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Bank Distress and the Borrowers' Productivity

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Abstract

In this paper, we propose a theoretical model in which a banking crisis (or bank distress) causes declines in the aggregate productivity. When borrowing firms need additional bank loans to continue their businesses, a high probability of bank failure discourages ex ante investments (i.e., "specialization") by the firms that enhance their productivity. In a general equilibrium setting, we also show that there may be multiple equilibria, in one of which bank distress continues and the borrowers' productivity is low, and in the other equilibrium, banks are healthy and the borrowers' productivity is high. We show that the bank capital requirement may be effective to eliminate the bad equilibrium and may lead the economy to the good equilibrium in which the productivity of borrowing firms and the aggregate output are both high and the probability of bank failure is low.

1 Introduction

The historical episodes of banking crises apparently showed that the bank distress causes deterioration of economic activities in the (very) short-run mainly due to liquidity shortage. There may exists an additional effect of bank failures that changes the economic structure possibly in the long-run, and deters economic growth. This paper examines

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the causes and consequences of bank failure and check how bank failures affect productivity growth. During the International Great Depression in the 1930s, many countries experienced banking crises and productivity declines. Cole, Ohanian and Leung (2005) examined data of 17 countries during the 1930s, and pointed out that there may be causal relationship between banking crises (or bank distress) and declines in the aggregate productivity. In the 1990s, the Japanese economy has experienced the decade-long bank distress and slowdown in the productivity growth. The bank distress seemed to cause a persistent deterioration of the economy as a whole, though a well-functioning financial market had been developed in Japan.[1]. What is the main mechanism which generates such relationship?

The main purpose of this paper is to reexamine the mechanism of banking panics and to show how the failures or panics affect productivity. We will show here that the effort choices among borrowing firms may affect the banking panics. In other words, a coordination failure among borrowers is a trigger for banking panics and productivity declines.

There are many papers which examine bank failures and panics. The causes of these crises have been debated. Some paper have shown the depositors' panics are the main factors for the crises[2] and some papers have shown that external shocks generate bank failures. Diamond and Rajan (2005) have focused on the external shocks on borrowing firms. They have shown that those shocks generate the liquidity shortages and introduce banking panics. Although those papers implicitly assumed that banking panics affect the economic conditions or macro performances, they have not examined the relation between the banking panics and productivity explicitly[3]. Hence it is not so clear how bank failures affect economic conditions. Even though there was a bank run, for example,

[1]For example, see Hayashi and Prescott [2002]

[2]The seminal paper is Diamond and Dybvig(1983).For example, Allen nad Gale(2000), Bhattacharya and Gale(1987) are related papers.

[3]Levine and Zervos (1998) have shown that banks and stock market provide different services and both of them contribute to economic growth. From this result we can infer that bank failures deter economic growth.They did not examine, however, this possibility explicitly.

new banks might be able to offer alternative financial services.

In this paper, we are going to show another mechanism which generates bank panics. This paper focuses on the behaviors of borrowing firms. In this sense, this paper is related to Diamond and Rajan (2005). Crucial difference between this paper and Diamond and Rajan (2005) is that this paper assumes the productivity conditions of firms are endogenously determined, although Diamond and Rajan (2005) have assumed there are exogenous random variables in the borrowing sector. By treating them endogenous, it becomes possible to get the following important insights. First, we can derive another reason of banking panics. This paper stresses the coordination failure of borrowers. Of course, we do not deny the reasons those previous papers have explored. We will show there is another possibility. It might seem strange that borrowers affect the condition of a bank since they have already borrowed from the bank. If the borrowing firms may require additional investments or liquidities, however, it becomes natural that conditions of other borrowing firms affect the balance sheet of the lending bank and the incentive of a borrowing firm. We will stress this relation in this paper. Second, it becomes easier to explain the relation between banking panics and economic productivity. Since the productivity of each firm becomes endogenous, we can examine the productivity and bank panic directly. The possibility of bank failure decreases the incentive of borrowing firms and decreases the productivity of borrowing firms.

To explain these points, we use a theoretical model in which bank distress causes a decline in the productivity of the borrowing firms, even though there exists a well-functioning financial market. The model is a modified version of the models of Diamond and Rajan(2005) and Holmstrom and Tirole (1998). We assume that a firm can enhance its own productivity by costly investment, which may be interpreted as investment in effort for specialization. (Thus we call this ex ante investment or effort choice "specialization.") The firm needs to borrow from a bank to start the business, and it also needs an additional investment to continue its business if the firm is hit by a shock at the interim period. From the aspect of specific skill as explored by Diamond and Rajan(2001, 2005), we assume the return of the project is not perfectly verifiable and seizable to lenders.

Hence, as in Holmstrom and Tirole's model, the firms cannot borrow additional fund in the financial market when the incumbent banks fail.

In this setting, if there is a positive probability of bank failures, a borrowing firm cannot get the necessary additional loan and must shut down its business with the positive probability. Hence a high probability of bank failure is expected, a firm expects high probability of shut down and low expected return on the specialization. Since we assume the specialization by individual firms enhances its productivity, a lower specialization leads to a lower level of productivity. In other words, we can show that a banking crisis leads to the less specialization of the borrowing firms, and the decline of aggregate productivity.

Next we embed this partial equilibrium model into a general equilibrium setting, in which consumers, as depositors and bank shareholders, provide funds to firms through banks. Modeling the general equilibrium economy, we endogenize the probability of bank failure, and show that the economy may end up with two steady state equilibria: A good equilibrium and a bad equilibrium. The key point is the following externality effect among borrowers. The less specialization does not only decrease its productivity but also increases the probability of bank failure. This means the level of specialization has the externality effect to the other borrowing firms through the change of the bank failure probability. Moreover, if those low productivities are anticipated by depositors, they will demand resources immediately and generate bank runs as stressed by Diamond and Rajan (2005). Hence, in the good equilibrium, firms choose the highest level of specialization which generates the high aggregate productivity and low probability of bank failure. In the bad equilibrium, however, firms choose the lowest level of specialization, the aggregate productivity is low, and the probability of bank failure is high.

In this general equilibrium model, we conduct numerical experiments in which we impose capital requirement for banks. The capital requirement policy has an effect, which eliminates the bad equilibrium for a certain range of parameter values. Therefore, the capital requirement policy may have an impact that increases the aggregate productivity of the economy and lowers the probability of banking crisis, through enhancing the specialization by firms. Our result implies that the bank capital requirement may have a significant impact on the social welfare as a whole by affecting the aggregate productivity, while the existing literature on this topic tend to stress the moral hazard or adverse selection problems in the banking sector (see, for example, Morrison and White, 2005).

The organization of this paper is as follows. In Section 2, we present a simple example to show our propositions intuitively and we construct the partial equilibrium version of our model in Section 3. In Section 4, we embed the partial equilibrium model in the general equilibrium in which bank failure possibility is endogeously determined. We describe the model with and without the bank capital requirements. In Section 5, we demonstrate numerical examples of the general equilibrium model. Section 6 provides concluding remarks.

2 Simplified Example

To clarify the basic structure of our model, we demonstrate a simplified model in this section. There are N firms which have a potential investment opportunity. This investmet requires 1 input at date 1 and will generate $R > 1$ at date 3. Only fR is seizable for banks, however, since R is not perfectly verifiable. In other words, $(1-f)R$ becomes the benefit of each firm under any type of contracts. This investment opportunity may require additional investments by an idiosyncratic and independent shock at date 2. With probability q, the additional investment ρ becomes necessary. For simplicity, we assume that the firm generates 0 at date 3 if this additional investment was not implemented. The market interest rate is supposed to be zero for simplicity.

One crucial assumption is that R is dependent upon the effort level, s, of each firm. Each firm can choose s^H or s^L and the private cost for choosing s^H (s^L) is C (0). Naturally $R(s^H)$ is higher than $R(s^L)$. The effort level is observable and is chosen at date 1 before the loan contract is made. Since all firms choose their effort levels simultaneously, they cannot coordinate their choice over s_H or s_L: Thus there is a possibility of coordination failure. It is assumed that

$$fR(s^H) > 1 + q\rho.$$

In other words, this lending is profitable for a bank as long as the firm chooses s^H, even though it can seize only $fR(s^H)$ and it has to pay the additional investment cost ρ. Here we assume, however,

$$fR(s^H) < \rho.$$

In this setting, as explored by Holmstrom and Tirole(1998), it is difficult to get ρ in the market after the shock at date 2. But a bank can offer a credit line contract at date 1 which guarantees to supply ρ when the additional investment is necessary.

Moreover, as long as there is no bank failure and

$$(1 - f)R(s^H) - C > (1 - f)R(s^L),$$

each firm has an incentive to choose s^H. Hence it is a Nash equilibrium that all firms choose s^H.

If the effort choice affects a possibility of bank failure, however, there may exist another equilibrium. Suppose that a bank will fail at date 2 with probability ν_c and this probability is a decreasing function of the effort level of the borrowing firms. When all firms choose s^H, ν_c becomes *0*, but it becomes very high when all firms choose s^L. If a bank has failed at date 2, the borrowing firm cannot get ρ after the shock and R becomes 0 even though the firm has chosen s^H. In this situation, ν_c becomes high and it may becomes very difficult to get ρ if $N - 1$ firms have chosen s^L. Hence another firm cannot have an incentive to choose s^H. More rigorously, if

$$\{1 - q\nu_c(s_i^H, s_{-i}^L)\}(1 - f)R(s^H) - C < \{1 - q\nu_c(s_i^L, s_{-i}^L)\}(1 - f)R(s^L),$$

there is another equilibrium in which all firms choose s^L where $\nu_c(s_i^H, s_{-i}^L)$ is the probability of bank failure when firm i chooses s^H and the other firms choose s^L. In other words there are multiple equilibria.

This result is intuitive explanation of our propositions. From the next section, we formulate a more rigorous model to examine this intuition.

3 Partial equilibrium model: Exogenous probability of bank run

In this section, we consider a partial equilibrium model in which the probability of bank run is exogenously given. In the next section, we embed this model in a general equilibrium economy in which consumers provide funds to firms through banks and the probability of bank run is endogenously determined.

The economy is a simplified version of Holmstrom and Tirole's (1998) model, in which there are continua of banks and firms. Measures of firms and banks are normalized to one. The economy continues only one period, and agents can choose their actions two times: at the beginning of the period and at the middle of the period. Following Holmstrom and Tirole (1998), we assume that banks have all the bargaining power over firms, and they maximize the expected return on the loans to firms, making firms break-even.

3.1 Firm

Firms are indexed by i, where $i \in [0, 1]$. At the beginning of the period, firm i chooses its level of specialization, s_i, where $s_i \in [0, 1]$. We assume that s_i is observable. Specialization incurs private cost ξs_i for the firm. After s_i is chosen, firm i borrows X_i units of consumer goods from a bank, and invest X_i in its production project. At the middle of the period, a macro shock $\nu \in [\underline{\nu}, 1]$ and an idiosyncratic shock $\rho_i \in \{0, \rho\}$ hit the economy. $\rho_i = 0$ with probability $1 - q$, and $\rho_i = \rho > 0$ with probability q. The macro shock ν indicates the success probability of a firm's project (see equations (1) and (2) below). If $\rho_i = \rho$, firm i needs to invest additional fund ρX_i at the middle of the period in order to continue the project. Otherwise, the project must be shut down leaving the liquidation value, $(1 - \delta)X_i$. If firm i successfully finance ρX_i or $\rho_i = 0$, it can continue the project.

In order to consider the situation in which the return of the project is not perfectly seizable to investors, we consider the following moral hazard situation. After firm i chooses to continue the project, the firm faces an opportunity to shirk. If the firm shirks, it enjoys private benefit, $b(s_i)$, but the output, y_i, at the end of the period becomes

$$y_i = \begin{cases} (r+s_i)X_i, & \text{with prob. } \nu - \underline{\nu}, \\ (1-\delta)X_i, & \text{with prob. } 1-\nu+\underline{\nu}, \end{cases} \tag{1}$$

where $r > 1$. If the firm does not shirk and works diligently, it does not obtain private benefit, but the output at the end of the period becomes

$$y_i = \begin{cases} (r+s_i)X_i, & \text{with prob. } \nu, \\ (1-\delta)X_i, & \text{with prob. } 1-\nu. \end{cases} \tag{2}$$

Therefore, if a firm shirks, the success probability of its project is lowered by $\underline{\nu}$. The private benefit for the firm of shirking, $b(s_i)$ may be increasing in the level of specialization, s_i. Note that the specialization directly increases the output. Thus the average of s_i can be interpreted as "aggregate productivity." in this model. We assume a weakly convex benefit:

$$b(s_i) = \frac{b'}{2}s_i^2 + c's_i, \tag{3}$$

where $b' \geq 0$ and $c' > 0$. In order to give the incentive to no-shirking, lenders have to abandon a part of the output as will be explained bellow.

3.2 Bank failure and debt contract

We assume that if $\nu \leq \nu_c$, the bank run occurs and all banks are shut down, where ν_c is an exogenous parameter in this section. (The bank run is endogenized in the general equilibrium setting in Section 4.)

A bank solves the following problem:

$$\max_{R_f(i)} \int_{\nu_c}^{1} \left[\nu\left\{r+s_i-R_f(i)\right\}+(1-\nu)(1-\delta)\right] df(\nu) - 1 - q\rho, \tag{4}$$

subject to

$$\underline{\nu}R_f(i) \geq b(s_i), \tag{5}$$

where $f(\nu)$ is the p.d.f. for ν and $R_f(i)$ is the final payment to firm i. $R_f(i)$ must be determined such that firm i gets better expected payment when it works diligently than when it shirks. Binding (5) gives

$$R_f(i) = \frac{b(s_i)}{\underline{\nu}} \equiv \frac{b}{2}s_i^2 + cs_i, \tag{6}$$

where $b = b'/\underline{\nu}$ and $c = c'/\underline{\nu}$.

3.3 Firm's problem

Anticipating R_f in (6), firm i chooses s_i before it borrows from a bank. We assume that the loan contract between the bank and the firm survives even if the bank fails: The firm must repay retaining R_f in its hand as long as output is produced; and if firm i is hit by the idiosyncratic shock ρ_i after the bank failed, the firm cannot obtain the necessary fund for the additional investment and its output becomes $(1-\delta)X_i$, all of which is to be repaid to the creditor. (When the bank failed, the creditor of the loan contract is a group of bank depositors. See Section 4 for details.) Therefore, firm i solves

$$\max_s \int_{\nu_c}^{1} (\frac{b}{2}s_i^2 + cs_i)\nu df(\nu) + \int_{\underline{\nu}}^{\nu_c} (1-q)(\frac{b}{2}s_i^2 + cs_i) df(\nu) - \xi s_i. \tag{7}$$

We assume that

$$(r + s - \frac{b}{2}s^2 - cs)\nu_c + (1-\nu_c)(1-\delta) < \rho, \quad \text{for all } s \in [0,1]. \tag{8}$$

This assumption ensures that a firm cannot finance ρ in the financial market when the bank fails. We also assume that

$$E(\nu)\left(r + s - \frac{b}{2}s^2 - cs\right) + (1 - E(\nu))(1-\delta) > 1 + q\rho, \quad \text{for all } s \in [0,1]. \tag{9}$$

This assumption ensures that a bank will commit to providing credit line of ρ before ν is revealed in case of the liquidity shock. In the case where ν follows a uniform distribution, i.e., $f(\nu) = \frac{1}{1-\underline{\nu}}$, the firm's problem can be rewritten as:

$$\max_{s_i} \frac{1 - q\nu_c^2 - (1-q)\underline{\nu}^2}{2(1-\underline{\nu})} \left[\frac{b}{2}s_i^2 + cs_i\right] - \xi s_i. \tag{10}$$

The derivative of the objective function in (10) with respect to s_i is

$$\frac{1 - q\nu_c^2 - (1-q)\underline{\nu}^2}{2(1-\underline{\nu})} [bs_i + c] - \xi. \tag{11}$$

Obviously, if ν_c, the probability of bank failure, is large, the equilibrium value of s_i may be 0, the lower bound, and that if ν_c is small, s_i may be 1, the upper bound.

3.4 Implication of the model

This partial equilibrium model implies that bank distress, i.e., a large ν_c, may lower the level of specialization of the borrowing firms and therefore may lead to a lower level of the aggregate productivity of the economy. A higher probability of bank failure implies a higher probability that the borrowing firm fails to continue the business, since the firm cannot obtain additional funds if the bank fails. Therefore, the expected return on the ex ante specialization for the firm becomes lower if ν_c is higher, and it chooses the lowest level of the specialization. Since the specialization enhances productivity, a bank distress causes the productivity declines in our model. In this sense, our theory seems successful in explaining productivity declines observed during banking crises, such as the episodes of the US Great Depression (see Cole and Ohanian [1999] and Ohanian [2001]) and the lost decade in Japan in the 1990s (see Hayashi and Prescott [2002]). Our model may be regarded as one explanation for the conjecture by Cole, Ohanian, and Leung (2005) that the banking crises may have some causal linkage with the productivity declines in the International Great Depression.

4 General equilibrium model

We can embed the model of the previous section in the general equilibrium setting and endogenously determine the probability of bank failure, ν_c. The summary of the structure of the model is as follows: The firms choose the degree of specialization, s, taking ν_c as given; the banks choose ν_c, taking s and the market rate of interest, R, as given; and R is determined as an outcome of the general equilibrium. Therefore, the equilibrium of this model can be regarded as a Nash equilibrium in a simultaneous game in which firms choose s and banks choose ν_c, taking the other players' actions and R as given. (Although we used a term of "simultaneous" game, timing of the game is that banks choose ν_c after s is chosen by firms. Our theoretical and numerical results in this paper do not change even if the banks are the Stackelberg leader, i.e., if the banks can precommit to ν_c before firms choose s, taking the best response of the firms into account, See footnote 6.) Similar to the previous section, we assume that the economy continues only one period. There are continua of consumers, firms, and bank-managers, whose measures are normalized to one. A consumer is given E units of the consumer goods as endowment at the beginning of the period. Consumers can either invest the endowment in bank capital, C, or put it into the banks as deposits, D:

$$C + D \leq E. \tag{12}$$

The relationship among bank-managers, bank-capital (consumers), and depositors (consumers) is similar to that in Diamond and Rajan's (2000) model. A bank-manager has relation-specific technology to collect on loans from firms, but he can threaten the bank-capital and the depositors that he will walk away without collecting the loans unless he is paid more (the hold-up problem). To prevent the hold-up problem by the bank-managers, the bank-capital and the depositors set the deposit contract as the demandable deposit. Therefore, the depositors can withdraw their deposit at any time they like during the period. Since bank deposit is demandable, depositors run on banks if the bank-manager threaten the depositors by offering a renegotiation to lessen the payoff of the depositors, and the bank run destroys the bank-manager's surplus. Anticipating this result, the bank-manager cannot invoke renegotiation under demandable deposit. To make bank deposit demandable is the optimal design to ensure that the rate of return to bank deposit is high and to increase the funds deposited in banks. (In the equilibrium, bank runs may not occur.)

This contractual arrangement may have a side-effect when a macro shock ν is introduced in the economy: Under demandable deposit contracts, bank runs occur when the macro shock ν is less than a certain threshold value, ν_c. The feature that a macro shock triggers a bank run is the same as Allen and Gale's (1998) optimal finanical crisis model.

Bank runs: We assume the following for the payoffs of the agents in the event of bank run. When the bank run occurs, the ownership of bank assets is transferred to the groups of depositors. Thus, the bank capital obtains zero. A borrowing firm produces $(r + s_i)X_i$ if it is not hit by the idiosyncratic shock ρ, while it can produce $(1 - \delta)X_i$ if it is hit by the shock ρ, since it cannot obtain the additional investment which is necessary to continue production. Therefore, a firm gets $R_f(i)$ with probability $(1-q)$ and zero with probability q. We assume that the depositors get $(1-\delta)X + q\rho X + L$, which is the sum of the liquidation value of bank lending, $(1-\delta)X$, the remaining liquid asset, L, and $q\rho X$, which was kept for lending to the firms who will be hit by the shock ρ. Here we implicitly assumed that a firm's output that exceeds $(1-\delta)X_i$ is simply vanished as a dead weight loss due to the resource-consuming negotiations among depositors (or rent-seeking activities). This inability of depositors is consistent with the assumption that only the bank-managers have relation-specific technology to collect the full value of the bank loans.

4.1 Bank-capital's problem

A bank-capital, i.e., a coalition of consumers who invest C into a bank, takes the market rate of interest, R, and the level of the borrower's specialization, s_i, as given. The bank-capital chooses the deposit, D, that they borrow, the investment in a safe asset, L, the investment in the (risky) firms, X, the deposit rate, R_d, the rate of final payment to firm i, $R_f(i)$, and the threshold value of the macro shock that triggers a bank run, ν_c. Safe asset L is just storage of the consumer goods. Thus one unit of L can be converted to one unit of consumer goods at any time. X is the loan to firms, which is invested in the production projects by the borrowers. We assume that a bank lends to infinitely many firms so that the idiosyncratic risk, ρ_i is perfectly diversified for the bank. Therefore, a bank that lends X to firms must lend $q\rho X$ additionally to the firms at the interim period when the idiosyncratic liquidity shocks are revealed. (We assume that (9) is satisfied, that is the commit to the credit line ρX is ex ante optimal for a bank.) A bank run occurs if $\nu < \nu_c$. If $\nu = \nu_c$, it must be the case that depositors are indifferent whether to run on the bank or not to run. This condition is equivalent to $R_d D = [\nu_c\{r + s_i - R_f(i)\} + (1-\nu_c)(1-\delta)]X + L$. Finally, we assume that the bank-capital can obtain only the fraction θ (< 1) of the total surplus without the help of the bank-manager who has the relation-specific technology of collecting loans. This assumption implies that the total surplus of the bank is divided by a bargaining such that θ goes to the bank-capital and $1-\theta$ to the bank-manager.

Therefore, the bank-capital solves

$$\max_{D,L,X,\nu_c,R_d,R_f} \theta \int_{\nu_c}^{1} \left(\left[\nu\{r+s_i-R_f(i)\}+(1-\nu)(1-\delta) \right] X + L - R_d D \right) f(\nu) d\nu, \tag{13}$$

subject to

$$D + C = L + (1+q\rho)X, \tag{14}$$

$$\{(1-\delta)X + q\rho X + L\} \cdot \text{Prob}(\nu < \nu_c) + R_d D \cdot \text{Prob}(\nu \geq \nu_c) \geq RD, \tag{15}$$

$$R_d D = [\nu_c\{r+s_i-R_f(i)\}+(1-\nu_c)(1-\delta)]X + L, \tag{16}$$

$$\underline{\nu} R_f(i) \geq b(s_i), \tag{17}$$

where (14) is the balance-sheet identity for the bank, and (15) is the participation constraint for depositors. Incentive compatibility for firm i implies

$$R_f = \frac{b}{2} s_i^2 + c s_i. \tag{18}$$

4.2 Solution to the bank's problem

We focus on the case where the macro shock, ν, follows the uniform distribution over $[\underline{\nu}, 1]$, i.e., $f(\nu) = \frac{1}{1-\underline{\nu}}$. We define s as the average level of specialization of the borrowers of the bank. We focus on the symmetric equilibrium in which all firms choose the same specialization: $s_i = s$. The reduced form of the bank-capital's problem is

$$\max_{X,L,\nu_c} \frac{\theta}{2(1-\underline{\nu})} [1-\nu_c]^2 \left[r + s - \frac{b}{2} s^2 - cs - (1-\delta) \right] X, \tag{19}$$

subject to

$$g(\nu_c, R)X + (R-1)L \leq RC, \tag{20}$$

where

$$g(\nu_c, R) \equiv (1+q\rho)R - (1-\delta) - \frac{(\nu_c - \nu)}{1-\underline{\nu}} q\rho - \frac{1-\nu_c}{1-\underline{\nu}} \nu_c \left[r + s - \frac{b}{2} s^2 - cs - (1-\delta) \right]. \tag{21}$$

We assume and justify later that $g(\nu_c, R) \geq 0$ and $R > 1$. Then we get the solution:

$$L = 0, \tag{22}$$

$$X = \frac{RC}{g(\nu_c, R)}, \tag{23}$$

$$\nu_c = \frac{r + s - \frac{b}{2}s^2 - cs + q\rho - (1-\delta) - 2(1-\underline{\nu})(1+q\rho)R + 2(1-\underline{\nu})(1-\delta) - 2\underline{\nu}q\rho}{r + s - \frac{b}{2}s^2 - cs - q\rho - (1-\delta)}. \tag{24}$$

4.3 General equilibrium

In the general equilibrium, the arbitrage condition between the return rate of bank capital and the market rate of interest determines the value of R:

$$\frac{\theta}{2(1-\underline{\nu})}[1-\nu_c]^2 \left[r + s - \frac{b}{2}s^2 - cs - (1-\delta)\right] X = RC, \tag{25}$$

where $X = \frac{RC}{g(\nu_c, R)}$. This condition determines R (for given s). Finally, given $R(s)$ by (25) and $\nu_c(s)$ by (24), firm's problem (10) determines the value of s in the general equilibrium. Note that firm i chooses s_i to solve (10), taking $\nu_c(s)$ as given, where s is the average level of specialization for the borrowers of the bank. Since the objective function of firms is quadratic, the solution must be a corner solution: $s_i = 1$ if $\nu_c(s)$ is small and $s_i = 0$ if $\nu_c(s)$ is large. Therefore, either $s = 1$ or $s = 0$ in the equilibrium.[4] Note that s and ν_c can be regarded as the outcome of a simultaneous game between firms and banks: Firms choose s, taking ν_c as given; and banks choose ν_c, taking s and R as given. In Section 5 we will show numerical examples.

4.4 A model with the bank capital requirements

In this subsection, we consider the economy where a capital requirement is imposed by the government. The capital requirments have become a major part of the banking regulation recently. We will show in the numerical experiments in Section 5 that the capital requirements may be effective to improve social welfare by eliminating the bad equilibrium or by leading the economy to the good equilibrium.

The banks in this economy are subject to the following constraint: $X \leq \lambda C$.

[4] The equilibrium values of bank capital, C, and bank deposit, D, are deteremined by $C + D = E = L + X$, (22), and (23).

Bank: The bank's problem is reduced to

$$\max_{\nu_c} \frac{\theta}{2(1-\underline{\nu})}(1-\nu_c)^2 \left[r + \left(1 - \frac{b}{2}s - c\right) s - (1-\delta)\right] X, \tag{26}$$

subject to

$$X \leq \lambda C, \tag{27}$$

$$-g(\nu_c)X - (R-1)L + RC \geq 0. \tag{28}$$

We define $\tilde{R}$ and $g(\tilde{\nu}_c)$ as the solutions in the case where there is no capital requirements, that is, the equilibrium values in the previous subsection. If it holds that

$$\lambda < \frac{\tilde{R}}{g(\tilde{\nu}_c)}, \tag{29}$$

$X = \lambda C$ should hold in the equilibrium. We assume $\lambda C < \frac{\tilde{R}C}{g(\tilde{\nu})}$ here.[5] The problem is reduced to

$$\max_{X,\nu_c} \frac{\theta}{2}\frac{(1-\nu_c)^2}{1-\underline{\nu}}\lambda \left[r + (1 - \frac{b}{2}s - c)s - (1-\delta)\right], \tag{30}$$

subject to

$$g(\nu_c) \leq \frac{R}{\lambda}. \tag{31}$$

Thus, under our assumption, since the constraint should be binding,

$$g(\nu_c) = \frac{R}{\lambda}. \tag{32}$$

Solving (32) gives us $\nu_c(R,s)$.

General equilibrium: In a general equilibrium, by the arbitrage condition,

$$\frac{\theta}{2}\frac{(1-\nu_c)^2}{1-\underline{\nu}}\left[r + (1 - \frac{b}{2}s - c)s - (1-\delta)\right]\lambda = R. \tag{33}$$

This condition gives $R(s)$. Given $R(s)$ and $\nu_c(R,s)$, the first-order condition for the firm's problem, (11), gives the equilibrium value of s. There may be unique equilibrium or multiple equilibria, depending on the parameter values.

[5] In our numerical examples in Section 5, we checked that this assumption holds.

5 Numerical example

In this section, we show some numerical examples.

5.1 From multiple equilibria to good equilibrium

In the first example, there are multiple equilibria ($s = 0$ and $s = 1$) if the bank capital requirement is not imposed; and imposition of the capital requirement can eliminate the bad equilibrium in which $s = 0$, and the good equilibrium in which $s = 1$ becomes the unique equilibrium.

We employ the values of parameters as in Table 1. In this case, the equilibrium of

r	δ	C	θ	$\bar{s}$	ξ	ρ	$\underline{\nu}$	b	c	q	λ
5	.5	2	.5	1	.32	3	.3	.1	.5	.3	1.5

Table 1: Parameter Values (1)

the basic model in which no capital requirement is imposed is as in Table 2. There are multiple equilibria. We find that these two equilibria are stable.[6]

s	R	ν_c	C/X
0	1.4586	.4667	.3134
1	1.5673	.4545	.3356

Table 2: Result (1) - Without Capital Requirement

However, if we introduce the bank capital requirement, there is unique equilibrium with $s = \bar{s} = 1$ as in Table 3. This unique equilibrium is also stable. Therefore, we can make

[6] We checked whether the banks' payoff can be improved if the banks choose $\nu_c(\hat{s})$ and the firms choose $\hat{s}$, where $\hat{s} = 1 - s^*$ and s^* ($= 0$ or 1) is the value in the equilibrium, while the market rate of interest is fixed at the equilibrium value, R^*. If there is such a possibility and the banks are the Stackelberg leader, the banks have an incentive to deviate from the equilibrium, and therefore the equilibrium is unstable. (Note that banks want to deviate, taking $R = R^*$ as given, while R will change from R^* if they actually deviate.) If there is no such $\hat{s}$ that improves the banks' payoffs, we call the equilibrium stable.

s	R	ν_c	C/X
1	1.2994	.3000	.6667

Table 3: Result (1) - With Capital Requirement

the good equilibrium the unique equilibrium using the capital requirement. The reason why the capital requirement is effective to eliminate the bad equilibrium is simply that a bank with more capital is less susceptible to a bank run: Suppose that X and L are fixed and that C increases, i.e., D decreases; condition (16) implies that ν_c decreases in this case; and therefore, the derivative of the objective function of the firm's problem, (11), implies that the equilibrium value of s is more likely to be one, the upper bound.

5.2 From bad equilibrium to multiple equilibria

In the second example, there exists only the bad equilibrium in which $s = 0$ if the bank capital requirement is not imposed; and imposition of the capital requirement can generate the good equilibrium in which $s = 1$, and there become multiple equilibria. We employ the values of parameters as in Table 4. In this case, the equilibrium of the

r	δ	C	θ	$\bar{s}$	ξ	ρ	$\underline{\nu}$	b	c	q	λ
5	.5	2	.5	1	.46	3	.2	.3	.5	.3	1.5

Table 4: Parameter Values (2)

basic model in which the capital requirement is not imposed is as in Table 5. There is a unique equilibrium in which $s = 0$. We find that this equilibrium is stable. However,

s	R	ν_c	C/X
0	1.3684	.4667	.2923

Table 5: Result (2) - Without Capital Requirement

if we introduce the capital requirement, there are multiple equilibria as in Table 6. The good equilibrium with $s = 1$ is stable, while the bad equilibrium with $s = 0$ is unstable.[7]

s	R	ν_c	C/X
0	1.2475	.2315	.6667
1	1.3258	.2364	.6667

Table 6: Result (2) - With Capital Requirement

Therefore, we can say that introducing the capital requirement gives us a chance that we can shift the economy to the good equilibrium from the bad one.

5.3 A case where capital requirements do not matter

In the previous two examples, introduction of the capital requirement has a good effect to increase the productivity and output in the economy, since the policy makes (a chance of) realization of the good equilibrium. However, if we employ the following values of paramters as in Table 7, the capital requirement does not change the equilibrium values of specialization: There are multiple equilibria both in the cases with and without the capital requirement.

r	δ	C	θ	$\bar{s}$	ξ	ρ	$\underline{\nu}$	b	c	q	λ
5	.5	2	.5	1	.24	3	.3	.3	.1	.3	1.5

Table 7: Parameter Values (3)

For these parameter values, the equilibria of the basic model in which the capital requirement is not introduced are as in Table 8. There are multiple equilibria. We find that these equilibria are stable. Even if we introduce the capital requirement, there are also multiple equilibria as in Table 9. Only the good equilibrium with $s = 1$ is stable in this case. Therefore, if the banks move taking the firms actions as given, introducing

[7] If the banks are the Stackelberg leaters, the good equilibrium is the unique equilibrium in this economy.

s	R	ν_c	C/X
0	1.4586	.4667	.3134
1	1.6402	.4476	.3488

Table 8: Result (3) - Without Capital Requirement

s	R	ν_c	C/X
0	1.1812	.3000	.6667
1	1.3781	.3000	.6667

Table 9: Result (3) - With Capital Requirement

the capital requirement does not resolve the multiplicity, while if the banks are the Stackelberg leaders, only the good equilibrium survives in the economy with capital requirement.

5.4 The bank-induced instability and the capital requirements

Our general equilibrium model has interesting implications. The banks in this economy provides insurance for the idiosyncratic liquidity shocks to firms, as those in Holmstrom and Tirole's (1998) model. This insurance function enables firms to undertake production projects and ex ante specialization, and thus increases the aggregate productivity of the economy. If the economy is not subject to the macro shock, ν, the existence of banks leads the economy to the good equilibrium. This is consistent with well known result in the literature that financial deepening is relevant to or even crucial for the economic growth (see Levine [1997]). Our model implies that if there exists the macro shock, the existence of banks may generate multiple equilibria on the premise that the banks are subject to bank runs à la Diamond and Rajan (2000). In this case, the economy may become instable and fluctuate between the good and the bad equilibria. Therefore, the existence of banks is good in that they provide insurance and generate the good equilibrium, but is not sufficiently good in that they cannot necessarily eliminate the bad equilibrium if the economy is subject to the macro shock, causing a large instability in the economy.

The possibility of bank runs decreases the expected return on specialization for the firms, and thus increases the instability. The capital requirement policy in this model can be regarded as a complement to the financial sector, which eliminates or reduces the instability of the economy. The capital requirement changes the equilibrium composition of C and D, so that the bank run is less likely to occur. Therefore, the bank capital requirement may raise the aggregate productivity through reducing the probability of bank run, ν_c, and enhancing the specialization by the firms.

6 Conclusion

We introduced the borrowers' choice of specialization into Holmstrom and Tirole's (1998) model, and showed that the specialization is negatively affected by bank distress. A high probability of bank failure discourages the borrowers' specialization ex ante, and lowers the aggregate productivity of the economy. Our theory seems successful in explaining productivity declines observed during banking crises, such as the episodes of the Great Depression in the 1930s and the lost decade in Japan in the 1990s.

The general equilibrium version of our model also provides a potential motive for the bank capital requirements. The model implies that the bank capital requirements may be able to lead the economy to the good equilibrium where firms choose a higher level of specialization. The bank regulation may be effective to enhance the aggregate productivity through reducing the bank-induced instability, or eliminating the bad equilibrium. Multiplicity of equilibria or the bank-induced instability may be important in explaining large business fluctuations associated with banking crises, especially in the emerging markets. The effectiveness of the capital requirements in reducing the bank-induced instability may be worth studying further to deepen our understanding of the necessity of bank regulations.

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