CIRJE-F-269

A Theory of Commodity Tax Reform under Revenue Constraint

Tatsuo Hatta
The University of Tokyo

March 2004

A Theory of Commodity Tax Reform under Revenue Constraint

By

Tatsuo Hatta

Center for Spatial Information Science

And

Center for International Research on the Japanese Economy

University of Tokyo

e-mail: hatta@csis.u-tokyo.ac.jp

Abstract

Despite the development of the optimal taxation theory , few of the practicing tax economists question the traditional wisdom that making tax rates flat will lead to a more efficient tax system. Practicing tax economists seem to have an intuition that even if the uniform tax structure may not be the most efficient, it may be a close approximation. The present paper survey the literature that provides theoretical under-pinnings for the practitioner's intuition. Also, the paper simplifies the statements and proofs of theorems in literature.

□. Introduction

Most economists used to advocate uniform commodity taxation under the presumption that the elasticity of the labor supply is negligible. Indeed, Musgrave's celebrated textbook on public finance (Musgrave, 1959), for example, mentions Ramsey only once, and that in a footnote. Over the last thirty years, however, the impressive growth of the field of optimal taxation theory has reminded us that the uniform commodity tax is theoretically groundless in an economy with an elastic labour supply. Among theorists of public economics, the old doctrine of uniform commodity taxation has been replaced by the theory of optimal taxation.

But practising tax economists have been reluctant to accept the prescriptions of optimal taxation theory. The major post-war tax debates that led to the 1982 US tax reform or the 1989 Japanese reform illustrate this reluctance. In these debates, few questioned the traditional wisdom that making tax rates flat by eliminating deductions and exclusions will lead to a more efficient tax system. Indeed, "Level the playing field" was the slogan of the 1982 US tax reform. Practising tax economists seem to have an intuition that, even if the uniform tax structure is not absolutely efficient, it is sufficiently efficient for practical purposes in the real world. The problem is that they have been relying on this intuition rather than on analysis.

Hatta (1986), Hatta and Haltiwanger (1986), Fukushima and Hatta (1989), and Hatta (1993) attempted to provide the theoretical underpinnings for the practitioner's intuition in a model that is perfectly consistent with optimal tax theory. . They described practical routes through which tax reforms aimed at uniformity could be carried out, starting from an arbitrarily given tax structure.

However, each of the above papers focuses on different aspects of tax reforms. Moreover, results in this literature are stated in terms of ad valorem tax rates based on producers' prices.

The aim of the present paper is to survey this literature in a unified framework and to simplify considerably the statements and proofs of the theorems in the literature by stating them in terms of an ad valorem tax rate based on consumers' prices.

Section 2 reviews the issues and Section 3 outlines the model. Section 4 gives the basic formula of tax reforms showing their welfare impacts, and Section 5 discusses the Inverse Elasticity Rule in terms of the basic formula.

Section 6 presents the basic results in terms of this literature, Section 7 shows how to proceed beyond the initial stage of tax reform, and Section 8 discusses broad-based taxes within a group of close substitutes. Section 9 relates the empirical estimation of optimal tax rates to the analysis of the present paper, Section 10 discusses the equity issue, and Section 11 concludes.

2. Revenue-constrained optimum tax rules: the issues

Suppose that one of the goods consumed by a country is leisure, [1] and call goods other than leisure "commodities". Then the phrase "all commodities" will not include leisure, while the phrase "all goods" will cover all commodities and leisure. The excise tax structure with an equal tax rate on all goods is called a *proportional tax*, while that on all commodities is called a *uniform tax*. Thus, a proportional tax is a combination of a uniform tax and a tax on leisure at an equal rate.

Under a proportional tax, the budget plane of any consumer becomes parallel to the consumer's budget plane before taxes. This combination of excise taxes amounts to a lump-sum tax, since a tax-payer cannot then reduce the amount of his tax obligation by changing the consumption mix. A proportional tax, therefore, attains the optimum under the constraint of raising a given amount of government revenue. [2]

It is not feasible, however, to impose a proportional tax on all goods, because leisure is untaxable. A tax-payer has a strong incentive to understate his leisure consumption; by doing so he is able to reduce his claimed tax base for the leisure tax. Thus, the tax office cannot gather precise information on the leisure consumption of each tax-payer, and hence cannot levy taxes on leisure.

[1] Note that the word "leisure" here is used in the sense of non-working hours.
[2] A uniform tax encourages leisure consumption, but a tax on leisure consumption neutralizes that effect. Hence a proportional tax can attain an optimum.

The tax office, however, *can* obtain data on a tax-payer's wage income. In the real economy, therefore, taxes are imposed on wage income, rather than on leisure consumption. An important difference between the two taxes is that a tax on leisure discourages leisure consumption, while a tax on wage income encourages it. This is because the leisure tax is an excise tax on the total consumption of leisure, while the wage tax is an excise tax on the net supply of leisure, i.e. an excise tax on the net consumption of leisure with a negative rate.

Unlike a tax on leisure, a tax on wage income reinforces the over-consumption of leisure caused by commodity taxes. Therefore, a simultaneous imposition of taxes on all commodities and wage income cannot attain the optimum under the constraint of raising a given government revenue, because it necessarily enlarges the distortion in the leisure-commodity consumption choice. This distortion is ultimately created by the non-availability of a leisure tax.

However, the distortion can be partially mitigated by making the commodity tax structure non-uniform. For example, imposing a high tax rate on complements of leisure (e.g. yachts and concerts) and subsidizing substitutes for leisure (e.g. dish-washers and microwave ovens) will reduce leisure consumption, serving to counteract the over-consumption of leisure induced by the commodity taxes in general. But the type of non-uniformity of tax rates among commodities, which reduces the distortion in the leisure-commodity consumption choice, will create new distortions in the choice among commodities, as is eloquently stated by Musgrave and Musgrave (1980,p286): "A tax on Pintos can be avoided by buying some other car; a tax on cars in general can be avoided, if less conveniently, by taking buses or by flying; but a general sales tax can be avoided only by consuming less and saving." In making commodity tax rates non-uniform, therefore, a policy-maker faces a trade-off between distortion in the leisure-commodity consumption choice and distortions among commodities. The optimal tax structure is one that strikes a balance in this trade-off. Optimal tax theory characterizes the structure of such taxes.

In reality, it is not easy to estimate the optimal tax rate precisely, because that would require a precise estimate of demand parameters. The present paper shows how to approximate the optimal rate and how to reform an arbitrarily given tax structure towards such an optimal rate.

3. The model

Consider an economy with only one consumer, who consumes leisure and n commodities. The consumer is assumed to be a perfect competitor and his compensated demand function for leisure and commodities is expressed as $x = x(q,u)$, where u is his utility level, $x' = (x^0, x^1, \Lambda\ , x^n)$ is his net demand vector and $q' = (q^0, q^1, \Lambda\ q^n)$ is his price vector, with the superscript 0 denoting variables relating to leisure. Thus, his demand vector satisfies his budget constraint

$$q'x = 0, \qquad (1)$$

which may be rewritten as $q^1x^1 + \Lambda\ + q^nx^n = q^0(-x^0)$. The last equality states that his expenditure equals wage income, since $-x^0$ denotes net supply of his labour. We assume that $x^0 < 0$ and $x^j > 0$ for j = 1, 2,…, n. The first inequality simply states that the consumer is a net supplier of leisure.

The government provides a fixed vector $g = (g^0, g^1, \Lambda\ , g^n)$ of public goods, financing it by ad valorem excise taxes on net consumption. The excise tax on the net consumption of leisure amounts to a wage tax.

Technology can be represented by a linear production possibility frontier, and in equilibrium we have

$$p'x + p'g = 0, \qquad (2)$$

where $p' = (p^0, p^1, ..., p^n)$ and $p^0 = 1$ Then p^i (i=1,…, n.) represents the marginal cost of producing a commodity in terms of labour-hours used. For simplicity, we say that p is the *vector of marginal costs*.

Equation (2) may be rewritten as

$$p^1x^1 + \ldots + p^nx^n + p^0g^0 + p^1g^1 + \ldots + p^ng^n = -x^0$$

Then, the LHS represents the demand for labour and the RHS represents the supply for labour. Since producers are perfect competitors, p must be proportional to the vector of producers' prices in equilibrium. We choose the unit of measurement so that this factor of proportionality is 1.

Let t_i be the ad valorem tax rate of the i-th good relative to its consumer price q^i. Then we have

$$(1-t_i)q^i = p^i \quad \text{for i = 0, 1,…, n.} \tag{3}$$

The positive wage tax rate implies $t_0 < 0$, since the consumer receives less than what is paid by the producer. Noting that p is fixed in our model, we have

$$q = q(t),$$

where $t' = (t_0,\ t_1, \ldots,\ t_n)$ is the vector of tax rates. There are no other taxes in our model. In particular, there is no lump—sum tax.

We assume that the government spends all its tax revenue on the purchase of the public good. Hence $(q-p)x = p'g$. But this is already implied by (1) and (2).

Letting $r = p'g$ and substituting the compensated demand function for x in (1) and (2), we obtain

$$q'x(q,u) = 0 \quad \text{and} \quad p'x(q,u) + r = 0$$

Substituting (4) for q in these two equations, we have

$$m(q(t),u) = 0 \tag{5}$$

$$p'x(q(t),u) + r = 0 \tag{6}$$

where $m(q,u) \equiv q'x(q,u)$. These two equations contain $n+3$ variables, i.e. t, u and r. When $n+1$ of them are exogenously given, these equations can determine the equilibrium values of the remaining two variables. For example, when $t_0,...,t_{n-1}$ and r are given, the system of equations (5) and (6) can determine the equilibrium values of t_n and u.

4. The welfare effect of a revenue-neutral tax reform

4.1 Decomposition

To examine the welfare effect of a revenue-neutral commodity tax reform, we first analyse a formula that shows whether or not a particular revenue-neutral tax reform improves efficiency.

Throughout the rest of this section, we make the following assumption:

(A1) The tax rates of goods 1 and n are both revenue-increasing.

This means that raising the tax rate of commodity 1 (commodity n), keeping all other tax rates constant, increases the revenue.

<u>Lemma (Hatta, 1986)</u>: Under (A1), efficiency is improved by an increase in t_1 accompanied by a revenue-neutralizing decrease in t_n if and only if [3]

$$N^{1n} \square 0 \tag{7}$$

<u>where</u>

$$N^{1n} = \sum_{j=0}^{n} \eta_j^1 \cdot t_j - \sum_{j=0}^{n} \eta_j^n \cdot t_j \tag{8}$$

<u>and</u>

---

[3] Lemma 1 is obtained by examining the effect of an increase in t_1 upon u in (5) and (6) while treating u and t_n as the only endogeneous variables.In Lemma 1 of Hatta(1986),the excise tax rate t' was defined by $q^i = (1+t^i)p^i$,unlike t_i in (3) above.This explains the apparent difference between my definition of N^{la} here and that in Hatta (1986). Note that the following hold:
$q^i(1-t_i) = p_i; q^i = (1+t_i)p^i; (t^n - t^i)/(t^n+1) = (t_n - t_i)q^i / p^i$

$$\eta_j^i \equiv \frac{\partial x^i}{\partial q^j}\frac{q^j}{x^i}.$$

The RHS of (8) can be decomposed in a number of ways depending on the purpose of the decomposition. Noting that

for any i,

$$\sum_{j=0}^{n} \eta_j^i = 0 \tag{9}$$

equation (8) may be rewritten as[4]

$$N^{1n} = \left[(t_n - t_0)\eta_0^n - (t_1 - t_0)\eta_0^1\right] + \left(\sum_{j=1}^{n}(t_n - t_j)\eta_j^n + \sum_{j=1}^{n}(t_j - t_1)\eta_j^1\right). \tag{10}$$

The term η_0^1 in the RHS of (10) represents the wage elasticity of compensated demand for the first commodity, while η_1^n represents the elasticity of compensated demand for the n th commodity with respect to the price of the first commodity.

The first term of the RHS of (10) may be called the *leisure substitution term*, while the second term may be called the *commodity substitution term*. The welfare effect of the tax reform can be studied by signing the two terms.

4.2 Uniform tax as the initial condition.

In deriving policy implications from (7) and (10), first consider the situation where the initial commodity tax rates are equal. Then the second term in (10) vanishes, and the first term alone dictates the direction of the welfare effect. Since $t_1 = t_n$ holds in this situation, this reform improves efficiency if and only if

$$\eta_0^n - \eta_0^1 > 0. \tag{11}$$

This implies that a higher tax rate should be imposed on complements of leisure than on substitutes of leisure. This reform works because it discourages the consumption of leisure, mitigating the distortions created by the

[4] This is the counterpart of (31) in Hatta (1986).

non-availability of the leisure tax. This observation, due to Corlett and Hague (1953), crucially depends upon the assumption that the initial tax rate structure is uniform. In the two-commodity and leisure case that Corlett and Hague considered, however, this local examination was sufficient to enable them to conclude that a complement of leisure should be taxed at a higher rate than a substitute of leisure. More generally, the commodity that is less substitutable for leisure should be taxed at a higher rate, regardless of the magnitude of those substitability between the two commodities. In an economy with more than two commodities, however, local examination of a uniform commodity tax structure does not give useful qualitative information about the nature of the optimum commodity tax structure. Nor does it tell us how to proceed with a tax reform once we change the tax structure of an arbitrarily chosen commodity pair starting from uniformity.

In order to determining the desirable direction of a tax reform when a non-uniform initial tax structure is given, we have to depart from the assumption of Corlett and Hague (1953) that the initial tax rate structure is uniform.

5. How useful is the inverse elasticity rule?

It is useful to examine the implication of (10) for optimal tax rules, especially the Inverse Elasticity Rule, before discussing tax reforms.

5.1 Optimal tax structure

A tax vector t is called an *optimal tax structure* if it maximizes u in the model of (5) and (6) for a fixed r. Thus, an optimal tax structure is the solution for t of the following model:

$$\max_{t,u} \square @ u \square @$$

$$\text{subject to} \quad px(q(t),u)p + r = 0,$$

$$m(q(t),u) = 0$$

Optimal tax rules are obtained from the optimality condition of this problem. In view of Lemma 1, $N^{1n} = 0$ holds if the tax vector t is optimal.

5.2 The inverse wage elasticity rule

When cross elasticities among commodities are zero, i.e. when

$$\eta_i^j = o$$

if $i \neq j$ and if $i, j \geq 1$, (12)

the commodity substitution term in (10) vanishes, and hence the leisure substitution term in (10) must also vanish at the optimal tax structure, since □ then. Thus, the optimal tax rates for t_1 and t_n are attained at the levels where

$$(t_n - t_0)\eta_0^n = (t_1 - t_0)\eta_0^1$$

holds, regardless of the tax rates of other commodities.Define the *intrinsic tax rate* of the commodity *i* by $\tau_0^i = t_i - t_o$. Then the above equality may be rewritten as $\tau_0^n \eta_0^n = \tau_0^1 \eta_0^1$.

When the optimal tax structure is attained without keeping any commodity tax rate fixed, similar results hold for all commodity pairs, and we have

$$\tau_0^1 \eta_0^1 = \tau_0^2 \eta_0^2 = \ldots = \tau_0^n \eta_0^n \quad (13)$$

at the optimal tax structure. Thus the intrinsic tax rate of a commodity is inversely proportional to the wage elasticity of demand for that commodity at the optimum when (12) holds. This may be called the Inverse Wage Elasticity Rule, which is Proposition 4 of Hatta (1993).

5.3 Inverse elasticity rule

Among the optimal tax rules, the Inverse Elasticity Rule (in terms of own elasticities of demand) is the best known and has been applied widely in practice. I now show that the own elasticities of demand are not as important as is generally believed in ranking tax rates.

Under (12), it also holds that[5]

$$\eta_0^i = -\eta_i^i . \tag{14}$$

Thus, (13) may be rewritten as

$$\tau_1\eta_0^0 = \tau_2\eta_2^2 = \ldots = \tau_n\eta_n^n \qquad (15□$$

In words, the intrinsic commodity tax rate is inversely proportional to the own demand elasticity of that commodity. This is called the Inverse Elasticity Rule and is due to Ramsey (1927).[6] This implies that a lower tax rate should be imposed on a commodity with higher demand elasticity when condition (12) holds.

In the real world, a commodity with a high demand elasticity is usually a strong substitute for another commodity. But the Inverse Elasticity Rule only considers the situation where even a highly demand elastic commodity has no substitutes among commodities; such a commodity has a high demand elasticity only because it is closely substitutable for leisure. It is not easy to come up with an example of such a commodity. The Inverse Elasticity Rule, therefore, does not have the general applicability as would first appear.

5.4 Wage elasticity rule vs own elasticity rule

---

[5] This follows directly by applying (12) to $\eta_0^i + \eta_1^i + \ldots\ldots + \eta_n^i = 0$.

[6] This rule is usually derived for the case of t_0=0, where $\tau_0^i = t_i$ hold for all *i*. Ramsey himself derived this formula for uncompensated elasticities.

In Section 4.2, we noted that in the two commodity economy, the commodity with the higher wage elasticity will necessarily have the lower optimal tax rate, even when the two commodities are substitutable. Unless the two commodities are independent, however, the ranking of the optimal tax rates can be the opposite of what is implied by the Inverse Elasticity Rule: the commodity with the higher own demand elasticity can have the higher optimum tax rate.[7]

This may be seen most clearly by considering a tax reform in a economy with two-substitutable commodities when a uniform tax is imposed initially, i.e. when

$$t_1 = t_2 . \qquad (16)$$

We know from (10) that under this initial condition increasing t_1 and decreasing t_2 improves welfare if and only if

$$\eta_0^2 - \eta_0^1 > 0 . \qquad (17)$$

Inequality (17) may be rewritten in terms of own elasticities η_1^1 and η_2^2 as $(\eta_2^2 - \eta_1^1) > (\eta_2^1 - \eta_1^2)$. This holds even if $\eta_2^2 - \eta_1^1 < 0$, as long as $\eta_2^1 - \eta_1^2 < 0$. This implies that raising the tax rate of the commodity with the higher own elasticity can improve efficiency when the two commodities are not independent.

The ranking of optimal tax rates implied by the Inverse Wage Elasticity Rule is not dependent upon the assumption of zero cross elasticities among commodities. On the other hand, the ranking implication of the Inverse Elasticity Rule follows from that of the Inverse Wage Elasticity Rule and the fact that (14) holds under the assumption of zero cross elasticities among commodities. In this sense, the qualitative implication of the Inverse Wage Elasticity Rule is robust, unlike the qualitative implication of the Inverse Elasticity rule.

6. The first stage of tax reform toward uniformity

---

7 This follows from (9).

Optimal tax rules are obtained from the optimality conditions, which implicitly contain tax rates as variables. Optimal tax rules give insightful characterizations of optimal tax rates,[8] but they do not give guidance as to the direction in which the tax structure should be reformed, unless the exact point estimates of demand elasticities of all goods is known.

The theory of revenue-constrained tax reform, unlike the theory of optimal tax, gives explicit criteria about whether or not a particular tax reform can improve efficiency without requiring precise point estimates of the demand elasticities of all goods, but instead only the signs and the relative magnitudes of demand elasticities of a selected number of goods.

Let us now consider the situation where the initial tax structure is non-uniform, satisfying

$$t_1 < t_2 \le \Lambda \le t_{n-1} < t_n,.$$
$$t_0 < 0 \qquad (18)$$

Under (18), the commodity substitution term in (10) is positive if the following conditions are satisfied:[9]

(A2) (a) The first commodity is substitutable for all other commodities. (b) The n th commodity is substitutable for all other commodities.

In other words, the commodity substitution term is positive when commodities with extreme tax rates are substitutable for all other commodities.

Lemma and (10) immediately yield the following proposition.

---

[8] e.g. Sandmo (1976), Auerbach (1986), and Hatta (1993).

[9] Condition (a) implies that $t_1 < t_i$ and $x_i^1 > 0$ for $i = 2,\ldots,n,$ while condition (b) implies that $t_i < t_n$ and $x_i^n > 0$ for $i = 1,\ldots,n-1$.

Proposition 1 (Hatta, 1986, p.106): Consider the initial tax structure satisfying (18). Then efficiency is improved by an increase in t_1 accompanied by a simultaneous reduction in t_n so as to maintain the initial revenue level if

$$(t_n - t_0)\eta_0^n \geq (t_1 - t_0)\eta_0^1 \tag{19}$$

holds in addition to (A1) and (A2).

Under (18), inequality (19) is automatically satisfied if the n th good is more substitutable for leisure than the first good, i.e. if

$$\eta_0^n > \eta_0^1 . \tag{20}$$

Thus, the revenue-constrained squeezing of extreme commodity tax rates improves efficiency when (18), (A2) and (20) are satisfied. This may be viewed as a generalization of the Corlett-Hague (1953) result to the situation where the initial commodity tax structure is not uniform.

Even if (20) is violated, however, (19) can still be satisfied under (18). When $t_0 = -0.3$, $t_1 = 0$ and $t_n = 0.3$, for example, we have $(t_n - t_0)/(t_1 - t_0) = 1.50$; (19) is satisfied even when η_0^1 is greater than η_0^n by 50 %. When the initial commodity tax rates are divergent, condition (19) is hardly restrictive.

Besides, (19) is only a part of a set of sufficient conditions for efficiency improvement, and hence, even if it is violated, the squeezing of the tax rates can improve efficiency under condition (A2). This is because, even when the leisure substitution term is negative, the whole of expression (10) may be positive owing to the positivity of the commodity substitution term. Thus, when commodities are substitutable, the commodity substitution term *pulls* tax rates towards uniformity.

These observations imply the following.

(A4)Unless commodity 1 is strongly complementary with leisure or commodity n is strongly substitutable for leisure, the sign of the commodity substitution term can easily dominate the sign of the leisure substitution term.

Moreover, other things being equal, the value of the commodity substitution term becomes larger, and the "pulling power" stronger, as the gap between t_n and t_1 increases. When the initial commodity tax structure is divergent, therefore, a tax change towards uniformity is likely to improve welfare regardless of the relative magnitudes of η_0^1 and η_0^2, as long as commodities are substitutable.

Practitioners of tax policy usually recommend the equalization of tax rates so as to reduce distortions among commodities, without paying much attention to wage elasticities. The positivity of the commodity substitution term justifies their hunch.

Now note that condition (A2) can be generalized. For example, the first half of the commodity substitution term can be positive, i.e.

$$\sum_{j=1}^{n}(t_n - t_j)\eta_j^n > 0 \qquad (21)$$

even if $\eta_j^n < 0$ for some j. As long as this expression as a whole is positive, tax rate squeezing will improve efficiency.

Thus, we need a new definition of substitution between a commodity and a composite commodity. We say *good* i *is substitutable for the composite good* J *with* t_k *as its reference rate* if

$$\sum_{j \in J}\left|t_k - t_j\right|\eta_j^i > 0 \qquad (22)$$

holds.[10] Under this condition, the compensated demand for good i is increased when prices of all goods in J are increased in proportion to the distance between t_k and the tax rate of each good.

Proposition 1 holds even when (A2) is replaced by the following condition.

(A3)The first commodity is substitutable for the composite good consisting of all other commodities with t_1 as its reference rate, and the n -th commodity is substitutable for the composite good consisting of all other commodities with t_n as its reference rate.

This condition may hold even if good n has a strong complement. For example, suppose that the highest tax rate is imposed on wine, and that wine is a complement of wine glasses but is a substitute for all other commodities. Then (A3) is satisfied as long as wine is "on average" substitutable for other commodities, in the sense that wine is substitutable for the composite good consisting of all other commodities, with the tax rate of wine as its reference rate.

6. Next stages of tax reform towards uniformity

As the extreme tax rates are squeezed, (18) will eventually be violated; either $t_1 = t_2$ or $t_{n-1} = t_n$ will be reached. Suppose that the former takes place first. Then a joint increase in t_1 and t_2 will improve efficiency when conditions similar to those of Proposition 1 hold.

Proposition 2 (Fukushima and Hatta, 1989): Consider the initial tax structure satisfying

$$t_1 = \Lambda = t_k < t_{k+1} < \Lambda < t_m < t_{m+1} = \Lambda = t_n ,$$

$$\square @ t_0 < 0 . \qquad (23)$$

[10] See Figure 1 of Fukushima and Hatta(1989) for an illustration of this concept.

Then efficiency will be improved by an equal increase of $t_1,...,t_k$ accompanied by a simultaneous reduction of $t_{m+1},...,t_n$ so as to maintain the initial revenue level if: (i′) the tax rates to be changed are revenue increasing, (ii′) any good whose rate is to be changed is substitutable for the composite good consisting of all other goods, and (iii′)

$$(t_n - t_0)\eta_0^H > (t_1 - t_0)\eta_0^L$$

holds, where $\eta_0^L \equiv \Sigma_{i=1}^k c^i \eta_0^i$, $\eta_0^H \equiv \Sigma_{i=m+1}^n c^i \eta_0^i$, and $c_i = p^i x^i / \Sigma_{i=1}^k p^i x^i$.

When these conditions are satisfied, efficiency will be successively improved by applying this proposition at each step, thereby squeezing the extreme tax rate towards uniformity and bringing the tax structure closer to uniformity.

As the reform progresses and as more and more commodities share the extreme tax rates, the magnitudes of η^L and η^H are likely to become closer, for then the effects of idiosyncratically strong substitutes or complements of leisure upon the magnitudes of η^L and η^H will be diluted. In view of $t^n > t^1$, therefore, assumption (iii′) becomes more likely as the reform progresses.[11]

Thus, we can expect that, in an economy where substitutability dominates, a squeezing of the extreme tax rates towards uniformity is likely to improve efficiency monotonically.

As a result of such efficiency-improving tax reforms, uniformity will be eventually attained. Even then, however, there is still room for improvement from the uniform tax structure. Efficiency will be further improved by applying the Corlett & Hague rule to the commodity pairs that flagrantly violate Proposition2 (ii).

[11] Even when (iii) is not satisfied at a reform stage, the tax reform towards uniformity can proceed for most tax rates, leaving behind some other rates. See Fukushima and Hatta (1979).

8. Grouping of commodities

If a much higher tax rate is imposed on chicken than on fish, a consumer will be able to avoid paying a substantial amount of tax by substituting fish for chicken, regardless of the tax rates of the commodities other than this pair, because fish and chicken are close substitutes. An economist's intuition would be that a revenue-neutral reduction of the tax rate differential of this pair, leaving all other tax rates intact, would improve efficiency.

The problem, of course, is that reducing the distortion between the two goods in question will, in general, cause some increase in the distortions between the two goods and other goods. Thus, the increase in the tax rate of fish increases the distortion between fish and goods with tax rates on the latter being lower than the tax on fish. We need to define the circumstances under which the positive welfare effect of reducing the distortion between two close substitutes dominates the negative effects arising from the accompanied increase in distortions. Hatta and Haltiwanger (1986) established an empirically testable criterion of strong substitutability of a commodity pair by which the squeezing of their tax rates improves efficiency, when all other tax rates are kept constant.

Before establishing the criteria in Proposition 3, we need to introduce a few definitions.

Consider the initial tax structure satisfying

$$t_1 \leq t_2 \leq \Lambda \leq t_a \leq t_{a+1} \leq \Lambda \leq t_{b-1} \leq t_b \leq \Lambda \leq t_n \tag{24}$$

We say that *goods a and b are strong substitutes relative to the goods with tax rates lower than t^a* when following holds:

$$\sum_{j=0}^{a} (t_j - t_a)\eta_j^a + (t_b - t_a)\eta_b^a > 0\,. \tag{25}$$

When this inequality holds, the increase in the compensated demand for x^a caused by an increase in q^b is so large that it outweighs the countervailing effect caused by the reduction in all of the tax rates lower than t_a. We say that *goods a and b are strong substitutes relative to the goods with tax rates higher than t_a w*hen the following holds:

$$(t_b - t_a)\eta_a^b + \sum_{j=b+1}^{n} (t_b - t_j)\eta_j^b < 0 \tag{26}$$

When this inequality holds, the increase in the compensated demand for x^b caused by an increase in q^a is so large that it outweighs the countervailing effect caused by the reduction in all of the tax rates higher than t_b.

Finally, we say that *goods a and b are strong substitutes relative to the outliers* if (25) and (26) are both satisfied, i.e. if a and b are strong substitutes relative to the goods with lower tax rates than t_a as well as to the goods with higher tax rates than t_b.

We are now in a position to state and prove the following proposition.

<u>*Proposition 3.*</u> (Hatta and Haltiwanger, 1986): Suppose that the initial tax structure in the model of (□) and (6) satisfies (24).

Suppose also that goods a and b are commodities and that t_a and t_b are both revenue-increasing. Then efficiency is improved by an increase in t_a, accompanied by a revenue-offsetting reduction in t_b, if the following condition are satisfied:

(i) Good a is a substitute for all goods with a higher tax rate than t_a and good b is a substitute for all goods with a lower rate than t_b.

(ii) Goods a and b are strong substitutes relative to the outliers.

☐Proof. The theorem will be proved by establishing that the assumptions made imply inequality (7). As in (8), we have

$$N^{ab} = \sum_{j=0}^{n} \eta_j^a t_j + \left(-\sum_{j=0}^{n} \eta_j^b t_j\right). \tag{27}$$

In view of (9), the first term on the RHS is rewritten as

□

where the last inequality follows from the fact that the first term is positive from (ii) and the last is positive from (i) under (24). Similarly, the second term of (27) can be shown to be positive.☐

Q.E.D.

One of the best known results of optimal taxation theory is that the lowest tax rates should be imposed on the commodities with the highest price elasticities of demand. The underlying intuition is that this will minimize the tax-created deviation from the pre-tax, i.e., undistorted, allocation. The present analysis shows that, if the price elasticity of a commodity is extremely high because it has close substitutes, a high tax rate on this good way create little distortion in resource allocation, as long as a uniform tax rate is imposed on the good and its close substitutes. This also implies that the optimal tax rate for such a good may be quite high.

This suggests a two-step implementation of tax reforms towards uniformity, starting at an initially arbitrarily divergent tax system.

First, bundle together close substitutes and make the tax rates within each bundle uniform, while leaving the tax rates across different bundles different. For example, equate the tax rate for all food items, while also equating the tax rates for all clothing items. Each bundle may be called a *compound commodity.*

Second, apply the extreme tax rate squeezing rule, treating each compound commodity as if it were a real commodity. Compound commodities are likely to be substitutable for each other. For example, food as a whole is substitutable for clothing. Thus, the sufficient conditions for efficiency improvement are likely to be satisfied at each successive squeezing of the extreme tax rates.

Substitutability among composite commodities is generally weak, and the so-called inverse elasticity rule becomes useful when it is applied to composites. Hence it is conceivable to apply the Inverse Elasticity Rule after the first step above. Then we make the tax rates of the bundles inversely proportional to their demand elasticities.

However, there may not be a composite with high demand elasticity; the demand elasticities may not be dramatically different across composites. Then we will have the situation where one tax rate is imposed on all the composite commodities and another analogous rate is imposed on the composite factors – a situation similar to uniform commodity taxation.

The second step above then becomes useful in guiding us towards a uniform tax structure.

9. Empirical estimation of optimal tax rates

The literature of optimal taxation emphasized the non-uniformity of optimal commodity tax rates. The celebrated example is Atkinson and Stiglitz (1972) ,who numerically derived optimal tax rates from the parameter estimates of direct addilog demand functions given by Houthakker (1960). The last column of Table 1 shows their estimates of optimal tax rates for Canada, as an example. This demonstrates that optimal tax rates are quite divergent.

However, Fukushima and Hatta (1980) showed that Atkinson and Stiglitz's estimate of the divergent optimal tax rates is due to latter's assumption of unrealistically high elasticity of labor supply. When the elasticity of labor supply is high, wage elasticities of commodities can be widely divergent. And hence optimal tax rate can be divergent, according to the theoretical analysis of the present paper.

Fukushima and Hatta (1989) established that when the elasticity of labor supply is changed to realistic values in the Atkinson-Stiglitz model, the optimal tax rates become much closer to uniformity. Table 1 reproduces their estimates of the optimal tax rates derived from the same estimates of the demand parameters, but under various assumed values of wage elasticities of compensated labor supply that Atkinson and Stiglitz used. The last two rows of the table show the gain obtained by adopting the uniform rather than the optimal tax structure of the same revenue as a percentage of the government revenue. The size of the gain varies greatly with the assumed level of labor supply elasticity. As the table shows, the gain is 135.9% of government revenue when the compensated wage elasticity of labor is assumed to be 3.35. It shows that if the compensated wage elasticity of labor supply is 0.5 or less, the efficiency cost of adopting a uniform tax structure rather than the optimal one is less than 5% of the government revenue. Thus the gains obtained from the tax rates optimal rather than uniform will be relatively small when a realistic value of elasticity of labor supply is assumed. [12]Asano and Fukushima (2001) also empirically shows that the optimal tax structure is close to uniformity using Japanese data.

[12] See Gruber and Saez (2002) for recent estimates of the elasticity of taxable income.

X : Equity and Commodity Taxation

In the present paper, we did not take equity considerations into account when discussing commodity taxation reform. There are three major reasons for this.

First, in order to attain equity through taxation, far more powerful instruments than commodity taxation are available, such as progressive income taxes, land taxes, or inheritance taxes. If we delegate the attainment of equity to such taxes, then the role of commodity taxation is to restore neutrality violated by the absence of taxation of leisure consumption itself.

Second, when leisure is separable in the utility function and ability is the only source of income differences, the optimum combination of a non-linear income tax and linear commodity taxes results in a uniform commodity tax rates, as Atkinson and Stiglitz (1976) and Deaton (1981) have shown. Under separability assumption, optimal commodity tax structure in a single consumer economy is uniform. Hence their results show that redistributional concern does not affect the optimal tax structure when an additional policy instrument of non-linear income tax is available

Third, when tax reform is viewed as part of the entire set of microeconomic policies of a government (including anti-trust policy, trade liberalization, and cost-benefit analysis), it should be carried out independently of short run equity considerations as long as each reform satisfies Samuelson's (1950) criterion of efficiency improvement. An efficiency improving policy may reduce social welfare due to income distribution effects, but a series of many efficiency improving reforms will have offsetting income distribution effects, and it is likely to bring about an improvement in social welfare in the long run. On the contrary, requiring each policy to monotonically improve will severely social welfare limit policy options and may prevent maximization of long run social welfare.[13]

If the cumulative distributional effects of efficiency improving reforms are clearly skewed, the government should revise the progressivity of the income tax rather than abandon the efficiency improving

[13] See Hatta (1986, fn. 16) for the remarks by Corden (1981) and Hicks (1941) on this.

policies altogether.

There are also two technical reasons for not considering equity in the present paper. First, empirical evidence suggests that the redistribution capacity of commodity taxation is extremely limited (Sah, 1983 and Ray, 1986). Second, estimated egalitarian optimal tax rates are sensitive to the specification of the functional forms of the utility function (Ray, 1986), and economists have only limited ability to identify the functional form of the utility function.

XI. Conclusion

Distortions that induce over-consumption of leisure are inevitable under any combination of taxes on wage and commodities. The theory of optimum commodity taxation states that the efficiency-seeking commodity tax rates should be differentiated so as to offset the stimulating effect of wage taxation upon labor supply. But the differentiated commodity tax rates create new non-neutrality among the commodities themselves. This implies that when commodity tax rates are excessively differentiated, the welfare loss from increased distortion among these commodities would more than offset the welfare gain from the reduced distortion between leisure and other commodities. Fukushima and Hatta's numerical computations demonstrated that only a small welfare loss results from adopting an optimal commodity taxation structure rather than a uniform one with an equal revenue yield when realistic values of labor supply elasticity are assumed.

Following Fukushima and Hatta (1989), the rough and ready policy implication of our theoretical and computational analysis may be summarized as follows: "Differentiation of tax rates among *broad categories of goods* like clothing and housing is ineffective in reducing non-neutrality between commodities and leisure because their cross-elasticities with leisure simply cannot be sufficiently divergent under realistic values of the wage elasticity of labor supply. In finer categories of commodities, there are a small number of obvious complements (e.g., summer homes, yachts, and golf equipment) and substitutes (e.g., washing machines, vacuum cleaners, and microwave ovens) of leisure. Their tax rates should be differentiated a la Corlett and Hague. For the vast majority of commodities that are not particularly strongly related to leisure, however, their tax rates should be made uniform."

References

Asano, S. and T. Fukushima (2001) "Some Empirical Evidence on Demand System and Optimal Commodity Taxation", Discussion Paper 94-01 (revised), Faculty of Economics, Osaka University.

Atkinson, A. B. and J. E. Stiglitz (1972) "The Structure of Indirect Taxation and Economic Efficiency," *Journal of Public Economics,* Vol. 1, pp. 97-119.

_____ (1976) "The Design of Tax Structure: Direct and Indirect Taxation", *Journal of Public Economics*, Vol. 6, pp.55-75.

Auerbach, A. J. (1986) "The Theory of Excess Burden and Optimal Taxation", in A.J. Auerbach and M. Feldstein (eds), *Handbook of International Economics*, Vol. 1, North Holland.

Corden, W. M. (1981) "The Normative Theory of International Trade", in R. W. Jones and P.B. Kenen (eds.), *Handbook of International Economics*, Vol. I, Amsterdam: North-Holland.

Corlett, W. C. and D. C. Hague (1953) "Complementarity and the Excess Burden of Taxation," *Review of Economic Studies,* Vol. 21, pp. 21-30.

Dahl, H., S. Devarajan and S. van Wijinbergen (1994) "Revenue-Neutral Tariff Reform: Theory and an Application to Cameroon," *Economic Studies Quarterly,* Vol. 45, No. 3, pp. 213-226.

Deaton, A. S. (1984) "Optimal Taxes and the Structure of Preferences", *Econometrica,* Vol. 49, pp. 1245-1260.

Fukushima, T. and T. Hatta (1989) "Why Not Tax Uniformly rather than Optimally?" *Economic Studies Quarterly,* Vol. 40, pp. 220-248.

Gruber, J. and E. Saez (2002) "The Elasticity of Taxable Income: Evidence and Implications", *Journal of Public Economics*, Vol. 84, pp. 1-32.

Hatta, T. (1986) "Welfare Effect of Changing Commodity Tax Rates toward Uniformity," *Journal of Public Economics,* Vol. 29, pp. 99-112.

_____ (1993) "Four Basic Rules of Optimal Commodity Taxation," in Ali M. El-Agraaed (ed.), *Public and International Economics,* New York: St. Martins Press, pp. 125-147.

_____ and J. Haltiwanger (1986) "Tax Reform and Strong Substitutes," *International Economic Review,* Vol. 27, pp. 303-315.

_____ and Y. Ogawa (2003) "A Theory of Optimal Tariffs under a Revenue Constraint", CIRJE Discussion Paper, No. F202, Department of Economics, University of Tokyo.

Hicks, J. (1941) "The Rehabilitaion of Consumer's Surplus", *Review of Economics Studies*, Vol. 9, pp. 108-116; reprinted in K. J. Arrow and T. Scitovsky (eds.), *Readings in Welfare Economics*, Homewood, Ill.: Richard Irwin, 1969.

Houthakker, M. R. (1960) "Additive Preference", *Econometrica*, Vol. 28, pp. 244-257.

Mirrlees, J. A. (1987) "Optimal Taxation," *Handbook of Mathematical Economics,* Vol. 2, Amsterdam: North-Holland.

Mitra, P. K. (1994) "Protective and Revenue Raising Trade Taxes: Theory and an Application to India," *Economic Studies Quarterly,* Vol. 45, No.3, pp. 265-287.

Musgrave, R. A. (1959) *The Theory of Public Finance*, New York: McGraw-Hill.

____ and P. Musgrave, (1980) *Public Finance in Theory and Practice*, 3rd edition, New York: McGraw-Hill.

Ramsey, F. (1927) "A Contribution to the Theory of Taxation," *Economic Journal,* Vol.37, pp. 47-61.

Ray, R. (1986) "Sensitivity of 'Optimal' Commodity Tax Rates to Alternative Demand Functional Form: An Econometric Case Study of India", *Journal of Public Economics*, Vol. 31, pp. 253-268.

Sah, R. K. (1983) "How Much Redistribution is Possible through Commodity Taxes?" *Journal of Public Economics*, Vol. 20, pp. 89-101.

Sandmo, A. (1976) "Optimal Taxation – An Introduction to the Literature," *Journal of Public Economics,* Vol. 6, pp. 37-54.

Acknowledgement

I would like to express my sincere thanks to Professor Yoshitomo Ogawa of Kinki University for his many useful comments.