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Soft-Budget Constraints and Local Expenditures

Abstract
This paper investigates how the soft budget constraint with grants from the central government to local governments tends to exaggerate inefficient local expenditures. We first develop a theoretical model, which explains soft budget problem in a multi-government setting. We then show that in Japan’s case local governments implemented inefficient public investments and hence the bad outcome of soft budget problem occurred in the 1990s.

JEL classification: E6, H5, H6

Keyword: Soft budget constraint, local expenditures, central government, local government
1. Introduction

This paper investigates how local governments increase inefficient local public expenditures by highlighting the soft-budget constraint of grants from the central (or federal) government to subnational governments (hereafter local governments). Namely, this paper will analyze theoretically the soft-budget effect of intergovernmental financing on local expenditures by developing a simple game between the two governments with the overlapping tax bases between them and then will evaluate empirically the growing dependence on transfers for covering local public investment from the viewpoint of intergovernmental financing for Japan's case.

It is well recognized that if local governments face soft budget constraints, they will have an incentive to over-spend, over-borrow, and/or pay insufficient attention to the quality of the investments that their borrowing finances. Such over-spending/borrowing can occur through the common pool mechanism. See, for example, Wildasin (1997, 2004), Goodspeed (2002), Akai and Sato (2005) among others. That is, the natural conjecture is that if the central government imposes hard budget constraints, inefficient investment should not arise. However, recently Besfamille and Lockwood (2004) show that hard budget constraints can be too hard and discourage investment that is socially efficient. Namely, they point out the possibility that the hard budget constraint over-incentives the soft budget constraint to provide effort by penalizing it too much for project failure, thus leading ultimately to the possibility that socially efficient projects may not be undertaken.

In this paper without incorporating any uncertainty or imperfect information of effort with respect to public investment and other government activities, we develop a simple game theoretic model of the central and local governments, which shows that while the hard budget constraint does not necessarily realize the first best solution, the soft budget constraint may or may not be better off than the hard budget constraint case, depending on the initial size of hard budget constraint and the optimizing behavior of central government with respect to intergovernmental transfers.

We pay attention to the vertical externality of shared tax bases between the central and local governments. Multileveled government normally means some commonality of tax base between central and local governments. As a result the tax base may overlap and shared tax bases create another type of common pool problem. It is now well recognized that vertical externalities are likely to leave local taxes too high. This is because each local government unduly discounts the pressure on central government's spending it creates by raising its own tax rate. See Keen and Kotsogiannis (2002), Keen (1998), and Wilson (1999) among others. In this paper we do not consider such vertical/horizontal tax competition between central and local governments and would simply assume that tax rates are given for central and local
governments. Rather, we would like to focus on another inefficiency of local expenditures due to overlapping tax bases.

By assuming that the tax share is exogenously given, we incorporate two sources of inefficiency. First, the distribution of public spending between the central and local governments is not necessarily determined optimally. If the tax share to the central government is too high, the size of local public spending is too low (and vice versa). Second, local public investment may have a positive vertical externality effect. Namely, if an increase in local expenditure on infrastructure stimulates macroeconomic activities, it may enlarge the overlapping tax base, which would then increase taxes for the central government at the given share of tax base between two governments. This is a positive spillover of vertical externality. In this sense, the non-cooperative Nash equilibrium level of local public investment is too low.

Under these two inefficiencies in a two period model the benevolent central government may have an incentive to stimulate local expenditure in period 2 by means of additional grants ex post. However, such additional grants could produce soft budget problems by increasing grants when the local government spends and borrows more in period 1 because local borrowing more in period 1 means a decline in local public goods in period 2 and hence upsets the central government's optimal allocation strategy.

This paper consists of five sections. In Section 2, we develop a theoretical model of the central and local governments and investigate the subgame perfect equilibrium to explore the soft-budget problem with vertical externality of overlapping tax bases. Then we summarize Japanese intergovernmental financing policy in the recent years and discuss soft-budget constraints between the central and local governments in Japan's case in Section 3. Based on our theoretical model, we implement time series analyses concerning local public finance and macroeconomic activities in Section 4. Namely, we investigate how the bad outcome of soft-budget constraints is relevant in the real economy by empirically studying the impact of intergovernmental grants on local expenditures in Japan's case. Finally, we present some concluding remarks in Section 5.

2. Analytical Model of Central and Local Governments

2.1 Model

We develop a two-period intergovernmental financing model of two governments, the central government (or CG), the lower-level local government (or LG) in a small open economy, in order to explain how local public expenditures may be over-provided under the soft-budget constraint. For simplicity, we consider the representative local government, and do not consider the free-riding and/or spillover effects within local governments. This is just an assumption for simplicity. There
are many papers to explore the horizontal and vertical externalities due to non-cooperative competition among local governments. See Wilson (1999) among others. As shown in Appendix, the analytical results would be qualitatively the same even if we consider non-cooperative behavior of multi-local governments. Moreover, in Japan’s case, many local governments behave cooperatively and their behavior may be summarized by the representative local government (the Ministry of Home Affairs), which is in particular a good approximation of Japan’s case.

The representative local government (LG) provides local public goods \( g_t \) and the central government (CG) provides nation-wide public goods \( G_t \) in each period. Each public good is beneficial and its utility is given by a twice-continuously differentiable and strictly quasi-concave function. Moreover, we assume that all goods are normal ones. The relative price of each good is set to be unity for simplicity. Thus, the social welfare \( W \), which reflects the representative agent’s preferences over public goods, is given by

\[
W = u(G_1) + v(g_1) + \delta[u(G_2) + v(g_2)]
\]

where \( 0 < \delta < 1 \) is a discount factor. For simplicity, private consumption is assumed to be fixed and hence we only consider the utility from public goods.

The local government also conducts public investment \( k \) in period 1, which has a productive effect of raising tax revenue in period 2. Let \( Y_t \) represent total tax revenue of the two governments in period \( t \) (\( t = 1, 2 \)). We assume that \( Y_1 \) is exogenously given but \( Y_2 \) is dependent on public works conducted by the local government in period 1. \( Y_2 = Y_1 + f(k) \). \( k \) denotes local public investment in period 1, which would increase total tax revenue of period 2. Investment product function \( f(\cdot) \) satisfies the standard Inada condition: \( f'(\cdot) > 0, f''(\cdot) < 0 \). For simplicity we do not consider public investment by the central government. We do not also consider pork barrel spending by the local government. As shown in DelRossi and Inman (1999), it is well recognized that pork barrel projects are too high due to subsidies from the central government caused by local governments’ political demand. It is theoretically obvious that wasteful projects are too high due to the political pressure. In this paper we consider nation-wide beneficial local public investment. In a multi-local government setting local public investment does not have spillover effects over regions. Still it has the vertical externality effect on the central government’s tax revenue. Nevertheless, we show that public investment may be over-provided under the soft budget constraint.

Next, we specify each government’s budget constraint. Both central and local governments levy taxes on overlapping economic activities. Since the tax base is overlapping, the tax revenue may be shared by the two governments. We set \( \beta \) as local government’s portion of total tax revenue, \( 0 < \beta < 1 \). The central government gains a portion of the total tax revenue. Thus \( 1 - \beta \) means share of the central government to total tax revenue. The share parameter \( \beta \) is assumed to be
exogenously given and constant over time.

The period-by-period budget constraints of CG are given as follows,

\[ B = G_1 + Z_1 - (1 - \beta)Y_1 \]  
\[ G_2 + Z_2 + (1 + r)B = (1 - \beta)Y_2 \]

where \( Z_t \) is grants from the central government to the local government in period \( t \). \( B \) is the central government debt. \( r > 0 \) is the exogenously given world interest rate.

The period-by-period budget constraints of LG are given as follows,

\[ D = g_1 + k - Z_1 - \beta Y_1 \]  
\[ g_2 + (1 + r)D = Z_2 + \beta Y_2 \]

where \( D \) is the local government debt.

From (2) and (3) we can rewrite the intertemporal budget constraints of the central and local government, respectively, as follows.

\[ \begin{align*}
G_1 + \frac{G_2}{1 + r} &= (1 - \beta)Y_1 + \frac{(1 - \beta)Y_2}{1 + r} - Z_1 - \frac{Z_2}{1 + r} \\
g_1 + \frac{g_2}{1 + r} + k &= \beta Y_1 + \frac{\beta Y_2}{1 + r} + Z_1 + \frac{Z_2}{1 + r}
\end{align*} \]

2.2 Unitary Government

First of all, we investigate the Pareto efficient first best allocation in this model as a benchmark. Since we do not incorporate any uncertainty or imperfect information with respect to public investment and other government activities in the unitary system, unitary government, consolidating CG and LG, could attain the first best by allocating optimally the total tax revenues among nation-wide public goods and local public goods in each period. Namely, the unitary government, who implements the optimal allocation \( \{G_t, g_t, k\} \), maximizes social welfare (1) subject to the following overall feasibility constraint

\[ Y_1 + \frac{Y_2}{1 + r} = G_1 + \frac{G_2}{1 + r} + g_1 + \frac{g_2}{1 + r} + k \]

which is obtained from (2-3) and (3-3) by eliminating \( \beta \) and \( Z_1, Z_2 \).

First order conditions of this optimization problem are as follows,

\[ \begin{align*}
u_{G_1} - \mu &= 0 \\
\delta u_{G_2} - \frac{\mu}{1 + r} &= 0 \quad \text{where} \quad u_{G_t} = \frac{\partial u(G_t)}{\partial G_t} \\
v_{g_1} - \mu &= 0 \\
\delta v_{g_2} - \frac{\mu}{1 + r} &= 0 \quad \text{where} \quad v_{g_t} = \frac{\partial v(g_t)}{\partial g_t}
\end{align*} \]
\[ \mu \left\{ \frac{f'(k)}{1 + r} - 1 \right\} = 0 \]

\( \mu \) is the Lagrangian multiplier of equation (4). From these conditions we have

\[ v_{g1} = u_{G1} \] (5-1)

\[ u_{G2} = v_{g2} \] (5-2)

\[ \frac{u_{G1}}{u_{G2}} = \frac{v_{g1}}{v_{g2}} = (1 + r)\delta \] (5-3)

\[ f'(k) = 1 + r \] (5-4)

The above optimality conditions (5-1,..4) and the feasibility condition (4) determine the Pareto efficient allocation as the benchmark case. Conditions (5-1)(5-2) mean that the marginal benefit of public goods is equalized between CG and LG. Condition (5-3) governs the standard (inter-temporal) optimal allocation of public spending between two periods. Finally, condition (5-4) is the standard first-best criterion of public investment.

### 2.3 Outcome in a Decentralized System

We now investigate whether the first best solution is obtained in a multi-government non-cooperative world where central and local governments decide their policy variables non-cooperatively. Suppose first of all the fully (or isolated) decentralized Nash equilibrium at the exogenously given \( \beta > 0 \), where there is no intergovernmental transfer between CG and LG; \( Z_1 = Z_2 = 0 \). CG maximizes (1) subject to (2-3) by choosing nation wide public goods while assuming local public goods fixed. Similarly, LG maximizes (1) subject to (3-3) by choosing local public goods and investment, while assuming nation-wide public goods fixed. Then, first order conditions of this Nash non-cooperative equilibrium are as follows,

\[ u_{G1} - \Psi = 0 \]

\[ \delta u_{G2} - \frac{\Psi}{1 + r} = 0 \]

\[ v_{g1} - \psi = 0 \]

\[ \delta v_{g2} - \frac{\psi}{1 + r} = 0 \]

\[ \psi \left\{ \frac{f'(k)\beta}{1 + r} - 1 \right\} = 0 \]

where \( \Psi \) and \( \psi \) are the Lagrangian multipliers of equations (2-3) and (3-3) with \( Z_1 = Z_2 = 0 \), respectively. From these conditions we have
Condition (6-1), which is the same as (5-3), implies that relative (intertemporal) allocation between $g_1$ and $g_2$ as well as relative (intertemporal) allocation between $G_1$ and $G_2$ is efficient. But the levels of these public goods and local investment are not necessarily provided optimally. In other words, conditions (5-1)(5-2) do not necessarily hold since the total levels of public goods, $G_1 + \frac{G_2}{1+r}$ and $g_1 + \frac{g_2}{1+r}$, are arbitrarily set, depending on the exogenous parameter, $\beta$.

Moreover, (6-2) means that $k$ is under-provided due to the overlapping tax base; $\beta < 1$. Condition (5-4) does not hold. Since the local government does not take into account the positive spillover effect of increasing the overlapping tax base on public goods provided by the central government, local public investment provided by the local government is not sufficient and total tax revenue shared by both governments in period 2 is inefficiently low.

There are two sources of inefficiency in the decentralized system. First, $\beta$ is not necessarily set at the optimal level and hence the allocation of public spending between CG and LG is not determined optimally. In this paper we do not consider the possibility of choosing $\beta$ optimally. Second, there is a vertical externality of public investment due to the overlapping tax base. So long as $\beta < 1$, (6-2) implies that $k$ is too low.

It should be noted that although the first distortion may be corrected by a lump-sum transfer between CG and LG, the second distortion cannot be corrected by a lump-sum transfer alone. We need a non-lump sum intergovernmental transfer scheme from the central government to the local government to stimulate public investment $k$. We also consider the possibility of making additional lump-sum grants from CG to LG in period 2, and it may create the soft budget problem. In section 2.4 we develop a simple game between CG and LG to explore the bad outcome of soft budget problems for Japan's case.

### 2.4 Non-Lump Sum Transfer

#### 2.4.1 Analytical Framework of IGT

From now on we consider an intergovernmental grant transfer scheme (IGT) from CG to LG. Generally, $Z_1, Z_2$ may be functions of $g_1, g_2, k, Y_1$, and $Y_2$. An example of the simplest scheme would be
\[ \hat{p} k = Z_1 \]  \hspace{1cm} (7-1) \\
\[ 0 = Z_2 \]  \hspace{1cm} (7-2) 

where the transfer in period 1 is increasing with local public investment, while there is no transfer in period 2. We will assume this simple scheme in the following game. In section 3.3 we consider another example of IGT scheme, which is more relevant for Japan's case. The analytical results are qualitatively the same as in the simple scheme of (7.1)(7.2).

The local government, which receives IGT Grants, faces budget constraint (3-1), (3-2), (7-1) and (7-2). Then we have as the local government budget constraint in the IGT system
\[ g_1 + (1 - \hat{p})k + \frac{1}{1 + r} g_2 = \beta Y_i + \frac{1}{1 + r} \beta \{Y_i + f(k)\} \]  \hspace{1cm} (8)

2.4.2 Game between Two Governments
2.4.2.1 Structure of the games

We consider the following two games;

Game I)

CG is the leader and LG is the follower. The game is done at the beginning of period 1. Namely, at the first stage CG determines IGT parameter \( \hat{p} \) and public goods \( G_1, G_2 \) with regarding \( g_1 \) and \( g_2 \) fixed, and then at the second stage LG determines its expenditures, \( g_1, g_2, k \).

Game II)

LG is the leader and CG is the follower. And, IGT parameter \( \hat{p} \) and public goods \( G_1, G_2 \) are exogenously set at the levels given by solutions of Game I. Namely, the first stage of this game is done at the beginning of period 1, where LG determines its expenditures of period 1, \( g_1, k \). Then, the second stage of this game is done at the end of period 1, where CG chooses additional grants, A, as well as \( G_2, g_2 \). In this sense this game becomes a soft-budget game.

The relation between these two games is explained as follows. Note that \( \beta \) is not necessarily set at the optimal level. Hence, when Game I is over and the second period comes, the central government may not want to commit to the determined level of \( \beta \) at the beginning of period 2 and now may want to transfer an additional amount \( A \) in period 2 to maximize social welfare ex-post.

2.4.2.2 Game I
Second Stage

In Game I LG faces the hard-budget constraint. We first investigate the optimizing behavior of LG at the second stage, which occurs at the beginning of the first period. As explained above, LG regards nation-wide public goods as fixed when LG maximizes utility (1). It follows that the local government maximizes the objective function (1) subject to (8) at given levels of tax share parameter $\beta$, IGT parameter, $\hat{p}$, and nation wide public goods, $G_1$ and $G_2$.

Therefore, the first order conditions with respect to its policy variables, $g_1$, $g_2$, and $k$ are respectively given as follows,

\[ v_{g_1} - \lambda = 0 \] 
\[ \delta v_{g_2} - \frac{1}{1 + r} \lambda = 0 \] 
\[ \lambda (1 - \hat{p} - \frac{1}{1 + r} \beta f'(k)) = 0 \]

where $\lambda$ is the Lagrange multiplier of constraint (8). From these conditions we have

\[ \frac{v_{g_1}}{v_{g_2}} = (1 + r)\delta \] 
\[ f'(k) = \frac{1 - \hat{p}}{\beta} (1 + r) \]

Equation (10-1) governs the intertemporal allocation of $g_1$ and $g_2$, and equation (10-2) determines $k$ at given levels of tax share parameter $\beta$ and IGT parameter, $\hat{p}$.

From these conditions (10-1,2) and the budget constraint (8), we may derive the response functions of local government.

\[ g_1 = \Gamma_1(\beta, \hat{p}) \] 
\[ g_2 = \Gamma_2(\beta, \hat{p}) \] 
\[ k = K(\beta, \hat{p}) \]

An increase in $\hat{p}$ will raise local expenditures. An increase in $\beta$ would also stimulate local expenditures. These results are intuitively appealing.

First Stage

At the first stage of Game I, the central government maximizes the national welfare (1) subject to its budget constraints (2-3), (7-1,2), and the response functions of local government (11-1,2,3) by choosing IGT parameter, $\lambda$, as well as nation-wide public spending $G_1$, $G_2$ and public debt $B$. Although the central government can effectively control local expenditures based on (11) by choosing the IGT parameter, it may not realize the first best allocation by setting policy variables
appropriately. Namely, the central government may not attain (5-1) - (5-4) at the same time by choosing $G_1, G_2, \hat{p}$ appropriately.

This is because the size of tax share in each period is exogenously given as $\beta$ at an arbitrary level, and hence the present value of central government's spending $G_1 + \frac{1}{1 + r}G_2$ is also exogenously given by (2-3) at an arbitrary level. From (11-3) CG can affect $k$. Namely, if $\hat{p}$ is chosen to meet the following condition,

$$\beta = 1 - \hat{p}$$

(5-4) is attained. Then, the marginal benefit of $k$ is equal to the marginal cost of $k$ for LG, and the first best level of $k$ is attained. But, if $k$ is given at the first best level associated with (5-4), then $Z_1$ is also given, and hence the total size of $G_1 + \frac{1}{1 + r}G_2$ cannot be changed any more by using the IGT parameter.

Hence, optimality conditions (5-1) - (5-4) are not necessarily realized at the subgame perfect solution. If $\beta$ is too low, $G_1$ and $G_2$ are too high, while $g_1, g_2$ are too low (and vice versa). In such a case, it would be desirable to raise a transfer from CG to LG in period 1 (and vice versa). However, this option is not available in our setting. It is assumed that CG cannot choose $\beta$ optimally. In reality CG would have a difficulty of evaluating $v(g_1), v(g_2)$ perfectly at the beginning of period 1 due to asymmetric information. Our assumption of exogenously given $\beta$ may capture this reality.

2.4.2.3 Game II

Second Stage

In Game II after CG knows $v(g_1)$ well, at the beginning of period 2 CG may not want to commit to the initial level of $\beta$. CG may effectively change $\beta$ by creating grants to LG ex post. Thus, LG faces the soft-budget constraint. We first investigate the optimizing behavior of CG at the second stage of the game, which occurs at the beginning of the second period. After LG determines local expenditures, $g_1$ and $k$, in period 1, CG may effectively choose its public spending $G_2$ and $g_2$ subject to the budget conditions (2-2) and (3-2) by creating an additional grant, $A$, appropriately in period 2.

The budget constraint of the central government in period 2 is rewritten as

$$G_2 + (1 + r)B = (1 - \beta)Y_2 - A$$

(2-2)

Similarly, the budget constraint of the local government in period 2 is rewritten as

$$g_2 + (1 + r)D = \beta Y_2 + A$$

(3-2')

From (2-2') and (3-2') eliminating $A$ gives the relevant overall budget constraint in period 2 as
By choosing \( A \) ex post in period 2, the central government may in fact choose the allocation of \( G_2 \) and \( g_2 \) under the above overall constraint \((13)\) to maximize the social welfare in period 2, \( u(\hat{G}_2) + v(g_2) \). Thus, the first-order condition is given by

\[
\frac{\partial u}{\partial g_2} = \frac{\partial v}{\partial g_2}
\]

(5-2)

From the above optimality condition \((5-2)\) and the budget constraints \((2-2')\), \((3-2')\), at given levels of local expenditures \( g_1 \) and \( k \), which are chosen in period 1, we may derive the optimal response of \( A, g_2 \) (and hence \( G_2 \)) of the central government as functions of \( g_1 \) and \( k \), respectively.

\[
A = J(g_1, k)
\]

(14-1)

\[
g_2 = P(g_1, k)
\]

(14-2)

Considering \((3-1)\) and \((7-1)\), \( D \) is determined by \( g_1 \) and \( k \). We have \( dD = dg_1 + (1 - \hat{p})dk \). By totally differentiating the budget conditions \((2-2')\) and \((13)\) and the optimality condition \((5-2)\), we have

\[
dG_2 + dg_2 + (1 + r)(dg_1 + (1 - \hat{p})dk) = f'(k)dk
\]

\[
(1 - \eta)dG_2 = \eta dg_2
\]

\[
dG_2 = (1 - \beta)f'(k)dk - dA
\]

where \( \eta \equiv \left| \frac{\partial u}{\partial G_2} \right| / \left| \frac{\partial u}{\partial g_2} \right| + \left| \frac{\partial v}{\partial G_2} \right| \) means the relative evaluation of \( G_2 \) compared with \( g_2 \). It is assumed for simplicity that \( 0 < \eta < 1 \) is constant. Then, considering \((2-2')\), we have as the property of response functions

\[
J_g = \frac{\partial A}{\partial g_1} = \frac{\partial G_2}{\partial g_1} = \eta(1 + r) > 0
\]

(15-1)

\[
J_k = \frac{\partial A}{\partial k} = \frac{\partial G_2}{\partial k} + (1 - \beta)f'(k) = \eta(1 + r)(1 - \hat{p}) + (1 - \beta)f'(k) - \eta f'(k)
\]

(15-2)

\[
P_g = \frac{\partial g_2}{\partial g_1} = -(1 - \eta)(1 + r) < 0
\]

(15-3)

\[
P_k = \frac{\partial g_2}{\partial k} = -(1 - \eta)((1 + r)(1 - \hat{p}) - f'(k))
\]

(15-4)

As shown in \((15-2)\), the sign of \( J_k \) is generally ambiguous. If \( 1 - \beta > \eta \), then \( J_k > 0 \). That is, if the marginal valuation of \( G_2 \) is relatively small and \( 1 - \beta \) is too high, \( g_2 \) is too low compared with \( G_2 \), and hence the central government would react to increase \( A \) in order to maximize social welfare.

Intuition is as follows. When \( k \) is increased by issuing more debt \( D \), \( g_2 \) is decreased from \((3-2)\), which is not good for the central government since it would
like to realize the optimality condition (5-2) to raise social welfare. Thus, the central government has an incentive to make additional subsidies to the local government in period 2 to raise the ex post level of $g_2$. We may call the case of $J_g > 0$ the typical soft-budget case. Moreover, (15-1) shows another outcome of the soft budget constraint. An increase in $g_1$ results in more grants $A$ from the central government. $J_g > 0$

A key part of the model is the interaction between the central government and the local government. The central government intends to allocate revenues to equalize marginal gains of public goods between the central and local governments. The central government’s benevolent incentives result in creating a soft budget constraint by creating additional grants in period 2 when the local government borrows more in period 1 because local borrowing more in period 1 means a decline in local public goods in period 2 and hence upsets the central government’s optimal allocation strategy. The central government intends to maximize the social welfare in period 2 by making additional grants in period 2 in response to local borrowing for more public investment and public goods in period 1.

First Stage
We now investigate the optimizing behavior of the local government at the first stage of Game II, which occurs at the beginning of period 1. The local government's budget constraint (8) is effectively binding here only under the condition that the central government changes $A$ in response to local expenditures of period 1, as summarized by equation (14-1,2). Namely, the effective budget constraint for the local government is given by

$$g_1 + \frac{P(g_1, k)}{1+r} + (1-\hat{p})k = \beta Y_1 + \frac{\beta [Y_1 + f(k)]}{1+r} + \frac{J(g_1, k)}{1+r}$$

(16)

The local government maximizes the objective function (1) subject to (16) at given levels of tax share parameter $\beta$, IGT parameter $\hat{p}$, and nation wide public goods, $G_1$ and $G_2$. Actually, response functions (14-1,2) means that LG can affect $G_2$ by choosing $g_1$ and $k$. But we assume that LG always considers nation-wide public goods, $G_1$ and $G_2$ as given at its optimization.

Therefore, the first order condition with respect to its policy variables, $g_1$ and $k$, are respectively given as follows,

$$v_{g_1} + \delta v_{g_2} P_g - (1+\frac{P_g - J_g}{1+r})\omega = 0$$

(17-1)

$$v_{g_2} P - \omega (1-\hat{p}) + \frac{P_g}{1+r} - \frac{\beta}{1+r} f'(k) - \frac{J_g}{1+r} = 0$$

(17-2)

where $\omega (>0)$ is the Lagrange multiplier of constraint (16). Equations (17-1,2)
govern the allocation of \( g_1 \) and \( k \) at a given level of tax share parameter \( \beta \) and IGT parameter, \( \hat{p} \). Substituting (15-1,3) into (17-1), we have

\[ v_{g_1} = \delta v_{g_2}(1-\eta)(1+r) \]  

(18-1)

Thus, the optimality condition between \( g_1 \) and \( g_2 \) given by (5-3) is not realized here at the subgame perfect solution. If CG did not make additional grants \( A \), the optimizing behavior of LG could have attained condition (5-3) with respect to \( g_1 \) and \( g_2 \). When LG takes into account the response functions of CG (14-1,2), it would effectively reduce the marginal cost of raising \( g_1 \), stimulating \( g_1 \) in period 1. (18-1) means that \( g_1 \) is too high, compared with \( g_2 \) and \( G_2 \). An increase in \( g_1 \) would result in an increase in \( A \), which has a positive income effect on local expenditures. This is a plausible result of the soft budget constraint.

Next, substituting (15-2,4) into (17-2), we have

\[ (1+r)(1-\hat{p}) = f' \]  

(18-2)

Since the IGT parameter \( \hat{p} \) is positive and less than 1, (18-2) means \( 1+r > f' \). It follows that at the subgame perfect solution \( k \) is too high in Game II.

2.4.2. Comments

Several comments are useful. First, in Game II LC is the leader and CG is the follower. We assume that IGT parameter \( \hat{p} \) and public goods \( G_1 \) are exogenously set at the levels given by solutions of Game I. In other words, CG does not change IGT parameter \( \hat{p} \) and public goods \( G_1 \) given by Game I. In reality it is not easy to change IGT parameters often, which would likely be relevant in Japan’s case. Wildasin (1997) assumes the similar behavior of CG in his model of a game between CG and LG.

Second, we could regard the second stage of Game II as the third stage of Game I. In such a case, as the first stage of Game II, we may consider an additional game, in which the central government now maximizes the national welfare (1) subject to its budget constraints (2-1,2), (7-1,2), and the response functions of local government based on (17-1,2) by choosing optimally IGT parameter, \( \hat{p} \), as well as nation-wide public spending, \( G_1 \), and public debt \( B \) at the beginning of the first period. It is interesting to note that CG cannot still attain the optimality conditions with respect to \( g_1 \), \( g_2 \) and \( k \), (5-3) and (5-4) by setting the IGT parameter. And hence it cannot realize the overall optimality conditions given by (5-1,2,3,4). This is because (18-1) (18-2) always mean that \( g_1 \) and \( k \) are too high.

Third, welfare in Game II could be lower than welfare in Game I, depending on the initial size of \( \beta \). Due to the soft-budget constraint, \( g_1 \) is too high. Also, \( k \) is too high and local public investment is done inefficiently. We may say
that the soft-budget effect would stimulate inefficient local public expenditures.

2.4.3 Soft Budget Problem

We call Game I the hard-budget game, while Game II the soft-budget game. When the central government commits to a predetermined value of $\beta$ as the leader of intergovernmental game between central and local governments, the local government is subject to the hard budget constraint. However, in this case the central government may not attain the first best by choosing the IGT parameter appropriately unless $\beta$ happens to be optimal. Considering asymmetric information of $v(g_1), v(g_2)$, the central government would not likely set $\beta$ optimally from the beginning in reality even if it can choose $\beta$.

On the other hand, when the central government cannot commit to a predetermined value of $\beta$ in period 2 and may add new grants $A$ after the local government determines its expenditures, the local government is subject to the soft budget constraint. Namely, when the local government raises local expenditures in period 1, the central government has an incentive to support such larger local expenditures by creating additional subsidies to the local government after the central government knows more about $v(g_1)$ in period 2. It follows that in such a game the local government has a strong incentive to increase the local expenditures in period 1. The central government may respond to such demand in period 2. In Game II, the central government overthrows the commitment if the marginal valuation of $G_2$ is relatively small and/or a predetermined level of tax share parameter $\beta$ is too high at the subgame perfect solution. We have shown that in Game II the central government may have an incentive to make additional grant $A$ in period 2 ex post and if so, in Game II $k$ and $g_1$ may well be too high.

Moreover we have shown the possibility of negative welfare effect of soft budget constraint. While the hard budget constraint does not always attain the first best, the soft budget constraint may deteriorate social welfare by inducing inefficient expenditure due to an additional grant. Our analytical result suggests that either constraint could be better or worse, depending of the initial level of $\beta$. If the hard budget constraint is too hard in the sense that $\beta$ is too low compared with the first best solution, the soft budget outcome would be better, and vice versa.

It should also be noted that if local public expenditures are too high in Game II, it would depress macroeconomic activities as well. In such a case the rate of return on public expenditures becomes very low, resulting in lower GDP compared with the first best allocation. As to Japan's case, local expenditures and grants from CG to LG actually increased in the 1990s. If the bad outcome of Game II is more relevant with Japan's case than Game I, the soft budget constraint may produce serious problems to Japan's economy. We would like to confirm this
conjecture by employing some empirical studies for Japan’s case.

3. Japan’s Intergovernmental Financing and Soft-Budget Problem
3.1 Fiscal Policy in the 1990s

Before conducting empirical studies, let us briefly explain Japan’s fiscal policy and intergovernmental fiscal system. After a “bubble economy” was broken in 1991, Japan did not grow much because of severe economic and financial situation. Responding to political pressures of interest groups, the central government employed some measures for stimulating the aggregate demand. Namely, the Japanese government implemented increases in public investment (based on the traditional Keynesian counter-cyclical policy). However, these counter-cyclical measures were not so effective, resulting in an increase in the huge fiscal deficit. According to previous studies, e.g. Ihori, Doi, and Kondo (2001) and Ihori, Nakazato, and Kawade (2003), Japan’s public investment, sharply increased in the early 1990s, was not effective to enhance economic growth. Ihori and Kondo (2001) among others show that Japan’s public investment was over-provided in the recent years. It is one of reasons why “the Lost Decade” was triggered.

Japan’s central government provides heavy financial support to local governments, amounting to about 5% of GDP every fiscal year. In the 1990s, the government deficits in Japan increased rapidly because local governments in the rural and agricultural area got a lot of transfers mainly in the form of public works. Agriculture-related public capitals and fishing ports and measures for flood control and conservation of forests were accumulated too much. In comparison with other countries’ figures, we may say that local governments in Japan have larger privileges, wasteful local expenditures, than in other countries.

3.2 Intergovernmental Financing

The ratio of national taxes to local taxes within the total tax burden borne by Japanese citizens is approximately 2 to 1, but in order to achieve balanced finances among all prefectures, a fixed percentage of national taxes are provided as so called “The Local Allocation Tax (hereafter LAT)” Grants to local governments. The central government reserves a certain ratio of national tax revenue in the General Account as a common fund for local governments. In the General Account of the central government, LAT Grants distribution amounts to a certain percentage of national tax revenues that are determined by the Local Allocation Tax Law. It includes 32% of the revenue from the personal income tax and the liquor tax, 35.8% of the revenue from the corporate income tax and 29.5% of the revenue from the consumption tax, and 25% of the revenue from the tobacco tax.

The Grants for LAT are transferred from the General Account to the Special Account for Allocation and Transfer Taxes. Then the LAT Grants are
allocated from the Special Account to local governments. The central government distributes these funds to each local government according to its fiscal needs and local revenue sources, based on a detailed scheme determined by the central government (the Ministry of Home Affairs).

3.3 Japan’s Soft-Budget Problem and LAT Grants

In Japan’s case, in place of (7-1) and (7-2) we could specify the non-lump sum intergovernmental grant transfer structure (IGT) as an approximation of LAT grants as follows

\[
\begin{align*}
1 \quad h \cdot g_1 - \theta \beta Y &= Z_1 \\
2 \quad h \cdot g_2 + p \cdot k - \theta \beta Y &= Z_2 
\end{align*}
\]

where \( h \) is coefficient for local public spending, \( p \equiv (1+r) \hat{p} \) is coefficient for public capital (investment expenditure), and \( \theta \) is coefficient for local tax revenue (\( \theta \beta Y \)).

In Japan’s case both \( h \cdot g_1 \) and \( h \cdot g_2 + p \cdot k \) mean “Standard Fiscal Need” (SFN) in period 1 and period 2 respectively, and \( \theta \beta Y \) means “Standard Fiscal Revenue” (SFR). SFN is increasing with local expenditures, while SFR is increasing with local taxes. IGT Grants just cover the shortfall, SFN minus SFR. Thus, grants from the central government are increasing with local expenditures and decreasing with local taxes.

In Japan it is plausible to assume that the tax share (\( \beta \)) is exogenously given all through the analysis in order to explore the role of IGT system with the soft budget problem. In principle, as explained above, the supply side of IGT Grants is given by a portion of national taxes, which is determined by a national law. There is thus an exogenous limit on the size of transfer in each period, \( Z_1, Z_2 \), respectively.

\[
\begin{align*}
Z_1 &= \bar{Z}_1 \\
Z_2 &= \bar{Z}_2 
\end{align*}
\]

Equations (19-1) and (19-2) are hard budget constraints.

Since \( h \) and \( \theta \) are now available in addition to \( p \), CG can effectively change the allocation of public spending between CG and LG if it may choose \( \bar{Z}_1, \bar{Z}_2 \) as well. Thus, we need the exogenously given limit on the size of transfers, \( \bar{Z}_1, \bar{Z}_2 \). Otherwise, if CG can also choose \( \bar{Z}_1, \bar{Z}_2 \) optimally at the first stage of the game, then the first best may be attained. In order to make the comparison of the hard budget constraint and the soft budget constraint meaningful, we focus on the

\[\text{1} \quad \text{This specification of the formulation is empirically confirmed by Hayashi (2000) and Doi (2002) for Japan’s case.}\]
second-best case where $Z_1, Z_2$ are not optimally chosen. In reality CG may not have accurate information of $v(\cdot)$ at the beginning of period 1. Our assumption that $Z_1, Z_2$ are not optimally chosen captures this aspect, and is relevant for Japan’s case.

Also, we assume that LG does not take into account the constraints (19-1) and (19-2) at its optimization. Namely, LG does not think that it faces the ceiling constraint of $Z_i$ given by (19-1) and (19-2). Rather, LG thinks that it can obtain grants as much as possible by raising local expenditures, $g_i$ and $k$, and local public debt, $D$, at given IGT parameters. Then the analytical results are qualitatively the same as in section 2.

Now, the effective budget constraint for the local government is given by

$$(1-h)g_i + \frac{(1-h)P(g_i,k)}{1+r} + (1-\hat{p})k$$

$$= (1-\theta)\beta Y_i + \frac{(1-\theta)\beta[Y_i + f(k)]}{1+r} + \frac{J(g_i,k)}{1+r} \quad (16)'$$

Note that (3-1) is replaced by

$$D = (1-h)g_i + k - (1-\theta)\beta Y_i$$

and, (15-1) and (15-3) are replaced by

$$J_g = \eta(1-h)(1+r)$$

$$P_g = -(1-\eta)(1-h)(1+r)$$

under the LAT Grant system. Then, in place of (18-1) and (18-2) we have

$$v_{g_1} = \delta v_{g_2}(1-h)(1-\eta)(1+r) + \omega(1-h)(1-\eta)h$$

$$- v_{g_2}(1-\eta)(1+r - f')$$

$$- \frac{\omega}{1+r} [(1+r)(1-\eta)h - \hat{p}) - f''[(1-\eta)h - \theta\beta)] = 0 \quad (18-1)'$$

$$- v_{g_2}(1-\eta)(1+r - f')$$

$$- \frac{\omega}{1+r} [(1+r)(1-\eta)h - \hat{p}) - f''[(1-\eta)h - \theta\beta)] = 0 \quad (18-2)'$$

The main difference occurs when we regard the second stage of Game II as the third stage of Game I. In such a case, if CG can choose $h, \theta, \hat{p}, G_1$, CG may attain the overall optimality conditions given by (5-1,2,3,4) by setting IGT parameters so that (18-1)' and (18-2)' are consistent with the first-best optimality conditions. However, in Game II we assume that IGT parameters $h, \hat{p}, \theta$ are exogenously set at the levels given by subgame perfect solutions of Game I. In Japan’s reality it is not easy to change IGT parameters often.

Regarding the additional grant $A$, in Japan’s case if the central government intends to increase the LAT Grants ex post, it could raise the local allocation tax rate or increase the rates of national taxes themselves, and hence can raise the LAT Grants. It, however, would take some time to adopt these measures. In Japan’s case, the central government mainly increases borrowing at the Special Account of...
Allocation and Transfer Taxes ex post facto, in response to excessive local expenditures by local governments. When the central government is politically weak, such a response would be easier to take, but it would create a "soft budget" problem with LAT Grants.

Local government bonds as well as national government bonds rapidly increased in the 1990s, as shown in Figure 1. As the outstanding of local bonds increased from 52 trillion yen at the end of fiscal 1990 to 130 trillion yen at the end of fiscal 1999, borrowing of the Special Account for Allocation and Transfer Taxes increased. This borrowing is incurred to spend LAT Grants. The increase was from 1.5 trillion yen at the end of fiscal 1990 to 30 trillion yen at the end of fiscal 1999. By the end of fiscal 1999, the total outstanding of these bonds and borrowing was 506 trillion yen (from 222 trillion yen at the end of fiscal 1990). Especially, borrowing of the Special Account for Allocation and Transfer Taxes was a peculiar phenomenon in the 1990s.

Local expenditures, mainly local public works, were heavily financed by local government bonds. "Unsubsidized" local public investment, which was implemented by local governments based on LAT grants and debt issuance without receiving any matching grants from the central government, was dramatically expanded. Ratio of total amount of local public investment (called it “ordinary construction expenses” in Japan's local public finance) to GDP soared to the highest level (over 6%) in the second half of 20th century.

It should be noted that the central government admitted to issue these local bonds so that LAT Grants would also cover the resulting future repayment expenditure of these local bonds. Local governments can raise funds for applicable projects even without sufficient repayment ability. By doing so, they may enjoy the benefits of these public projects, while they do not have to pay for them, just depending on future national tax revenues, including tax revenues collected in other local governments (through the LAT Grants scheme) for debt repayment. Ratio of LAT Grants to cover local bonds repayment to those expenses on settlement basis increased in the 1990s. Then it seems to trigger off an increase in LAT Grants, and lead to a further increase in borrowing for LAT Grants.

4. Empirical Analysis

2 Precisely speaking, Standard Fiscal Need, which explained below, includes repayment expenditure for depopulated area development bonds, revenue resource support bonds, and revenue decrease compensation bonds.

3 Doi (2002a) theoretically examines economic aspects of this scheme. Sato (2002), Akai, Sato, and Yamashita (2003), and Akai (2006) theoretically and empirically described defects of LAT Grants. They explored the free-riding behavior among local governments in the LAT system.
4.1 Estimation

In this section, we investigate how serious the bad outcome of soft budget problem may be observed or not in Japan's Local Allocation Tax Grants. In order to confirm empirically such a soft-budget phenomenon, it is important to note causality relationship between local expenditure and additional grants mainly by borrowing for LAT Grants in Japan's case, theoretically explored in Section 2. When local governments increase expenditures, in particular inefficient public goods and investment, their fiscal shortfalls increase in the future. When the central government could not commit to prohibiting borrowing at the Special Account for Allocation and Transfer Taxes to cover their shortfall, local governments intend to increase expenditures more.

Therefore, if we can observe the reaction functions (14-1,2), which mean that an increase in local expenditures leads to an increase in grants mainly in borrowing of the Special Account for Allocation and Transfer Taxes, we confirm the soft budget constraint with LAT Grants, and hence Game II would be relevant. On the contrary, if we do not have such a causality relationship, then Game I would be relevant.

The Granger causality test using the conventional VAR analysis is adequate to confirm empirically the above relationship. The similar Granger causality tests have been implemented in previous studies of revenue-expenditure nexus. As an example in Japan's case, Ihori, Doi, and Kondo (2001) and Doi and Ihori (2002) analyzed causalities between revenues and expenditures. In this section we empirically study the soft budget constraint in Japan focusing mainly on borrowing of the Special Account for the Grants, which is the first attempt to verify soft budget problems in Japan.

We employ the following data. First, data of Local Allocation Tax Grants, denotes $Z$, and the borrowing outstanding of the Special Account for Allocation and Transfer Taxes, denotes $L$, which is the accumulative sum of borrowing, can be collected from settlement statistics of the central government. Since Japan's local fiscal measures, explained in Section 3, are taken once a year, we collect these data annually. We employ the data of local public investment from “Annual Report on National Accounts.” Public investment (gross capital formation) of local governments, denotes $k$, is used. Unfortunately, these data are available only since fiscal 1970. Hence the sample period in this paper is restricted by one of these data. Also since tax revenue is almost proportional to GDP, GDP is used as a proxy of $Y$ in this analysis. We use these variables deflated by these deflator.

According to the results of the augmented Dickey-Fuller unit root tests, the orders of integration of the four variables, $\ln L$, $\ln Y$, $\ln Z$, and $\ln k$, are all one. Hence, we can estimate the ordinal VAR model using first difference of these variables to implement the Granger causality test. We analyze a VAR model with the following
four endogenous variables: $\Delta \ln L$, $\Delta \ln Y$, $\Delta \ln Z$, and $\Delta \ln k$. Sample period is fiscal 1975-2004.

Before estimating the VAR equation, we consider simultaneous effects of these variables. In order to identify the VAR model, we implement block exogeneity tests of four variables. Table 1, the result of the exogeneity tests, suggests that ordering the variables as {$\Delta \ln k$, $\Delta \ln Z$, $\Delta \ln L$, $\Delta \ln Y$}.

We estimate the VAR equation, which lag length equals two determined by the Schwarz criterion. The result is reported in Table 2. We then implement the Granger causality tests based on the Wald statistics from the OLS estimators. The Wald statistics are reported in Table 3. Figure 2 summarizes the results of the Granger causality tests in Table 3.

According to these Granger causality tests, we observe that causality from $\Delta \ln k$ to $\Delta \ln L$ is strong. From results in 5% significant coefficients in Table 3, an increase in local expenditures leads to an increase borrowing of the Special Account for Allocation and Transfer Taxes in the next year.

Incidentally, we observe also the causality from $\Delta \ln Y$ to $\Delta \ln L$. A decline in GDP growth leads to an increase in the borrowing at the Special Account for Allocation and Transfer Taxes. It suggests that when a decline in GDP growth reduces tax revenues, the central government responds to maintain or increase the amount of LAT Grants by using borrowing for the grants in spite of a decrease in tax revenues. From the theoretical analysis of section 2 it is easy to show that a decrease in $Y_1$ would raise additional grants $A$ in Game II.

Namely, if we consider a change in $Y_1$ in Game II, we have

$$dG_2 + dg_2 + (1+r)(-\beta dY_1) = dY_1$$

$$(1-\eta)dG_2 = \eta dg_2$$

$$dG_2 = (1-\beta)dY_1 - dA$$

Hence,

$$J_{Y1} = \frac{\partial A}{\partial Y_1} = 1 - \beta - \eta[1 + (1+r)\beta]$$

which is likely negative. The above empirical finding is consistent with this analytical result.

4.2 Impulse Response

In addition we estimate impulse response functions based on the above VAR model. The estimated impulse responses to one standard deviation shock in the four variables VAR {$\Delta \ln k$, $\Delta \ln Z$, $\Delta \ln L$, $\Delta \ln Y$} are reported in Figure 3. The decomposition method is Cholesky's decomposition with degree of freedom correction. The decomposition ordering is $\Delta \ln k$, $\Delta \ln Z$, $\Delta \ln L$, and $\Delta \ln Y$. Figure 3
displays the impulse response of $\Delta \ln k$ to a one standard deviation shock in other variables in the VAR. A shock of an increase in local public investment leads to increases in borrowing, Local Allocation Tax Grants, but a decrease in GDP growth. In other words, an increase in local public investment depresses GDP growth.

The above results suggest that borrowing for LAT Grants, which became larger in response to excessive local public investment caused by the soft budget constraint of local governments, aggravated Japan's GDP growth. The soft budget constraint caused a lot of serious problems in Japan's economy.

5. Concluding Remarks

In this paper, we have investigated theoretically and empirically the soft-budget constraint with grants from the central government to the local government by clarifying the vertical externality of local expenditures due to overlapping tax bases between two governments using a two-period model. The central government's benevolent incentive results in creating a soft budget constraint by increasing grants when the local government spends and borrows more in period 1 because local borrowing more in period 1 means a decline in local public goods in period 2 and hence upsets the central government's optimal allocation strategy. We have shown the possibility of negative welfare effect of soft budget constraint.

If the hard budget constraint is too hard compared with the first best solution, the soft budget outcome would be better, and vice versa. Our analytical result means that either constraint could be better or worse, depending on the size of initial hard budget. While the hard budget constraint does not always attain the first best, the soft budget constraint may well deteriorate social welfare by inducing an additional increase in transfers from the central government to the local government. It is interesting to note that, as far as the outcome of public investment is concerned, the size of public investment becomes too high and hence the soft budget problem exaggerates local expenditures.

As to Japan's case, productivity of public investment at the local level declined much in the 1990s. Section 4 has empirically shown that Japan's local governments actually implemented inefficient public investments due to the soft budget constraint, compatible with the analytical results in section 2. Local governments implemented inefficient local expenditures much. The soft-budget constraint is one of the reasons why GDP growth slowed down in the decade.

In this paper we have not explicitly incorporated any political aspects to explain local governments' aggressive behavior. Doi and Ihori (2002)'s empirical evidence indicates that lobbying activities of local interest groups was exaggerated in the 1990s. Namely, an increase in local and/or national taxes resulted in an increase in subsidies of local interest groups. If the marginal benefit of local
expenditures rises, it induces a further increase in lobbying activities to seek for more privileges and larger deficits, while it reduces national-wide public goods. Such movements were actually observed in the 1990s when the Japanese economy suffered from a slow-down of economic growth. In short, under such a soft-budget constraint the financial resources needed by local governments are transferred from the central government to local governments in response to the demand of local governments.

Incorporating political aspects into the theoretical model of intergovernmental finance would be a very useful extension for Japan's case. Such an extension would reinforce the basic results of the present paper. Politically, more representatives in the ruling party, the Liberal Democratic Party (LDP) for postwar period, have been seated for the rural regions. People in the rural regions have more representatives in the ruling party than in the urban regions. The ruling party exerts an influence to decide the national budget. So the representatives for the rural regions, who are affected by local interest groups and voters, put political pressure to distribute more grants to the rural regions. Representatives of the Diet appeal to the cabinet or the central bureaucrats to distribute more in their own regions. A region where more representatives in the ruling party are elected for did obtain more subsidies from the central government throughout the period. Allocation of region-specific privileges in the form of subsidies or public works from the central government has been mainly determined by the political factor. See Ihori and Itaya (2003).

Finally, reforming the intergovernmental transfer system so that the central government can commit not to making additional grants is crucial for solving the bad outcome of soft budget problem. Also it is useful to attain the desirable overlapping tax share in such a way that the central and local governments may collect taxes to finance their first-best levels of spending.
Appendix: Multiple local governments

Suppose there are $n$ local governments. If we define the total amount of local public goods as $g_1, g_2$ and each local government’s supply of public goods as $g_1^i, g_2^i$, then we have

$$g_1 = \sum_{i=1}^{n} g_1^i, \quad g_2 = \sum_{i=1}^{n} g_2^i$$  \hspace{1cm} (A1)

The social welfare (1) is now rewritten as

$$W' = u(G_1) + v(g_1^i) + \delta[u(G_2) + v(g_2^i)]$$  \hspace{1cm} (A2)

where $W'$ is the social welfare in the representative agent in region $i$.

$$W = \sum_{i=1}^{n} W'^i$$

We may define other variables of local governments as in (A1). Then the budget constraints of CG and LG are the same as in the text. For simplicity suppose all local governments are identical. It follows that in the section of 2.2 the first best conditions are given by

$$v_{g_1} = nu_{G_1}$$  \hspace{1cm} (A3-1)

$$u_{g_2} = nv_{g_2}$$  \hspace{1cm} (A3-2)

and (5-3)(5-4). (A3-1) and (A3-2) correspond to the well-known Samuelson condition of the pure public good, $G$. We have analytically the same results as in sections 2.3 and 2.4.1.

Regarding the game between CG and LG, we may assume that each LG behaves non-cooperatively and regards other LG’s choice variables given. Then, the analytical results in section 2.4.2 are the same as in the text. For example, (13) may be rewritten as

$$G_2 + g_2^i + \sum_{j \neq i} g_2^j + (1 + r)(B + D^i + \sum_{j \neq i} D^j) = Y_2$$  \hspace{1cm} (A4)

Then, central government’s response functions are given as

$$A^i = J^i(g_1^i, k^i)$$  \hspace{1cm} (A5-1)

$$g_1^i = P^i(g_1^i, k^i)$$  \hspace{1cm} (A6-2)

Similarly, we have

$$dG_2 + dg_2^i + (1 + r)(dg_1^i + (1 - \hat{p})dk^i) = f'(k^i)dk^i$$

$$(1 - \eta)dG_2 = \eta dg_2^i$$

$$dG_2 = (1 - \beta)f'(k^i)dk^i - dA^i$$

Hence, we have (15-1,2,3,4) as in the text.
References


Table 1

Block Exogeneity Tests
Wald statistics

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The above parentheses indicate the p-values of the hypothesis: All coefficients with respect to other endogenous variables equal zero.
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<td></td>
<td>(0.830)</td>
<td>(1.419)</td>
<td>(-2.474)</td>
<td>(0.424)</td>
</tr>
<tr>
<td>( \Delta \ln Y_{t-2} )</td>
<td>1.032</td>
<td>-0.628</td>
<td>-0.888</td>
<td>-0.273</td>
</tr>
<tr>
<td></td>
<td>(1.200)</td>
<td>(-0.646)</td>
<td>(-0.169)</td>
<td>(-1.407)</td>
</tr>
<tr>
<td>intercept</td>
<td>0.062</td>
<td>0.039</td>
<td>2.246</td>
<td>0.071</td>
</tr>
<tr>
<td></td>
<td>(0.732)</td>
<td>(0.414)</td>
<td>(4.364)</td>
<td>(3.726)</td>
</tr>
<tr>
<td>( t )</td>
<td>-0.039</td>
<td>0.016</td>
<td>-0.330</td>
<td>-0.019</td>
</tr>
<tr>
<td></td>
<td>(-1.232)</td>
<td>(0.440)</td>
<td>(-1.689)</td>
<td>(-2.635)</td>
</tr>
<tr>
<td>( t^2 )</td>
<td>0.00447</td>
<td>-0.00416</td>
<td>0.01188</td>
<td>0.00291</td>
</tr>
<tr>
<td></td>
<td>(1.001)</td>
<td>(-0.826)</td>
<td>(0.436)</td>
<td>(2.885)</td>
</tr>
<tr>
<td>( t^3 )</td>
<td>-0.00018</td>
<td>0.00027</td>
<td>0.00023</td>
<td>-0.00016</td>
</tr>
<tr>
<td></td>
<td>(-0.770)</td>
<td>(1.050)</td>
<td>(0.167)</td>
<td>(-3.077)</td>
</tr>
</tbody>
</table>

**log of likelihood function**: 203.925  
**Schwarz criterion**: -7.700  
**std. err. of regression**: 0.056 0.063 0.343 0.013  
**adj. R\(^2\)**: 0.420 0.0084 0.595 0.591
Table 3

Granger causality tests
Wald statistics

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Independent variables</th>
<th>Δlnk</th>
<th>ΔlnZ</th>
<th>ΔlnL</th>
<th>ΔlnY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δlnk</td>
<td>0.370 (0.831)</td>
<td>1.170 (0.557)</td>
<td>2.027 (0.363)</td>
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<td></td>
</tr>
<tr>
<td>ΔlnZ</td>
<td>3.730 (0.155)</td>
<td>1.043 (0.594)</td>
<td>2.539 (0.281)</td>
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<td></td>
</tr>
<tr>
<td>ΔlnL</td>
<td>6.054 (0.048)</td>
<td>3.051 (0.218)</td>
<td>6.120 (0.047)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔlnY</td>
<td>2.199 (0.333)</td>
<td>5.226 (0.073)</td>
<td>11.136 (0.004)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The above parentheses indicate the p-values of the hypothesis: The independent variable does not Granger-cause the dependent variable.
Figure 1

Government Debt Outstanding in Japan

Government Debt to GDP Ratio
Figure 2

Granger causality tests

$\Delta \ln Y \leftrightarrow \Delta \ln L$

$\Delta \ln k \leftrightarrow \Delta \ln Z$

Legend: 1% significance
5% significance
10% significance
Figure 3
Accumulated Response of Each Variable

Accumulated Response of $dk$ to Cholesky
One S.D. Innovations

Accumulated Response of $dZ$ to Cholesky
One S.D. Innovations

Accumulated Response of $dL$ to Cholesky
One S.D. Innovations

Accumulated Response of $dY$ to Cholesky
One S.D. Innovations

Cholesky Ordering: $\Delta \ln k$, $\Delta \ln Z$, $\Delta \ln L$, $\Delta \ln Y$