is essentially a policy that stabilizes the inflation rate at zero.

3.2.3 The individual effects of cyclical variation in technology and variation in idiosyncratic risk

We have seen in Tables 1 and 2 that variation in technology and variation in idiosyncratic risk play distinct roles in our results and these roles vary with the persistence of the shocks to the effective discount factor. To further explore the effects of these two shocks consider Table 3 which reports results for a scenario where the effective discount factor is assumed to follow (43) or alternatively (44) but where variation in $\epsilon_{z,t}$ or z_t are not allowed to affect the state of technology. In terms of the original incomplete-markets economy, we are considering a situation where the variance of idiosyncratic risk fluctuates over time, but the aggregate state of technology is constant.

The first column of Table 3 reports results for the specification where the effective discount factor is given by (43) and thus i.i.d. Under this assumption cyclical variation in idiosyncratic risk produces negligible costs of business cycles. The costs of pursuing strict inflation targeting are also tiny. This is quite striking because from Table 1 we know that the combination of variation in technology and countercyclical variation in risk produces large costs of business cycles as compared to the case where there is only variation in technology. Thus, in this case, we need both shocks to generate a sizable cost of business cycles. To see this, note that the specification (43) implies that the inequality shock $\epsilon_{z,t}$ only affects $\tilde{\beta}_{t-1,t}$, the effective discount factor between $t-1$ and t. Future discount factors, $\tilde{\beta}_{s,s+1}$ for $s \geq t$, are not affected. Since $\epsilon_{z,t}$ does not affect technology in this experiment, it does not have a first-order effect on individuals' choices made in period t. That is, in this case, the cost of business cycles is small because business cycles generated by $\{\epsilon_{z,t}\}$ are very small.

Consider now the case where the effective discount factor follows (44) and is thus persistent.

For this specification cyclical variation in idiosyncratic risk is very important. Now the costs of business cycles are quite large. They are 11 percent when there is only cyclical variation in the effective discount factor as compared to 12 percent when there is also variation in the state of technology. Thus, in this specification, fluctuations in the state of technology are not essential to produce the sizable cost of business cycles. In addition, the welfare costs of inflation-targeting actually increase. However, the overall size of the costs of inflation targeting continue to be very small (0.0075 percent).

In terms of the cost of business cycles, what distinguishes the two cases in Table 3 is whether or not the effective discount factor is serially correlated. Consider (44) in which z_t is serially correlated. In this case the effective discount factor is also serially correlated. In particular, when $\gamma_c > 1$, an increase in inequality in period t (i.e., an increase in z_t) raises both $\tilde{\beta}_{t-1,t}$ and the expected value of $\tilde{\beta}_{t,t+1}$. The rise in the expected value of $\tilde{\beta}_{t,t+1}$ tends to reduce consumption today, C_t, and thus lowers the period-t utility flow. On the other hand, the increase in $\tilde{\beta}_{t-1,t}$ increases the weight on the period-t utility flow in the lifetime utility evaluation. Thus, a rise in inequality in period t lowers the current utility flow and, at the same time, increases its weight in the lifetime utility, which tends to make a recession a more miserable event. This is why we have a sizable cost of business cycles in this case even though the aggregate productivity is made constant. We can see that the serial correlation in the effective discount factor is crucial in this argument. Without it, a rise in inequality in period t would not cause a decline in C_t.

4 Conclusion

We conclude by briefly discussing the robustness of our results to some of our modeling assumptions. We have limited attention to technology shocks. Galí and Rabanal (2004) provide empirical evidence that suggests that technology shocks are not an important source of business cycle fluctuations. It is straightforward to extend the model to allow for other aggregate shocks to the markup, and/or government purchases. If either of these shocks is correlated with the variance of idiosyncratic risk instead, our results will still go through.

In this paper we have focused on sticky prices. With sticky wages, however, stabilizing the price level ceases to be optimal even in the representative agent framework. In the Appendix we extend our basic model to incorporate sticky wages show that our aggregation result still holds. There, the optimal monetary policy no longer takes the form of price-level stabilization, but the optimal monetary policy for the incomplete markets economy is still identical to the optimal monetary policy in the corresponding representative agent economy with preference shocks.

We have also assumed that the shock to labor and capital income is perfectly correlated. Reducing this correlation from one enhances the ability of individuals to self insure against either type of risk and thus reduces the need for implicit insurance. However, at the same time, when this correlation is reduced government policy has differential effects on individuals. This opens up the possibility for government policy to provide insurance by manipulating the price system.

Finally, we have followed the convention in the New Keynesian literature and abstracted from modeling the demand for money. This abstraction facilitates the derivation of our aggregation result. However, it also rules out a channel for monetary policy to affect household decisions. Inflation is not a tax that affects labor supply in the cashless New Keynesian economy.

5 Appendix: Adding sticky wages

In this Appendix we generalize our benchmark model to allow for both sticky prices and sticky wages. The setting we consider closely follows the sticky-wage model of Schmitt-Grohé and Uribe (2005). However, we extend their model to allow for heterogeneous agents.

Suppose that preferences of each individual are specified in the same way as in the benchmark model:

$$u_{i,0} = E_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left[c_{i,t}^{\theta} (1 - l_{i,t})^{1-\theta} \right]^{1-\gamma} \tag{47}$$

But now assume that an individual supplies a continuum of differentiated labor services $j \in [0, 1]$. Total labor supply of individual i is given by

$$l_{i,t} = \int_0^1 l_{i,t}(j)\, dj$$

where $l_{i,t}(j)$ is the amount of type-j labor supplied by individual i. As in the benchmark model, individuals differ in terms of their labor productivity, $\eta_{i,t}$. In terms of efficiency units, the supply of type-j labor of individual i in period t is $\eta_{i,t} l_{i,t}(j)$ for all $j \in [0, 1]$.

The production technology and the model of price adjustment of each firm is the same as in the benchmark model but we now assume that firms use a composite labor input made from differentiated labor services. Assume that the production technology of firm n is

$$Y_{n,t} = z_t^{1-\alpha} K_{n,t}^{\alpha} H_{n,t}^{1-\alpha} - \Phi_t$$

$H_{n,t}$ denotes the amount of composite labor input used by firm n in period t:

$$H_{n,t} = \left(\int_0^1 H_{n,t}(j)^{1-\frac{1}{\tilde{\zeta}}}\, dj \right)^{\frac{\tilde{\zeta}}{\tilde{\zeta}-1}}$$

where $\tilde{\zeta}$ is the elasticity of substitution across different labor services, and $H_{n,t}(j)$ is the amount of type-j labor used by firm n:

$$H_{n,t}(j) = \int_0^1 \eta_{i,t} l_{i,n,t}(j)\, di \tag{48}$$

In the above expression $l_{i,n,t}(j)$ denotes the amount of type-j labor supplied by individual i to firm n in period t.

The nominal wage rate of type-j labor is given by $W_t(j)$, and $w_t(j) \equiv W_t(j)/P_t$ the real wage rate. Finally, let H_t denote the aggregate amount of the composite labor input:

$$H_t \equiv \int_0^1 H_{n,t}\, dn$$

Then aggregate demand for type-j labor can be expressed as

$$\int_0^1 H_{n,t}(j)\,dn = \left[\frac{w_t(j)}{w_t}\right]^{-\tilde{\zeta}} H_t$$

where $w_t \equiv W_t/P_t$ with

$$W_t \equiv \left(\int_0^1 W_t(j)^{1-\tilde{\zeta}}\,dj\right)^{\frac{1}{1-\tilde{\zeta}}}$$

In each period, wage rates and hours worked are determined by the "labor union," as in Schmitt-Grohé and Uribe (2005), among others. The nominal wage rate of each type of labor, $W_t(j)$, is set by the union in the Calvo way. In each period, each nominal wage rate is adjusted with probability $(1-\tilde{\xi})$. If $W_t(j)$ is not adjusted in period t, then it is equal to its value in the previous period, i.e., $W_t(j) = W_{t-1}(j)$. In each labor market j, the union takes H_t and W_t as exogenous. Given the wage rates, $W_t(j)$, the union next sets hours worked for each individual so as to satisfy the demands for each type of labor. For simplicity, we assume that the union allocates hours worked equally across individuals, that is,

$$l_{i,t}(j) = L_t(j), \qquad \text{for all } i \in [0,1].$$

Since the cross-sectional average of $\eta_{i,t}$ is unity for each period t, it follows from (48) that

$$\int_0^1 H_{n,t}(j)\,dn = L_t(j), \qquad \text{for each } i \in [0,1].$$

Thus the demand function for each type of labor that the union faces is expressed as

$$L_t(j) = \left[\frac{w_t(j)}{w_t}\right]^{-\tilde{\zeta}} H_t \tag{49}$$

It follows that the utility maximization problem for each individual is the same as in the benchmark model except that he/she no longer chooses hours worked by himself/herself. As in the benchmark model, the initial condition for each individual is assumed to satisfy $k_{i,-1} = \eta_{i,-1}K_{-1}$ and $s_{i,-1} = \eta_{i,-1}S_{-1}$ with $\int_0^1 \eta_{i,-1}\,di = 1$. Let $\{L_t(j) : j \in [0,1]\}_{t=0}^{\infty}$ denote the supply of type-j labor assigned by the union. Then, given prices, and the assigned hours worked, each individual i chooses $\{c_{i,t}, k_{i,t}, s_{i,t}\}$ so as to maximize $u_{i,0}$ in (47) subject to

$$l_{i,t} = \int_0^1 L_t(j)\,dj$$

$$c_{i,t} + k_{i,t} + s_{i,t} = \frac{\eta_{i,t}}{\eta_{i,t-1}}(R_{k,t}k_{i,t-1} + R_{s,t}s_{i,t-1}) + \eta_{i,t}\int_0^1 w_t(j)L_t(j)\,dj$$

with $k_{i,t}, s_{i,t} \geq 0$ for all t.

An aggregation result similar to Proposition 1 holds for the individual utility maximization problems. As in the benchmark model, consider a representative agent with preferences given by

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma}\nu_t \left[C_t^{\theta}(1-L_t)^{1-\theta}\right]^{1-\gamma} \tag{50}$$

where ν_t is as defined in the benchmark model, and

$$L_t = \int_0^1 L_t(j)\, dj$$

Given prices and assigned hours worked for each type of labor, $\{L_t(j) : j \in [0,1]\}$, the representative agent chooses a contingent plan $\{C_t, K_t, S_t\}$ so as to maximize utility (50) subject to the sequence of flow budget constraints:

$$C_t + K_t + S_t = R_{k,t}K_{t-1} + R_{s,t}S_{t-1} + \int_0^1 w_t(j)L_t(j)\, dj \tag{51}$$

and $K_t, S_t \geq 0$ with the initial condition $K_{-1}, S_{-1} > 0$. Now suppose that $\{C_t, K_t, S_t\}$ is the solution to the representative agent's utility maximization problem. And for each $i \in [0,1]$, let $c_{i,t} = \eta_{i,t}C_t$, $k_{i,t} = \eta_{i,t}K_t$, and $s_{i,t} = \eta_{i,t}S_t$. Then it is straightforward to show that the allocation $\{c_{i,t}, k_{i,t}, s_{i,t}\}$ constructed in this fashion is the solution to the utility maximization problem for each individual i. In addition, U_0 is proportional to the cross-sectional average of $\int_0^1 u_{i,0}\, di$ so that maximizing U_0 indeed maximizes average utility of all individuals.

Given this aggregation result, the utility maximization problem with sticky wages can be formulated in exactly the same way as the corresponding problem with a representative household in Schmitt-Grohé and Uribe (2005). Consider the following problem for the representative-agent/union:

$$\max_{\{C_t, K_t, S_t, (w_t(j), L_t(j): j\in[0,1])\}} U_0$$

subject to the flow budget constraints (51), short-selling constraints, the labor demand condition (49), and the Calvo-type restriction on wage adjustments:

$$w_t(j) = \begin{cases} \tilde{w}_t, & \text{with probability } 1 - \tilde{\xi}, \\ \frac{w_{t-1}(j)}{\pi_t}, & \text{with probability } \tilde{\xi}_t, \end{cases}$$

where $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate in period t. Here, $\tilde{w}_t$ denotes the real wage rate in period t chosen for those types of labor for which the union has the opportunity to re-optimize.

The objective function of the monetary authority in this problem factors in an analogous way to Proposition 2. It follows that incomplete markets is irrelevant in the following sense: the optimal monetary policy in our model with uninsured risk is identical to optimal monetary policy in the corresponding representative agent model. Price stabilization is no longer the optimal monetary policy for the reasons emphasized in Galí (2008).

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γ_c	0.7	0.7	2	2
b	0	-0.8	0	-0.8
Δ_{bc} (%)	-0.8191	-1.2983	2.0938	7.3301
Δ_{inf} (%)	0.0000	0.0000	0.0002	0.0006

Table 1: Welfare measures with permanent technology shocks

γ_c	0.7	0.7	2	2
b	0	-0.8	0	-0.8
Δ_{bc} (%)	-0.0171	-0.6191	-0.0073	12.2258
Δ_{inf} (%)	0.0000	0.0001	0.0000	0.0024

Table 2: Welfare measures with temporary technology shocks

	i.i.d.	persistent
Δ_{bc} (%)	0.0061	11.0914
Δ_{inf} (%)	0.0000	0.0075

Table 3: Welfare measures when the state of technology is constant ($\gamma_c = 2$ and $b = -0.8$). The column labeled "i.i.d." reports results for the case where the effective discount factor is i.i.d and the column labeled "persistent" reports results for the case where it is persistent.