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Repeated Games with Private Monitoring: Two Players

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Abstract

We investigate two-player infinitely repeated games where the discount factor is less than but close to unity. Monitoring is private and players cannot communicate. We require no condition concerning the accuracy of players' monitoring technology. We show the folk theorem for the prisoners' dilemma with conditional independence. We also investigate more general games where players' private signals are correlated only through an unobservable macro shock. We show that efficiency is sustainable for generic private signal structures when the size of the set of private signals is sufficiently large. Finally, we show that cartel collusion is sustainable in price-setting duopoly.

KEYWORDS: Discounted Repeated Games, Private Monitoring, Review Strategies, Folk Theorem, Macro Shock, Efficiency, Price-Setting Duopoly.

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[1] The earlier versions of Section 3 correspond to Matsushima (2000, 2001a), and the earlier version of Sections 4 and 5 corresponds to Matsushima (2002). I am grateful to Professor Dilip Abreu for encouraging me to complete the work. I thank the co-editor, anonymous referees, Mr. Satoru Takahashi, and Mr. Parag A. Pathak for their comments. I thank Mr. Daisuke Shimizu much for his careful reading. All errors are mine. The research for this paper was supported by Grant-In-Aid for Scientific Research (KAKENHI 15330036) from JSPS and MEXT of the Japanese government.

1. Introduction

We investigate two-player infinitely repeated games where the discount factor is less than but close to unity. Players can only imperfectly and *privately* monitor their opponents' actions. That is, players can only observe their noisy private signals that are drawn according to a probability function conditional on the action profile played. There exist no public signals and no public randomization devices, and players cannot communicate. The purpose of the paper is to clarify the possibility that players can make self-enforcing collusive agreements even if their private monitoring is far from perfect.

First, we assume that players' private signals are conditionally independent in the sense that players can obtain *no* information on what their opponents have observed by observing their own private signals when they choose a pure action profile. We show the folk theorem for the prisoners' dilemma in that every individually rational payoff vector can be sustained by a sequential equilibrium in the limit as the discount factor approaches unity. This result is permissive because we require *no* conditions regarding the accuracy of players' private signals.

The study of repeated games with private monitoring is relatively new. Many of the past works have assumed that monitoring is either perfect or public and have investigated only perfect public equilibria. See Pearce (1992) for the survey on repeated games with public monitoring, and see Kandori (2002) for a brief survey on repeated games with private monitoring. It is well known that under mild conditions, in the limit as the discount factor approaches unity, every individually rational payoff vector can be sustained by a perfect public equilibrium when monitoring is imperfect but public. See Fudenberg, Levine, and Maskin (1994), for example. Perfect public equilibrium requires that the past histories relevant to future play are common knowledge in every period. This common knowledge attribute makes equilibrium analyses tractable because players' future play can always be described as a Nash equilibrium.

As monitoring is *not* public in this paper, the problem is more delicate. When monitoring is private, it is inevitable that an equilibrium sustaining implicit collusion depends on players' private histories, and therefore, the past histories relevant to future play are not common knowledge. This makes equilibrium analyses much more difficult, especially in the discounting case, because players' future play may not be described as a Nash equilibrium. Even when a player is certain that a particular opponent has deviated, the other players will typically not share this certainty, and they will be unable to coordinate on an equilibrium that punishes the deviant in the continuation game. Hence, each player's anticipation on which strategies the other players will play may depend on her private history in more complicated ways. Nevertheless, a careful argument establishes the folk theorem for the prisoners' dilemma. Hence, we have the folk theorem with completely public signals on the one hand, and we have the folk theorem even with completely private signals on the other hand.

To the best of our knowledge, Radner (1986) is the first to examine repeated games with private monitoring, which, however, assumed no discounting.[2] Two papers by Matsushima (1990a, 1990b) appear to be the first to investigate the discounting case. Matsushima (1990a) proved an anti-folk theorem that it is impossible to sustain implicit collusion by Nash equilibria when private signals are conditionally independent and Nash equilibria are restricted to be independent of payoff-irrelevant private histories. The present paper establishes the converse result: the folk theorem *holds* when we use Nash equilibria that can depend on payoff-irrelevant private histories.

Matsushima (1990b) conjectured that a folk theorem type result could be obtained even with private monitoring and discounting when players can communicate by making publicly observable announcements. Subsequently, Kandori and Matsushima (1998) and Compte (1998) proved that the folk theorem with communication is valid. Communication synthetically generates public signals. Consequently, it is possible to conduct the standard dynamic analysis in terms of perfect public equilibria. The present paper assumes that players make *no* publicly observable announcements.

Interest in repeated games with private monitoring and no communication has been stimulated by a number of more recent papers, including Sekiguchi (1997), Bhaskar (1999), Ely and Välimäki (2002), and Piccione (2002). Sekiguchi (1997) investigated a restricted class of prisoners' dilemma based on the assumption that monitoring is almost perfect and that players' private signals are conditionally independent. Sekiguchi is the first to show that a Nash equilibrium payoff vector can approximate an efficient payoff vector even if players cannot communicate. By using public randomization devices, Bhaskar and Obara (2002) extended Sekiguchi's result to more general games.

Ely and Välimäki (2002) also considered repeated prisoners' dilemma where the discount factor is less than but close to unity. Ely and Välimäki investigated only a restricted class of two-state Markov equilibria that satisfy *interchangeability* in the sense that each player is indifferent between the collusive action and the defective action irrespective of her opponent's possible future strategy, and therefore, all combinations of players' possible continuation strategies are Nash equilibria.[3] This restriction drastically simplifies equilibrium analysis. Hence, Ely and Välimäki could show that the folk Theorem holds for the repeated prisoners' dilemma even if monitoring is private. Piccione (2002) independently introduced a similar idea by using dynamic programming techniques over infinite state spaces. The technical aspects of the present paper are closely related to Ely and Välimäki (2002), and to a lesser extent Piccione (2002). Both papers, however, investigated only the almost-perfect monitoring case, and most of their arguments rely heavily on this assumption.

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[2] See also Lehrer (1989) for the study of repeated games with no discounting and private monitoring. Fudenberg and Levine (1991) investigated infinitely repeated games with discounting and private monitoring in terms of epsilon-equilibria.

[3] A related idea is found in Piccione (2002). See also Obara (1999) and Kandori and Obara (2000).

Mailath and Morris (2002) investigated the robustness of perfect public equilibria when monitoring is almost public, i.e., each player can always discern accurately which private signal her opponent has observed by observing her own private signal. The present paper does *not* assume that monitoring is almost public.

Therefore, this paper has many substantial points of departure from the earlier literature. We assume that there are no public signals, players make no publicly observed announcements, and no public randomization devices exist. We do not require that monitoring is either almost perfect or almost public. Hence, the present paper can be regarded as one of the first works to provide affirmative answers to the possibility of implicit collusion with discounting when monitoring is truly imperfect, truly private, and truly conditionally independent.[4]

We construct *review strategies*, which are originated in Radner (1985) and cultivated by Matsushima (2001b), as follows. The infinite time horizon is divided into finite period intervals named review phases. In each review phase, each player counts the number of periods in which a particular event occurs. If the resultant number of the event occurring during the review phase is larger than a threshold level, the player will be likely to punish the opponent in the next review phase. Here, the player chooses the event as being ‘bad’ in that the probability of its occurrence is the smallest when the opponent plays collusively. According to the law of large numbers, a review strategy profile approximately induces the efficient payoff vector.

In order to simplify the equilibrium analysis, we require a review strategy profile to satisfy a weaker version of interchangeability à la Ely and Välimäki (2002): Each player is indifferent between the repeated collusive choices and the repeated defective choices during each review phase. Moreover, by choosing the threshold level to be as severe as possible, we can strengthen each player’s incentive not to deviate. The latter idea is basically originated in Matsushima (2001b).

Conditional independence can simplify the way to check whether the review strategy profiles are sequential equilibria. All we have to do is to show that there exists no strategy preferred to the review strategy that does not depend on private signal histories during the first review phase. Without conditional independence the problem may be more complicated, because there may exist a strategy preferred to the review strategy that does depend on private signal histories. When the private signal history observed by a player in the middle of the review phase implies that with high probability the opponent has already received many bad events and recognized that the review has failed, the player will have no incentive to choose collusively in the remainder of the review phase. Conditional independence, together with the pure action plays during the review phase, guarantees each player’s private signal to have no information about whether the opponent observed the bad event, and therefore, have no information about which strategy she will play from the next period. This *uninformative* property will be the driving force to incentivize players to play the review strategies.

[4] See Piccione (2002) and Ely and Välimäki (2000) for the discussion on the case that private monitoring is not almost perfect.

In many real economic situations such as price-setting oligopoly, however, it is not appropriate to assume conditional independence, because there may exist unobservable macro shocks through which players' private signals are correlated. Based on this observation, the latter part of this paper will show a sufficient condition under which players' private signals satisfy the uninformative property and the above review strategy construction works even if their private signals are not conditionally independent but imperfectly correlated. We assume that players' private signals are correlated *only* through an unobservable macro shock. We show that whenever the size of the set of private signals for each player is large relatively to the sizes of the set of macro shocks and the set of actions for the opponent, then efficiency is sustainable for *generic* private signal structures.

This efficiency result would provide the study of cartel oligopoly with the following substantial impact. The classical work by Stigler (1964) has pointed out that each firm may have the option of making *secret price cuts* in that it offers to consumers a sales price that is lower than the cartel price in secret from the other firms. Stigler then emphasized that secret price cuts would be the main course of preventing the firms' cartel agreement from being self-enforcing. Since Stigler did not provide a systematic analysis, we should carefully check to what extent his arguments was correct by making an appropriate model, which would be a discounted repeated game with correlated private monitoring. In contrast to Stigler's argument, our efficiency result implies that the full cartel agreement can be self-enforcing even if firms have the option of making secret price cuts.

The present paper will not investigate the three or more player case. Whether the ideas of equilibrium construction in this paper such as review strategies and interchangeability can be applied to the three or more player case should be carefully studied in future researches.

The organization of this paper is as follows. Section 2 shows the basic model. Section 3 shows the folk theorem for the prisoners' dilemma with conditional independence. Section 4 shows that efficiency is generically sustainable when the size of the set of macro shocks is sufficiently large. Section 5 applies this result to price setting duopoly. In Subsection 5.3, we will discuss the implications of this application in relation to the literature of price-maintenance by cartels.

2. The Model

A two-player infinitely repeated game with discounting is denoted by $\Gamma(\delta) = ((A_i, u_i, \Omega_i)_{i\in\{1,2\}}, \delta, p)$, where $\delta \in [0,1)$ is the common discount factor. In every period $t \geq 1$, players 1 and 2 play the component game defined by $(A_i, u_i)_{i\in\{1,2\}}$, where A_i is the finite set of actions for player $i \in \{1,2\}$, $A \equiv A_1 \times A_2$, and player $i's$ payoff function is given by $u_i : A \to R$, which satisfies the expected utility hypothesis. Let $a = (a_1, a_2)$ and $u(a) = (u_1(a), u_2(a)) \in R^2$. The set of *feasible* payoff vectors $V \subset R^2$ is defined as the convex hull of the set $\{v = (v_1, v_2) \in R^2 \mid u(a) = v \text{ for some } a \in A\}$. A payoff vector v is said to be *efficient* if $v \in V$ and there exists no $v' \in V / \{v\}$ such that $v' \geq v$. A mixed action for player i is denoted by $\alpha_i : A_i \to [0,1]$, where $\sum_{a_i \in A_i} \alpha_i(a_i) = 1$. Let Δ_i denote the set of mixed actions for player i. We denote $j = 1$ ($j = 2$) when $i = 2$ ($i = 1$, respectively). We define the *minimax point* $v^* \in V$ by

$$v_i^* = \min_{\alpha_j \in \Delta_j} [\max_{a_i \in A_i} \sum_{a_j \in A_j} u_i(a) \alpha_j(a_j)] \text{ for both } i \in \{1,2\}.$$

A payoff vector $v \in R^2$ is said to be *individually rational* if $v \in V$ and $v \geq v^*$. Let $V^* \subset V$ denote the set of individually rational payoff vectors.

At the end of every period, player i observes her *private* signal ω_i. The opponent $j \neq i$ cannot observe player $i's$ private signal ω_i. The finite set of private signals for player i is denoted by Ω_i. Let $\Omega \equiv \Omega_1 \times \Omega_2$ denote the set of private signal profiles. A signal profile $\omega \equiv (\omega_1, \omega_2) \in \Omega$ occurs with probability $p(\omega|a)$ when players choose the action profile $a \in A$. Let $p \equiv (p(\cdot \mid a))_{a\in A}$. Let (Ω, p) denote *a private signal structure*. A private signal structure associated with Ω is denoted by p instead of (Ω, p). Let $P = P(\Omega)$ denote the set of private signal structures associated with Ω. For convenience, we assume that the private signal structure $p \in P$ has *full support* in the sense that

$$p(\omega \mid a) > 0 \text{ for all } (a, \omega) \in A \times \Omega.$$

Let $p_i(\omega_i \mid a) \equiv \sum_{\omega_j \in \Omega_j} p(\omega \mid a)$. We regard $u_i(a)$ as the expected value defined as

$$u_i(a) = \sum_{\omega_i \in \Omega_i} \pi_i(\omega_i, a_i) p_i(\omega_i \mid a),$$

where $\pi_i(\omega_i, a_i)$ is the realized payoff for player i in the component game when player i chooses a_i and observes ω_i. Note that for every $a \in A$, each $i \in \{1,2\}$, and every $a_j' \in A_j$,

$$p_i(\cdot \mid a) \neq p_i(\cdot \mid a_i, a_j') \text{ if } u_i(a) \neq u_i(a_i, a_j').$$

A private signal structure $p \in P$ is said to be *conditionally independent* if

$$p(\omega \mid a) = p_1(\omega_1 \mid a) p_2(\omega_2 \mid a) \text{ for all } a \in A \text{ and all } \omega \in \Omega.$$

Conditional independence implies that for every $i \in N$ and every $\omega_i \in \Omega_i$, the opponent $j's$ private signal has *no* information about whether player i observed ω_i when players choose pure actions, i.e., for every $a \in A$, and every $\omega_j \in \Omega_j$,

$$p_i(\omega_i \mid a, \omega_j) = p_i(\omega_i \mid a),$$

where $p_i(\omega_i \mid a, \omega_j) = \dfrac{p(\omega \mid a)}{\sum_{\omega_i' \in \Omega_i} p(\omega_i', \omega_j \mid a)}$ is the probability of ω_i occurring conditional on (a, ω_j).

A *private history* for player i up to period t is denoted by $h_i^t \equiv (a_i(\tau), \omega_i(\tau))_{\tau=1}^t$, where $a_i(\tau) \in A_i$ is the action for player i in period τ, and $\omega_i(\tau) \in \Omega_i$ is the private signal for player i in period τ. The null history is denoted by h_i^0. The set of private histories for player i up to period t is denoted by H_i^t. A *strategy for player* i is defined as a function $s_i : \bigcup_{t=0}^{\infty} H_i^t \to \Delta_i$.[5] Player i chooses the action a_i with probability $s_i(h_i^{t-1})(a_i)$ in period t when h_i^{t-1} is realized. The *continuation* strategy of s_i, which player i plays after period t when h_i^{t-1} is realized, is denoted by $s_i |_{h_i^{t-1}}$. Let S_i denote the set of strategies for player i. Let $S \equiv S_1 \times S_2$. Player $i's$ normalized long-run payoff induced by a strategy profile $s \in S$ after period t when h_i^{t-1} is realized is given by

$$v_i(\delta, s, h_i^{t-1}) \equiv (1-\delta) E[\sum_{\tau=1}^{\infty} \delta^{\tau-1} u_i(a(\tau + t - 1)) \mid s, h_i^{t-1}],$$

where $E[\cdot \mid s, h_i^{t-1}]$ implies the expectation conditional on (s, h_i^{t-1}). Player $i's$ normalized long-run payoff induced by $s \in S$ is denoted by

$$v_i(\delta, s) \equiv v_i(\delta, s, h_i^0).$$

Let $v(\delta, s) \equiv (v_1(\delta, s), v_2(\delta, s))$.

A strategy profile $s \in S$ is said to be a *Nash equilibrium* in $\Gamma(\delta)$ if for each $i \in \{1,2\}$, and every $s_i' \in S_i$,

$$v_i(\delta, s) \geq v_i(\delta, s_i', s_j).$$

A strategy profile $s \in S$ is said to be a *sequential equilibrium* in $\Gamma(\delta)$ if for each $i \in \{1,2\}$, every $s_i' \in S_i$, every $t \geq 1$, and every $h_i^{t-1} \in H_i^{t-1}$,

[5] The earlier version of this paper, i.e., Matsushima (2000, 2001a), assumed that the set of private signal profiles is the continuum and showed the folk theorem for the prisoners' dilemma by using only pure strategies. The purification is possible for all parts of this paper when we replace the finite set of signal profiles with the continuum.

$$v_i(\delta, s, h_i^{t-1}) \geq v_i(\delta, s_i', s_j, h_i^{t-1}) .$$

Since the private signal structure has full support, it follows that the set of Nash equilibrium payoff vectors equals the set of sequential equilibrium payoff vectors.[6]

A payoff vector $v = (v_1, v_2) \in R^2$ is said to be *sustainable* if for every $\varepsilon > 0$ and every infinite sequence of discount factors $(\delta^m)_{m=1}^{\infty}$ satisfying $\lim_{m \to +\infty} \delta^m = 1$, there exists an infinite sequence of strategy profiles $(s^m)_{m=1}^{\infty}$ such that for every large enough m, s^m is a sequential equilibrium in $\Gamma(\delta^m)$ and

$$v_i - \varepsilon \leq v_i(\delta^m, s^m) \leq v_i + \varepsilon \text{ for both } i \in \{1,2\} .$$

Hence, v is sustainable if a sequential equilibrium payoff vector approximates it whenever players are sufficiently patient. Since the set of Nash equilibrium payoff vectors equals the set of sequential equilibrium payoff vectors, it follows that v is sustainable if a Nash equilibrium payoff vector approximates it whenever players are sufficiently patient. Note that the set of sustainable payoff vectors is compact and convex.

[6] With full support, by replacing the continuation strategy of any subgame off the equilibrium path with the best response, we can transform any Nash equilibrium into a sequential equilibrium.

3. Conditional Independence and Prisoners' Dilemma

This section considers the repeated prisoners' dilemma where for each $i \in \{1,2\}$, $A_i = \{c_i, d_i\}$, $c \equiv (c_1, c_2)$, $d \equiv (d_1, d_2)$,

$$u_i(c) = 1,\ u_i(d) = 0,\ u_i(d / c_j) = 1 + x_i > 1, \text{ and } u_i(c / d_j) = -y_i < 0,$$

where $x_1 x_2 \leq (1 + y_1)(1 + y_2)$. Note that the action profile d is the unique one-shot Nash equilibrium, $u(d) = (0,0)$ is the minimax point, and $u(c) = (1,1)$ is efficient and individually rational.

Theorem 1: *If p is conditionally independent, then the folk theorem holds, i.e., every individually rational payoff vector is sustainable.*

This theorem is in contrast to the anti-folk theorem provided by Matsushima (1990a). Matsushima showed that the repetition of the one-shot Nash equilibrium is the only Nash equilibrium if players' private signals are conditionally independent and only strategies satisfying independence of payoff-irrelevant histories are permitted. Here, a strategy profile s is said to be independent of payoff-irrelevant histories if for each $i \in \{1,2\}$, every $t = 1,2,\ldots$, every $h_i^t \in H_i$, and every $h_i'^t \in H_i$,

$$s_i|_{h_i^t} = s_i|_{h_i'^t} \text{ whenever } p_i(h_j^t | s, h_i^t) = p_i(h_j^t | s, h_i'^t) \text{ for all } h_j^t \in H_j,$$

where $p_i(h_j^t | s, h_i^t)$ is the probability anticipated by player i that the opponent j observes private history $h_j^t \in H_j$ when player i observes $h_i^t \in H_i$, given that players behave according to $s \in S$. The independence of payoff-irrelevant histories implies that whenever a player anticipates the same future strategy for the opponent then she plays the same strategy. In contrast to Matsushima (1990a), this paper shows the folk theorem when players' private signals are conditionally independent and strategies depending on payoff-irrelevant histories are permitted.

The proof of Theorem 1 will be shown in Appendix B. We will show the outline of the proof of the result that the efficient payoff vector $u(c)$ is sustainable as follows. Since $u_i(a) \neq u_i(a')$ for all $a \in A$ and $a' \in A / \{a\}$, we can choose the first and second *bad* private signals for each player i, denoted by ω_i^* and ω_i^{**}, respectively, that satisfy that

(1) $$p_i(\omega_i^* \mid c) < p_i(\omega_i^* \mid c_i, d_j) \text{ and } p_i(\omega_i^{**} \mid d_i, c_j) < p_i(\omega_i^{**} \mid d).$$

As we will see later, how many times player i observes the bad private signal during a fixed period interval will be crucial for the future punishment of the opponent j. Since p is conditionally independent, it satisfies the *uninformative* property in the sense that the opponent j's private signal has no information about whether player i observed the bad private signal, i.e., for every $a_j \in A_j$ and every $\omega_j \in \Omega_j$,

(2) $$p_i(\omega_i^* \mid c_i, a_j, \omega_j) = p_i(\omega_i^* \mid c_i, a_j) \text{ and } p_i(\omega_i^{**} \mid d_i, a_j, \omega_j) = p_i(\omega_i^{**} \mid d_i, a_j).$$

As we will see later, this uninformative property will be crucial for player $i's$ incentive to choose collusively.

Fix a sufficiently large integer $T > 0$ arbitrarily. We divide the infinite time horizon into infinitely many T period intervals, and call each interval a *review phase*. From inequalities (1), for each $i \in \{1,2\}$, we can choose a positive integer $M_i \in \{1,...,T\}$ satisfying that

$$p_i(\omega_i^* \mid c) < \frac{M_i}{T} < p_i(\omega_i^* \mid c_i, d_j) .$$

We specify a strategy $\bar{s}_i$ named the *collusive review strategy*, and another strategy $\underline{s}_i$ named the *defective review strategy*, for each player i in the following way.

According to the collusive review strategy $\bar{s}_i$, player i continues by choosing c_i collusively during the first review phase. When the number of periods in which the first bad private signal ω_i^* occurs is less than or equals M_i, the opponent j will pass player $i's$ review, and therefore, player i will continue choosing c_i during the next review phase and repeat the same strategy as $\bar{s}_i$ from period $T+1$. When this number is more than M_i, the opponent j will fail player $i's$ review, and therefore, player i will continue by choosing d_i defectively during the next review phase and repeat the same strategy as the defective review strategy $\underline{s}_i$, which will be specified later, from period $T+1$ (continue by choosing c_i collusively during the next review phase and repeat the same strategy as $\bar{s}_i$ from period $T+1$) with positive probability $\xi_i(c_i) \in [0,1]$ (probability $1-\xi_i(c_i)$, respectively). Here, for each $i \in \{1,2\}$, and every $a_i \in A_i$, the real number $\xi_i(a_i) \in [0,1]$ will be specified later.

According to the defective review strategy $\underline{s}_i$, player i continues by choosing d_i defectively during the first review phase. When player i never observes the second bad private signal ω_i^{**}, the opponent j will pass player $i's$ review, and therefore, player i will continue by choosing c_i collusively during the next review phase and repeat the same strategy as $\bar{s}_i$ from period $T+1$ (continue by choosing d_i defectively during the next review phase and repeat the same strategy as $\underline{s}_i$ from period $T+1$) with probability $1-\xi_i(d_i)$ (probability $\xi_i(d_i)$, respectively). When there exists a period during the first review phase in which player i observes ω_i^{**}, the opponent j will fail player $i's$ review, and therefore, player i will continue by choosing d_i defectively during the next review phase and repeat the same strategy as $\underline{s}_i$ from period $T+1$. Note that all continuation strategies of $\bar{s}_i$ and $\underline{s}_i$ that start from period $T+1$ are mixtures of $\bar{s}_i$ and $\underline{s}_i$.

It is straightforward from the law of large numbers that $v(\delta, \bar{s})$ and $v(\delta, \underline{s})$ approximate $u(c)$ and $u(d)$, respectively. The law of large numbers implies that when

$\bar{s} = (\bar{s}_1, \bar{s}_2)$ is played, it is almost certain that the number of periods in which ω_i^* occurs during the first review phase, divided by T, is close to $p_i(\omega_i^* \mid c)$. Since $\frac{M_i}{T} > p_i(\omega_i^* \mid c)$, the opponent j will almost certainly pass player $i's$ review. This implies that $v(\delta, \bar{s})$ approximates $u(c)$. The law of large numbers implies also that when $\underline{s} = (\underline{s}_1, \underline{s}_2)$ is played, it is almost certain that the number of periods in which ω_i^{**} occurs, divided by T, is close to $p_i(\omega_i^{**} \mid d)$. Since the threshold for this review equals zero and $p_i(\omega_i^{**} \mid d)$ is positive, the opponent j will almost certainly fail the review. This implies that $v(\delta, \underline{s})$ approximates $u(d)$.

The following three steps will show that $\bar{s}$, $(\bar{s}_1, \underline{s}_2)$, $(\underline{s}_1, \bar{s}_2)$, and $\underline{s}$ are Nash equilibria.

Step 1: The opponent j is indifferent to the choice between $\bar{s}_j$ and $\underline{s}_j$, whenever player i plays either $\bar{s}_i$ or $\underline{s}_i$, that is,

(3) $$v_j(\delta, \bar{s}) = v_j(\delta, \bar{s}_i, \underline{s}_j) \text{ and } v_j(\delta, \underline{s}) = v_j(\delta, \underline{s}_i, \bar{s}_j).$$

Proof Sketch: Suppose that $\bar{s}$ is played. Then, it is almost certain that the number of periods in which ω_i^* occurs, divided by T, is close to $p_i(\omega_i^* \mid c_i, d_j)$. Since $\frac{M_i}{T} < p_i(\omega_i^* \mid c_i, d_j)$, the opponent j will almost certainly fail player $i's$ review. Hence, the opponent j does not necessarily prefer $\underline{s}_j$ to $\bar{s}_j$ even if she can earn the short-term benefit from playing $\underline{s}_j$ instead of $\bar{s}_j$. In fact, by choosing $\xi_i(c_i)$ appropriately, we can make the opponent j indifferent to the choice between $\bar{s}_j$ and $\underline{s}_j$, i.e., $v_j(\delta, \bar{s}) = v_j(\delta, \bar{s}_i, \underline{s}_j)$. Suppose that $(\underline{s}_i, \bar{s}_j)$ is played. Then, the probability that ω_i^{**} occurs in all periods is $p_i(\omega_i^{**} \mid d_i, c_j)^T$. On the other hand, when $\underline{s}$ is played instead of $(\underline{s}_i, \bar{s}_j)$, this probability equals $p_i(\omega_i^{**} \mid d)^T$. Since $p_i(\omega_i^{**} \mid d_i, c_j) < p_i(\omega_i^{**} \mid d)$ and T is sufficiently large, $(\frac{p_i(\omega_i^{**} \mid d_i, c_j)}{p_i(\omega_i^{**} \mid d)})^T$ is close to zero. Hence, the opponent j does not necessarily prefer $\underline{s}_j$ to $\bar{s}_j$ even if she can earn the short-term benefit from playing $\underline{s}_j$ instead of $\bar{s}_j$. In fact, by choosing $\xi_i(d_i)$ appropriately, we can make the opponent j indifferent to the choice between $\bar{s}_j$ and $\underline{s}_j$, i.e., $v_j(\delta, \underline{s}) = v_j(\delta, \underline{s}_i, \bar{s}_j)$.

Q.E.D.

Straightforwardly from Step 1, we can check that each player i is indifferent to the choice among all strategies that induce her to make either the repeated choices of the action

c_i or the repeated choices of the action d_i during each review phase. This interchangeability, implied by Step 1, is closely related to Ely and Välimäki (2002). There, however, exists a difference between Ely and Välimäki and Step 1 of this paper. Ely and Välimäki require each player $i \in \{1,2\}$ to be indifferent to the choice between c_i and d_i in every period, whereas Step 1 requires each player $i \in \{1,2\}$ to be indifferent to the choice between the repeated choices of c_i and the repeated choices of d_i in every review phase. In other words, Ely and Välimäki investigated review strategies only in the case of $T = 1$ where equalities (3) imply that all strategies for player i *are the best replies to* $\bar{s}$, i.e.,

$$v_i(\delta, \bar{s}) = v_i(\delta, s_i, \bar{s}_j) \text{ for all } s_i \in S_i.$$

Hence, equalities (3) are sufficient for $\bar{s}$, $\underline{s}$, $(\underline{s}_1, \bar{s}_2)$ and $(\bar{s}_1, \underline{s}_2)$ to be Nash equilibria in the case where $T = 1$. By assuming that monitoring is almost perfect, Ely and Välimäki could prove that $u(c)$, $(u_1(d), u_2(c))$, $(u_1(c), u_2(d))$, and $u(d)$ are sustainable. Step 1 extends this idea to general cases with large T. In contrast to the case where $T = 1$, however, equalities (3) do not imply that $\bar{s}$, $\underline{s}$, $(\underline{s}_1, \bar{s}_2)$, and $(\bar{s}_1, \underline{s}_2)$ are Nash equilibria when $T \geq 2$. This is why we need the following two more steps for the proof of Theorem 1.

Let $\hat{S}_i \subset S_i$ denote the set of all strategies s_i for player i that are *history-independent* during the first review phase in the sense that there exists $(a_i(1), ..., a_i(T)) \in A^T$ such that

$$s_i(h_i^{t-1}) = a(t) \text{ for all } t = 1, ..., T \text{ and all } h_i^{t-1} \in H_i^{t-1},$$

and the continuation strategy after period $T+1$ is either the collusive review strategy or the defective review strategy, i.e., $s_i|_{h_i^T} \in \{\bar{s}_i, \underline{s}_i\}$ for all $h_i^T \in H_i^T$. Step 2 will show that each player i has no strict incentive to play any strategy in $\hat{S}_i / \{\bar{s}_i\}$. Let $\tilde{S}_i \subset S_i$ denote the set of all strategies s_i for player i that are *history-dependent* during the first review phase in the sense that the continuation strategy after period $T+1$ is either the collusive review strategy or the defective review strategy but there does not exist such $(a_i(1), ..., a_i(T)) \in A^T$ as above. Step 3 will show that each player i has no strict incentive to play any strategy in $\tilde{S}_i$.

Step 2: For each $i \in \{1,2\}$ and every $s_i \in \hat{S}_i$,

$$v_i(\delta, \bar{s}) \geq v_i(\delta, s_i, \bar{s}_j) \text{ and } v_i(\delta, \underline{s}) \geq v_i(\delta, s_i, \underline{s}_j).$$

Proof Sketch: Suppose that the opponent j plays $\bar{s}_j$. By letting $\frac{M_j}{T}$ as close to $p_j(\omega_j^* \mid c)$ as possible, we can make the increase in the probability of player i being punished sufficiently large even if she deviates from $\bar{s}_i$ by choosing d_i only in a small number of periods. Next, suppose that the opponent j plays $\underline{s}_j$. Since the threshold for the opponent $j's$ review equals zero, it follows that the increase in the probability of player i being punished is sufficiently large even if she deviates from $\bar{s}_i$ by choosing d_i only in a small number of periods. These large increases in probabilities of player $i's$ being punished will be the driving forces to prevent player i from playing any strategy in $\hat{S}_i / \{\bar{s}_i\}$.

Q.E.D.

The basic logic for Step 2 is closely related to Matsushima (2001b), which investigated repeated games with public monitoring where in every period players play T different prisoner-dilemma games and observe T different public signals at one time. Matsushima showed in a similar way to Step 2 that with sufficiently large T, the efficient payoff vector is sustainable even if the discount factor is far less than unity.[7]

Step 3: For each $i \in \{1,2\}$ and every $s_i \in \tilde{S}_i$,

$$v_i(\delta, \bar{s}) \geq v_i(\delta, s_i, \bar{s}_j) \text{ and } v_i(\delta, \underline{s}) \geq v_i(\delta, s_i, \underline{s}_j).$$

Proof Sketch: This step relies crucially on the uninformative property (3). When each player i plays $\bar{s}_i$ (plays $\underline{s}_i$), the opponent $j's$ private signal ω_j has no information about whether ω_i^* (ω_i^{**}, respectively) occurs. This implies that ω_j has no information about which strategy player i will play from the next period. Hence, during the first review phase, the best-response actions for each player can be chosen independently of which private signals she has ever observed. This implies that whenever there exists $s_i \in \tilde{S}_i$ such that

$$\text{either } v_i(\delta, \bar{s}) < v_i(\delta, s_i, \bar{s}_j) \text{ or } v_i(\delta, \underline{s}) < v_i(\delta, s_i, \underline{s}_j),$$

then we can find such s_i also in the set $\hat{S}_i$. Since we have already shown in Step 2 that no such s_i exists in $\hat{S}_i$, we have proved Step 3.

Q.E.D.

Remark: The uninformative property is crucial in proving Step 3. Suppose that the uninformative property does not hold, and there exists $\hat{\omega}_j \in \Omega_j$ such that

$$p_i(\omega_i^* \mid c, \hat{\omega}_j) > p_i(\omega_i^* \mid c_i, d_j),$$

[7] See also Kandori and Matsushima (1998), which investigated repeated games with private monitoring and with communication, and showed the folk theorem by using a similar technique.

Then, when the opponent j chooses c_j and observes $\hat{\omega}_j$, she will update the probability of player $i's$ observing ω_i^* so that it is higher than the probability when she chooses d_j. This implies that the opponent j will expect to fail player $i's$ review with high probability so that she will have strict incentive to stop choosing c_j from the next period. This contradicts the Nash equilibrium property.

Based on the arguments above, we can prove that $u(c)$, $(u_1(d), u_2(c))$, $(u_1(c), u_2(d))$, and $u(d)$ are sustainable.

4. Correlated Signals

This section investigates more general two-player repeated games where there may exist three or more actions for each player and the private signals are correlated.[8] Fix two action profiles $c = (c_1, c_2) \in A$ and $d = (d_1, d_2) \in A$ arbitrarily, where we assume that

(4) $$u_i(d_i, c_j) > u_i(c)\,,\ u_i(d) > u_i(c_i, d_j)\,,$$

(5) $$u_i(c) > \max_{a_i \in A_i} u_i(a_i, d_j)\,,$$

and the payoff vector $u(d)$ may be efficient. Note that the action profiles c and d in the prisoners' dilemma satisfy inequalities (4) and (5).

We assume that the private signal structure p is *decomposed into* two functions q and f_0 in the following way. In every period, after players choose $a \in A$, a *macro shock* θ_0 is randomly drawn according to the conditional probability function $f_0(\cdot \mid a) : \Xi_0 \to [0,1]$, where Ξ_0 is the finite set of macro shocks. Players cannot observe the realization of this shock. After players choose $a \in A$ and $\theta_0 \in \Xi_0$ occurs, the private signal profile $\omega \in \Omega$ is randomly drawn according to the conditional probability function $q(\cdot \mid a, \theta_0) : \Omega \to [0,1]$. Hence, the probability $p(\omega \mid a)$ of a private signal profile $\omega \in \Omega$ occurring when players choose $a \in A$ equals

$$p(\omega \mid a) = \sum_{\theta_0 \in \Xi_0} q(\omega \mid a, \theta_0) f_0(\theta_0 \mid a)\,.$$

Hence, the private signal structure p is described by the triplicate (Ξ_0, f_0, q). Let $q_i(\omega_i \mid a, \theta_0) \equiv \sum_{\omega_j \in \Omega_j} q(\omega \mid a, \theta_0)$ and $q_i(\psi_i \mid a, \theta_0) \equiv \sum_{\omega_i \in \Omega_i} \psi_i(\omega_i) q_i(\omega_i \mid a, \theta_0)$. We assume that players' private signals are correlated *only* through this unobservable macro shock. That is, we assume that for every $(a, \theta_0) \in A \times \Xi_0$, $q(\cdot \mid a, \theta_0)$ is *conditionally independent* in the sense that

$$q(\omega \mid a, \theta_0) = q_1(\omega_1 \mid a, \theta_0) q_2(\omega_2 \mid a, \theta_0) \text{ for all } \omega \in \Omega\,.$$

Condition LI: For each $i \in \{1,2\}$ and each $a_i \in \{c_i, d_i\}$, the collection of the probability functions on Ω_i given by $\{q_i(\cdot \mid a, \theta_0) \mid (a_j, \theta_0) \in A_j \times \Xi_0\}$ is *linearly independent* in the sense that there exists no function $e : A_j \times \Xi_0 \to R$ such that $(e(a_j, \theta_0))_{(a_j, \theta_0) \in A_j \times \Xi_0} \neq 0$ and

$$\sum_{(a_j, \theta_0) \in A_j \times \Xi_0} e(a_j, \theta_0) q_i(\cdot \mid a, \theta_0) = 0\,.$$

Theorem 2: *Under Condition LI, any individually rational payoff vector $v \in V^*$ is sustainable if $\max_{a_i \in A_i} u_i(a_i, d_j) \leq v_i \leq u_i(c)$ for both $i \in \{1,2\}$.*

Let $P^* = P^*(\Xi_0, \Omega)$ denote the set of private signal structures associated with Ξ_0 and Ω.

[8] See Ely and Välimäki (2000) for two-player games with three or more actions.

If

(6) $\qquad |\Omega_i| \geq |A_j| \times |\Xi_0|$ for both $i \in \{1,2\}$,

then Condition LI holds for generic private signal structures in P^*. Hence, Theorem 2 implies that the efficient payoff vector $u(c)$ is sustainable for generic private signal structures in P^* if the size of the set of private signals for each player is so large relatively to the sizes of the set of the opponent's actions and the set of macro shocks as to satisfy inequality (6).

The proof of Theorem 2 will be shown in Appendix A. The outline of this proof is as follows. For each $i \in \{1,2\}$, we define a random event on Ω_i as a function $\psi_i : \Omega_i \to [0,1]$. A random event ψ_i is interpreted as follows. Suppose that player i observes not only the private signal ω_i but also a real number x_i that is drawn according to the uniform distribution on the interval [0,1], which is independent of ω and a. We will say that the random event ψ_i occurs when player $i's$ observation (ω_i, x_i) satisfies $0 \leq x_i < \psi_i(\omega_i)$. Hence, we will say that the random event ψ_i occurs with probability $\psi_i(\omega_i)$ when player i observes the private signal ω_i. The probability that the random event ψ_i occurs when players choose $a \in A$ is given by

$$p_i(\psi_i \mid a) \equiv \sum_{\omega_i \in \Omega_i} \psi_i(\omega_i) p_i(\omega_i \mid a).$$

The probability that ψ_i occurs when players choose a and the opponent j observes ω_j is given by $p_i(\psi_i \mid a, \omega_j) \equiv \sum_{\omega_i \in \Omega_i} \psi_i(\omega_i) p_i(\omega_i \mid a, \omega_j)$.

Lemma 3: *Under Condition LI, for each $i \in \{1,2\}$, there exist four random events on Ω_i denoted by ψ_i^*, ψ_i^+, ψ_i^{**} and ψ_i^{++} that satisfy the following properties.*

(i) *For every $a_j \notin \{c_j, d_j\}$,*

$$p_i(\psi_i^* \mid c) < p_i(\psi_i^* \mid c_i, d_j) = p_i(\psi_i^* \mid c_i, a_j),$$
$$p_i(\psi_i^+ \mid c) = p_i(\psi_i^+ \mid c_i, d_j) < p_i(\psi_i^+ \mid c_i, a_j),$$
$$p_i(\psi_i^{**} \mid d_i, c_j) < p_i(\psi_i^{**} \mid d) = p_i(\psi_i^{**} \mid d_i, a_j), \text{ and}$$
$$p_i(\psi_i^{++} \mid d_i, c_j) = p_i(\psi_i^{++} \mid d) < p_i(\psi_i^{++} \mid d_i, a_j).$$

(ii) *For every $a_j \in A_j$, and every $\omega_j \in \Omega_j$,*

$$p_i(\psi_i^* \mid c_i, a_j, \omega_j) = p_i(\psi_i^* \mid c_i, a_j),$$
$$p_i(\psi_i^+ \mid c_i, a_j, \omega_j) = p_i(\psi_i^+ \mid c_i, a_j),$$
$$p_i(\psi_i^{**} \mid d_i, a_j, \omega_j) = p_i(\psi_i^{**} \mid d_i, a_j), \text{ and}$$
$$p_i(\psi_i^{++} \mid d_i, a_j, \omega_j) = p_i(\psi_i^{++} \mid d_i, a_j).$$

Proof: See Appendix C.

Property (i) implies that each of the random events is 'bad' in the sense that the probability of its occurrence is the lowest when the opponent chooses collusively. Property (ii) implies the uninformative property in the sense that the opponent $j's$ private signal has no information about whether the random events ψ_i^* and ψ_i^+ (ψ_i^{**} and ψ_i^{++}) on Ω_i occur or not, provided that player i chooses c_i (d_i, respectively).

The existence of the bad random events or private signals satisfying the uninformative property is crucial in the way of how to prove sustainability that was shown in Section 3, and therefore, guarantees it applicable to more general cases without conditional independence. For example, consider the prisoners' dilemma that is the same as in Section 3 except for the point that the private signal structure is not conditionally independent but satisfies Condition LI. Since c_i and d_i are the only available action choices for each player i, we can choose ψ_i^+ and ψ_i^{++} as being equivalent to the null event, i.e., $\psi_i^+(\omega_i) = \psi_i^{++}(\omega_i) = 0$ for all $\omega_i \in \Omega_i$. Construct the review strategies in the same way as in Section 3 except for the point that the bad private signals ω_i^* and ω_i^{**} for each player i are replaced with the bad random events ψ_i^* and ψ_i^{**}, respectively. Property (i) guarantees that we can prove Steps 1, 2, and 3 under Condition LI in the same way as in the conditionally independent case. The uninformative property implied by property (ii) guarantees that we can prove Step 4 under Condition LI in the same way as in the conditionally independent case.

5. Secret Price Cuts

This section investigates price-setting dynamic duopoly with product differentiation, and shows the possibility that cartel collusion is sustainable even if the price level for each firm is not observable to its rival firm and each firm has the option of making secret price cuts.

In Subsection 5.1, we will show the model of dynamic duopoly where consumers' demands are correlated only through a macro shock, and show that this model can be regarded as a special case of Section 4. In Subsection 5.2, we will find the collusive and defective price vectors denoted by c and d, respectively, which satisfy inequalities (4) and (5), and show that Theorem 2 can be applied to this dynamic duopoly. In Subsection 5.3, we will discuss the implications of this analysis in relation to the literature of price-maintenance by cartels.

5.1 Dynamic Duopoly

There exist two firms $i = 1,2$ that sell their differentiated products. For each $i \in \{1,2\}$, let $A_i = \{1,2,...,\overline{a}_i\}$ and $\Omega_i = \{1,2...,\overline{\omega}_i\}$, where $\overline{a}_i$ and $\overline{\omega}_i$ are positive integers. In every period t, each firm $i \in \{1,2\}$ chooses the price level $a_i(t) \in A_i$, which is not observable to its rival firm $j \neq i$. At the end of the period, firm i receives its sales level $\omega_i(t) \in \Omega_i$, which is not observable to its rival firm j and is regarded as firm $i's$ only available information about the rival firm's price choice $a_j(t)$. Firm $i's$ production capacity is limited so that $\overline{\omega}_i$ is regarded as the maximal amount up to that firm i can produce and supply at one time, and therefore, $\overline{\omega}_i$ is the upper bound of firm $i's$ sales level.

When firm i chooses the price level $a_i \in A_i$ and receives the sales level $\omega_i \in \Omega_i$, its instantaneous profit is given by $\pi_i(\omega_i, a_i) = a_i\omega_i - z_i(\omega_i)$, where $z_i(\omega_i)$ is the cost for firm $i's$ production. The expected instantaneous profit for firm i is given by

$$u_i(a) = \sum_{\omega_i \in \Omega_i} \{a_i\omega_i - z_i(\omega_i)\} p_i(\omega_i \mid a) \text{ for all } a \in A.$$

How each firm $i's$ sales level is to be determined is modeled as follows. There exist n consumers $h \in \{1,...,n\}$. Each consumer $h's$ utility depends on the macro shock θ_0 and her private shock θ_h. Let $\theta \equiv (\theta_0,...,\theta_n)$ denote a shock profile that is randomly drawn according to the conditional probability function $f(\cdot \mid a) : \Xi \to [0,1]$, where Ξ_h denotes the finite set of private shocks θ_h for each $h \in \{1,...,n\}$, and $\Xi \equiv \prod_{h=0}^{n} \Xi_h$ denotes the set of shock profiles. The shock profile $\theta(t) \in \Xi$ is not observable to the firms but each consumer $h \in \{1,...,n\}$ can observe (θ_0, θ_h). Let $l_h(i, a, \theta_0, \theta_h) \in R_+$ denote the amount that consumer h buys from firm i when she observes (θ_0, θ_h) and the firms choose the price vector a. The total demand level for firm i is given by $D_i(a,\theta) \equiv \sum_{h=1}^{n} l_h(i, a, \theta_0, \theta_h)$. Hence, the sales level for firm i is given by

$$\omega_i(a,\theta) \equiv \max\{\omega_i \in \Omega_i \mid \omega_i \le D_i(a,\theta)\}.$$

Let $\omega(a,\theta) \equiv (\omega_1(a,\theta),\omega_2(a,\theta)) \in \Omega$.

We assume that each consumer *never* buys from both firms at one time, and which firm she buys from depends only on the price vector $a \in A$. That is, for every $h \in \{1,...,n\}$, and every $a \in A$, there exists $\iota(h,a) \in \{1,2\}$ such that

$$l_h(i,a,\theta_0,\theta_h) = 0 \text{ if } i \ne \iota(h,a).$$[9]

Example: The following example of consumers' utilities satisfies the above assumptions. When each consumer h buys $l_h(i)$ amount from firm i for each $i \in \{1,2\}$, her utility is given by

$$\{v_h(1) - a_1\}l_h(1) + \{v_h(2) - a_2\}l_h(2),$$

where $v_h(1)$ and $v_h(2)$ are positive real numbers. Consumer $h's$ budget depends on (θ_0,θ_h) and is given by $I_h(\theta_0,\theta_h) \in R_+$. Consumer h maximizes her utility with the budgetary constraint, where she expects to buy as many as she wants. Hence, her demand vector $(l_h(1,a,\theta_0,\theta_h), l_h(2,a,\theta_0,\theta_h))$ must be the solution to the problem given by

$$\max_{(l_h(1),l_h(2))\in R_+^2} [\{v_h(1) - a_1\}l_h(1) + \{v_h(2) - a_2\}l_h(2)]$$

subject to

$$a_1 l_h(1) + a_2 l_h(2) \le I_h(\theta_0,\theta_h).$$

Hence, we can choose $\iota(h,a) \in \{1,2\}$ by

$$\iota(h,a) = 1 \text{ if and only if } v_h(1) - a_1 \ge v_h(2) - a_2,$$

and $(l_h(1,a,\theta_0,\theta_h), l_h(2,a,\theta_0,\theta_h))$ by

$$l_h(i,a,\theta_0,\theta_h) = \frac{I_h(\theta_0,\theta_h)}{a_i} \text{ if } i = \iota(h,a) \text{ and } v_h(i) - a_i \ge 0,$$

and

$$l_h(i,a,\theta_0,\theta_h) = 0 \text{ if either } i \ne \iota(h,a) \text{ or } v_h(i) - a_i < 0.$$

The expected instantaneous profit for each firm $i \in \{1,2\}$ equals

$$u_i(a) = \sum_{\theta\in\Xi} \{a_i\omega_i(a,\theta) - z_i(\omega_i(a,\theta))\} f(\theta \mid a).$$

Let $\Xi_{-h} \equiv \prod_{h'\ne h} \Xi_{h'}$, $\theta_{-h} \equiv (\theta_{h'})_{h'\ne h} \in \Xi_{-h}$, $\Xi_{-0-h} \equiv \prod_{h'\notin\{0,h\}} \Xi_{h'}$,

$$f_0(\theta_0 \mid a) \equiv \sum_{\theta_{-0}\in\Xi_{-0}} f(\theta \mid a),\ f_{-0}(\theta_{-0} \mid a,\theta_0) \equiv \frac{f(\theta \mid a)}{f_0(\theta_0 \mid a)}, \text{ and}$$

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[9] We can permit it to depend on the macro shock θ_0, but not on the private shock θ_h.

$$f_h(\theta_h \mid a,\theta_0) \equiv \sum_{\theta_{-0-h} \in \Xi_{-0-h}} f(\theta_{-0} \mid a,\theta_0) \quad \text{for all} \quad h \in \{1,...,n\}.$$

For every $\omega \in \Omega$, every $a \in A$, and every $\theta_0 \in \Xi_0$, let

$$q(\omega \mid a,\theta_0) = \sum_{\theta_{-0} \in \Xi_{-0}: \omega(a,\theta)=\omega} f_{-0}(\theta_{-0} \mid a,\theta_0).$$

Hence, the private signal structure p is given by

$$p(\omega \mid a) = \sum_{\theta \in \Xi: \omega(a,\theta)=\omega} f(\theta \mid a) \quad \text{for all} \quad \omega \in \Omega \quad \text{and all} \quad \theta \in \Xi,$$

and is decomposed into q and f_0 in the same way as in Section 4.

We assume that the consumers' demands are correlated *only* through the macro shock θ_0. That is, we assume that for every $(a,\theta_0) \in A \times \Xi_0$, $f_{-0}(\cdot \mid a,\theta_0)$ is *conditionally independent* in the sense that

$$f_{-0}(\theta_{-0} \mid a,\theta_0) = \prod_{h \in \{1,...,n\}} f_h(\theta_h \mid a,\theta_0) \quad \text{for all} \quad \theta_{-0} \in \Xi_{-0}.$$

The following proposition states that firms' sales levels are correlated only through the macro shock θ_0. Hence, the price-setting duopoly specified in this section can be regarded as a special case of Section 4.

Proposition 4: *In the price-setting duopoly specified above,* $q(\cdot \mid a,\theta_0)$ *is conditionally independent for every* $(a,\theta_0) \in A \times \Xi_0$.

Proof: For each $i \in \{1,2\}$, we denote by $N(i,a)$ the set of all consumers h satisfying that $\iota(h,a) = i$. Since $\omega_i(a,\theta)$ does not depend on θ_h for every $h \notin N(i,a)$, we can write $\omega_i(a,\theta_0,(\theta_h)_{h \in N(i,a)})$ instead of $\omega_i(a,\theta)$. For each $i \in \{1,2\}$, let

$$q_i(\omega_i \mid a,\theta_0) = \sum_{\substack{(\theta_h)_{h \in N(i,a)} \in \prod_{h \in N(i,a)} \Xi_h: \\ \omega_i(a,\theta_0,(\theta_h)_{h \in N(i,a)})=\omega_i}} \{ \prod_{h \in N(i,a)} f_h(\theta_h \mid a,\theta_0) \}.$$

Note

$$q(\omega \mid a,\theta_0) = \sum_{\substack{\theta_{-0} \in \Xi_{-0}: \\ \omega(a,\theta)=\omega}} \{ \prod_{h=1}^{n} f_h(\theta_h \mid a,\theta_0) \}$$

$$= \sum_{\substack{\theta_{-0} \in \Xi_{-0}: \\ \omega(a,\theta)=\omega}} \{ \prod_{h \in N(1,a)} f_h(\theta_h \mid a,\theta_0) \prod_{h \in N(2,a)} f_h(\theta_h \mid a,\theta_0) \}$$

$$= [\sum_{\substack{(\theta_h)_{h \in N(1,a)} \in \prod_{h \in N(1,a)} \Xi_h: \\ \omega_1(a,\theta_0,(\theta_h)_{h \in N(1,a)})=\omega_1}} \{ \prod_{h \in N(1,a)} f_h(\theta_h \mid a,\theta_0) \}] [\sum_{\substack{(\theta_h)_{h \in N(2,a)} \in \prod_{h \in N(2,a)} \Xi_h: \\ \omega_2(a,\theta_0,(\theta_h)_{h \in N(2,a)})=\omega_2}} \{ \prod_{h \in N(2,a)} f_h(\theta_h \mid a,\theta_0) \}]$$

$$= q_1(\omega_1 \mid a,\theta_0) q_2(\omega_2 \mid a,\theta_0),$$

which implies that $q(\cdot \mid a,\theta_0)$ is conditionally independent.

Q.E.D.

5.2. Price Cartel

Fix the collusive price vector $c \in A$ and the defective price vector $d \in A$ arbitrarily. From Theorem 2 and Proposition 4, it follows that the collusive payoff vector $u(c)$ is sustainable if Condition LI holds and the price vectors c and d satisfy inequalities (4) and (5). Hence, with inequalities (4) and (5), whenever each firm $i's$ production capacity $\overline{\omega}_i$ is so large relatively to the sizes of the set of macro shocks and the set of price levels for the rival firm's product as to satisfy that

(7) $$\overline{\omega}_i + 1 \geq (\overline{a}_j + 1)|\Xi_0| \text{ for each } i \in \{1,2\},$$

then the collusive payoff vector $u(c)$ is sustainable for generic private signal structures in P^*.

This subsection will show that we can choose the price vectors c and d satisfying inequalities (4) and (5) in the following way. For convenience of our arguments, for each $i \in \{1,2\}$, we will replace $A_i = \{0,...,\overline{a}_i\}$ with the closed interval $[0, \overline{a}_i]$. We assume that for each $i \in \{1,2\}$, u_i is increasing with respect to a_j. Let $r_i : A_j \to A_i$ denote the best response function defined by

$$u_i(r_i(a_j), a_j) \geq u_i(a) \text{ for all } a \in A.$$

We assume that r_i is increasing and there exists a Nash equilibrium $a^* \in A$ where $a_i^* = r_i(a_j^*)$ for each $i \in \{1,2\}$. We also assume the single-peakedness of u_i with respect to a_i in the sense that for each $i \in \{1,2\}$ and every $a_j \in A_j$, $u_i(\cdot, a_j)$ is increasing (decreasing) with respect to $a_i \in A_i$ when $0 \leq a_i < r_i(a_j)$ ($r_i(a_j) < a_i \leq \overline{a}_i$, respectively).

Choose c satisfying that

$$c > a^*, \ u(c) > u(a^*), \text{ and } c_i > r_i(c_j) \text{ for each } i \in \{1,2\}.$$

Since u_i is increasing with respect to a_j, we can choose $d_j \in (r_j(c_i), c_j)$ satisfying that

(8) $$u_i(r_i(d_j), c_j) > u_i(c) > u_i(r_i(d_j), d_j).$$

It is straightforward from inequalities (8) that c and d satisfy inequalities (5). Since $r_i(c_j) < d_i < c_i$ and $u_i(\cdot, c_j)$ is decreasing with respect to $a_i \in A_i$ when $r_i(c_j) < a_i \leq \overline{a}_i$, it follows from inequalities (8) that $u_i(d_j, c_j) > u_i(c)$. Since $r_i(d_j) < r_i(c_j) < d_i < c_i$ and $u_i(\cdot, d_j)$ is decreasing with respect to $a_i \in A_i$ when $r_i(d_j) < a_i \leq \overline{a}_i$, it follows from inequalities (8) that $u_i(d) > u_i(c_i, d_j)$. Hence, the chosen price vectors c and d satisfy inequalities (4).

5.3. Discussion

We may not be able to make the defective payoff vector $u(d) \in R^2$ very low. Suppose that we make d equivalent to the one-shot Nash equilibrium a^* and choose c so as to satisfy $u(c) > u(d)$. Note that such c and d do not necessarily satisfy the first inequality of (4). Hence, we have to make d higher than a^*, and therefore, make the defective payoff vector $u(d)$ larger than the one-shot Nash equilibrium payoff vector $u(a^*)$. This implies that although a price war surely occurs on the equilibrium path of our dynamic duopoly, the fall in prices during the price war is not so drastic and this war goes on for a long time. This point is in contrast to the previous works such as Abreu, Pearce, and Stacchetti (1986), which showed that in quantity-setting dynamic oligopoly, a price war on the optimal symmetric equilibrium path goes on only one period, and induces firms to make very severe competitive prices.

Kandori and Matsushima (1998), Compte (1998), and Aoyagi (2002) have assumed that players could communicate, and showed their respective folk theorem. These works commonly concluded that communication enhances the possibility of self-enforcing cartel agreement. In real economic situations, however, the Anti-Trust Law prohibits communication among the firms' executives. The present paper does *not* allow firms to communicate. Hence, contrary to these works, we will conclude that *firms can make a self-enforcing cartel agreement even if their communication is severely regulated*.

The earlier work by Green and Porter (1984) has investigated cartel oligopoly in different ways from Stigler (1964) and this paper. In their model, not price but quantity is the only choice variable for each firm, which is not observable to the other firms. Each firm has to monitor the other firms' quantity choices through the market-clearing price. This price is observable to all firms, and fluctuates according to the exogenous macro shock that the firms cannot observe. Hence, unlike what might be Stigler's primary concern, Green and Porter did consider the public monitoring case. By using the trigger strategy construction, Green and Porter showed that business cycle takes place on the equilibrium path and cartel collusion can be self-enforcing.

The basic logic behind this paper has the following substantial difference from Green and Porter. In the public monitoring case such as Green and Porter, it is inevitable that the macro shock influences players' future behaviors because neither player can distinguish the impact of the opponent's deviation from that of the macro shock. In the private monitoring case such as this paper, however, players' future behaviors *never* depend on the macro shock because we choose a bad event so as not to have the probability of its occurrence depend on this shock.

This point is also in contrast to Rotemberg and Saloner (1986), which investigated repeated games with perfect monitoring where the market demand condition fluctuates according to the exogenous macro shock that is observable to the firms. In their paper the market price fluctuates counter-cyclically to the macro shock fluctuation between boom and recession, whereas in the present paper firms' pricing behaviors are never influenced by the macro shock fluctuation.

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Appendix A: Proof of Theorem 2

Let $a_i^t \equiv (a_i(\tau))_{\tau=1}^t$, $\omega_i^t \equiv (\omega_i(\tau))_{\tau=1}^t$, and $\omega_i^T(h_i^t) \equiv (\omega_i(\tau))_{\tau=t-T+1}^t$. When $a_i(\tau)=c_i$ ($a_i(\tau)=d_i$) for all $\tau \in \{1,...,t\}$, we will write $a_i^t \equiv c_i^t$ ($a_i^t \equiv d_i^t$, respectively). Let $f_i^*(r,t,a_j^t)$ ($f_i^+(r,t,a_j^t)$) denote the probability of ψ_i^* (ψ_i^+, respectively) occurring r times during the first t periods when (c_i^t,a_j^t) is chosen. Let $F_i^*(r,t,a_j^t) \equiv \sum_{r'=0}^{r} f_i^*(r',t,a_j^t)$ and $F_i^+(r,t,a_j^t) \equiv \sum_{r'=0}^{r} f_i^+(r',t,a_j^t)$. Let $f_i^{**}(r,t,a_j^t)$ ($f_i^{++}(r,t,a_j^t)$) denote the probability of ψ_i^{**} (ψ_i^{++}, respectively) occurring r times during the first t periods when (d_i^t,a_j^t) is chosen. Let $F_i^{**}(r,t,a_j^t) \equiv \sum_{r'=0}^{r} f_i^{**}(r',t,a_j^t)$ and $F_i^{++}(r,t,a_j^t) \equiv \sum_{r'=0}^{r} f_i^{++}(r',t,a_j^t)$. Fix an infinite sequence of positive integers $(T^m)_{m=1}^\infty$ arbitrarily, where $\lim_{m\to\infty} T^m = +\infty$.

Lemma A1: *There exist infinite sequences of integers* $(M_i^{*m})_{m=1}^\infty$, $(M_i^{+m})_{m=1}^\infty$, *and* $(M_i^{++m})_{m=1}^\infty$ *satisfying that*

$$(M_i^{*m}, M_i^{+m}, M_i^{++m}) \in \{0,...,T^m\}^3 \textit{ for all } m \geq 1,$$

(A1) $$\lim_{m\to\infty}(F_i^*(M_i^{*m},T^m,c_j^{T^m}), F_i^+(M_i^{+m},T^m,c_j^{T^m}), F_i^{++}(M_i^{++m},T^m,c_j^{T^m})) = (1,1,1),$$

(A2) $$\lim_{m\to\infty}\left(\frac{M_i^{*m}}{T^m}, \frac{M_i^{+m}}{T^m}, \frac{M_i^{++m}}{T^m}\right) = (p_i(\psi_i^* \mid c), p_i(\psi_i^+ \mid c), p_i(\psi_i^{++} \mid d)),$$

and

(A3) $$\lim_{m\to\infty}(T^m f_i^*(M_i^{*m},T^m,c_j^{T^m}), T^m f_i^+(M_i^{+m},T^m,c_j^{T^m}), T^m f_i^{++}(M_i^{++m},T^m,c_j^{T^m}))$$
$$= (+\infty,+\infty,+\infty).$$

Proof: Fix a positive real number z arbitrarily. For every sufficiently large m, there exists a positive integer r in the neighborhood of $T^m p_i(\psi_i^* \mid c)$ such that $T^m f_i^*(r,T^m,c_j^{T^m}) > z$. Hence, we can choose $(\varepsilon^m)_{m=1}^\infty$ and $(M_i^{*m})_{m=1}^\infty$ satisfying

$$\lim_{m\to\infty}\varepsilon^m = 0, \quad \lim_{m\to\infty} \sum_{r:\left|\frac{r}{T^m}-p_i(\psi_i^*|c)\right|<\varepsilon^m} f_i^*(r,T^m,c_j^{T^m}) = 1,$$

for every sufficiently large m,

$$\left|\frac{M_i^{*m}}{T^m} - p_i(\psi_i^* \mid c)\right| < \varepsilon^m, \quad T^m f_i^*(M_i^{*m},T^m,c_j^{T^m}) \geq z,$$

and for every r,

$$T^m f_i^*(r,T^m,c_j^{T^m}) \leq z \text{ if } T^m\{p_i(\psi_i^* \mid c) + \varepsilon(T)\} > r > M_i^{*m}.$$

Since $\lim_{m\to\infty}\{p_i(\psi_i^*\,|\,c)+\varepsilon^m-\frac{M_i^{*m}}{T^m}\}=0$, it follows that

$$\lim_{m\to\infty} F_i^*(M_i^{*m},T^m,c_j^{T^m})$$

$$\geq \lim_{m\to\infty}\{\sum_{r:\left|\frac{r}{T^m}-p_i(\psi_i^*|c)\right|<\varepsilon^m} f_i^*(r,T^m,c_j^{T^m}) - \sum_{r:\,p_i(\psi_i^*|c)+\varepsilon^m>\frac{r}{T^m}>\frac{M_i^{*m}}{T^m}} f_i^*(r,T^m,c_j^{T^m})\}$$

$$\geq 1-z\lim_{m\to\infty}\{p_i(\psi_i^*\,|\,c)+\varepsilon^m-\frac{M_i^{*m}}{T^m}\}=1.$$

Since we can choose z as large as possible, we have proved the existence of such a $(M_i^{*m})_{m=1}^{\infty}$. Similarly, we can prove the existence of such $(M_i^{+m})_{m=1}^{\infty}$, and $(M_i^{++m})_{m=1}^{\infty}$.

Q.E.D.

Choose any payoff vector $\underline{v}\in R^2$ satisfying that

$$\max_{a_i\in A_i} u_i(a_i,d_i)<\underline{v}_i<u_i(c) \quad \text{for each } i\in\{1,2\}.$$

From the above specifications, we can choose $(\delta^m,\bar{v}^m,\underline{v}^m,(\bar{\xi}_i^m,\underline{\xi}_i^m,\bar{\varsigma}_i^m,\underline{\varsigma}_i^m)_{i\in\{1,2\}})_{m=1}^{\infty}$ in the following way. Let $\gamma^m\equiv(\delta^m)^{T^m}$. For each $i\in\{1,2\}$,

$$0<(\bar{\xi}_i^m,\underline{\xi}_i^m,\bar{\varsigma}_i^m,\underline{\varsigma}_i^m)\in(0,\frac{1}{2})^4 \quad \text{for all } m\geq 1,$$

(A4) $$\lim_{m\to\infty}(\gamma^m,\bar{v}_i^m,\underline{v}_i^m)=(1,u_i(c),\underline{v}_i),$$

$$\lim_{m\to\infty}\frac{\gamma^m}{1-\gamma^m}\bar{\varsigma}_i^m=+\infty,\ \lim_{m\to\infty}\frac{\gamma^m}{1-\gamma^m}\underline{\varsigma}_i^m>0,\ \lim_{m\to\infty}\frac{\gamma^m}{1-\gamma^m}\underline{\xi}_i^m f_i^{**}(0,T^m,d_j^{T^m})=0,$$

and for every large enough m,

(A5) $$\bar{v}_j^m=u_j(c)-\frac{\gamma^m}{1-\gamma^m}[\bar{\xi}_i^m\{1-F_i^*(M_i^{*m},T^m,c_j^{T^m})\}$$
$$+\bar{\zeta}_i^m\{1-F_i^+(M_i^{+m},T^m,c_j^{T^m})\}](\bar{v}_j^m-\underline{v}_j^m)$$
$$=u_j(c_i,d_j)-\frac{\gamma^m}{1-\gamma^m}[\bar{\xi}_i^m\{1-F_i^*(M_i^{*m},T^m,d_j^{T^m})\}$$
$$+\bar{\zeta}_i^m\{1-F_i^+(M_i^{+m},T^m,d_j^{T^m})\}](\bar{v}_j^m-\underline{v}_j^m),$$

and

(A6) $$\underline{v}_j^m=u_j(d)+\frac{\gamma^m}{1-\gamma^m}\{\underline{\xi}_i^m f_i^{**}(0,T^m,d_j^{T^m})+\underline{\zeta}_i^m F_i^{++}(M_i^{++m},T^m,d_j^{T^m})\}(\bar{v}_j^m-\underline{v}_j^m)$$
$$=u_j(d_i,c_j)+\frac{\gamma^m}{1-\gamma^m}\{\underline{\xi}_i^m f_i^{**}(0,T^m,c_j^{T^m})+\underline{\zeta}_i^m F_i^{++}(M_i^{++m},T^m,c_j^{T^m})\}(\bar{v}_j^m-\underline{v}_j^m).$$

Let $w_i^*(r,t,\omega_i^t)$ ($w_i^{**}(r,t,\omega_i^t)$, $w_i^+(r,t,\omega_i^t)$, and $w_i^{++}(r,t,\omega_i^t)$) denote the probability of

ψ_i^* (ψ_i^{**}, ψ_i^+, and ψ_i^{++}, respectively) occurring r times during the first t periods when player i observes ω_j^t. Let $W_i^*(r,t,\omega_i^t) \equiv \sum_{r'=0}^{r} w_i^*(r',t,\omega_i^t)$, $W_i^+(r,t,\omega_i^t) \equiv \sum_{r'=0}^{r} w_i^+(r',t,\omega_i^t)$, and $W_i^{++}(r,t,\omega_i^t) \equiv \sum_{r'=0}^{r} w_i^{++}(r',t,\omega_i^t)$. We define

$$\overline{\rho}_i^m(\omega_i^{T^m}) \equiv 1 - \overline{\xi}_i^m \{1 - W_i^*(M_i^{*m}, T^m, \omega_i^{T^m})\} - \overline{\varsigma}_i^m \{1 - W_i^+(M_i^{+m}, T^m, \omega_i^{T^m})\},$$

and

$$\underline{\rho}_i^m(\omega_i^{T^m}) \equiv \underline{\xi}_i^m w_i^{**}(0, T^m, \omega_i^{T^m}) + \underline{\varsigma}_i^m W_i^{++}(M_i^{++m}, T^m, \omega_i^{T^m}).$$

We specify an infinite sequence of two strategy profiles $(\overline{s}^m, \underline{s}^m)_{m=1}^{\infty}$ as follows. For every $t \geq 1$, and every $h_i^{t-1} \in H_i$,

$$\overline{s}_i^m(h_i^{t-1})(a_i) = 0 \text{ and } \underline{s}_i^m(h_i^{t-1})(a_i) = 0 \text{ for all } a_i \notin \{c_i, d_i\}.$$

For every $t \in \{1,...,T^m\}$ and every $h_i^{t-1} \in H_i$,

$$\overline{s}_i^m(h_i^{t-1})(c_i) = 1 \text{ and } \underline{s}_i^m(h_i^{t-1})(d_i) = 1.$$

For every $k \in \{1,2,...\}$, every $t \in \{kT^m + 2, ..., (k+1)T^m\}$, and every $h_i^{t-1} \in H_i$,

$$\overline{s}_i^m(h_i^{t-1})(c_i) = 1 \text{ and } \underline{s}_i^m(h_i^{t-1})(c_i) = 1 \text{ if } a_i(t-1) = c_i,$$

and

$$\overline{s}_i^m(h_i^{t-1})(d_i) = 1 \text{ and } \underline{s}_i^m(h_i^{t-1})(d_i) = 1 \text{ if } a_i(t-1) \neq c_i.$$

For every $k \in \{1,2,...\}$, and every $h_i^{kT^m} \in H_i$,

$$\overline{s}_i^m(h_i^{kT^m})(c_i) = \overline{\rho}_i^m(\omega_i^{T^m}(h_i^{kT^m})) \text{ and } \underline{s}_i^m(h_i^{kT^m})(c_i) = \overline{\rho}_i^m(\omega_i^{T^m}(h_i^{kT^m}))$$
$$\text{if } a_i(kT^m) = c_i,$$

and

$$\overline{s}_i^m(h_i^{kT^m})(c_i) = \underline{\rho}_i^m(\omega_i^{T^m}(h_i^{kT^m})) \text{ and } \underline{s}_i^m(h_i^{kT^m})(c_i) = \underline{\rho}_i^m(\omega_i^{T^m}(h_i^{kT^m}))$$
$$\text{if } a_i(kT^m) = d_i.$$

According to $\overline{s}_i^m$, player i continues choosing c_i during the first T^m periods. When the number of periods in which ψ_i^* occurs is more than M_i^{*m}, she will play $\underline{s}_i^m$ from period $T^m + 1$ with probability $\overline{\xi}_i^m$. When the number of periods in which ψ_i^+ occurs is more than M_i^{+m}, she will play $\underline{s}_i^m$ with probability $\overline{\varsigma}_i^m$. When both numbers are more than their respective thresholds, she will play $\underline{s}_i^m$ with probability $\overline{\xi}_i^m + \overline{\varsigma}_i^m$. Otherwise, she will play $\overline{s}_i^m$. Hence, $\overline{\rho}_i^m(\omega_i^{T^m})$ is regarded as the probability of player $i's$ playing $\overline{s}_i^m$ collusively from period $T^m + 1$ when she observes $\omega_i^{T^m}$.

According to $\underline{s}_i^m$, player i continues choosing d_i during the first T^m periods. When the number of periods in which ψ_i^{**} occurs equals zero, she will play $\bar{s}_i^m$ from period T^m+1 with probability $\bar{\xi}_i^m$. When the number of periods in which ψ_i^{++} occurs is less than or equals M_i^{++m}, she will play $\bar{s}_i^m$ with probability $\underline{\varsigma}_i^m$. When both numbers are less than or equal their respective thresholds, she will play $\bar{s}_i^m$ with probability $\bar{\xi}_i^m+\underline{\varsigma}_i^m$. Otherwise, she will play $\underline{s}_i^m$. Hence, $\underline{\rho}_i^m(\omega_i^{T^m})$ is regarded as the probability of player $i's$ playing $\bar{s}_i^m$ collusively from period T^m+1 when she observes $\omega_i^{T^m}$.

Since

$$f_i^*(r,t,a_j^t)=E[w_i^*(r,t,\omega_i^{t-1})\,|\,c_i^t,a_j^t],\ \ f_i^{**}(r,t,a_j^t)=E[w_i^{**}(r,t,\omega_i^{t-1})\,|\,d_i^t,a_j^t],$$

$$f_i^+(r,t,a_j^t)=E[w_i^+(r,t,\omega_i^{t-1})\,|\,c_i^t,a_j^t], \text{ and}$$

$$f_i^{++}(r,t,a_j^t)=E[w_i^{++}(r,t,\omega_i^{t-1})\,|\,d_i^t,a_j^t],$$

it follows from equalities (A5) and (A6) that for every large enough m,

$$v_j(\delta^m,\bar{s}^m)=v_j(\delta^m,\bar{s}^m/\underline{s}_j^m)=\bar{v}_j^m \text{ and } v_j(\delta^m,\underline{s}^m)=v_j(\delta^m,\underline{s}^m/\bar{s}_j^m)=\underline{v}_j^m,$$

and therefore,

$$\lim_{m\to\infty}v_j(\delta^m,\bar{s}^m)=\lim_{m\to\infty}v_j(\delta^m,\bar{s}^m/\underline{s}_j^m)=u_j(c), \text{ and}$$

$$\lim_{m\to\infty}v_j(\delta^m,\underline{s}^m)=\lim_{m\to\infty}v_j(\delta^m,\underline{s}^m/\bar{s}_j^m)=\underline{v}_j^m.$$

We will show below that $\bar{s}^m$, $\underline{s}^m$, $(\bar{s}_1^m,\underline{s}_2^m)$, and $(\underline{s}_1^m,\bar{s}_2^m)$ are Nash equilibria for every large enough m. All we have to do is to prove that each player has no incentive to choose any strategy other than $\bar{s}_i^m$ and $\underline{s}_i^m$ whose continuation strategies after period T^m+1 are either $\bar{s}_i^m$ or $\underline{s}_i^m$. Let $\hat{S}_i(m)\subset S_i$ denote the set of strategies s_i for player i satisfying that $s_i|_{h_i^{T^m}}\in\{\bar{s}_i^m,\underline{s}_i^m\}$ for all $h_i^{T^m}\in H_i$ and the action choices during the first T^m periods are history-independent in the sense that there exists $\hat{a}_i^{T^m}=(\hat{a}_i(1),...,\hat{a}_i(T^m))$ such that $s_i(h_i^{t-1})(\hat{a}_i(t))=1$ for all $t\in\{1,...,T^m\}$. Property (ii) of Lemma 3 implies that when each player i plays $\bar{s}_i^m$ (plays $\underline{s}_i^m$), the opponent $j's$ private signal ω_j has no information about whether ψ_i^* and/or ψ_i^+ (ψ_i^{**} and/or ψ_i^{++}, respectively) occur. Hence, ω_j has no information about which strategy player i will play from the next period. Based on this observation, we can choose a best-response strategy whose action choices during the first T^m periods are history-independent. Hence, all we have to do is to show that $\bar{v}_j^m\geq v_j(\delta^m,\bar{s}_i^m,s_j)$ and $\underline{v}_j^m\geq v_j(\delta^m,\underline{s}_i^m,s_j)$ for all $s_j\in\hat{S}_j(m)$.

First, we will show that $\bar{v}_j^m\geq v_j(\delta^m,\bar{s}_i^m,s_j)$ for all $s_j\in\hat{S}_j(m)$. Fix $\tau\in\{0,...,T^m\}$ arbitrarily, and consider any strategy $s_j\in\hat{S}_j(m)$ satisfying that

$$\hat{a}_j(t) \in \{c_j, d_j\} \text{ for all } \tau \in \{0,...,T^m\}, \text{ and } \hat{a}_j(t) = d_j \text{ for } \tau \text{ periods.}$$

In this case, $f_i^*(r, T^m, \hat{a}_j^{T^m})$ and $f_i^{**}(r, T^m, \hat{a}_j^{T^m})$ do not depend on the detail of $\hat{a}_j^{T^m}$ except for the number $\hat{\tau}$, and therefore, we can write $f_i^*(r, T^m, \tau)$ and $f_i^{**}(r, T^m, \tau)$ instead of $f_i^*(r, T^m, \hat{a}_j^{T^m})$ and $f_i^{**}(r, T^m, \hat{a}_j^{T^m})$, respectively. Let $s_{j,\tau}^m \in \hat{S}_j(m)$ denote the strategy according to which player j chooses d_j during the first τ periods and c_j from period $\tau+1$ to period T^m. Since $u_j(c) < u_j(c_i, d_j)$, it follows

$$v_j(\delta^m, \bar{s}_i^m, s_{j,\tau}^m) \geq v_j(\delta^m, \bar{s}_i^m, s_j).$$

Hence, all we have to do is to prove that

$$\bar{v}_j^m \geq v_j(\delta^m, \bar{s}_i^m, s_{j,\tau}^m) \text{ for all } \tau \in \{0,...,T^m\}.$$

Define

$$B(\tau) \equiv u_j(c_i, d_j) - u_j(c) - \bar{\xi}_i^m \{ p_i(\psi_i^* \mid c/d_j) - p_i(\psi_i^* \mid c)\}$$

$$f_i^*(M_i^{*m}, T^m - 1, \tau - 1) \frac{\gamma^m}{1-\delta^m} (\bar{v}_j^m - \underline{v}_j^m),$$

Lemma 2 in Matsushima (2001b) implies that $f_i^*(r, T^m, \tau)$ is single-peaked with respect to $\tau \in \{0,...,T^m\}$, and therefore, $-B(\tau)$ is single-peaked with respect to τ. Since the difference of the probabilities of player $i's$ playing $\bar{s}_i^m$ from period T^m+1 between $(\bar{s}_i^m, s_{j,\tau}^m)$ and $(\bar{s}_i^m, s'^m_{j,\tau-1})$ equals

$$\bar{\xi}_i^m \{p_i(\psi_i^* \mid c/d_j) - p_i(\psi_i^* \mid c)\} f_i^*(M_i^{*m}, T^m - 1, \tau - 1),$$

the payoff difference $\frac{1}{1-\delta^m}\{v_j(\delta^m, \bar{s}_i^m, s_{j,\tau}^m) - v_j(\delta^m, \bar{s}_i^m, s_{j,\tau-1}^m)\}$ is equivalent to

$$(\delta^m)^{\tau-1}\{u_j(c_i, d_j) - u_j(c)\} - \bar{\xi}_i^m \{ p_i(\psi_i^* \mid c/d_j)$$

$$- p_i(\psi_i^* \mid c)\} f_i^*(M_i^{*m}, T^m - 1, \tau - 1) \frac{\gamma^m}{1-\delta^m} (\bar{v}_j^m - \underline{v}_j^m).$$

Equalities (A4) imply that for every large enough m, $(\delta^m)^{\tau-1}$ is approximated by 1, and therefore, this payoff difference is approximated by $B(\tau)$. Hence, from the single-peakedness of $-B(\tau)$ and $v_j(\delta^m, \bar{s}^m) = v_j(\delta^m, \bar{s}_i^m, \underline{s}_j^m)$, it follows that whenever $\bar{v}_j^m < v_j(\delta^m, \bar{s}_i^m, s_{j,\tau}^m)$ for some $\tau \in \{0,...,T^m\}$, then it must hold that $\bar{v}_j^m < v_j(\delta^m, \bar{s}_i^m, s_{j,1}^m)$. Hence, all we have to do is to prove that $\bar{v}_j^m \geq v_j(\delta^m, \bar{s}_i^m, s_{j,1}^m)$. Property (i) of Lemma 3 implies $F_i^+(M_i^{+m}, T^m, d_j^{T^m}) = F_i^+(M_i^{+m}, T^m, c_j^{T^m})$, which, together with equalities (A1), implies $\lim_{m\to\infty} F_i^+(M_i^{+m}, T^m, d_j^{T^m}) = 1$. From the law of large numbers, inequality $p_i(\psi_i^* \mid c/d_j) > p_i(\psi_i^* \mid c)$, and equalities (A2), it follows $\lim_{m\to\infty} F_i^*(M_i^{*m}, T^m, d_j^{T^m}) = 0$. Hence, it follows from equalities (A4) and (A5) that

$$\lim_{m\to\infty}\frac{\gamma^m}{(1-\delta^m)T^m}\bar{\xi}_i^m=\lim_{m\to\infty}\frac{\gamma^m}{1-\gamma^m}\bar{\xi}_i^m\lim_{m\to\infty}\frac{\sum_{t=0}^{T^m-1}(\delta^m)^t}{T^m}=\frac{u_j(c_i,d_j)-u_j(c)}{u_j(c)-\underline{v}_j},$$

and therefore,

$$\lim_{m\to\infty}\frac{1}{1-\delta^m}\{v_j(\delta^m,\bar{s}_i^m,s_{j,1}^m)-v_j(\delta^m,\bar{s}^m)\}=u_j(c_i,d_j)-u_j(c)$$
$$-\frac{u_j(c_i,d_j)-u_j(c)}{u_j(c)-\underline{v}_j}\{p_i(\psi_i^*\mid c/d_j)-p_i(\psi_i^*\mid c)\}\lim_{m\to\infty}T^m f_i^*(M_i^{*m},T^m-1,0).$$

Since

$$f_i^*(M_i^{*m},T^m-1,0)=(1-\frac{M_i^{*m}}{T^m})\{1-p(\psi_i^*\mid c)\}^{-1}f_i^*(M_i^{*m},T^m,c^{T^m}),$$

it follows from equalities (A2) and (A3) that

$$\lim_{m\to\infty}\frac{1}{1-\delta^m}\{v_j(\delta^m,\bar{s}_i^m,s_{j,1}^m)-v_j(\delta^m,\bar{s}^m)\}=-\infty.$$

Hence, we have proved $\bar{v}_j^m\geq v_j(\delta^m,\bar{s}_i^m,s_{j,1}^m)$.

Consider any strategy $s_j\in\hat{S}_j(m)$ satisfying that $\hat{a}_j(t)\notin\{c_j,d_j\}$ for τ periods during the first T^m periods. By replacing $(M_i^{*m})_{m=1}^{\infty}$ with $(M_i^{+m})_{m=1}^{\infty}$, we can prove in the same way as above that there exists $s_j'\in\hat{S}_i(m)$ such that $\hat{a}_j(t)\in\{c_j,d_j\}$ for all $\tau\in\{0,...,T^m\}$, and $v_j(\delta^m,\bar{s}_i^m,s_j')\geq v_j(\delta^m,\bar{s}_i^m,s_j)$.

Next, we will show that $\underline{v}_j^m\geq v_j(\delta^m,\underline{s}_i^m,s_j)$ for all $s_j\in\hat{S}_j(m)$. Fix $\tau\in\{0,...,T^m\}$ arbitrarily, and consider any strategy $s_j\in\hat{S}_j(m)$ satisfying that $\hat{a}_j(t)\in\{c_j,d_j\}$ for all $\tau\in\{0,...,T^m\}$, and $\hat{a}_j(t)=d_j$ for τ periods. Note from property (i) of Lemma 3 that

$$F_i^{++}(M_i^{++m},T^m,\hat{a}_j^{T^m})=F_i^{++}(M_i^{++m},T^m,c_j^{T^m}).$$

Note also that

$$f_i^{**}(0,T^m,\tau)=q^{T^m-\tau}f_i^{**}(0,T^m,T^m)\ \text{ where }\ q\equiv\frac{1-p_i(\psi_i^{**}\mid d_i,c_j)}{1-p_i(\psi_i^{**}\mid d)}>1.$$

Hence, it follows from equalities (A6) that

$$v_j(\delta^m,\underline{s}^m/\bar{s}_{j,\tau-1}^m)-v_j(\delta^m,\underline{s}^m/\bar{s}_{j,\tau}^m)$$
$$=(\frac{1}{\delta^m})^{T^m-\tau}(\frac{(1-\delta^m)\gamma^m}{1-\gamma^m})\{u_j(d_i,c_j)-u_j(d)\}$$
$$+q^{T^m-\tau}(q-1)\frac{\gamma^m}{1-\gamma^m}\underline{\xi}_i^m f_i^{**}(0,T^m,T^m)\,(\bar{v}_j^m-\underline{v}_j^m).$$

Given that m is sufficiently large, we can assume

$$1 < \frac{1}{\delta^m} < q .$$

Note that

$$(\frac{(1-\delta^m)\gamma^m}{1-\gamma^m})\{u_j(c_i, d_j) - u_j(d)\} < 0 \text{, and}$$

$$(q-1)\frac{\gamma^m}{1-\gamma^m}\underline{\xi}_i^m f_i^{**}(0, T^m, T^m)\ (\overline{v}_j^m - \underline{v}_j^m) > 0 .$$

Hence, it follows that $-v_j(\delta^m, \underline{s}^m / \overline{s}_{j,\tau}^m)$ is single-peaked with respect to $\tau \in \{0,..., T^m\}$. This, together with $v_j(\delta^m, \underline{s}^m / \overline{s}_{j,0}^m) = v_j(\delta^m, \underline{s}^m / \overline{s}_{j,T^m}^m)$, implies

$$v_j(\delta^m, \underline{s}^m) \geq v_j(\delta^m, \underline{s}^m / \overline{s}_{j,\tau}^m) \quad \text{for all} \quad \tau \in \{1,..., T^m\} .$$

Consider any strategy $s_j \in \hat{S}_j(m)$ satisfying that $\hat{a}_j(t) \notin \{c_j, d_j\}$ for τ periods during the first T^m periods. In the same way, we can prove that there exists $s_j' \in \hat{S}_i(m)$ such that $\hat{a}_j(t) \in \{c_j, d_j\}$ for all $\tau \in \{0,..., T^m\}$, and $v_j(\delta^m, \underline{s}_i^m, s_j') \geq v_j(\delta^m, \underline{s}_i^m, s_j)$.

From these observations, we have proved that $\overline{s}^m$, $\underline{s}^m$, $\overline{s}^m / \underline{s}_j^m$, and $\underline{s}^m / \overline{s}_j^m$ are Nash equilibria for every large enough m. Since we can choose $\underline{v}_i$ as close to $\max_{a_i} u_i(a_i, d_j)$ as possible for each $i \in \{1,2\}$ and the set of sustainable payoff vectors is convex, we have completed the proof of Theorem 2.

Appendix B: Proof of Theorem 1

Let $z^{[1]} \equiv (0, \frac{1+y_1+x_2}{1+y_1})$ and $z^{[2]} \equiv (\frac{1+y_2+x_1}{1+y_2}, 0)$. The set of individually rational payoff vectors equals the convex hull of the set $\{(1,1),(0,0),z^{[1]},z^{[2]}\}$. Note that the set of sustainable payoff vectors is convex, and we can prove as a corollary of Theorem 2 that (1,1) and (0,0) are sustainable. Hence, all we have to do is to prove that $z^{[1]}$ and $z^{[2]}$ are sustainable.

Consider $z^{[1]}$. Fix an infinite sequence of positive integers $(T^m)_{m=1}^{\infty}$ arbitrarily, where $\lim_{m\to\infty} T^m = +\infty$. In the same way as in Lemma A1, we choose an infinite sequence of positive integers $(M_2^{**m})_{m=1}^{\infty}$ satisfying that

$$M_2^{**m} \in \{0,...,T^m\} \text{ for all } m \geq 1, \lim_{m\to\infty} F_2^{**}(M_2^{**m}, T^m, c_1^{T^m}) = 1,$$

$$\lim_{m\to\infty} \frac{M_2^{**m}}{T^m} = p_2(\omega_2^{**} \mid d/c_1), \text{ and } \lim_{m\to\infty} T^m f_2^{**}(M_2^{**m}, T^m, c_1^{T^m}) = \infty.$$

We choose a positive real number $b > 0$ arbitrarily, which is less than but close to $\frac{1}{1+y_1}$. Let

$$\tilde{v} \equiv b(-y_1, 1+x_2) + (1-b)(1,1),$$

which approximates $z^{[1]}$ and $\tilde{v}_1 > z_1^{[1]} = 0$. Choose an infinite sequence $(\delta^m, \bar{s}^m, \underline{s}^m)_{m=1}^{\infty}$ satisfying that $\bar{s}^m$, $(\bar{s}_1^m, \underline{s}_2^m)$, $(\underline{s}_1^m, \bar{s}_2^m)$, and $\underline{s}^m$ are Nash equilibria in $\Gamma(\delta^m)$,

$$\lim_{m\to\infty} \gamma^m = 1-b \text{ where } \gamma^m \equiv (\delta^m)^{T^m},$$

$$\lim_{m\to\infty} u(\delta, \bar{s}^m) = u(c), \text{ and } \lim_{m\to\infty} u(\delta, \underline{s}^m) = u(d).$$

We specify an infinite sequence of strategy profiles $(\hat{s}^m)_{m=1}^{\infty}$ by

$$\hat{s}^m(h_2^{t-1}) = (c_1, d_2) \text{ if } 1 \leq t \leq T^m,$$

$$\hat{s}_1^m|_{h_1^{T^m}} = \bar{s}_1^m \text{ for all } h_1^{T^m} \in H_1,$$

for every $h_2^{T^m} \in H_2$,

$$\hat{s}_2^m|_{h_2^{T^m}} = \bar{s}_2^m \text{ if there exist at most } M_2^{**} \text{ periods such that } \omega_2(t) = \omega_2^{**},$$

and

$$\hat{s}_2^m|_{h_2^{T^m}} = \underline{s}_2^m \text{ otherwise.}$$

According to $\hat{s}^m$, players choose (c_1, d_2) during the first T^m periods. From period $T^m + 1$, player 1 will certainly play $\bar{s}_1^m$, whereas player 2 will play $\bar{s}_2^m$ ($\underline{s}_2^m$) if player 1 passes the review of player 2 (fails the review, respectively). Note that

$$v_1(\delta^m, \hat{s}^m) = (1-\gamma^m)(-y_1) + \gamma^m[F_2^{**}(M_2^{**m}, T^m, T^m) v_1(\delta^m, \bar{s}^m)$$

$$+ \{1 - F_2^{**}(M_2^{**m}, T^m, T^m)\} v_1(\delta^m, \bar{s}_1^m, \underline{s}_2^m)],$$
$$v_2(\delta^m, \hat{s}^m) = (1 - \gamma^m)(1 + x_2) + \gamma^m v_2(\delta^m, \bar{s}^m),$$

and therefore,

$$\lim_{m\to\infty} v(\delta^m, \hat{s}^m) = b(-y_1, 1 + x_2) + (1 - b)(1,1) = \tilde{v} .$$

We will omit the proof that $\hat{s}^m$ is a Nash equilibrium, because it can be done in the way that is based on the law of large numbers and is the same as or even simpler than Theorem 2. Since we can choose $\tilde{v}$ as close to $z^{[1]}$ as possible, we have proved that $z^{[1]}$ is sustainable. Similarly we can prove that $z^{[2]}$ is sustainable.

Appendix C: Proof of Lemma 3

Fix $i \in \{1,2\}$ arbitrarily. Since $\{q_i(\cdot | a, \theta_0) | (a_j, \theta_0) \in A_j \times \Xi_0\}$ is linearly independent for all $a_i \in \{c_i, d_i\}$, there exist ψ_i^*, ψ_i^+, ψ_i^{**}, and ψ_i^{++} such that for every $a_j \notin \{c_j, d_j\}$,

$$p_i(\psi_i^* | c) < p_i(\psi_i^* | c_i, d_j) \le p_i(\psi_i^* | c_i, a_j),$$
$$p_i(\psi_i^+ | c) = p_i(\psi_i^+ | c_i, d_j) < p_i(\psi_i^+ | c_i, a_j),$$
$$p_i(\psi_i^{**} | d_i, c_j) < p_i(\psi_i^{**} | d) \le p_i(\psi_i^{**} | d_i, a_j),$$
$$p_i(\psi_i^{++} | d_i, c_j) = p_i(\psi_i^{++} | d) < p_i(\psi_i^{++} | d_i, a_j),$$

for every $(a_j, \theta_0) \in A_j \times \Xi_0$,

$$p_i(\psi_i^* | c_i, a_j, \theta_0) = p_i(\psi_i^* | c_i, a_j),$$
$$p_i(\psi_i^+ | c_i, a_j, \theta_0) = p_i(\psi_i^+ | c_i, a_j),$$
$$p_i(\psi_i^{**} | d_i, a_j, \theta_0) = p_i(\psi_i^{**} | d_i, a_j), \text{ and}$$
$$p_i(\psi_i^{++} | d_i, a_j, \theta_0) = p_i(\psi_i^{++} | d_i, a_j).$$

Note that these random events satisfy property (i). Since $q(\cdot | a, \theta_0)$ is conditionally independent, it follows that

$$p_i(\omega_i | a, \omega_j) = \frac{\sum_{\theta_0 \in \Xi_0} q_1(\omega_1 | a, \theta_0) q_2(\omega_2 | a, \theta_0) f(\theta_0 | a)}{\sum_{\theta_0 \in \Xi_0} q_j(\omega_j | a, \theta_0) f(\theta_0 | a)},$$

and therefore, for every $\psi_i \in \Psi_i$,

$$p_i(\psi_i | a, \omega_j)$$
$$= \sum_{\omega_i \in \Omega_i} \frac{\psi_i(\omega_i) \sum_{\theta_0 \in \Xi_0} q_1(\omega_1 | a_i, c_j, \theta_0) q_2(\omega_2 | a_i, c_j, \theta_0) f(\theta_0 | a_i, c_j)}{\sum_{\theta_0 \in \Xi_0} q_j(\omega_j | a_i, c_j, \theta_0) f(\theta_0 | a_i, c_j)}$$
$$= \frac{\sum_{\theta_0 \in \Xi_0} p_i(\psi_i | a, \theta_0) q_j(\omega_j | a_i, c_j, \theta_0) f(\theta_0 | a_i, c_j)}{\sum_{\theta_0 \in \Xi_0} q_j(\omega_j | a_i, c_j, \theta_0) f(\theta_0 | a_i, c_j)}.$$

Since $p_i(\psi_i^* | c_i, a_j, \theta_0) = p_i(\psi_i^* | c_i, a_j)$, it follows that

$$p_i(\psi_i^* | c_i, a_j, \omega_j) = p_i(\psi_i^* | c_i, a_j).$$

Similarly, we can show that

$$p_i(\psi_i^+ | c_i, a_j, \omega_j) = p_i(\psi_i^+ | c_i, a_j),$$
$$p_i(\psi_i^{**} | d_i, a_j, \omega_j) = p_i(\psi_i^{**} | d_i, a_j), \text{ and}$$
$$p_i(\psi_i^{++} | d_i, a_j, \omega_j) = p_i(\psi_i^{++} | d_i, a_j).$$

These equalities imply property (ii).

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