CIRJE-F-640

Modelling Conditional Correlations for Risk Diversification in Crude Oil Markets

Chia-Lin Chang
National Chung Hsing University

Michael McAleer
Erasmus University Rotterdam
and Tinbergen Institute
and CIRJE, Faculty of Economics, University of Tokyo

Roengchai Tansuchat
Maejo University
and Chiang Mai University

August 2009

Modelling Conditional Correlations for Risk Diversification in Crude Oil Markets

Chia-Lin Chang

Department of Applied Economics
National Chung Hsing University
Taichung, Taiwan

Michael McAleer

Econometric Institute
Erasmus School of Economics
Erasmus University Rotterdam
and
Tinbergen Institute
The Netherlands
and
Center for International Research on the Japanese Economy (CIRJE)
Faculty of Economics
University of Tokyo

Roengchai Tansuchat

Faculty of Economics
Maejo University
Thailand
and
Faculty of Economics
Chiang Mai University
Thailand

August 2009

Abstract

This paper estimates univariate and multivariate conditional volatility and conditional correlation models of spot, forward and futures returns from three major benchmarks of international crude oil markets, namely Brent, WTI and Dubai, to aid in risk diversification. Conditional correlations are estimated using the CCC model of Bollerslev (1990), VARMA-GARCH model of Ling and McAleer (2003), VARMA-AGARCH model of McAleer et al. (2009), and DCC model of Engle (2002). The paper also presents the ARCH and GARCH effects for returns and shows the presence of significant interdependences in the conditional volatilities across returns for each market. The estimates of volatility spillovers and asymmetric effects for negative and positive shocks on conditional variance suggest that VARMA-GARCH is superior to the VARMA-AGARCH model. In addition, the DCC model gives statistically significant estimates for the returns in each market, which shows that constant conditional correlations do not hold in practice.

Keywords: Conditional correlations, crude oil spot prices, forward prices, futures prices, risk diversification.

JEL Classifications: C22, C32, G17, G32

1. Introduction

Crude oil is arguably the world's most influential physical commodity as it provides energy for all kinds of human activities in the form of refined energy products, such as liquefied petroleum gases (LPGs), gasoline and diesel. Consequently, crude oil is a dynamically traded commodity that affects many economies. For instance, Sadorsky (1999) found that oil price volatility shocks have asymmetric effects on the economy, namely changes in oil prices affect economic activity, but changes in economic activity have little impact on oil prices, so that oil price fluctuations have large macroeconomic impacts. Guo and Kliesen (2005) argued that changes in oil prices affect aggregate economic activity through changes in the dollar price of crude oil (relative price change), and increases in uncertainty regarding future price.

Substantial research has been conducted on the volatility of spot, forward and futures prices. Models of crude oil price volatility can be univariate or multivariate. In the former case, Fong and See (2002) examined the temporal behaviour for daily returns for crude oil futures using a Markov switching model of conditional volatility. Lanza et al. (2006) used the AR(1)-GARCH(1,1) and AR(1)-GJR(1,1) models to estimate conditional volatility based on forward and futures returns. Manera et al. (2006) used univariate ARCH and GARCH models to estimate spot and forward returns. Standard diagnostic tests also showed that the AR(1)-GARCH(1,1) and AR(1)-GJR(1,1) specifications were statistically adequate for both the conditional mean and conditional variance.

Sadorsky (2006) investigated the forecast performance of a large number of models. The fitted model for heating oil and natural gas volatility was TGARCH, whereas GARCH was used for crude oil and unleaded gasoline volatility. Lee and Zyren (2007) calculated historical volatility and GARCH models to compare the historical price volatility behaviour of crude oil, motor gasoline and heating oil in U.S. markets since 1990. They combined the shifting variable in GARCH and TARCH models to capture the response from changes in OPEC's pricing behaviour. Narayan and Narayan (2007) modelled crude oil price volatility using daily data by using the EGARCH model to gauge two features of crude oil price volatility, namely asymmetry and the persistence of shocks.

For the multivariate conditional volatility model, Lanza et al. (2006) modelled conditional correlations in the WTI oil forward and future returns using the CCC model of Bollerslev (1990) and DCC model of Engle (2002). They found that DCC could vary dramatically, being negative in four of ten cases and close to zero in another five cases. Only in the case of dynamic volatilities of the three-month and six-month future returns was the range of variation relatively narrow. Manera et al. (2006) estimated DCC in the returns for Tapis oil spot and one-month forward prices using CCC, VARMA-GARCH model of Ling and McAleer (2003), VARMA-AGARCH model of McAleer et al. (2009), and DCC, and also tested and compared volatility specifications.

Trojani and Audrino (2005) proposed a multivariate tree-structured DCC model by incorporating multivariate thresholds in conditional volatilities and correlations. They found in some Monte Carlo simulations that the model was able to capture GARCH-type dynamics and a complex threshold structure in conditional volatilities and correlations. In the empirical data for international equity markets, the estimated conditional volatilities were strongly influenced by GARCH and multivariate threshold effects. They concluded that conditional correlations were determined by simple threshold structures, whereas no GARCH-type effects could be identified.

The purpose of this paper is to estimate univariate and multivariate conditional volatility models for the returns on spot, forward and futures prices for Brent, WTI and Dubai to aid in risk diversification in crude oil markets. The remainder of the paper is organized as follows. Section 2 discusses the univariate and multivariate GARCH models to be estimated. Section 3 explains the data, descriptive statistics and unit root tests. Section 4 describes the empirical estimates and some diagnostic tests of the univariate and multivariate models. Section 5 provides some concluding remarks.

2. Econometric models

2.1 Univariate conditional volatility models

Following Engle (1982), consider the time series $y_t = E_{t-1}(y_t) + \varepsilon_t$, where $E_{t-1}(y_t)$ is the conditional expectation of y_t at $t-1$ time and ε_t is the associated error. The generalized autoregressive conditional heteroskedastity (GARCH) model of Bollerslev (1986) is given as follows:

$$\varepsilon_t = \sqrt{h_t}\eta_t, \qquad \eta_t \sim N(0,1) \tag{1}$$

$$h_t = \omega + \sum_{j=1}^{p} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{2}$$

where $\omega > 0$, $\alpha_j \geq 0$ and $\beta_j \geq 0$ are sufficient conditions to ensure that the conditional variance $h_t > 0$. The parameter α_j represents the ARCH effect, or the short run persistence of shocks to returns, and β_j represents the GARCH effect, where $\alpha_j + \beta_j$ measures the persistence of the contribution of shocks to return *i* to long run persistence.

Equation (2) assumes that the conditional variance is a function of the magnitudes of the lagged residuals and not their signs, such that a positive shock ($\varepsilon_t > 0$) has the same impact on conditional variance as a negative shock ($\varepsilon_t < 0$) of equal magnitude. In order to accommodate differential impacts on the conditional variance of positive and negative shocks, Glosten et al. (1992) proposed the asymmetric GARCH, or GJR model, which is given by

$$h_t = \omega + \sum_{j=1}^{r} \left(\alpha_j + \gamma_j I\left(\varepsilon_{t-j}\right) \right) \varepsilon_{t-j}^2 + \sum_{j=1}^{s} \beta_j h_{t-j} \tag{3}$$

where

$$I_{it} = \begin{cases} 0, & \varepsilon_{it} \geq 0 \\ 1, & \varepsilon_{it} < 0 \end{cases}$$

is an indicator function to differentiate between positive and negative shocks. When $r = s = 1$, sufficient conditions to ensure the conditional variance, $h_t > 0$, are $\omega > 0$, $\alpha_1 \geq 0, \alpha_1 + \gamma_1 \geq 0$ and $\beta_1 \geq 0$. The short run persistence of positive and negative shocks are given by α_1 and $\left(\alpha_1 + \gamma_1\right)$, respectively. When the conditional shocks, η_t, follow a symmetric distribution, the short run persistence is $\alpha_1 + \gamma_1/2$, and the contribution of shocks to expected long-run persistence is $\alpha_1 + \gamma_1/2 + \beta_1$.

In order to estimate the parameters of model (1)-(3), maximum likelihood estimation is used with a joint normal distribution of η_t. However, when η_t does not follow a normal distribution, or the conditional distribution is not known, quasi-MLE (QMLE) is used to maximize the likelihood function.

Bollerslev (1986) showed the necessary and sufficient condition for the second-order stationarity of GARCH is $\sum_{i=1}^{r} \alpha_i + \sum_{i=1}^{s} \beta_i < 1$. For the GARCH(1,1) model, Nelson (1991) obtained the log-moment condition for strict stationary and ergodicity as $E\left(\log\left(\alpha_1 \eta_t^2\right) + \beta_1\right) < 0$, which is important in deriving the statistical properties of the QMLE. For GJR(1,1), Ling and McAleer (2002a, 2002b) presented the necessary and sufficient condition for $E\left(\varepsilon_t^2\right)<\infty$ as $\alpha_1+\gamma_1/2+\beta_1<1$. McAleer et al. (2007) established the log-moment condition for GJR(1,1) as $E\left(\log\left(\alpha_1+\gamma_1 I\left(\eta_t\right)\eta_t^2+\beta_1\right)\right)<0$, and showed that it is sufficient for consistency and asymptotic normality of the QMLE.

2.2 Multivariate conditional volatility models

The typical specification underlying the multivariate conditional mean and conditional variance in returns is given as follows:

$$y_t = E\left(y_t \middle| F_{t-1}\right)+\varepsilon_t \tag{4}$$

$$\varepsilon_t = D_t\eta_t$$

where $y_t=\left(y_{1t},...,y_{mt}\right)'$, $\eta_t=\left(\eta_{1t},...,\eta_{mt}\right)'$ is a sequence of independently and identically distributed (i.i.d.) random vectors, F_t is the past information available to time *t*, $D_t = diag\left(h_1^{1/2},...,h_m^{1/2}\right)$, *m* is the number of returns, and $t=1,...,n$, (see Li, Ling and McAleer (2002), McAleer (2005), and Bauwens et al. (2006)). The constant conditional correlation (CCC) model of Bollerslev (1990) assumes that the conditional variance for each return, h_{it}, $i=1,..,m$, follows a univariate GARCH process, that is

$$h_{it}=\omega_i+\sum_{j=1}^{r}\alpha_{ij}\varepsilon_{i,t-j}^2+\sum_{j=1}^{s}\beta_{ij}h_{i,t-j} \tag{5}$$

where α_{ij} represents the ARCH effect, or short run persistence of shocks to return *i*, and β_{ij} represents the GARCH effect, or the contribution of shocks to return *i* to long run persistence, namely $\sum_{j=1}^{r}\alpha_{ij}+\sum_{j=1}^{s}\beta_{ij}$.

The conditional correlation matrix of CCC is $\Gamma=E\left(\eta_t\eta_t'\middle|F_{t-1}\right)=E\left(\eta_t\eta'\right)$, where $\Gamma=\left\{\rho_{it}\right\}$ for $i,j=1,...,m$. From (4), $\varepsilon_t\varepsilon_t'=D_t\eta_t\eta' D_t$, $D_t=\left(\text{diag}\ Q_t\right)^{1/2}$, and $E\left(\varepsilon_t\varepsilon_t'\middle|F_{t-1}\right)$ $=Q_t=D_t\Gamma D_t$, where Q_t is the conditional covariance matrix. The conditional correlation matrix is defined as $\Gamma=D_t^{-1}Q_tD_t^{-1}$, and each conditional correlation coefficient is estimated from the standardized residuals in (4) and (5). Therefore, there is no multivariate estimation involved for CCC, which involves *m* univariate GARCH models, except in the calculation of the conditional correlations.

Although the CCC specification in (5) is a computationally straightforward "multivariate" GARCH model, it assumes independence of the conditional variances across returns and does not accommodate asymmetric behaviour. In order to incorporate interdependencies, Ling and McAleer (2003) proposed a vector autoregressive moving average (VARMA) specification of the conditional mean in (4), and the following specification for the conditional variance:

$$H_t = W + \sum_{i=1}^{r} A_i \vec{\varepsilon}_{t-i} + \sum_{j=1}^{s} B_j H_{t-j} \tag{6}$$

where $H_t = \left(h_{1t},...,h_{mt}\right)'$, $\vec{\varepsilon} = \left(\varepsilon_{1t}^2,...\varepsilon_{mt}^2\right)'$, and *W*, A_i for $i = 1,..,r$ and B_j for $j = 1,..,s$ are $m \times m$ matrices. As in the univariate GARCH model, VARMA-GARCH assumes that negative and positive shocks have identical impacts on the conditional variance. In order to separate the asymmetric impacts of the positive and negative shocks, McAleer, Hoti and Chan (2009) proposed the VARMA-AGARCH specification for the conditional variance, namely

$$H_t = W + \sum_{i=1}^{r} A_i \vec{\varepsilon}_{t-i} + \sum_{i=1}^{r} C_i I_{t-i} \vec{\varepsilon}_{t-i} + \sum_{j=1}^{s} B_j H_{t-j} \tag{7}$$

where C_i are $m \times m$ matrices for $i = 1,..,r$, and $I_t = \text{diag}\left(I_{1t},...,I_{mt}\right)$, where

$$I_{it} = \begin{cases} 0, & \varepsilon_{it} > 0 \\ 1, & \varepsilon_{it} \le 0 \end{cases}$$

If $m = 1$, (6) collapses to the asymmetric GARCH, or GJR, model. Moreover, VARMA-AGARCH reduces to VARMA-GARCH when $C_i = 0$ for all *i*. If $C_i = 0$ and A_i and B_j are diagonal matrices for all *i* and *j*, then VARMA-AGARCH reduces to the CCCmodel. The parameters of model (4)-(7) are obtained by maximum likelihood estimation (MLE) using a joint normal density. When η_t does not follow a joint multivariate normal distribution, the appropriate estimator is QMLE.

Unless η_t is a sequence of iid random vectors, or alternatively a martingale difference process, the assumption that the conditional correlations are constant may seen unrealistic. In order to make the conditional correlation matrix time dependent, Engle (2002) proposed a dynamic conditional correlation (DCC) model, which is defined as

$$y_t \left| \Im_{t-1} \right. \sim (0, Q_t) \ , \quad t = 1,2,...,n \tag{8}$$

$$Q_t = D_t \Gamma_t D_t, \tag{9}$$

where $D_t = \text{diag}\left(h_{1t},...,h_{kt}\right)$ is a diagonal matrix of conditional variances, and $\Im_t$ is the information set available to time *t*. The conditional variance, h_{it}, can be defined as a univariate GARCH model as follows:

$$h_{it} = \omega_i + \sum_{k=1}^{p} \alpha_{ik} \varepsilon_{i,t-k} + \sum_{l=1}^{q} \beta_{il} h_{i,t-l} . \tag{10}$$

If η_t is a vector of i.i.d. random variables, with zero mean and unit variance, Q_t in (9) is the conditional covariance matrix (after standardization, $\eta_{it} = y_{it} \big/ \sqrt{h_{it}}$). The η_{it} are used to estimate the dynamic conditional correlations, as follows:

$$\Gamma_t = \left\{(diag(Q_t)^{-1/2}\right\} Q_t \left\{(diag(Q_t)^{-1/2}\right\} \tag{11}$$

where the $k \times k$ symmetric positive definite matrix Q_t is given by

$$Q_t = (1-\theta_1-\theta_2)\bar{Q} + \theta_1 \eta_{t-1}\eta'_{t-1} + \theta_2 Q_{t-1} \tag{12}$$

in which θ_1 and θ_2 are scalar parameters to capture the effects of previous shocks and previous dynamic conditional correlations on the current dynamic conditional correlation, and θ_1 and θ_2 are non-negative scalar parameters. As Q_t is conditional on the vector of standardized residuals, (12) is a conditional covariance matrix, and $\bar{Q}$ is the $k \times k$ unconditional variance matrix of η_t. For further details, and critique of the DCC model, see Caporin and McAleer (2009).

3. Data

The data used in this paper are daily synchronous closing price of spot, forward and futures crude oil prices from three major crude oil markets, namely Brent, WTI and Dubai. The 4,659 price observations from 2 January 1991 to 10 November 2008 are obtained from the DataStream database service. The returns of crude oil prices *i* of market *j* at time *t* in a continuous compound basis are calculated as $r_{ij,t} = \log\left(P_{ij,t} \big/ P_{ij,t-1}\right)$, where $P_{ij,t}$ and $P_{ij,t-1}$ are the closing prices of crude oil price *i* of market *j* for days t and $t-1$, respectively. The univariate and multivariate conditional volatility models are estimated using the EViews 6 econometric software package.

The descriptive statistics for the crude oil returns series are summarized in Table 1. The sample mean is quite small, but the corresponding variance of returns is much higher. Both negative skewness and high kurtosis suggest that returns are not distributed normally.

Similarly, the null hypothesis of normality is also rejected for the sample return series by the Jarque-Bera(J-B) test lagrange multiplier statistics.

The logarithms of crude oil prices are plotted in Figure 1. It is clear that there is substantial clustering of volatilities, such that a turbulent trading day tends to be followed by another turbulent day, while a tranquil period tends to be followed by another tranquil period.

[Insert Tables 1-2 here]

[Insert Figure 1 here]

The empirical results of the unit root tests for the sample returns in each market are summarized in Table 2. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are used to test for unit roots in the individual series. The large negative values in all cases indicate rejection of the null hypothesis at the 1% level, such that all returns are stationary.

4. Empirical Results

Univariate estimates of the conditional volatilities, GARCH(1,1) and GJR(1,1), with different conditional mean equation models based on spot, forward and futures returns in each market, are given in Tables 3-5, which report the respective QMLE and the Bollerslev-Woodridge (1992) robust *t*-ratios. The log-moment and second moment conditions are also presented to confirm the statistical properties of the estimates. The second moments of GARCH(1,1) and GJR(1,1), namely $\alpha_1+\beta_1$ and $\alpha_1+\gamma_1/2+\beta_1$, are less than 1, and the estimated log-moments of GARCH(1,1) and GJR(1,1), which are given as $E\left(\log\left(\alpha_1\eta_t^2+\beta_1\right)\right)$ and $E(\log(\alpha_1+\ \gamma_1 I\left(\eta_t\right)\eta_t^2+\beta_1))$, respectively, are less than 0, so the QMLE are consistent and asymptotically normal (see McAleer (2005) and McAleer et al. (2007)).

The univariate GARCH estimates for Brent are given in Table 3. The coefficients in the mean equations in Panel 3a are not all statistically significant. The mean equation of AR(1)-GARCH(1,1) is significant only for forward returns, while ARMA(1,1)-GARCH(1,1) is significant in all returns series. In addition, the coefficient in the conditional variance equations for both AR(1)-GARCH(1,1) and ARMA(1,1)-GARCH(1,1) are all significant. Consequently, ARMA(1,1)-GARCH(1,1) is preferred to AR(1)-GARCH(1,1).

In the case of the asymmetric GARCH(1,1) model in Panel 3b, only the coefficients in the mean equation for ARMA(1,1) are significant. The estimates of the asymmetric effect for the univariate model are not statistically significant, except for spot returns.

The results for univariate estimation of the WTI market are reported in Table 4. The robust *t*-ratios show that the ARMA(1,1)-GARCH(1,1) specification for all returns is statistically adequate in both the conditional mean and conditional variance equations, but the coefficients in the conditional mean equation of AR(1)-GARCH(1,1) are insignificant. The univariate GJR models are presented in Panel 4b in Table 4, where only the forward returns for ARMA-GARCH model are significant. However, asymmetry between negative and positive shocks on the conditional variance is not observed.

For the Dubai market in Table 5, the coefficients in the mean equation for spot and forward returns in Panels 5a and 5b are significant only for AR(1)-GARCH(1,1) and AR(1)-GJR(1,1). Panel 5a shows that the coefficients in the conditional variance equation for AR(1)-GARCH(1,1) are all statistically significant, whereas in Panel 5b, the conditional variance coefficients are significant only in spot returns. These results show that there is an asymmetric effect between negative and positive shocks on the conditional variance.

[Insert Tables 3-5 here]

Table 6 presents the constant conditional correlations for the spot, forward and futures returns in each market using the CCC model based on univariate GARCH(1,1) estimates. Three returns in the Brent and WTI markets in Panels 6a and 6b provide six conditional correlations, while two returns in the Dubai market in Panel 6c give one conditional correlation. The highest estimated conditional correlation in the Brent market is 0.940, namely between the standardized shocks to the volatility of the spot and forward returns. In the case of the WTI market, the highest estimated conditional correlation for Brent is 0.883, namely between the standardized shocks to the volatility of spot and futures returns, and futures and forward returns. The conditional correlation between the shocks to spot and forward returns for the Dubai market is 0.936.

[Insert Table 6 here]

[Insert Figure 6 here]

The estimates of the dynamic conditional correlations and the descriptive statistics for DCC across the shocks to returns in each market are presented in Table7, Panels 7a and 7b, respectively. Based on the Bollerslev and Wooldridge (1992) robust *t*-ratios, the estimates of the DCC parameters, $\hat{\theta}_1$ and $\hat{\theta}_2$, in each market are always statistically significant. This indicates that the assumption of constant conditional correlation for all shocks to returns is not supported empirically. In addition, the mean of the dynamic conditional correlations of each pair is identical to the constant conditional correlation estimates reported in Table 6. The short run persistence of shocks on the dynamic conditional correlations is greatest for WTI at 0.264, while the largest long run persistence of shocks to the conditional correlations is for Brent, namely 0.995 = 0.027 + 0.968.

[Insert Tables 7-10 here]

The corresponding multivariate estimates for the VARMA(1,1)-GARCH and VARMA(1,1)-AGARCH models for each market are given in Tables 8-10. It is clear from Table 8, Panel a, that the forward returns are significant only for ARCH and GARCH, while the spot and futures returns are only significant for ARCH. Moreover, there are significant interdependences in the conditional volatility between spot and forward returns, and between spot and futures returns. The results in Panel b show that the ARCH and GARCH effects are significant in the conditional volatility model for spot, forward and futures returns. There are also significant interdependences in the conditional volatility between spot and futures returns. In addition, as the asymmetric effects for each return in Panel 8a are insignificant, if follows that VARMA-GARCH model dominates its asymmetric counterpart, VARMA-AGARCH.

Table 9, Panel a, for Brent presents the VARMA-GARCH model, in which the ARCH and GARCH effects are significant in the conditional volatility model for spot, forward and futures returns. Also present are the spillover effects across the spot, forward and futures returns. In contrast, Panel 9b shows that the ARCH and GARCH effects are insignificant, except for the GARCH effect for forward returns. In addition, the asymmetric spillover effects for each of the returns is not statistically significant, such that VARMA-AGARCH is dominated by VARMA-GARCH.

Table 10 presents the VARMA-GARCH and VARMA-AGARCH estimates for Dubai. It is clear that the ARCH and GARCH effects for spot and forward returns are significant, and there is a significant display of interdependences in the conditional volatilities between the spot and forward returns. In Panel 10b, the ARCH and GARCH effects are statistically significant only for forward returns, but the ARCH effect is significant for spot returns. There is also the presence of interdependences between spot and forward returns, while the asymmetric spillover effects for each of the returns is insignificant. Consequently, VARMA-GARCH is preferred to VARMA-AGARCH.

5. Conclusion

This paper estimated four multivariate volatility models, namely CCC, DCC, VARMA-GARCH and VARMA-AGARCH, for the spot, forward and futures returns for three major benchmark international crude oil markets, namely Brent, WTI and Dubai. The returns for the period 2 January 1991 to 10 November 2008 were estimated using multivariate conditional volatility and conditional correlation models. Both the univariate ARCH and GARCH components of the GARCH(1,1) and GJR(1,1) models were significant for all returns, whereas most of the estimated asymmetric effects for GJR(1,1) were not significant.

The calculated constant conditional correlations across the conditional volatilities of returns using the CCC model were high. The paper also presented the ARCH and GARCH effects for returns, and significant interdependences in the conditional volatilities across returns in each market. The estimates of volatility spillovers and asymmetric effects for negative and positive shocks on the conditional variances suggested that the VARMA-GARCH model was superior to the asymmetric VARMA-AGARCH. In addition, the estimates of the DCC model for returns in each market were statistically significant. In short, constant conditional correlations were not supported in the empirical examples. Such estimates of the dynamic conditional correlations of shocks to returns associated with spot, forward and futures prices can be used as an aid to risk diversification in crude oil markets.

Acknowledgements

The authors wish to thank Felix Chan and Abdul Hakim for providing the computer programs. For financial support, the first author is most grateful to the National Science Council, Taiwan, the second author wishes to thank the Australian Research Council, and the third author acknowledges the Energy Conservation Promotion Fund, Ministry of Energy, Faculty of Economics, Maejo University, and Faculty of Economics, Chiang Mai University.

References

Bauwens, L., S. Laurent and J. Rombouts, 2006, Multivariate GARCH models: A survey. *Journal of Applied Econometrics* 21, 79-109.

Bollerslev, T., 1986, Generalised autoregressive conditional heteroscedasticity. *Journal of Econometrics* 31, 307-327.

Bollerslev, T., 1990, Modelling the coherence in short-run nominal exchange rate: A multivariate generalized ARCH approach. *Review of Economics and Statistics* 72, 498-505.

Bollerslev, T. and J. Wooldridge, 1992, Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. *Econometric Reviews* 11, 143-172.

Caporin, M. and M. McAleer, 2009, Do we really need both BEKK and DCC? A tale of two covariance models. Available at SSRN: http://ssrn.com/abstract=1338190.

Engle, R.F., 1982, Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica* 50, 987-1007.

Engle, R., 2002, Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics* 20, 339-350.

Fong, W. and K. See, 2002, A Markov switching model of the conditional volatility of crude oil futures prices. *Energy Economics* 24, 71-95.

Glosten, L., R. Jagannathan and D. Runkle (1992), On the relation between the expected value and volatility of nominal excess return on stocks. *Journal of Finance* 46, 1779-1801.

Guo, H. and K. Kliesen, 2005, Oil price volatility and U.S. macroeconomics activity. *Federal Reserve Bank of St. Louis Review* 87, 669-683.

Lanza, A., M. Manera, and M. McAleer, 2006, Modeling dynamic conditional correlations in WTI oil forward and future returns. *Finance Research Letters* 3, 114-132.

Lee, T. and J. Zyren, 2007, Volatility relationship between crude oil and petroleum products. *Atlantic Economic Journal* 35, 97-112.

Li, W.K., S. Ling and M. McAleer, 2002, Recent theoretical results for time series models with GARCH errors. *Journal of Economic Surveys* 16, 245-269. Reprinted in M. McAleer and L. Oxley (eds.), *Contributions to Financial Econometrics: Theoretical and Practical Issues*, Blackwell, Oxford, 2002, pp. 9-33.

Ling, S. and M. McAleer, 2002a, Stationary and the existence of moments of a family of GARCH processes. *Journal of Econometrics* 10, 505-531

Ling, S. and M. McAleer, 2002b, Necessary and sufficient moment conditions for the GARCH(r,s) and asymmetric power GARCH(r,s) models. *Econometric Theory* 18, 722-729.

Ling, S. and M. McAleer, 2003, Asymptotic theory for a vector ARMA-GARCH model, *Econometric Theory* 19, 278-308.

Manera, M., M. McAleer, and M. Grasso, 2006, Modelling time-varying conditional correlations in the volatility of Tapis oil spot and future returns. *Applied Financial Economics* 16, 525-533.

McAleer, M., 2005, Automated inference and learning in modeling financial volatility. *Econometric Theory* 21, 232-261.

McAleer, M., F. Chan, S. Hoti and O. Lieberman, 2008, Generalized autoregressive conditional correlation. *Econometric Theory* 24, 1554-1583.

McAleer, M., F. Chan and D. Marinova, 2007, An econometric analysis of asymmetric volatility: Theory and application to patents. *Journal of Econometrics* 139, 259-284.

McAleer, M., S. Hoti, and F. Chan, 2009, Structure and asymptotic theory for multivariate asymmetric conditional volatility. *Econometric Reviews* 28, 422-440.

Narayan, P. and S. Narayan, 2007, Modelling oil price volatility. *Energy Policy* 35, 6549-6553.

Nelson, D.B., 1991, Conditional heteroscedasticity in asset returns: A new approach. *Econometrica* 59, 347-370.

Sadorsky, P., 1999, Oil price shocks and stock market activity. *Energy Economics* 21, 449-469.

Sadorsky, P., 2006, Modeling and forecasting petroleum futures volatility. *Energy Economics* 28, 467-488.

Trojani, F. and F. Audrino, 2005, A general multivariate threshold GARCH model with dynamic conditional correlations. Discussion Paper no. 2005-04, Universität St. Gallen, 1-44.

Table 1. Descriptive statistics for crude oil price returns

Returns	Mean	Max	Min	S.D.	Skewness	Kurtosis	Jarque-Bera
rbresp	0.043	15.164	-12.601	2.347	-0.0007	5.341	686.6157
rbrefor	0.043	12.044	-12.534	2.146	-0.141	4.939	480.941
rbrefu	0.043	12.898	-10.946	2.212	-0.124	4.934	476.538
rwtisp	0.043	15.873	-13.795	2.412	-0.129	6.479	1524.764
rwtifor	0.042	13.958	-12.329	2.316	-0.182	5.204	625.414
rwtifu	0.043	14.546	-12.939	2.349	-0.151	6.318	1390.425
rdubsp	0.043	14.705	-12.943	2.199	-0.179	5.844	1029.861
rdubfor	0.040	13.767	-12.801	2.115	-0.308	5.718	973.0103
rtapsp	0.038	11.081	-10.483	2.000	-0.183	5.373	722.053
rtapfor	0.038	12.071	-12.869	2.076	-0.289	5.567	867.187

Table 2. Unit root tests for returns

Returns	ADF test			Phillips-Perron test		
	None	Constant	Constant and Trend	None	Constant	Constant and Trend
rbresp	-54.264*	-54.274*	-54.265*	-54.301*	-54.298*	-54.291*
rbrefor	-57.076*	-57.092*	-57.083*	-57.088*	-57.100*	-57.091*
rbrefu	-57.944*	-57.958*	-57.949*	-57.901*	-57.919*	-57.909*
rwtisp	-41.065*	-41.079*	-41.073*	-55.652*	-55.677*	-55.667*
rwtifor	-56.618*	-56.626*	-56.617*	-56.697*	-56.715*	-56.705*
rwtifu	-55.872*	-55.881*	-55.872*	-56.011*	-56.030*	-56.020*
rdubsp	-59.130*	-59.145*	-59.135*	-59.090*	-59.129*	-59.119*
rdubfor	-59.664*	-59.677*	-59.667*	-59.542*	-59.573*	-59.564*
rtapsp	-59.059*	-59.072*	-59.062*	-58.955*	-58.956*	-58.947*
rtapfor	-59.949*	-59.961*	-59.951*	-59.747*	-59.775*	-59.766*

Note: * significant at 1%.

Table 3. Univariate volatility models of crude oil returns for Brent

Panel 3a. AR(1)-GARCH(1,1) and ARMA(1,1)-GARCH(1,1) estimates

Returns	Mean equation			Variance equation			Log-Moment	Second moment	AIC	SIC
	c	AR(1)	MA(1)	$\hat{\omega}$	$\hat{\alpha}$	$\hat{\beta}$				
Spot	0.042	0.026		**0.035**	**0.050**	**0.944**	-0.0043	0.994	4.265	4.272
	1.468	1.648		**3.395**	**5.847**	**110.768**				
	0.041	**-0.807**	**0.825**	**0.031**	**0.048**	**0.947**	-0.0037	0.995	4.264	4.273
	1.452	**-5.601**	**5.964**	**3.112**	**5.629**	**110.849**				
Forward	0.038	**-0.032**		**0.032**	**0.050**	**0.943**	-0.0046	0.993	4.103	4.109
	1.491	**-1.978**		**3.657**	**5.897**	**110.737**				
	0.038	**0.608**	**-0.642**	**0.031**	**0.049**	**0.944**	-0.0043	0.993	4.102	4.110
	1.575	**3.365**	**-3.681**	**3.523**	**5.799**	**109.983**				
Futures	0.028	-0.021		**0.034**	**0.057**	**0.937**	-0.0048	0.994	4.142	4.149
	1.059	-1.291		**3.760**	**7.451**	**126.898**				
	0.029	**0.736**	**-0.760**	**0.032**	**0.056**	**0.938**	-0.0046	0.994	4.141	4.149
	1.175	**4.459**	**-4.787**	**3.653**	**7.275**	**125.755**				

Panel 3b. AR(1)-GJR(1,1) and ARMA(1,1)-GJR(1,1) estimates

zReturns	Mean equation			Variance equation				Log-Moment	Second moment	AIC	SIC
	c	AR(1)	MA(1)	$\hat{\omega}$	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\beta}$				
Spot	0.023	0.025		**0.035**	**0.031**	**0.031**	**0.947**	-0.0031	0.994	4.262	4.270
	0.803	1.594		**3.317**	**3.139**	**2.249**	**118.816**				
	0.022	**-0.799**	**0.816**	**0.030**	**0.029**	**0.029**	**0.951**	-0.0026	0.995	4.261	4.271
	0.794	**-5.190**	**5.520**	**3.039**	**3.076**	**2.225**	**121.227**				
Forward	0.032	-0.033		**0.032**	**0.043**	0.012	**0.944**	-0.0031	0.993	4.103	4.111
	1.277	-1.960		**3.564**	**3.035**	0.755	**102.014**				
	0.032	**0.597**	**-0.632**	**0.031**	**0.043**	0.011	**0.945**	-0.0029	0.994	4.102	4.111
	1.371	**3.249**	**-3.556**	**3.438**	**3.095**	0.751	**101.950**				
Futures	0.036	-0.019		**0.035**	**0.065**	-0.014	**0.935**	-0.0029	0.993	4.141	4.150
	1.402	-1.200		**3.752**	**4.952**	-0.898	**118.747**				
	0.035	**0.742**	**-0.765**	**0.033**	**0.063**	-0.012	**0.937**	-0.0027	0.994	4.141	4.150
	1.482	**4.448**	**-4.765**	**3.644**	**4.988**	-0.874	**118.017**				

Notes: (1) The two entries for each parameter are their respective parameter estimates and Bollerslev and Wooldridge (1992) robust *t*- ratios.
(2) Entries in bold are significant at 5%.

Table 4. Univariate volatility models of crude oil returns for WTI

Panel 4a. AR(1)-GARCH(1,1) and ARMA(1,1)-GARCH(1,1) estimates

Returns	Mean equation			Variance equation			Log-Moment	Second moment	AIC	SIC
	c	AR(1)	MA(1)	$\hat{\omega}$	$\hat{\alpha}$	$\hat{\beta}$				
Spot	0.0212	-0.017		**0.055**	**0.061**	**0.931**	-0.0063	0.992	4.346	4.353
	0.683	-0.986		**3.363**	**5.634**	**83.514**				
	0.024	**0.842**	**-0.871**	**0.050**	**0.059**	**0.933**	-0.0057	0.992	4.344	4.352
	0.965	**9.754**	**-11.201**	**3.296**	**5.586**	**86.009**				
Forward	0.033	-0.022		**0.040**	**0.055**	**0.937**	-0.0050	0.992	4.246	4.253
	1.216	-1.367		**3.711**	**6.810**	**116.961**				
	0.032	**-0.572**	**0.561**	**0.037**	**0.053**	**0.940**	-0.0045	0.993	4.246	4.254
	1.160	**-2.327**	**2.265**	**3.489**	**6.633**	**117.146**				
Futures	0.037	-3.43E-05		**0.041**	**0.058**	**0.935**	-0.0051	0.993	4.250	4.257
	1.330	-0.002		**3.812**	**6.203**	**107.793**				
	0.037	**-0.957**	**0.959**	**0.042**	**0.059**	**0.934**	-0.0053	0.993	4.251	4.259
	1.342	**-30.672**	**31.052**	**3.884**	**6.273**	**107.696**				

Panel 4b. AR(1)-GJR(1,1) and ARMA(1,1)-GJR(1,1) estimates

Returns	Mean equation			Variance equation				Log-Moment	Second moment	AIC	SIC
	c	AR(1)	MA(1)	$\hat{\omega}$	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\beta}$				
Spot	0.029	-0.016		**0.055**	**0.067**	-0.012	**0.931**	-0.0039	0.992	4.346	4.354
	1.005	-0.916		**3.306**	**3.865**	-0.656	**80.209**				
	0.029	-0.362	0.356	**0.054**	**0.067**	-0.013	**0.931**	-0.0038	0.992	4.346	4.356
	0.999	-1.080	1.057	**3.193**	**3.842**	-0.697	**79.394**				
Forward	0.027	-0.022		**0.039**	**0.048**	0.011	**0.939**	-0.0031	0.993	4.246	4.254
	0.988	-1.383		**3.671**	**3.673**	0.769	**112.825**				
	0.026	**-0.555**	**0.543**	**0.035**	**0.047**	0.010	**0.941**	-0.0028	0.993	4.246	4.255
	0.947	**-2.177**	**2.118**	**3.444**	**3.573**	0.709	**112.522**				
Futures	0.029	-0.001		**0.041**	**0.050**	0.014	**0.936**	-0.0030	0.993	4.250	4.258
	1.049	-0.049		**3.748**	**4.038**	1.018	**105.812**				
	0.028	-0.520	0.529	**0.037**	**0.049**	0.013	**0.938**	-0.0027	0.994	4.250	4.259
	1.027	-1.004	1.027	**3.554**	**3.965**	0.965	**106.468**				

Notes: (1) The two entries for each parameter are their respective parameter estimates and Bollerslev and Wooldridge (1992) robust *t*- ratios. (2) Entries in bold are significant at the 95% level

Table 5. Univariate volatility models of crude oil returns for Dubai

Panel 5a. AR(1)-GARCH(1,1) and ARMA(1,1)-GARCH(1,1) estimates

Returns	Mean equation			Variance equation			Log-Moment	Second moment	AIC	SIC
	c	AR(1)	MA(1)	$\hat{\omega}$	$\hat{\alpha}$	$\hat{\beta}$				
Spot	**0.053**	**-0.064**		**0.045**	**0.052**	**0.938**	-0.0059	0.99	4.156	4.163
	2.162	**-4.122**		**3.384**	**6.448**	**106.264**				
	0.053	0.329	**-0.397**	**0.044**	**0.052**	**0.938**	-0.0059	0.99	4.156	4.164
	2.286	1.776	**-2.197**	**3.386**	**6.397**	**106.082**				
Forward	**0.052**	**-0.068**		**0.039**	**0.054**	**0.937**	-0.0057	0.991	4.065	4.072
	2.206	**-4.344**		**3.691**	**6.885**	**113.271**				
	4.072	**0.399**	**-0.469**	**0.038**	**0.054**	**0.937**	-0.0056	0.991	4.064	4.073
	2.367	**2.529**	**3.084**	**3.659**	**6.833**	**113.141**				

Panel 5b. AR(1)-GJR(1,1) and ARMA(1,1)-GJR(1,1) estimates

Returns	Mean equation			Variance equation				Log-Moment	Second moment	AIC	SIC
	c	AR(1)	MA(1)	$\hat{\omega}$	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\beta}$				
Spot	0.036	**-0.067**		**0.046**	**0.031**	**0.030**	**0.944**	-0.0045	0.99	4.153	4.162
	1.478	**-4.162**		**3.391**	**2.874**	**2.412**	**107.349**				
	0.038	0.323	**-0.393**	**0.042**	**0.031**	**0.029**	**0.944**	-0.0045	0.999	4.153	4.163
	1.610	1.800	**-2.246**	**3.405**	**2.907**	**2.457**	**107.267**				
Forward	0.039	**-0.069**		**0.040**	**0.038**	0.024	**0.939**	-0.0045	0.989	4.064	4.072
	1.659	**-4.334**		**3.758**	**3.152**	1.866	**105.081**				
	0.040	0.387	**-0.458**	**0.039**	**0.038**	0.024	**0.940**	-0.0044	0.99	4.063	4.073
	1.829	2.445	**-2.996**	**3.745**	**3.249**	1.878	**105.641**				

Notes: (1) The two entries for each parameter are their respective parameter estimates and Bollerslev and Wooldridge (1992) robust *t*- ratios.
(2) Entries in bold are significant at 5%.

Table 6. Constant conditional correlations (CCC) based on GARCH(1,1)

Panel 6a: Brent

Returns	rbresp	rbrefor	rbrefu
rbresp	1.000		
rbrefor	0.940	1.000	
rbrefu	0.784	0.783	1.000

Panel 6b: WTI

Returns	rwtisp	rwtifor	rwtifu
rwtisp	1.000		
rwtifor	0.837	1.000	
rwtifu	0.883	0.883	1.000

Panel 6c: Dubai

Returns	rdubsp	rdubfor
rdubsp	1.000	
rdubfor	0.936	1.000

Table 7. Dynamic conditional correlations (DCC) based on GARCH(1,1)

Panel 7a. Estimates of *Q*

Returns	$\hat{\theta}_1$	$\hat{\theta}_2$
rbresp_rbrefor_rbrefu	0.027	0.968
	5.140	135.802
rwtisp_rwtifor_rwtifu	0.264	0.446
	9.544	14.070
rdubsp_rdubfor	0.095	0.894
	3.321	26.858

Note: Two entries for each parameters are their respective estimate and robust *t*-ratio.

Panel 7b. Descriptive statistics

Returns	**Mean**	**Max**	**Min**	**S.D.**	**Skewness**	**Kurtosis**
rbresp_rbrefor	0.939	0.991	0.648	0.041	-2.315	11.474
rbresp_rbrefu	0.782	0.951	0.267	0.113	-1.077	3.803
rbrefor_rbrefu	0.785	0.957	0.272	0.113	-1.087	3.861
rwtisp_rwtifor	0.837	0.989	-0.346	0.113	-3.894	25.590
rwtisp_rwtifu	0.883	0.995	-0.423	0.099	-4.625	32.601
rwtifor_rwtifu	0.882	0.992	-0.272	0.093	-4.705	35.334
rdubsp_rdubfor	0.941	0.998	-0.131	0.106	-4.135	24.456

Table 8. VARMA-GARCH and VARMA-AGARCH models for Brent

Panel a. VARMA(1,1)-GARCH(1,1)

Returns	ω	α_{bresp}	α_{brefor}	α_{brefu}	β_{bresp}	β_{brefor}	β_{brefu}
rbresp	**0.034**	0.018	-0.011	**0.049**	**0.962**	0.005	**-0.028**
	(4.085)	(1.735)	(-0.509)	**(3.163)**	**(79.990)**	(0.231)	**(-2.140)**
rbrefor	0.215	-0.019	-0.033	**0.147**	**0.407**	-0.164	**0.487**
	(1.390)	(-0.690)	(-1.377)	**(4.703)**	**(3.179)**	(-0.882)	**(2.777)**
rbrefu	-0.002	**-0.040**	**0.071**	**0.046**	**0.095**	-0.026	**0.854**
	(-0.079)	**(-9.420)**	**(3.656)**	**(2.465)**	**(3.252)**	(-0.472)	**(16.441)**

Panel b.VARMA(1,1)-AGARCH(1,1)

Returns	ω	α_{bresp}	α_{brefor}	α_{brefu}	γ	β_{bresp}	β_{brefor}	β_{brefu}
rbresp	0.030	0.001	-0.011	0.048	0.026	0.967	0.005	-0.027
	3.870	0.129	-0.535	3.336	2.395	101.050	0.229	-2.155
rbrefor	0.105	-0.014	-0.017	0.105	0.032	0.160	0.644	0.043
	1.934	-0.619	-0.436	3.608	0.929	2.379	5.101	0.760
rbrefu	0.012	-0.031	0.057	0.049	-0.011	0.062	-0.031	0.897
	0.630	-2.677	2.654	2.466	-0.626	2.624	-0.711	21.709

Notes: Entries in bold are significant at 5%.

Table 9. VARMA-GARCH and VARMA-AGARCH models for WTI

Panel a. VARMA(1,1)-GARCH(1,1)

Returns	ω	α_{rwtisp}	α_{wtifor}	α_{wtifu}	β_{wtisp}	β_{wtifor}	β_{wtifu}
rwtisp	0.005	0.041	**0.113**	-0.016	**0.640**	0.202	0.058
	(0.062)	(0.818)	**(2.331)**	(-0.279)	**(4.001)**	(1.184)	(0.643)
rwtifor	**0.026**	-0.006	**0.020**	**0.031**	0.009	**0.979**	**-0.036**
	(5.365)	(-1.311)	**(1.976)**	**(2.669)**	(1.452)	**(186.055)**	**(-4.697)**
rwtifu	-0.010	-0.013	0.064	0.038	0.046	0.141	**0.728**
	(-0.179)	(-1.851)	(1.829)	(1.075)	(1.534)	(0.876)	**(4.583)**

Panel b.VARMA(1,1)-AGARCH(1,1)

Returns	ω	α_{wtisp}	α_{wtifor}	α_{wtifu}	γ	β_{wtisp}	β_{wtifor}	β_{wtifu}
rwtisp	-0.007	0.012	**0.119**	-0.011	0.045	**0.607**	0.237	0.058
	(-0.078)	(0.314)	**(2.395)**	(-0.195)	(0.843)	**(3.805)**	(1.349)	(0.596)
rwtifor	**0.026**	-0.004	0.017	**0.029**	0.006	0.007	**0.979**	**-0.035**
	(5.641)	(-0.960)	(1.277)	**(2.448)**	(0.743)	(1.178)	**(185.808)**	**(-4.502)**
rwtifu	-0.008	-0.012	0.062	0.029	0.023	0.041	0.146	**0.727**
	(-0.146)	(-1.760)	(1.757)	(0.676)	(0.658)	(1.380)	(0.978)	**(4.948)**

Notes: Entries in bold are significant at 5%.

Table 10. VARMA-GARCH and VARMA-AGARCH models for Dubai

Panel a. VARMA(1,1)-GARCH(1,1)

Returns	ω	α_{dubsp}	α_{dubfor}	β_{dubsp}	β_{dubfor}
rdubsp	**0.035**	0.004	**0.051**	**0.976**	**-0.038**
	(6.403)	(0.524)	**(4.672)**	**(106.169)**	**(-4.757)**
rdubfor	0.093	0.050	0.012	0.220	0.665
	(1.070)	(1.069)	(0.260)	(0.598)	(1.526)

Panel b.VARMA(1,1)-AGARCH(1,1)

Returns	ω	α_{dubsp}	α_{dubfor}	γ	β_{dubsp}	β_{dubfor}
rdubsp	**0.032**	-0.011	**0.051**	**0.021**	**0.975**	**-0.031**
	(5.510)	(-1.123)	**(5.409)**	**(2.421)**	**(106.637)**	**(-3.650)**
rdubfor	0.084	0.040	0.002	0.037	0.139	**0.758**
	(1.653)	(0.884)	(0.052)	(1.164)	(1.016)	**(4.639)**

Notes: Entries in bold are significant at 5%.

Figure 1
Returns of daily spot, forward and futures returns for Brent, WTI and Dubai

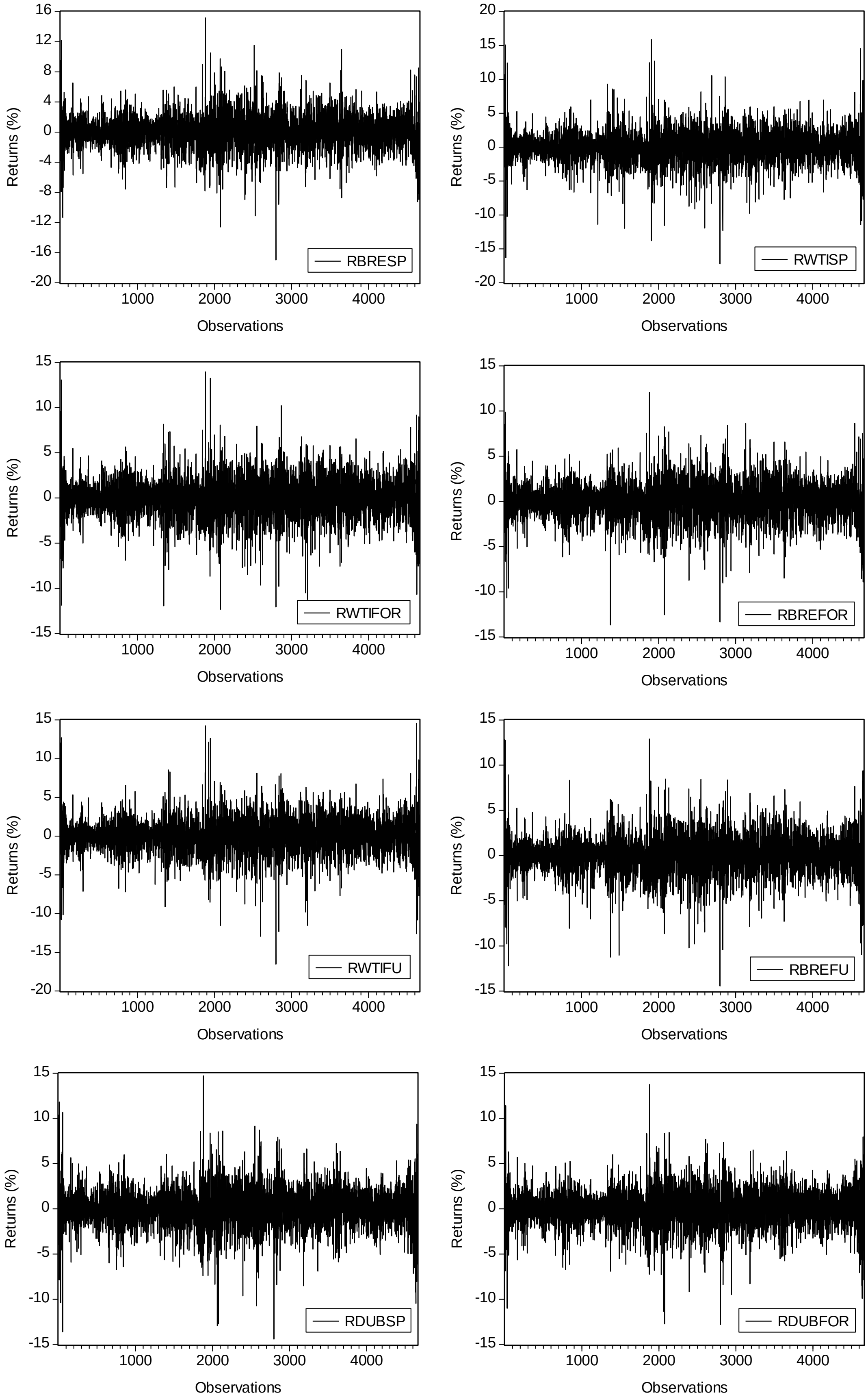

Figure 2
Dynamic conditional correlations