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Customer Lifetime Value and RFM Data:
ACCOUNTING YOUR CUSTOMERS: ONE BY ONE

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ABSTRACT

A customer behavior model that permits the estimation of customer lifetime value (CLV) from standard RFM data in "non-contractual" setting is developed by extending the hierarchical Bayes (HB) framework of the Pareto/NBD model (Abe 2008). The model relates customer characteristics to frequency, dropout and spending behavior, which, in turn, is linked to CLV to provide useful insight into acquisition. The proposed model (1) relaxes the assumption of independently distributed parameters for frequency, dropout and spending processes across customers, (2) accommodates the inclusion of covariates through hierarchical modeling, (3) allows easy estimation of latent variables at the individual level, which could be useful for CRM, and (4) provides the correct measure of errors. Using FSP data from a department store and a CD chain, the HB model is shown to perform well on calibration and holdout samples both at the aggregate and disaggregate levels in comparison with the benchmark Pareto/NBD-based model.

Several substantive issues are uncovered. First, both of our datasets exhibit correlation between frequency and spending parameters, violating the assumption of the existing Pareto/NBD-based CLV models. Direction of the correlation is found to be data dependent. Second, useful insight into acquisition is gained by decomposing the effect of change in covariates on CLV into three components: frequency, dropout and spending. The three components can exert influences in opposite directions, thereby canceling each other to produce less effect as the total on CLV. Third, not accounting for uncertainty in parameter estimate can cause large bias in measures, such as CLV and elasticity. Its ignorance can potentially have a serious consequence on managerial decision making.

Keywords: CRM, Acquisition, hierarchical Bayes, Pareto/NBD, MCMC

1. INTRODUCTION

Customer lifetime value (CLV) is one of the most important concepts in customer relationship management (CRM). To compute CLV, a customer retention rate, often denoted as *r* (Berger and Nasr 1998; Hughes 2000; Jain and Singh 2002), is required. Under "contractual" setting, customer dropout is clear, and therefore, the retention rate can be computed easily. Under "non-contractual" setting, however, the timing of customer dropout is not obvious. Customers do not declare the fact that they become inactive, but simply stop conducting business with the firm. This situation is also referred to as "unobserved customer attrition" by Blattberg, Kim and Neslin (2008). So, without knowing customer dropout (and hence the retention rate) in "non-contractual" setting, how do we compute CLV?

While scoring models work well in practice, Colombo and Jiang (1999) point out the weakness of scoring models as (1) falling short on generating explanatory insight and (2) treating customer heterogeneity as noise. Fader, Hardie and Lee (2005) (hereafter referred to as FHL) describe the problems associated with scoring models when estimating CLV: (1) they ignore periods 3, 4, 5, failing to capture the dynamics of buyer behavior well into the future, (2) data must be split into two or more periods in order to calibrate the model, and (3) they fail to recognize that different "slices" of the data will yield different values of the behavior variables, resulting in different parameter estimate. Malthouse and Blattberg (2005) report the difficulty of predicting CLV with scoring models by asserting the 20-55 and 80-15 rules: (1) of the future top 20% customers, approximately 55% will be misclassified, and (2) of the future bottom 80%, approximately 15% will be misclassified. The disappointing predictive accuracy is exacerbated by the fact that their scoring models include a highly flexible (in the sense of nonlinearity) neural network. These studies seem to imply that a fruitful direction for estimating CLV in "non-contractual" setting is to construct a *probability* model based on a sound behavior theory.

A popular approach along this direction is to use a Pareto/NBD model (Schimittlein, Morrison and Colombo 1978, hereafter referred to as SMC), whereby a customer being “alive” or “dead” is inferred from recency-frequency data through simple assumptions on purchase behavior. Some of the CLV research that utilize a Pareto/NBD model include FHL, Reinartz and Kumar (2003), and Smittlein and Peterson (1994).

The objectives of this research are two folds. First, in light of the previous argument, we attempt to estimate CLV in “non-contractual” setting with a Pareto/NBD-based behavior model using only standard recency, frequency, and monetary-value (RFM) data. We limit our focus on standard RFM data, because RFM analysis is used extensively in industry, implying that rather rich information on a customer is condensed into these three simple statistics. Even if firms may not keep the entire purchase history of each customer, most firms in CRM collect, at least, their customers’ RFM data. The second objective is to obtain useful implications for prospective customers, as CLV often includes the notion of not only retention but also acquisition (Berger and Nasr 1998; Blattberg and Deighton 1995, Blattberg, Getz and Thomas 2001). To seek insight into acquisition from the analysis of existing customers, some customer characteristics (e.g., demographics) are used to relate to RFM data (behavior), and hence CLV.

1.1. Conceptualization of the Model

While the detail will be discussed in the next section, Table 1 highlights our methodology in comparison with that of SP and FHL. For recency-frequency data, both SP and FHL adopt a Pareto/NBD model that presumes Poisson purchase and random dropout (a constant hazard rate) processes whose parameters are independently distributed as gamma. For monetary-value data, SP posit a normal-normal model, whereby purchase amounts on different occasions within a customer is normally distributed with the mean following a normal distribution in order to capture customer heterogeneity. FHL use a gamma-gamma model, whereby the normal distributions within and across customers in SP are replaced by gamma distributions. Both methodologies can be characterized as an individual-level behavior model whose parameters are compounded with a mixture distribution to capture customer heterogeneity, which, in turn, is estimated by an empirical Bayes method. An empirical Bayes method, in general, utilizes MLE for population-level parameters of the mixture distribution, which specifies the prior for individual-level parameters that are updated in a Bayesian manner.

Insert Table 1 about here.

The proposed methodology posits the same *behavioral* assumptions as SP and FHL, yet customer heterogeneity is captured through a more general mixture distribution to account for dependence among the three behavior processes: purchase, dropout, and spending. Our approach extends the Hierarchical Bayes framework of the Pareto/NBD model on customer transaction (Abe 2008) to purchase amount, whereby (1) the analytical part of the heterogeneity mixture distribution is replaced by a simulation method, and (2) unobservable measures, such as a customer lifetime (survival) and an active/inactive indicator, are incorporated into the model as latent variables. By avoiding analytical aggregation, the approach leads to a simpler and cleaner model that offers various advantages.

First, the proposed model relaxes the assumption of independently distributed parameters, λ, μ, and η, respectively, for purchase, dropout, and spending processes across customers, which is made in the Pareto/NBD with normal-normal (SP) or gamma-gamma (FHL) spending model. Managerially, this assumption restricts that shopping frequency, lifetime, and spending per trip are not related. One might speculate certain relationship, for instance, frequent shoppers tend to live longer and/or spend higher amounts (Reinartz, Thomas and Kumar 2005; Thomas, Reinartz and Kumar 2003). The proposed HB model accommodates a more general correlated mixture distribution with ease, because aggregation of heterogeneous customers is carried out by a simulation method. In the empirical section, we will indeed find that models by SP and FHL should not be applied to both of our datasets since the independence assumption does not hold. Our method not only accommodates correlation among parameters, but also allows the performing of statistical inference on the independence assumption.

Second, hierarchical models, whereby customer-specific parameters, λ, μ, and η, are a function of covariates, can be constructed and estimated with ease.

(a) This implies that, even in the absence of RFM data, customer characteristics can be used to predict purchase, dropout, and spending behavior, and thus CLV to certain extent. Operationally, such models can be quite useful when seeking prospective customers with high CLV for acquisition.

(b) Substantively, such hierarchical models can shed light on interesting yet conflicting findings in CRM. For example, what are characteristics of loyal (long lifetime) customers, and whether loyal customers spend more? Previous research investigated such issues with a two-step approach: lifetime duration is first estimated to identify loyal customers, and then customer characteristics (explanatory variables) are related to the lifetime duration (dependent variable). A hierarchical model, whose dropout parameter is a function of customer characteristics, can be estimated in one step, providing the correct measure of error for statistical inference.

Third, in the HB model, posterior distributions of purchase rate λ, dropout rate μ, and spending parameter η are constructed at the individual level by MCMC draws as a byproduct of the estimation. Thus, any of their statistics, such as mean and variance, can be computed by simple algebra, as discussed in Abe (2008).[1] It is also straightforward to obtain a distribution

[1] In a Pareto/NBD model, the expressions for the posterior density of an individual level λ, μ, and η involve complicated integration that cannot be reduced. Furthermore, as a new statistic based on λ, μ, and η is required (such as a mean, a variance, a survival probability, the

of any individual statistic that is a function of λ, μ, and η, such as a survival probability and CLV, by evaluating the function for each MCMC draw of λ, μ, and η. For this reason, the model provides useful individual level information for CRM, such as ranking customers according to frequency, lifetime, spending, CLV, etc.

Forth, a Bayesian framework using MCMC simulation provides a posterior distribution of parameters being estimated rather than point estimate, thereby providing a correct measure of error necessary for statistical inference, even with a small sample. It also has advantage over the Pareto/NBD-based model estimated by empirical Bayes, which overestimates the precision because the same data is used twice, once for constructing the likelihood function describing customer specific behavior and the other for estimating the prior (mixture distribution). It will be shown that ignoring uncertainty can lead to biased statistics such as CLV, and therefore, resulting in incorrect managerial decisions.

In the next section, the proposed HB model is described and compared against the Pareto/NBD-based model, followed by an explanation of the model estimation using an MCMC method in conjunction with a data augmentation technique. Then, empirical analyses using data from frequent shopper program of a department store and a music CD chain is presented. Finally, conclusions and future directions are discussed.

2. PROPOSED MODEL OF CLV

2.1. Model Assumptions

This section describes the assumptions of the HB model.

Individual Customer

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expected number of transaction and CLV), an analytical expression must be derived each time from the posterior distributions of λ, μ, and η. In actual computation, numerical evaluation of this integration (i.e., non-standard hypergeometric function of various kinds) must be repeated for each statistic and for each customer. This makes the computation of the individual level statistics difficult in Pareto/NBD models.

Assumption 1: Poisson purchases. While active, each customer makes purchases according to a Poisson process with rate λ.

Assumption 2: Exponential lifetime. Each customer remains active for a lifetime, which has an exponentially distributed duration with dropout rate μ.

Assumption 3: Lognormal spending. Within each customer, amounts of spending on purchase occasions are distributed as lognormal with location parameter η.

Assumptions 1 and 2 are identical to the behavioral assumptions of a Pareto/NBD model. Because their validity has been studied by other researchers (FHL 2005; Reinartz and Kumar 2000, 2003; SMC 1987; SP 1994), justification is not provided here for brevity. Assumption 3 is specified because (1) the domain of spending is positive, and (2) inspection of the distributions of spending amounts within customers reveals a skewed shape resembling lognormal. As described previously, SP and FHL assume normal and gamma, respectively, to characterize the distribution of spending amounts *within* a customer.

Heterogeneity across Customers

Assumption 4: Individuals' purchase rates λ, dropout rates μ, and spending parameters η follow a multivariate lognormal distribution.

Assumption 4 permits correlation among purchase, dropout, and spending parameters. Because Assumption 4 implies that $\log(\lambda)$, $\log(\mu)$, and $\log(\eta)$ follow a multivariate normal, estimation of the variance-covariance matrix is tractable using a standard Bayesian method. A Pareto/NBD combined with either normal-normal (SP) or gamma-gamma (FHL) spending model posits independence among the three behavioral processes. Both SP and FHL acknowledge the restriction on their models and perform an extensive assumption check on their data to validate their application.

2.2. Mathematical Notations

For recency and frequency data, we will follow the standard notations {x, t_x, T}, used by SMC, FHL, and Abe (2008). τ is an unobserved customer lifetime. For spending, s denotes the amount of spending on a purchase occasion. Using these mathematical notations, the previous assumptions can be expressed as follows.

(1) $$P[x \mid \lambda] = \begin{cases} \dfrac{(\lambda T)^x}{x!} e^{-\lambda T} & \text{if } \tau > T \\ \dfrac{(\lambda \tau)^x}{x!} e^{-\lambda \tau} & \text{if } \tau \le T \end{cases} \qquad x = 0,1,2,..$$

(2) $$f(\tau) = \mu\, e^{-\mu \tau} \qquad \tau \ge 0$$

(3) $$\log(\mathrm{s}) \sim N(\log(\eta), \omega^2) \qquad s > 0$$

(4) $$\begin{bmatrix} \log(\lambda) \\ \log(\mu) \\ \log(\eta) \end{bmatrix} \sim MVN\left(\theta_0 = \begin{bmatrix} \theta_\lambda \\ \theta_\mu \\ \theta_\eta \end{bmatrix}, \Gamma_0 = \begin{bmatrix} \sigma_\lambda^2 & \sigma_{\lambda\mu} & \sigma_{\lambda\eta} \\ \sigma_{\mu\lambda} & \sigma_\mu^2 & \sigma_{\mu\eta} \\ \sigma_{\eta\lambda} & \sigma_{\eta\mu} & \sigma_\eta^2 \end{bmatrix} \right)$$

where N and MVN denote univariate and multivariate normal distributions, respectively. ω^2 is the variance of spending amounts *within* a customer.

2.3. Expressions for Transactions, Sales, and CLV

Given the individual level parameters for (λ, μ), the expected number of transactions in the time period of u, $E[X(u)|\lambda, \mu]$, becomes

(5) $$E[X(u) \mid \lambda, \mu] = \lambda E[\psi] = \frac{\lambda}{\mu}\left(1 - e^{-\mu u}\right) \qquad \text{where } \psi = \min(\tau, u).$$

The expected sales during this period u is simply the product of the expected number of transactions shown in Equation (5) and the expected spending E[s] as

(6) $$E[sales(u) \mid \lambda, \mu, \eta, \omega] = E[s \mid \eta, \omega] E[X(u) \mid \lambda, \mu] = \eta e^{\omega^2/2} \frac{\lambda}{\mu}\left(1 - e^{-\mu u}\right) .$$

For CLV, we define "value" to be synonymous with "revenue" because margin and cost information is unknown in this study. The general formula of CLV for an individual customer under a continuous time framework, as appropriate for a Pareto/NBD model, is expressed as

$$CLV = \int_0^{\infty} V(t)R(t)D(t)dt \,,$$

where $V(t)$ is the customer's value (revenue) at time t, $R(t)$ is the survival function (the probability that a customer remains active until at least t), and $D(t)$ is a discount factor reflecting the present value of money received at time t (FHL, Rosset et al. 2003). Translating to our Assumptions 1~3, they imply $V(t)= \lambda \mathrm{E}[s]$ where $\mathrm{E}[s]=\eta \exp(\omega^2/2)$ from the definition of lognormal, and $R(t)=\exp(-\mu t)$. With continuously compounded discounting of an annual interest rate d, $D(t)=\exp(-\delta t)$, where $\delta=\log(1+d)$ with the time unit being a year. Therefore, our CLV reduces to the following simple expression.

(7) $$CLV = \int_0^{\infty} V(t)R(t)D(t)dt = \int_0^{\infty} \lambda\eta e^{\omega^2/2} e^{-\mu t} e^{-\delta t} dt = \frac{\lambda\eta e^{\omega^2/2}}{\mu+\delta}$$

Hence, if we could somehow estimate λ, μ, η, ω for each customer from RFM data, we can compute CLV as in Equation (7).

2.4. Incorporating Customer Characteristics

To gain insight into acquisition, we would like to relate customer characteristic variables for customer i, d_i (a $K \times 1$ vector) to customer specific parameters λ_i, μ_i, and η_i. A straightforward extension of Assumption 4 expressed in equation (4) results in a multivariate regression specification as follows.

(8) $$\begin{bmatrix} \log(\lambda_i) \\ \log(\mu_i) \\ \log(\eta_i) \end{bmatrix} \sim MVN\left(\theta_i = Bd_i, \Gamma_0 = \begin{bmatrix} \sigma_\lambda^2 & \sigma_{\lambda\mu} & \sigma_{\lambda\eta} \\ \sigma_{\mu\lambda} & \sigma_\mu^2 & \sigma_{\mu\eta} \\ \sigma_{\eta\lambda} & \sigma_{\eta\mu} & \sigma_\eta^2 \end{bmatrix} \right)$$

where B is a $3 \times K$ matrix of coefficients. When d_i contains a single element of 1 (i.e., no characteristic variables), the common mean, $\theta_0=\theta_i$ for all customers i, is estimated.

2.5. Elasticities

Useful implications can be obtained from computing elasticities of CLV with respect to (a) λ, μ, and η, and (b) characteristic variables d_i. From equation (7),

$$E_{\lambda}^{CLV} = \frac{\partial CLV / CLV}{\partial \lambda / \lambda} = 1, \qquad E_{\mu}^{CLV} = -\frac{\mu}{\mu+\delta}, \qquad E_{\eta}^{CLV} = 1, \tag{9}$$

implying that one percent increase in the purchase rate or spending parameter causes one percent increase in CLV, whereas one percent decrease in the dropout rate leads to less than one percent increase in CLV with the magnitude depending on the discount rate δ. Under a high interest rate, the impact of prolonging lifetime on CLV is not as rewarding since future customer value would be discounted heavily.

The effect of customer characteristics on CLV can be decomposed into frequency, dropout, and spending processes to provide further insight. Defining d_{ik} as the k-th (continuous) characteristic of customer i, the elasticity becomes

$$\begin{aligned} E_{d_{ik}}^{CLV} &= \frac{\partial CLV_i / CLV_i}{\partial d_{ik} / d_{ik}} = \left[\frac{\partial CLV_i}{\partial \lambda_i}\frac{\partial \lambda_i}{\partial d_{ik}} + \frac{\partial CLV_i}{\partial \mu_i}\frac{\partial \mu_i}{\partial d_{ik}} + \frac{\partial CLV_i}{\partial \eta_i}\frac{\partial \eta_i}{\partial d_{ik}} \right] \frac{d_{ik}}{CLV_i} \\ &= \left[b_{\lambda k} - \frac{b_{\mu k}\mu_i}{\mu_i + \delta} + b_{\eta k} \right] d_{ik} = Ef_{d_{ik}}^{CLV} + Ed_{d_{ik}}^{CLV} + Es_{d_{ik}}^{CLV} \end{aligned} \tag{10}$$

where b_{lk} ($l \in \{\lambda,\mu,\eta\}$, k=1,..,K) denotes $(l, k)^{th}$ element of matrix B.

3. ESTIMATION

In the previous section, analytical expressions for the customer process of frequency, dropout, and spending, are derived from the basic behavioral Assumptions 1, 2, and 3. To account for customer heterogeneity, the HB approach does not require the aggregate analytical expressions compounded by the mixture distribution.

3.1. Estimation of the Transaction (RF) Component

The transaction component is identical to the HB extension of the Pareto/NBD model proposed by Abe (2008). It is estimated by MCMC simulation through a data augmentation method. Because information about customer *i* being active (z_i=1) or not at time T, and if not, the dropout time ($y_i<T_i$) is unknown, z_i and y_i are considered as latent variables, which are randomly drawn from their posterior distributions. The detail is described in Abe.

3.2. Estimation of the Spending (M) Component

As explained at the beginning of this paper, we limit our behavior data to recency, frequency, and monetary-value (RFM), so that the model can be implemented even if a firm does not keep the complete purchase history of each customer. The RFM data for customer *i* are denoted as (x_i, t_{xi}, T_i, as_i), where x_i, t_{xi} and T_i are defined as in SMC and as_i stands for average spending per purchase occasion. Without the knowledge of spending variation *within a customer* from one purchase to another, however, there is no means to infer the variance of logarithmic spending ω^2, specified in equation (3), from RFM data alone. Here, we assume that ω^2 is known (for example, from past data) and common across customers.[2]

Assumption 3 allows adoption of standard normal conjugate updating in Bayesian estimation, whereby the posterior mean is a precision weighted average of the sample and the prior means. For this method to work, however, we need the mean of log(spending)s (or equivalently, the logarithm of the geometric mean of spending amounts) from each customer, whereas the M part of RFM data provides only the arithmetic mean of spendings as_i. The appendix shows that the sample mean of log(spending)s, can be approximated by the average spending as_i and ω^2, as in Equation (11).

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[2] To characterize variation in spending, we could have assumed that either ω^2, the variance in logarithmic spending, or the variance in the natural scale of spending is known. We posited the former because it seemed more reasonable to think that the magnitude of spending variation grows as the spending level increases, and inspection of the data supported our speculation.

$$(11) \qquad \frac{1}{x_i}\sum_{m=1}^{x_i}\log(s_{im}) \cong \log(as_i) - \frac{1}{2}\left(\omega^2 + \frac{\omega^4}{4}\right)$$

3.3. Prior Specification

Let us denote the customer specific parameters as φ_i = [log(λ_i), log(μ_i) log(η_i)]', which is normally distributed with mean $\theta_i=Bd_i$ and variance-covariance matrix Γ_0 as in Equation (8). Our objective is to estimate parameters { φ_i, y_i, z_i, $\forall i$; B, Γ_0} from observed RFM data {x_i, t_{xi}, T_i, as_i; $\forall i$}. In the HB framework, the prior of individual-level parameter φ_i corresponds to the population distribution $MVN(Bd_i, \Gamma_0)$. The priors for the hyperparameters B and Γ_0 are chosen to be multivariate normal and inverse Wishart, respectively.

$$vec(B) \sim MVN(b_{00}, \Sigma_{00}), \qquad \Gamma_0 \sim IW(\nu_{00}, \Gamma_{00})$$

These distributions are standard conjugate priors for multivariate regression models. Constants (b_{00}, Σ_{00}, ν_{00}, Γ_{00}) are chosen to provide very diffuse priors for the hyperparameters.

3.4. MCMC Procedure

We are now in a position to estimate parameters { φ_i, y_i, z_i, $\forall i$; B, Γ_0} using an MCMC method. To estimate the joint density, we sequentially generate each parameter, given the remaining parameters, from its conditional distribution until convergence is achieved. The algorithm can be found in Abe (2008).

4. EMPIRICAL ANALYSIS

4.1. FSP data for a department store

We now apply the proposed model to real data. The first dataset contains shopping records for 400 members of a frequent shopper program (FSP) at a department store in Japan. The first and the second 26 weeks of the data are used for model calibration and validation, respectively. They are the same data used by Abe (2008), and their detail can be found in his paper. As discussed in Section 3.1, the variance of log(spending)s within customers, ω^2, is assumed to be known. It was estimated to be 0.895 from the calibration data.

4.1.1. Model Validation

The MCMC steps were put through 15,000 iterations, of which the last 5,000 were used to construct the posterior distribution of parameters. The convergence was monitored visually and checked with the Geweke test (Geweke 1992). The dispersion of the proposal distribution in the Metropolis-Hastings algorithm was chosen such that the acceptance rate stayed at about 40% to allow even drawing from the probability space (Gelman et al. 1995).

Insert Table 2 about here.

Table 2 shows the result of various models that include different covariates. Before attempting to interpret the result, let us first discuss the model validation focusing on Model 3, which has the best marginal loglikelihood, as shown in the last row. The performance of Model 3 was evaluated with respect to the number of transactions and spending, obtained from Equations (5) and (6), respectively, in comparison with the benchmark Pareto/NBD-based model. The expected number of transactions, predicted by the Pareto/NBD, was multiplied by his/her average spending as_i to come up with customer *i*'s spending.

Insert Figure 1 about here.

Figure 1 shows the time-series tracking for the cumulative number of repeat purchases. Both models provide good fit in calibration and forecast in validation, which are separated by the vertical dashed line. With respect to the mean absolute percent errors (MAPE) between predicted and observed weekly cumulative purchases, the HB model performed better for validation (1.3% vs. 1.9%) and comparable for calibration (2.5% vs. 2.5%).

Insert Table 3 about here.

Fit statistics at the disaggregate level provide more stringent performance measures. Table 3 compares the correlation and mean squared error (MSE) between prediction and observation with respect to the number of transactions and total spending at the individual customer level during calibration and validation periods. While both models offer similar performance, the HB is slightly superior in predicting spending.

Insert Figures 2 and 3 about here.

Figure 2 shows the predicted number of transactions during the validation period, averaged across individuals, conditional on the number of purchases made during the calibration period. Figure 3 compares the predicted total spending during the validation period in the similar manner. Both figures demonstrate the superiority of the HB model over the Pareto/NBD-based model visually.

In sum, the HB model seems to fit and predict well in comparison with the Pareto/NBD-based model, in terms of the number of transactions and spending both at the aggregate and disaggregate levels. However, the difference of the two models is minor.

4.1.2. Interpretation of the Model Estimation

Having established the validity of the HB model, let us now examine Table 2 to interpret the estimation result. FOOD, the fraction of store visits on which food items were purchased and a proxy for store accessibility, is the most important covariate with the significant positive and negative coefficients for $\log(\lambda)$ and $\log(\eta)$, respectively, at the 5% level. The result is also supported by the Pareto/NBD model of Abe (2008), whereby "average spending" covariate was found to be significantly negative for $\log(\lambda)$. Managerially, food buyers tend to shop more often and spend a smaller amount on each shopping trip. This finding is consistent with the story told by a store manager in that, although food buyers spend a smaller amount on each shopping trip, they visit the store often enough to be considered as vital. Another significant covariate is AGE for log(η), implying that older customers tend to spend more at each shopping trip. This is hardly surprising either. Older people with lower income would shop in discount stores close to where they live rather than venture out or bother to visit a department store located in a busy shopping district, like this retailer.

Let us now turn our attention to the relationship among purchase rate λ, dropout rate μ, and spending parameter η. To check whether the independence assumption of Pareto/NBD is satisfied, correlation of Γ_0 must be tested on the intercept-only model (Model 0) but not the covariate model. This is because, if covariates explain the correlation among λ, μ and η completely, then no correlation remains in the error term as captured by Γ_0. First, we see that the correlation between log(λ) and log(μ) is not significantly different from 0, implying that the assumption of Pareto/NBD holds here. Second, correlation between log(λ) and log(η) is significantly negative (-0.28), the fact that is consistent with the FOOD variable having opposite signs on log(λ) and log(η) in Model 3. Figure 4 presents the scatter plot of the mean values of the individual λ_i and η_i (i=1,..,400). One can visually observe the correlation.

Insert Figure 4 about here.

Hence, the assumption of the independence between transaction (RF) and spending (M) components in the Pareto/NBD based model (SP and FHL) is violated in this dataset. For researchers using the SP and FHL models, the finding emphasizes the importance of verifying the independent assumption (as was done in SP and FHL). Managerially, this negative correlation implies that a frequent shopper tends to spend a smaller amount on each trip. No correlation is found between shopping frequency and his/her lifetime or between lifetime and per-trip spending.

4.1.3. Customer Lifetime Value

Table 4 presents nine customer-specific statistics for the top and bottom 10 customers in terms of CLV, along with the average, minimum, and maximum for the entire sample of 400 customers: posterior means of λ_i, μ_i and η_i, expected lifetime, survival rate after one year, the probability of being active at the end of the calibration period, an expected number of transactions (using equation (5)) and expected total spending during the validation period (using equation (6)), and CLV (using equation (7)). In computing CLV, an annual interest rate of 15% (δ = 0.0027 per week) was assumed.

Insert Table 4 about here.

There exists much heterogeneity across customers despite the use of the Bayesian shrinkage estimation. The mean expected lifetime, after incorporating discounting, is 8.6 years with the maximum and minimum of 20.0 and 1.4 years, respectively. The probability of being active at the end of the calibration period ranges from 0.19 to 1.00 with the average being 0.93. Over the validation period of 26 weeks, the expected number of transactions is 15.9 times with the total amount of 67,000 yen on average (divide by 100 to convert to the approximate US dollars). CLV ranges from 30,000 yen to 8.5 million yen with the average being about 0.59 million yen.[3]

Insert Figure 5 about here.
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[3] Let us emphasize that the posterior means of λ_i and μ_i, and hence, an expected lifetime cannot be obtained easily from the Pareto/NBD model. Furthermore, the remaining statistics are claimed by SMC and FHL as their main results, with complicated expressions (equations (11)-(13) and (22) of SMC and equation (2) of FHL) requiring multiple evaluations of various hypergeometric functions. With the HB model, these statistics can be computed from the MCMC draws of λ, μ, and η, using simple algebraic expressions of (5)~(7), and by taking their means over the draws.

Figure 5 shows a gain chart (solid line), in which customers are sorted according to the decreasing order of CLV and the cumulative CLV (y-axis, where the total CLV is normalized to 1) is plotted against the number of customers (x-axis). In addition, two gain charts are plotted. The dash-dotted line is based solely on recency criterion, whereby customers are sorted in the order of increasing recency (from most recent to least recent). The dotted line is a gain chart based on customers being ordered according to the sum of the three rankings of recency, frequency and monetary value. The 45 degree dashed line corresponds to the cumulative CLV for randomly ordered customers. This figure implies that recency criterion alone is not sufficient to identify good customers, despite many companies use this criterion. On the other hand, combined use of the three criteria (recency, frequency and monetary-value), even with the naïve equal weighting scheme, seems to provide rather accurate ordering of CLV. This finding strongly supports the wide use of RFM analysis and regression-type scoring models among practioners for identifying good customers.

4.1.4. Elasticity of Covariates on CLV

Another advantage of our Bayesian approach is that these statistics reflect the uncertainty in parameter estimates. Ignoring their uncertainty and computing various statistics from their point-estimates, say MLE, as if parameters are deterministic, could produce biased result, leading to incorrect managerial decisions. The point is illustrated in Table 5, which shows the decomposition of the elasticity of CLV with respect to each covariate into frequency, dropout, and spending components. To account for parameter uncertainty, elasticity is computed for each set of the 5000 MCMC draws of b_{lk} and μ_i according to Equation (10), which is then averaged over the 5000 draws and 400 customers. When the posterior mean of b_{lk} and μ_i is directly substituted into Equation (10) (bottom table) instead of averaging over MCMC draws (top table), elasticity with respect to the dropout component is overestimated (because of nonlinearity in μ_i), even if customer heterogeneity is accounted for.

Insert Table 5 about here.

To visualize the impact of covariates, the solid line in Figure 6 plots the value of log(CLV) for different values of a covariate when the other two covariates are fixed at their mean values. These graphs are computed using the mean estimate of the coefficients of Model 3 shown in Table 2, assuming that all covariates are continuous. For the FEMALE covariate, therefore, it should be interpreted as how log(CLV) varies when the gender mixture is changed from the current level of 93.3% female, while keeping the other two covariates unchanged. The dotted vertical line indicates the mean value of the covariate under consideration. Both FOOD and AGE have strong influences on log(CLV), whereas FEMALE exerts a very weak influence.

Insert Figure 6 about here.

Figure 6 also attempts to decompose the influence of covariates on log(CLV) into three components: frequency, dropout and spending. Taking logarithm of the basic formula of CLV in Equation (7) results in the following summation expression.

$$\begin{aligned}\log(CLV) &= -\log(\mu+\delta)+\log(\lambda)+\log(\eta)+\omega^2/2 \\ &= \left[-\log(\mu+\delta)+c_d\right]+\left[\log(\lambda)+c_f\right]+\left[\log(\eta)+c_s\right]+c' \\ &= \left[\text{Dropout component}\right]+\left[\text{Frequency component}\right]+\left[\text{Spending component}\right]+c'\end{aligned}$$

The graph can be interpreted as stacking these three components, dropout, frequency, and spending, from top to bottom, to constitute the overall log(CLV). To account for the scale differences among these components, c_d, c_f, and c_s are chosen such that each component is normalized to 1 at the mean value of the covariate. Therefore, log(CLV)=3 at the dotted vertical line.

The direction and magnitude of the effect of each covariate on the three components are consistent with the signs of the posterior means b_{lk} ($l \in \{\lambda,\mu,\eta\}$, k=1,..,K). Increasing the fraction of food buyers improves dropout (lengthen lifetime) and frequency, but decreases spending per trip with net increase in the overall CLV. Increasing the fraction of elderly people increases the spending without much influence on dropout and frequency, thereby resulting in net increase in the overall CLV. Increasing the fraction of female leads to little improvement in all three components and, hence, a negligible increase in the overall CLV.

Elasticity decomposition, shown in Figure 6 and Table 5, provides managers with useful insight into acquisition. An effort to manipulate certain customer characteristics might impact dropout, frequency, and spending components in opposite directions, thereby canceling each other to produce less effect as the total on CLV. For example, much of the improvement in frequency, from increasing the fraction of food buyers, is negated by the decline in spending, and only the dropout improvement provides the net contribution to CLV, as can be seen from Table 5 and the near flat dashed line of Figure 6. On the other hand, an effort to increase the proportion of elderly people is met with the boost in CLV, due to increased spending per trip with only a small negative influence on frequency.

To build effective acquisition strategy from these results, managers must make a fine balance between desired customer characteristics (i.e., demographics), desired behavioral profiles (i.e., dropout, frequency, and spending), responsiveness (elasticity) of the characteristic covariates on CLV, and acquisition cost of the desired target customers.

4.2. Retail FSP data for a Music CD chain

The second dataset is obtained from a FSP of a large chain for music CD. These data are also identical to the one used by Abe (2008). As for the first dataset, the MCMC steps were repeated 15,000 iterations and the last 5,000 were used to construct posterior distributions. Table 6 compares the performance of HB against Pareto/NBD for the number of transactions and spending at the customer level. The HB model is slightly superior to the Pareto/NBD in all criteria, and the visual plots, like Figures 2 and 3 shown for the department store, also confirm the fact.

Insert Tables 6 and 7 about here.

Table 7 reports the model estimation. Let us first examine Model 3, which results in the highest marginal loglikelihood, for significant explanatory variables. First, the amount of an initial purchase is positively significant on log(λ) and log(η), implying that customers with a larger trial purchase tend to buy more frequently and spend more per trip in the subsequent repeat purchases. Second, older customers appear to spend more per shopping trip.

Next, we turn our attention to the intercept model Model 0 for the relationship among λ, μ, and η. First, we see that the correlation between log(λ) and log(μ) is not significantly different from 0, implying that the assumption of Pareto/NBD holds here. Second, correlation between log(λ) and log(η) is significantly positive (0.14), the fact that is consistent with the initial purchase variable having the same significant signs on log(λ) and log(η). Once again, the independence assumption of the transaction and spending components in the Pareto/NBD based model (SP and FHL) is violated. This time, however, the sign is in the opposite direction, implying that the correlation between purchase frequency and spending per occasion is context dependent. While not the “average spending”, it is consistent with the Pareto/NBD model of Abe (2008) whereby “initial purchase amount” covariate has a significant positive sign for log(λ). Managerially, the correlation implies that frequent buyers spend more per shopping occasion.

Insert Table 8 and Figure 7 about here.

Table 8 shows the elasticity decomposition of CLV into frequency, dropout, and spending components. When parameter uncertainty is not accounted for, the dropout component is overestimated, as was the case for the department store data, by about 20%. Elasticity decomposition of CLV into the three components for varying levels of the three covariates is presented in Figure 7. A higher initial purchase amount is related to higher CLV by increasing frequency and increasing spending with almost no change in dropout. Older customers are associated with less frequency, reduced dropout, and higher spending per trip with the positive net contribution to CLV. Female customers are associated with less frequency, less dropout, less spending with the negative net contribution to CLV.

Insert Table 9 about here.

Finally, Table 9 presents nine customer-specific statistics for the top and bottom 10 customers in terms of CLV, along with the average, minimum, and maximum for the entire sample of 500 customers.

CONCLUSIONS

An individual behavior model that permits the estimation of CLV from standard RFM data in “non-contractual” setting was developed based on the HB framework. The model also related customer characteristics to frequency, dropout and spending behavior, which, in turn, were linked to CLV to provide useful insight into acquisition. The HB model posited three sound behavioral assumptions from previous research: (1) a Poisson purchase process, (2) a random dropout process (i.e., exponentially distributed lifetime), and (3) a lognormally distributed spending process, while accounting for customer heterogeneity in all three processes. Because, in the HB framework, heterogeneity was captured as a prior rather than through a mixture distribution, the entire modeling effort could bypass all the complications associated with aggregation, which was left to MCMC simulation. Using FSP data from a Japanese department store and a CD chain, the HB model was shown to perform well on calibration and holdout samples both at the aggregate and disaggregate levels in comparison with the benchmark Pareto/NBD-based model.

Methodological contributions of this research are [1] a development of the individual-level behavioral model for RFM data in the HB framework, in which an empirical Bayes approach was used previously, and [2] the MCMC method that combines a data augmentation technique to permit the estimation of individual level latent variables, such as an active/inactive indicator and dropout time in our study. The approach offers: (1) accommodation of correlated parameters, (2) ease of estimating latent variables at the individual level, (3) extension to hierarchical models incorporating covariates, and (4) estimation of the correct measure of errors. The proposed methodology has made it possible to address many issues suggested as future extensions by previous research (Blattberg et al. 2008, p.129; FHL 2005; Jain and Singh 2002; SMC 1987).

In addition, several substantive issues were uncovered.

- The independence between transaction (RF) and spending (M) components could be violated. Our datasets exhibited weak yet significant correlation between frequency and spending. Furthermore, the direction of the correlation is data dependent. Researchers must always check this assumption with their data before applying a Pareto/NBD-based model.
- Effect of the change in covariates on CLV is decomposed into three components: frequency, dropout and spending. They could have opposite signs, thereby canceling each other to produce less effect as the total on CLV. In our department store data, much of the improvement in frequency, from increasing the fraction of food buyers, was negated by the decline in spending, and only the dropout improvement contributed to the net increase in CLV.
- Not accounting for uncertainty in parameter estimate can cause bias in statistics, such as CLV and elasticity in our study, if the expression involving parameters is nonlinear. While

marketing has paid the universal attention to accommodating heterogeneity, that is not the case for parameter uncertainty. When computing elasticity, for example, some researchers simply substitute point-estimate, such as MLE, into the formula without bothering to account for their standard errors. Bayesian methods, in conjunction with sampling estimation techniques, are powerful means to address this uncertainty issue even under small samples because they do not rely on the asymptotic theories.

Finally, the model provides the following managerial implications when implemented on a CRM system that collects RFM data.

- The model outputs, besides a customer-specific survival rate, include individual level λ_i, μ_i and η_i, expected lifetime, the probability of being active, expected number of future transactions, expected total spending, and CLV, some of which would be difficult, if not impossible, to obtain from the Pareto/NBD-based model. These statistics fully recognize the difference among customers and can be useful in implementing effective customized marketing.
- Useful insight into acquisition can be gained from a simple formula of CLV (Equation (7)) and its elasticity decomposition into dropout, frequency, and spending components that are linked to customer characteristics (Equation (10)).
- When planning managerial actions based on a model, for example, by adding an optimization module, uncertainty in parameter estimation can easily be incorporated with the simulation-based MCMC method. Ignoring uncertainty could potentially have a serious consequence on decision making.

The current study is only the beginning for the stream of research toward understanding customer behavior in “non-contractual” setting. Possible extensions are synonymous to the limitations of the current model. First, while our assumption of Poisson purchase (i.e., exponentially distributed interpurchase time) and lognormal spending might to be suitable for simple RFM data, the model can be refined if further information on customers beyond RFM data is available. With customer's complete transaction history that includes interpurchase times and the amount of each transaction, one immediate refinement is to formulate customer-specific spending variance ω_i^2, updated in a Bayesian manner just like the mean η_i, from the data. Such a model can differentiate customers with different spending patterns, for example, food-only shoppers from food-and-occasional-large-ticket-item shoppers. Another refinement is to relax the assumption of the Poisson purchase process so that interpurchase time can take a more general form in distribution.

Second extension is to formulate a model in the structural modeling framework. If one wishes to incorporate the effect of marketing activities as the covariates for frequency, dropout, and spending, the endogeneity issue must be addressed.

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APPENDIX

Derivation of Equation (11)

We approximate the desired quantity as follows. Expanding $\log(s_i)$ around the arithmetic mean μ_s gives,

$$\log(s_i) \cong \log(\mu_s) + \frac{(s_i - \mu_s)}{\mu_s} - \frac{(s_i - \mu_s)^2}{2\mu_s^2}.$$

Taking its expectation,

$$E[\log(s_i)] = \log(\mu_s) - \frac{\sigma_s^2}{2\mu_s^2}$$

where σ_s^2 is a variance of the spending. Now, equating the left-hand-side with the sample mean of the logarithmic spendings as

$$E[\log(s_i)] \cong \frac{1}{x_i}\sum_{m=1}^{x_i}\log(s_{im}),$$

and replacing the expectation μ_s on the right-hand-side with its sample mean as_i follows

(12) $$\frac{1}{x_i}\sum_{m=1}^{x_i}\log(s_{im}) \cong \log(as_i) - \frac{\sigma_s^2}{2as_i^{\,2}}.$$

We now need to express variance σ_s^2 in terms of ω^2 as follows. Recall Assumption 3.

(3) $$\log(s) \sim \mathrm{N}(\log(\eta), \omega^2)$$

Define $u = \log(s)$ and $\mu = \log(\eta)$. Then $s(u) = e^u$, $E[s] = e^{u_0}$, where $u_0 = \mu + \frac{\omega^2}{2}$.

Expanding s(u) around u_0 becomes

$$s(u) = s(u_0) + (u - u_0)e^{u_0} = E[s] + (u - u_0)E[s].$$

Thus,

$$\begin{aligned}
\sigma_s^2 &\equiv E\left[(s(u) - E[s])^2\right] \\
&= E\left[(u - u_0)^2\right]E[s]^2 \\
&= E\left[\left(u - \mu - \frac{\omega^2}{2}\right)^2\right]E[s]^2 \\
&= E\left[(u-\mu)^2 + \frac{\omega^4}{4} - 2\frac{\omega^2}{2}(u-\mu)\right]E[s]^2 \\
&= \left[\omega^2 + \frac{\omega^4}{4}\right]E[s]^2
\end{aligned}$$

Replacing E[s] with its sample mean as_i and substituting in equation (12) leads to equation (11).

(11) $$\frac{1}{x_i}\sum_{m=1}^{x_i}\log(s_{im}) \cong \log(as_i) - \frac{1}{2}\left(\omega^2 + \frac{\omega^4}{4}\right)$$

Table 1. Comparison with the Existing Methods

Empirical Bayes Model

Data	**Model**	**Individual Behavior**	**Heterogeneity Distribution**
RF (recency-frequency)	Pareto/NBD (SMC 1987)	Poisson purchase (λ) Random dropout (μ)	λ ~ Gamma μ ~ Gamma λ and μ independent
M (monetary-value)	normal-normal (SP 1994)	Normal spending (mean θ)	θ ~ Normal θ, λ, μ independent
	gamma-gamma (FHL 2005)	Gamma spending (scale ν)	ν ~ Gamma ν, λ, μ independent

Proposed Hierarchical Bayes Model

Data	**Model**	**Individual Behavior**	**Heterogeneity Distribution**
RF (recency-frequency)	poisson/exponential	Poisson purchase (λ) Random dropout (μ)	λ, μ, η ~ MVL λ, μ, η correlated
M (monetary-value)	lognormal-lognormal	Lognormal spending (location η)	

Table 2. Estimation Results of Various Models

(Figures in parentheses indicate the 2.5 and 97.5 percentiles)

		Model 0	Model 1	Model 2	Model 3
Purchase rate λ	**Intercept**	-0.81 (-0.92, -0.71)	-1.96 (-2.28, -1.64)	-1.89 (-2.28, -1.52)	-2.03 (-2.52, -1.51)
	Food	---	1.45* (1.08, 1.82)	1.49* (1.09, 1.88)	1.50* (1.11, 1.89)
	Age	---	---	-0.19 (-0.83, 0.42)	-0.21 (-0.84, 0.40)
	Female (male=0)	---	---	---	0.15 (-0.20, 0.48)
Dropout Rate μ	**Intercept**	-6.13 (-7.10, -5.56)	-5.21 (-6.79, -4.14)	-4.94 (-6.19, -3.74)	-5.03 (-6.49, -3.57)
	Food	---	-1.54 (-3.42, 0.33)	-1.14 (-2.59, 0.43)	-1.09 (-2.66, 0.26)
	Age	---	---	-0.48 (-2.38, 1.38)	-0.34 (-2.35, 1.49)
	Female (male=0)	---	---	---	0.01 (-1.20, 1.38)
Spending Parameter η	**Intercept**	-3.66 (-3.74, -3.59)	-2.77 (-3.01, -2.54)	-3.23 (-3.51, -2.94)	-3.32 (-3.69, -2.94)
	Food	---	-1.13* (-1.40, -0.86)	-1.35* (-1.63, -1.06)	-1.35* (-1.63, -1.06)
	Age	---	---	1.19* (0.72, 1.65)	1.18* (0.73, 1.63)
	Female (male=0)	---	---	---	0.11 (-0.17, 0.38)
correlation(log(λ), log(μ))		-0.33 (-0.59, 0.01)	-0.27 (-0.52, 0.02)	-0.28 (-0.54, 0.05)	-0.24 (-0.51, 0.09)
correlation(log(λ), log(η))		-0.28* (-0.39, -0.17)	-0.14* (-0.26, -0.02)	-0.14* (-0.26, -0.01)	-0.14* (-0.26, -0.01)
correlation(log(μ), log(η))		-0.02 (-0.31, 0.26)	-0.10 (-0.37, 0.19)	-0.09 (-0.37, 0.17)	-0.05 (-0.33, 0.23)
marginal loglikelihood		-2105	-2088	-2084	-2078

*** indicates significance at the 5% level**

Table 3. Disaggregate Fit of Pareto/NBD and HB Models

		Pareto/NBD	HB
Spending			
Correlation	**validation**	0.80	0.83
	calibration	0.99	0.99
MSE	**validation**	0.39	0.35
	calibration	0.02	0.06
Transactions			
Correlation	**validation**	0.90	0.90
	calibration	1.00	1.00
MSE	**validation**	57.7	56.5
	calibration	1.22	3.92

Table 4. Customer-Specific Statistics for Top and Bottom 10 Customers

ID	mean(λ)	mean(μ)	mean(η)	Mean Expected lifetime (years)	1 year Survival rate	Probability of being active at the end of calibration	Expected number of transactions in validation period	Expected total spending in val. period ($x10^5$ yen)	CLV ($x10^5$ yen)
1	3.14	0.00213	0.068	20.0	0.908	1.000	79.4	8.48	85.3
2	1.94	0.00216	0.051	17.7	0.904	1.000	49.2	3.89	38.5
3	1.15	0.00250	0.088	15.4	0.893	0.999	28.9	3.97	38.1
4	2.54	0.00216	0.032	17.3	0.904	1.000	64.4	3.25	31.9
5	1.08	0.00379	0.079	8.9	0.844	0.997	26.7	3.31	27.7
6	2.27	0.00222	0.031	16.3	0.903	1.000	57.5	2.76	27.0
7	2.84	0.00233	0.024	15.0	0.897	0.999	71.7	2.69	25.7
8	3.88	0.00201	0.017	18.4	0.911	1.000	98.3	2.54	25.4
9	1.11	0.00231	0.061	14.5	0.897	0.996	27.8	2.64	25.3
10	0.87	0.00283	0.082	11.5	0.878	0.999	21.9	2.80	25.3
…	…	…	…	…	…	…	…	…	…
391	0.16	0.00750	0.014	5.4	0.749	0.791	2.9	0.06	0.6
392	0.12	0.00837	0.019	4.5	0.725	0.941	2.7	0.08	0.6
393	0.10	0.03670	0.045	1.4	0.466	0.435	0.7	0.05	0.6
394	0.15	0.00747	0.015	4.8	0.737	0.966	3.5	0.08	0.6
395	0.29	0.01245	0.010	3.2	0.652	0.371	1.6	0.02	0.5
396	0.10	0.03999	0.040	1.6	0.494	0.457	0.7	0.05	0.5
397	0.24	0.00550	0.007	6.2	0.790	0.953	5.6	0.06	0.5
398	0.20	0.00676	0.008	5.5	0.766	0.869	4.2	0.05	0.4
399	0.11	0.04342	0.031	1.5	0.472	0.432	0.7	0.04	0.4
400	0.14	0.01659	0.014	2.4	0.587	0.604	1.7	0.04	0.3
ave	0.66	0.00571	0.034	8.6	0.813	0.928	15.9	0.67	5.9
min	0.07	0.00196	0.006	1.4	0.466	0.187	0.5	0.01	0.3
max	3.88	0.04342	0.188	20.0	0.912	1.000	98.3	8.48	85.3

Table 5. Decomposition of CLV Elasticity into Three Components

Accounting for Parameter Uncertainty

	FOOD	AGE	FEMALE
Total	0.514	0.596	0.234
frequency: Ef^{CLV}	1.180	-0.112	0.136
dropout: Ed^{CLV}	0.396	0.086	-0.001
spending: Es^{CLV}	-1.062	0.622	0.099

Ignoring Parameter Uncertainty

	FOOD	AGE	FEMALE
Total	0.702	0.630	0.227
frequency: Ef^{CLV}	1.180	-0.112	0.136
dropout: Ed^{CLV}	0.584	0.120	-0.007
spending: Es^{CLV}	-1.062	0.622	0.099

* Note that elasticity for only dropout but neither frequency nor spending is different when uncertainty is ignored. This is because, as shown in equation (10), only μ enters the elasticity formula in a nonlinear fashion.

Table 6. Disaggregate Fit of Pareto/NBD and HB Models

		Pareto/NBD	HB
Spending			
Correlation	**validation**	0.47	0.62
	calibration	0.88	0.92
MSE	**validation**	2.81	2.28
	calibration	0.40	0.28
Transactions			
Correlation	**validation**	0.59	0.61
	calibration	0.95	0.95
MSE	**validation**	6.43	4.99
	calibration	2.14	1.66

Table 7. Estimation Result for the Music CD Chain Data

		Model 0	Model 2	Model 3
Purchase rate λ	**Intercept**	-2.11 (-2.19, -2.03)	-2.16 (-2.39, -1.95)	-2.10 (-2.34, -1.85)
	Initial amount	---	0.38* (0.12, 0.63)	0.37* (0.11, 0.63)
	Age	---	-0.24 (-0.84, 0.37)	-0.26 (-0.87, 0.34)
	Female (male=0)	---	---	-0.13 (-0.29, 0.03)
Dropout Rate μ	**Intercept**	-5.14 (-5.64, -4.72)	-5.10 (-5.88, -4.36)	-5.06 (-5.89, -4.34)
	Initial amount	---	0.14 (-0.82, 1.10)	0.02 (-1.09, 0.94)
	Age	---	-0.39 (-2.32, 1.40)	-0.15 (-1.84, 1.39)
	Female (male=0)	---	---	0.05 (-0.60, 0.64)
Spending Parameter η	**Intercept**	-1.18 (-1.22, -1.13)	-1.51 (-1.63, -1.38)	-1.49 (-1.63, -1.36)
	Initial amount	---	0.50* (0.36, 0.65)	0.50* (0.36, 0.64)
	Age	---	0.49* (0.13, 0.85)	0.48* (0.12, 0.84)
	Female (male=0)	---	---	-0.03 (-0.10, 0.05)
correlation ($\log(\lambda)$, $\log(\mu)$)		0.20 (-0.05, 0.44)	0.21 (-0.02, 0.43)	0.19 (-0.04, 0.42)
correlation ($\log(\lambda)$, $\log(\eta)$)		0.14* (0.01, 0.28)	0.10 (-0.04, 0.24)	0.10 (-0.05, 0.23)
correlation ($\log(\mu)$, $\log(\eta)$)		0.01 (-0.21, 0.22)	-0.01 (-0.23, 0.20)	-0.01 (-0.21, 0.21)
marginal loglikelihood		-2906	-2898	-2889

*** indicates significance at the 5% level**

Table 8. Decomposition of CLV Elasticity into Three Components

	Initial Amount	AGE	FEMALE
Total	0.302	0.102	-0.094
frequency: Ef^{CLV}	0.130	-0.081	-0.064
dropout: Ed^{CLV}	-0.005	0.032	-0.017
spending: Es^{CLV}	0.178	0.152	-0.013

Table 9. Decomposition of CLV Elasticity into Three Components (Music CD)

ID	mean(λ)	mean(μ)	mean(η)	Mean Expected lifetime (years)	1 year Survival rate	Probability of being active at the end of calibration	Expected number of transactions in validation period	Expected total spending in val. period ($x10^4$ yen)	CLV ($x10^4$ yen)
1	0.43	0.01104	0.777	7.1	0.649	0.993	9.7	7.87	43.0
2	0.28	0.01218	1.014	6.5	0.641	0.864	5.5	5.81	35.1
3	0.27	0.01073	0.875	6.8	0.654	0.979	6.1	5.55	30.5
4	0.41	0.01211	0.561	4.5	0.613	0.964	9.0	5.28	26.0
5	0.13	0.01136	1.409	6.1	0.660	0.824	2.4	3.51	22.7
6	0.65	0.01439	0.349	3.5	0.574	0.837	12.0	4.36	22.6
7	0.24	0.01204	0.784	4.2	0.622	0.753	3.9	3.23	20.5
8	0.17	0.00910	0.834	6.1	0.687	0.990	3.9	3.40	18.9
9	0.41	0.01129	0.399	4.3	0.629	0.995	9.2	3.85	18.6
10	0.16	0.01020	0.855	5.9	0.668	0.953	3.6	3.23	18.1
…	…	…	…	…	…	…	…	…	…
491	0.12	0.01454	0.123	4.6	0.612	0.743	1.9	0.25	1.6
492	0.11	0.01037	0.120	5.3	0.662	0.756	1.7	0.22	1.5
493	0.09	0.01172	0.134	5.5	0.649	0.725	1.5	0.21	1.5
494	0.09	0.01207	0.138	5.2	0.640	0.709	1.4	0.20	1.5
495	0.09	0.00887	0.118	5.9	0.689	0.960	2.1	0.26	1.5
496	0.10	0.00995	0.117	5.8	0.674	0.982	2.2	0.27	1.4
497	0.10	0.01250	0.120	5.4	0.641	0.792	1.8	0.22	1.4
498	0.10	0.00947	0.115	5.3	0.679	0.919	2.0	0.24	1.4
499	0.10	0.01158	0.118	4.8	0.644	0.824	1.8	0.23	1.4
500	0.11	0.01485	0.119	4.5	0.597	0.598	1.3	0.16	1.3
ave	0.14	0.01044	0.339	5.4	0.664	0.864	2.7	1.00	6.0
min	0.08	0.00751	0.115	3.3	0.525	0.352	0.8	0.16	1.3
max	0.65	0.02139	1.409	8.4	0.723	0.999	12.0	7.87	43.0

Figure 1. Weekly Cumulative Repeat Transaction Plot

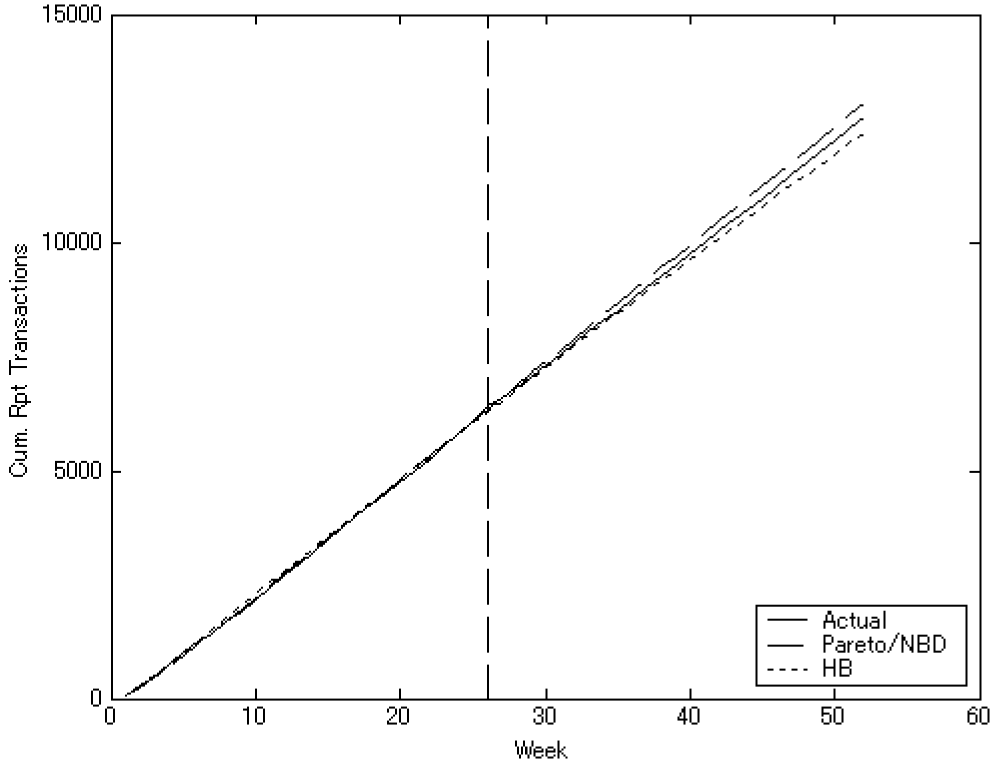


Figure 2. Conditional Expectation of Future Transactions

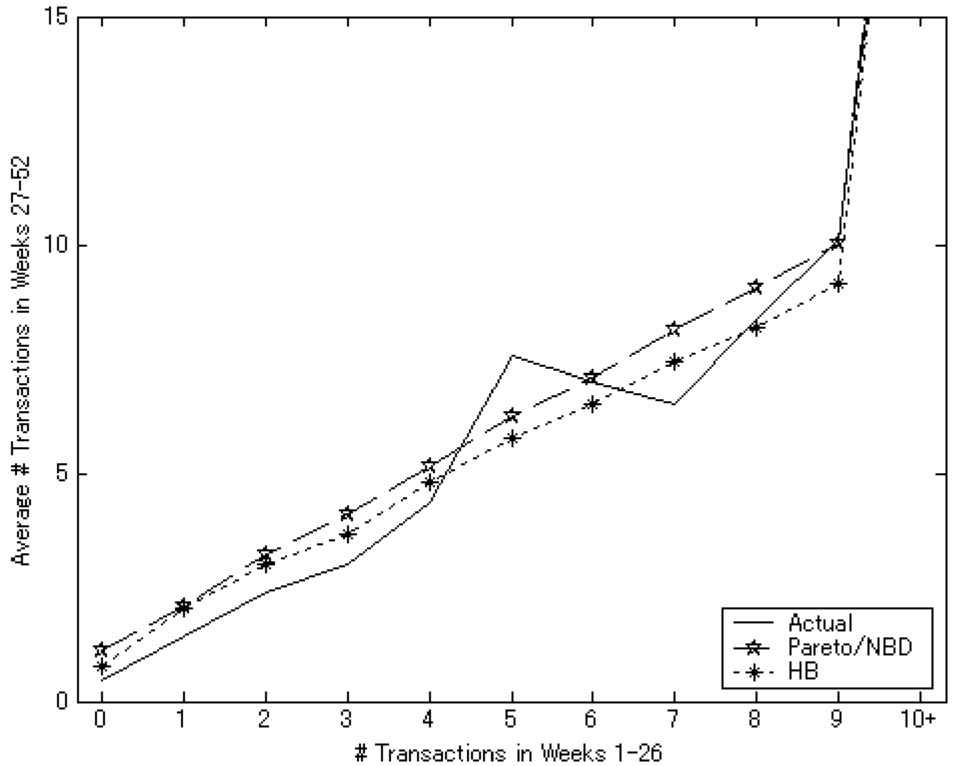


Figure 3. Conditional Expectation of Future Spending

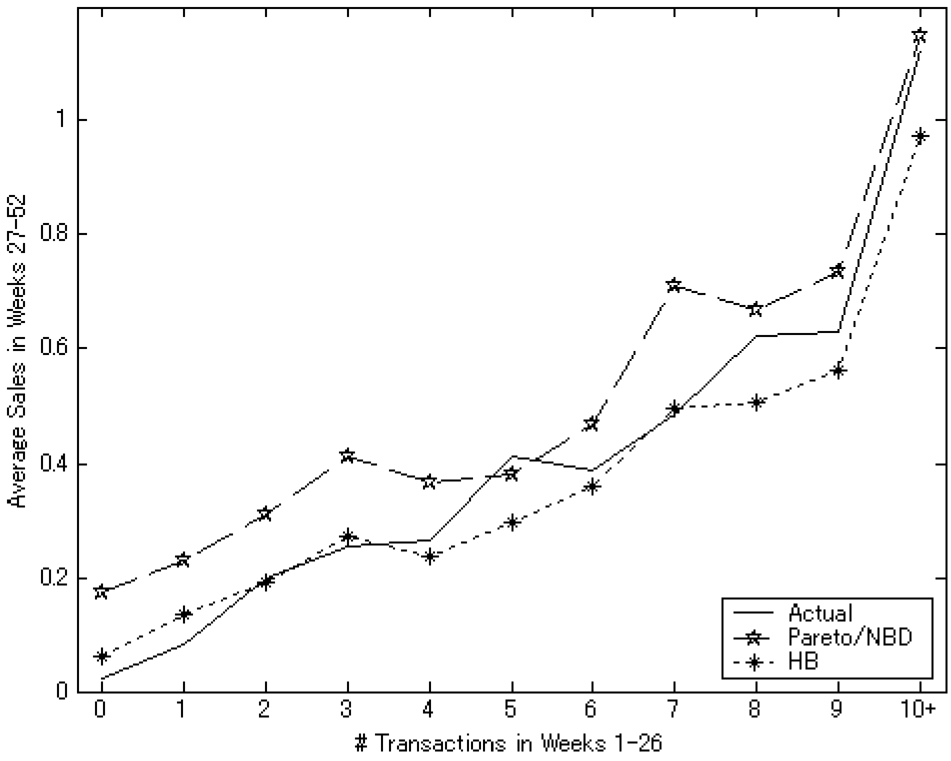


Figure 4. Scatter Plots of Posterior Means λ and η

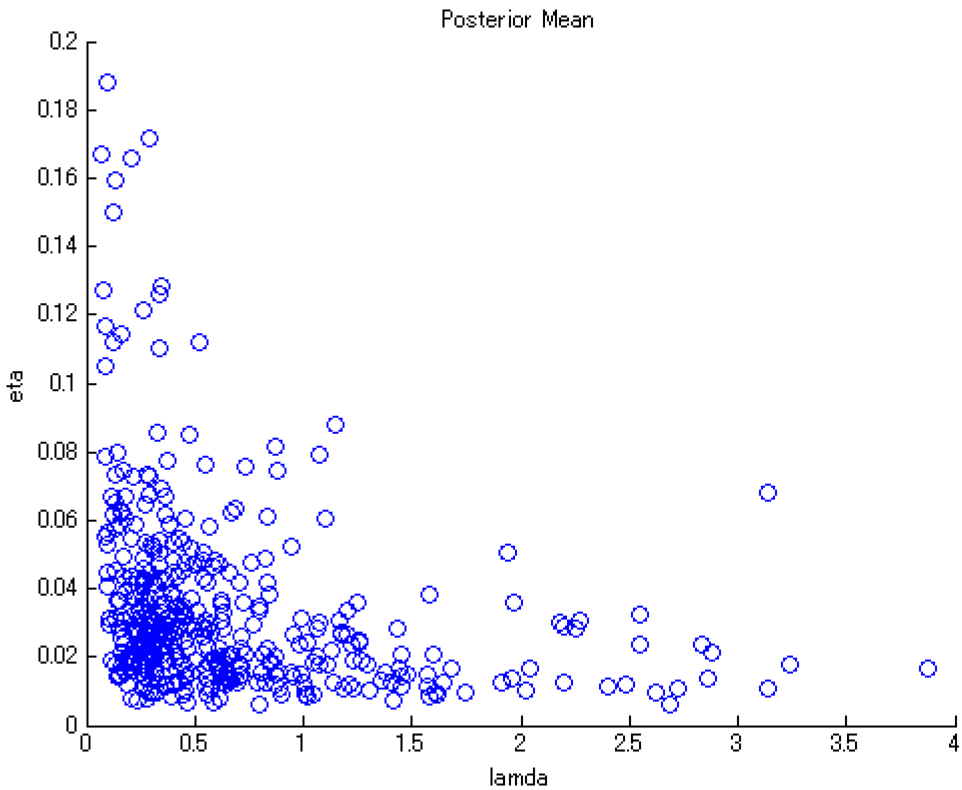


Figure 5. Gain Chart of CLV based on HB model and Simple Recency

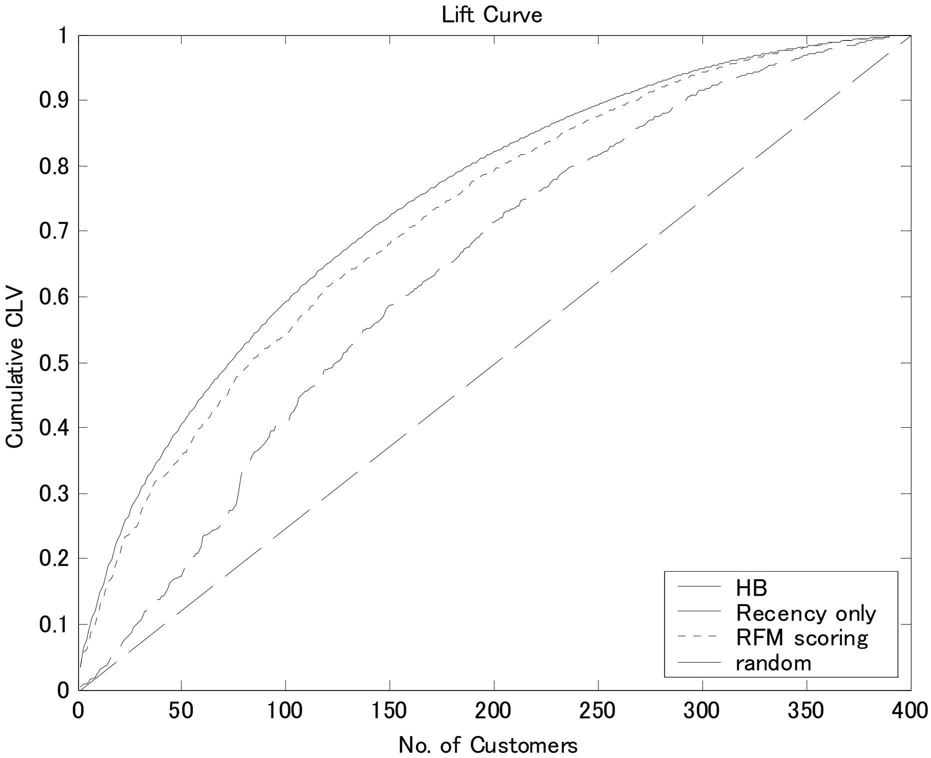
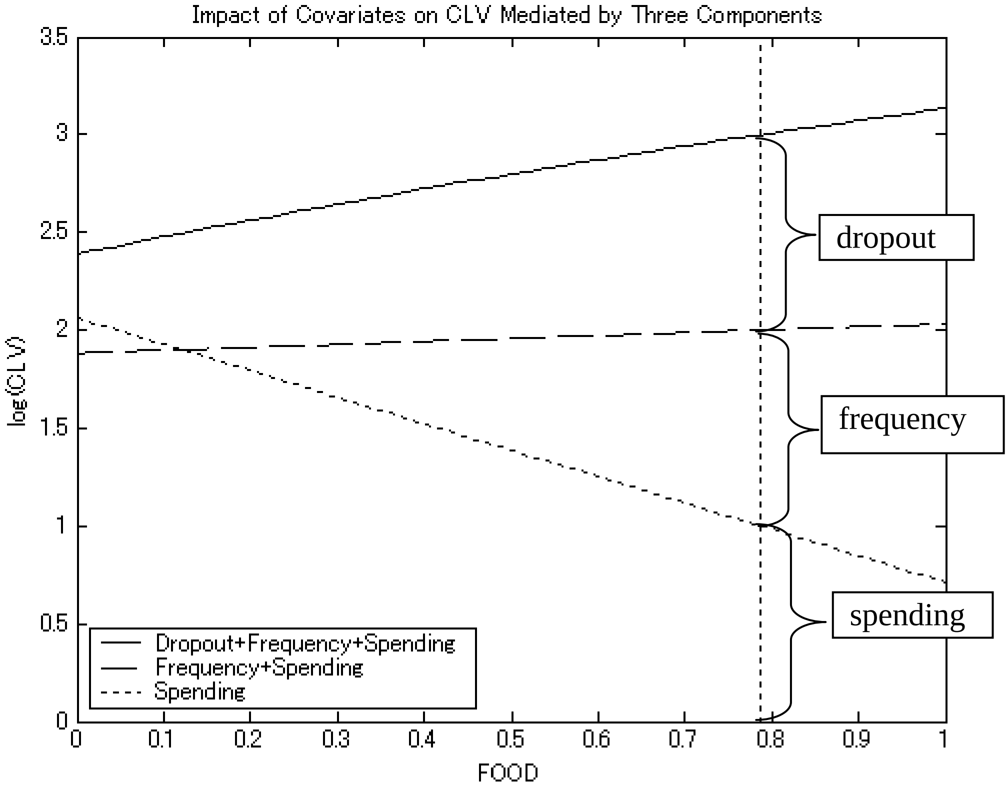
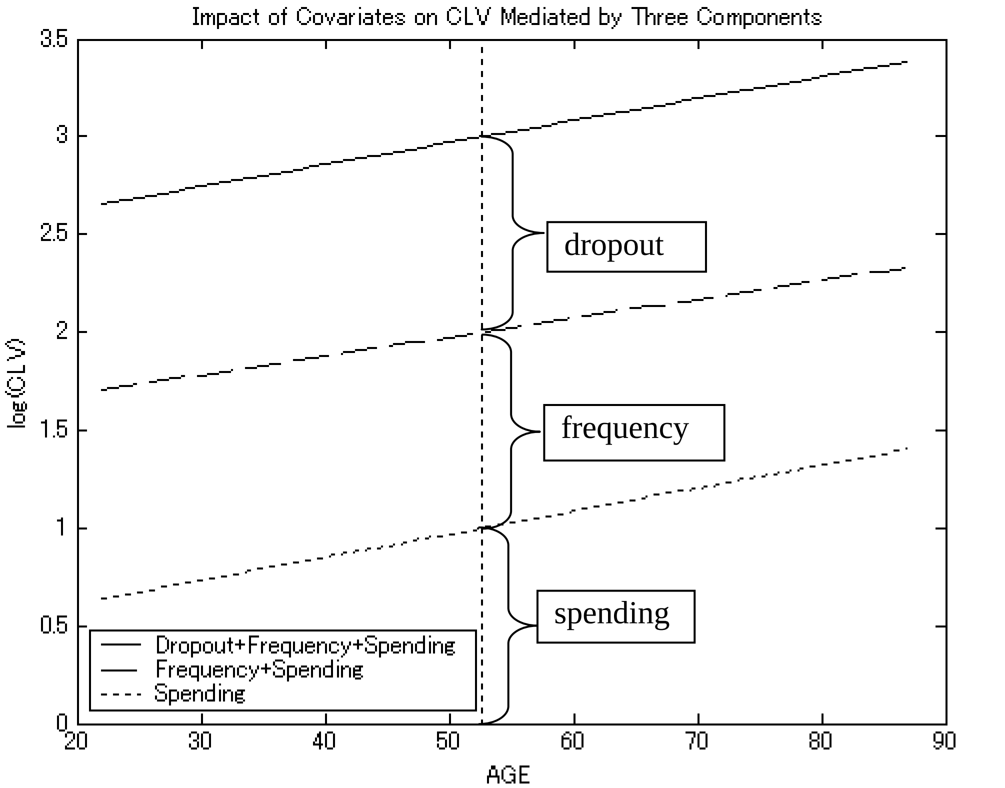


Figure 6. Impact of Covariates on CLV Decomposed into Three Components (Department store)





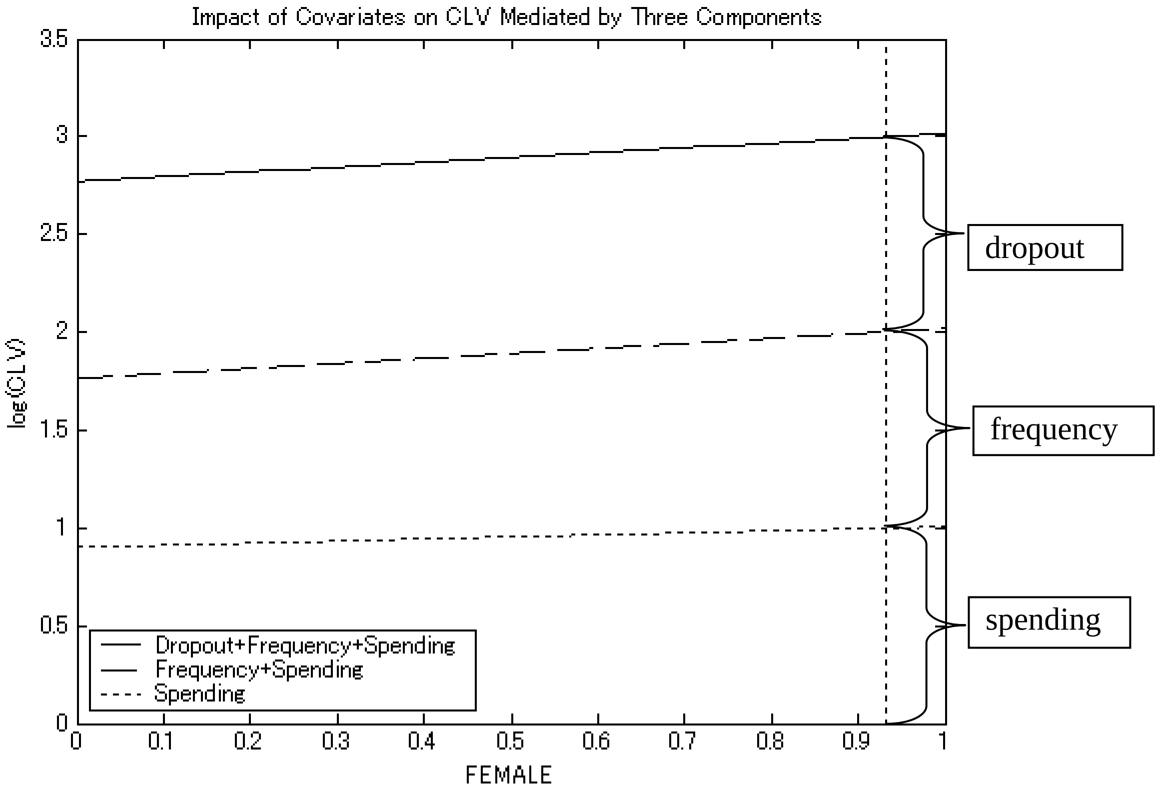
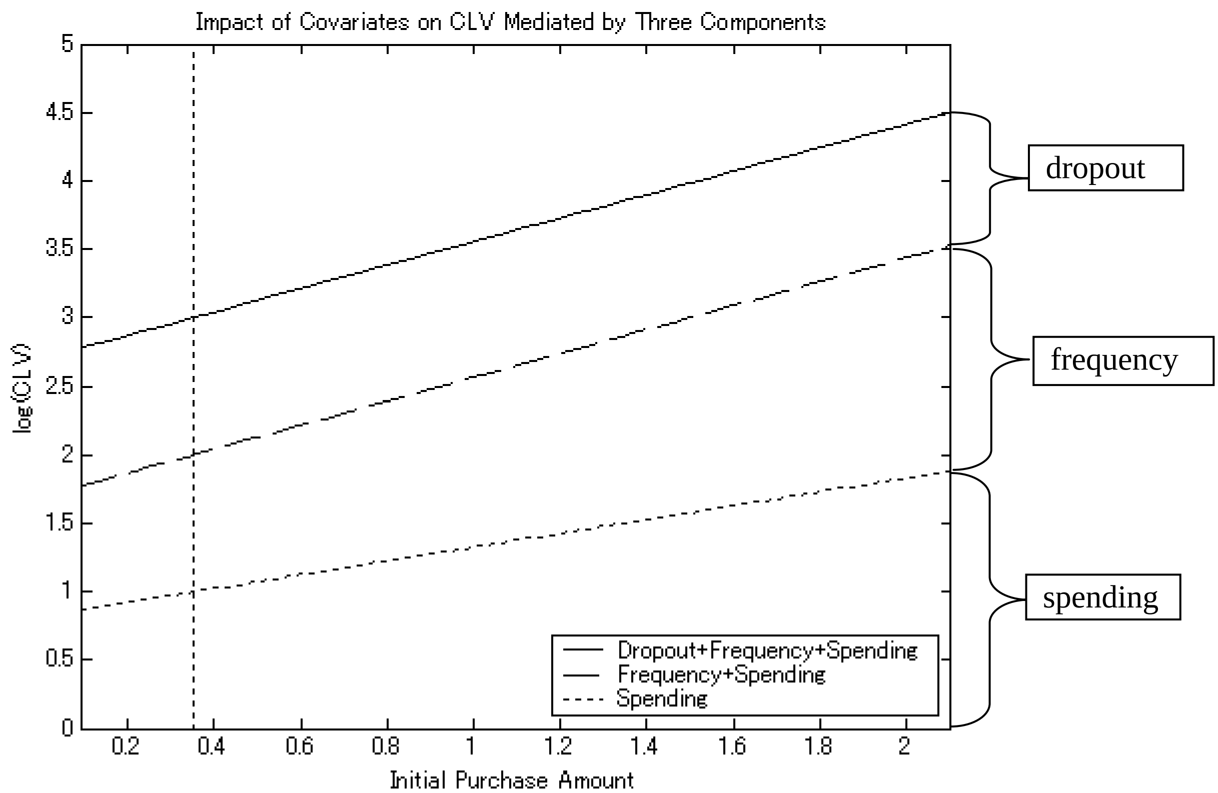
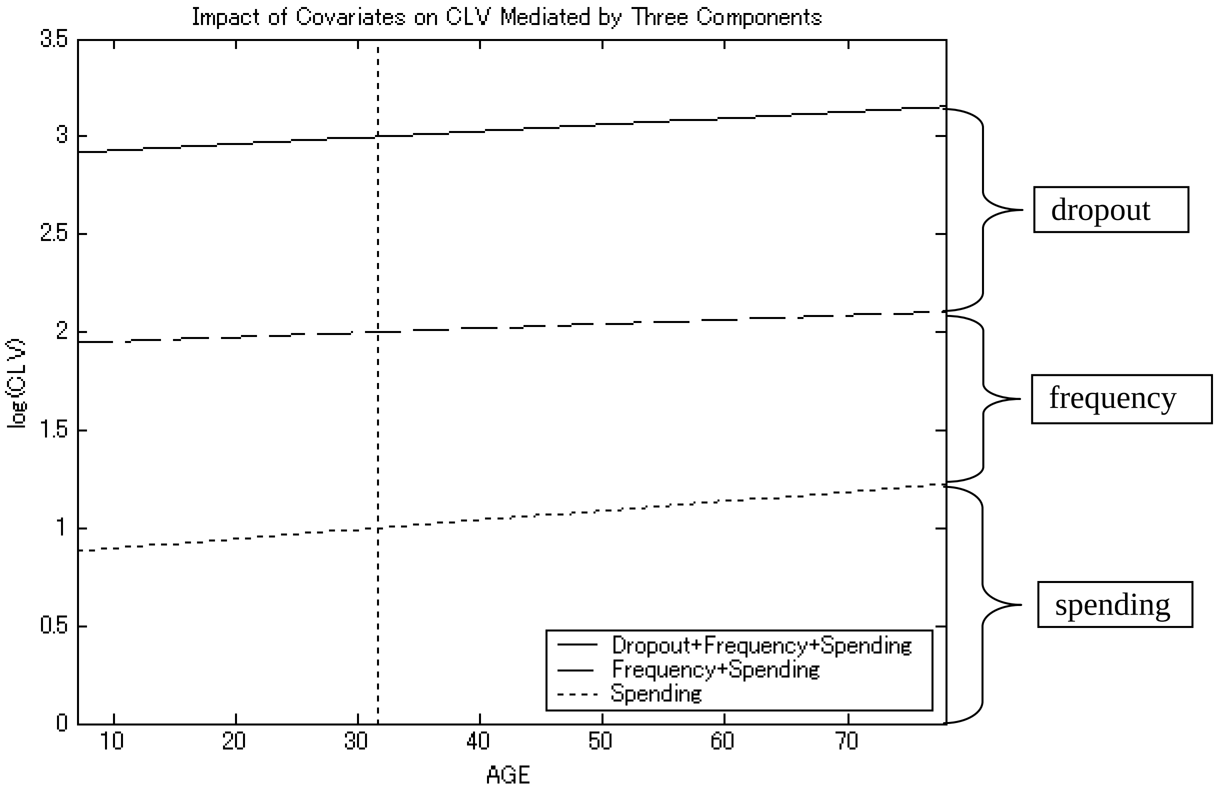
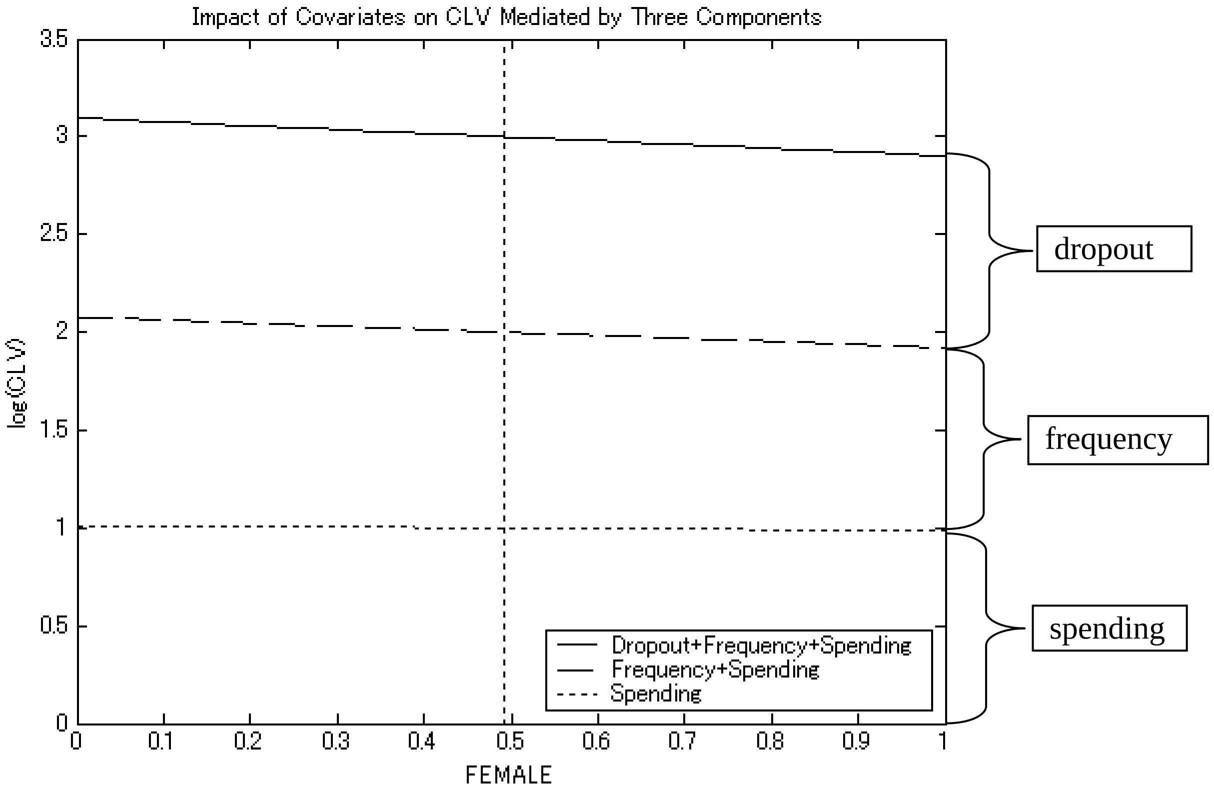


Figure 7. Impact of Covariates on CLV Decomposed into Three Components (Music CD)




Impact of Covariates on CLV Mediated by Three Components
log(CLV)
AGE
Dropout+Frequency+Spending
Frequency+Spending
Spending
dropout
frequency
spending


Impact of Covariates on CLV Mediated by Three Components
log(CLV)
FEMALE
Dropout+Frequency+Spending
Frequency+Spending
Spending
dropout
frequency
spending