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Masanao Aoki University of California, Los Angeles

> Hiroshi Yoshikawa The University of Tokyo

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# Uncertainty, Policy Ineffectiveness, and Long Stagnation of the Macroeconomy

Masanao AOKI,

Department of Economics, University of California, Los Angeles,

and

Hiroshi YOSHIKAWA\*

Faculty of Economics, University of Tokyo

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#### Abstract

The standard analysis in macroeconomics depends on the assumption of the representative agent. However, when the degree of uncertainty becomes significant, we cannot ignore a simple fact that the macroeconomy consists of a large number of heterogeneous agents. In this paper, we demonstrate the importance of the combinatory aspect. Specifically, the effectiveness of policy necessarily weakens as the degree of uncertainty rises. One might call this probrem "uncertainty trap". This may contribute to long stagnation of the macroeconomy.

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\* Aoki: Department of Economics, UCLA, Los Angels, CA 90095; Yoshikawa: Faculty of Economics, University of Tokyo, Tokyo. 113-0033, Japan. We would like to thank Professor Didier Sornette who gave us extremely valuable comments on the previous version of the paper. We are also grateful to Professor Toshihiro Shimizu for figure 1, and Ms. Tsukasa Atsuya of Center for International Research on the Japanese Economy, University of Tokyo for her resurch assistance.

# Uncertainty, Policy Ineffectiveness, and Long Stagnation of the Macroeconomy

History shows us that the economy can be trapped into long stagnation. In the nineteenth century, the British economy suffered from the Great Depression for almost a quarter of century (1873-96). The Great Depression in the 1930's attacked the whole world. And since the beginning of the 1990's, the Japanese economy has stagnated for more than a decade.

In every episode, various policies were discussed and tried. Yet the economy did not easily revive, and fell into long stagnation. Certainly, in each case, there must have been policy mistakes. Granted, it appears that once the economy is trapped into a deep depression, the effectiveness of standard policy measures weakens. Irving Fisher (1933), for example, in relation to his famous 'debt-deflation theory' made the following argument.

There may be equilibrium which, though stable, is so delicately poised that, after departure from it beyond certain limits, instability ensues, just as, at first, a stick may bend under strain, ready all the time to bend back, until a certain pointis reached, when it breaks. This simile probably applies when a debtor gets "broke," or when the breaking of many debtors constitutes a "crash," after which there is no coming back to the original equilibrium. To take another simile, such a disaster is somewhat like the "capsizing" of a ship which, under ordinary conditions, is always near stable equilibrium but which, after being tipped beyond a certain angle, a tendency to depart further from it. In this paper, we focus on a particular factor, namely *uncertainty*. Using a simple theoretical model, we show that mounting uncertainty *necessarily* weakens the effectiveness of macroeconomic policy. We certainly do not recommend policy makers to throw the mainstream macroeconomics textbooks away. However, in our view, the economy once facing great uncertainty *does* present economists and policy makers with the real difficulties the textbook remedies cannot easily handle.

We depart from the standard assumption of the representative agent, and take seriously the fact that the macroeconomy consists of a large number of heterogenous agents. For example, the number of households is of the order of  $10^7$ ; the number of firms is of the order of  $10^6$ . In analysing a system composed of such a large number of units, it is meaningless and impossible to pusue the precise behavior of each unit, because the economic constrains on each will differ, and even the objectives of the units are constantly changing in an idiosyncratic way. This does not mean that economic agents do not behave rationally or do not optimize their objective functions. They certainly do. Their rationaly may or may not be bounded, but this is not really essential for the purpose of macroeconomics. The point is that the pricise behavior of each agent is irrelevant. Rather, we need to recognize that microeconomic behavior is fundamentally stochastic, and so we need to resort to statistical methods to study a macroeconomy consisting of a large number of such agents(See Yoshikawa(2003)). Physicists call this approach coarse-graining.

James Tobin (1972, p.9), in his presidential address to the American Economic Association, proposes a notion of "stochastic macro-equilibrium." He argues that it is "stochastic, because random interesectoral shocks keep individual labor markets in diverse states of disequilibrium, macro-equilibrium, because the perpetual flux of particular markets produces fairly definite aggregate outcomes." Our approach is akin to what Tobin calles "a theory of stochastic macro-equilibrium."

The paper is organized as follows. Section 1 presents our model. Section

2 demonstrates the importance of uncertainty as a hindrance to macroeconomic policy. Section 3 offers concluding remarks. The appendix explains microeconomic foundations for the macro model in Section 1.

## 1 The Model

In this section, we present a theoretical model which shows the importance of *uncertainty* as a hindrance to the economy. The model is highly abstract, but is still useful in understanding policy ineffectiveness and long stagnation of the macroeconomy.

Suppose that there are N economic agents in the economy. There are K possible levels of production. Each agent, as a result of respective optimization, chooses one of K levels. To demonstrate our point, without loss of generality, we can assume that K is just two, "high" and "low". This assumption simplifies our presentation, though theoretically, the model does not have to be binary so long as K is finite. The "high" level of production is denoted by  $y^*$  whereas the "low" level by y ( $0 < y < y^*$ ).

If the number of economic agents which produce at the high leve,  $y^*$  is n  $(n=1, \ldots N)$ , then total output in the economy or GDP is

$$(1) Y = ny^* + (N-n)y$$

We denote the share of ecnomic agents which produce at  $y^*$  by x.

(2) 
$$x = \frac{n}{N} \quad (n = 1, \dots, N)$$

Using x, we can rewrite Y as follows:

(3) 
$$Y = N[xy^* + (1-x)y]$$

When N is large, x can be regarded as a real number  $(0 \le x \le 1)$ . Equation (3) shows that Y and x correspond to each other. While x fluctuates between 0 and 1, so does Y between Ny and Ny<sup>\*</sup>.

Changes in x are assumed to follow a jump Markov process. For a short period of time  $\Delta t$ , there are three possibilities; Namely, no economic agent changes its production level, or one either raises, or lowers its production level. This property is similar to the Poisson process, and is very robust in continuous time models. The process is then characterized by two transition rates, one from state y to  $y^*$  and the other from  $y^*$  to y. Once these two transition rates are given, they determine the model, and accordingly the (stochastic) dynamics it produces.

The probability that one economic agent producing at the low level, y raises its production to high level  $y^*$ , depends naturally on the number of agents currently producing at y, that is N(1 - x). Similarly, the transition rate from  $y^*$  to y depends on Nx.

Moreover, transition rates are assumed to be state-dependent in that N(1-x) and Nx are modified by  $\eta_1(x)$  and  $\eta_2(x)$ , respectively. Specifically, the transition rate from y to  $y^*$ , r is

(4) 
$$r = \lambda N(1-x)\eta_1(x) \qquad (\lambda > 0)$$

And, the transition rate from  $y^*$  to y, q is given by

(5) 
$$q = \mu N x \eta_2(x) \qquad (\mu > 0)$$

The transition rates r and q depend not only on the number of economic agents in each state, but also on  $\eta_1(x)$  and  $\eta_2(x)$ .  $\eta_1(x)$  and  $\eta_2(x)$  mean that the optimal strategy taken by each agent depends on the state of the economy, x or Y. For example, equation (4) means that a switch of strategy by an economic agent from "bear" who finds y as optimal, to "bull" who finds  $y^*$ as optimal depends on the share of bulls. Equation (5) means that the same is true for a switch of strategy from  $y^*$  to y. The state-dependent transition rates such as (4) and (5) mean the presence of externality. Peter Diamond (1982) gives an example of such externality in a search model. Here  $\eta_1(x)$  and  $\eta_2(x)$  are defined as

(6) 
$$\eta_1(x) = Z^{-1} e^{\beta g(x)} \qquad (\beta > 0)$$

(7) 
$$\eta_2(x) = 1 - \eta_1(x) = Z^{-1} e^{-\beta g(x)}$$

(8)  $Z = e^{\beta g(x)} + e^{-\beta g(x)}$ 

Equation (8) or Z simply makes sure that the sum of  $\eta_1(x)$  and  $\eta_2(x)$  is equal to one as it must be. At first sight, the above equations may look arbitrary or even odd. However, they are actually quite generic. The appendix explains how naturally equations (6) and (7) arise in microeconomic models of choice under uncertainty.

The function g(x) in (6) indicates how advantageous a switch of strategy from bear to bull is. The greater is g(x), the more advantageous is a switch from bear to bull, and vice versa. We assume that g(x) becomes zero at  $\bar{x}$ . Note that at  $\bar{x}$ ,  $\eta_1(\bar{x})$  and  $\eta_2(\bar{x})$  are both 1/2, and, therefore that a switch from y and y<sup>\*</sup>, and that from y<sup>\*</sup> to y are equally probable. We assume that g(x) has a stable critical value  $\bar{x}$  as shown in Figure 1.

Obviously, g(x) function plays an important role. We note that most of standard comparative static analyses can be interpreted as shifts of this g(x) function in our present analysis. Take the IS/LM analysis, for example. Suppose that a decline in profitability made the IS curve shift down. GDP or Y declines. This situation corresponds to the case where given x, economic agents now find more advantageous to switch from bull to bear, namely g(x) function shifts down to the left as shown in Figure 2 (a). The stable critical point moves to the left accordingly. Next, suppose that the authority lowered the interest rate to fight against this recession. The LM curve moves downward to the right leading Y to rise. This now corresponds to the case where thanks to the expansionary monetary policy, given x, economic agents find more advantageous than otherwise to switch from bear to bull. The g(x)function shifts up to the right as shown in Figure 2 (b). The economy returns from  $\bar{x}_2$  to  $\bar{x}_1$ , that is, recovers from recession. The other important parameter in transition rates is  $\beta$ . The appendix shows that  $\beta$  in equations (6) and (7) is a parameter which indicates the degree of uncertainty facing economic agents. Suppose, for example, that the pay off facing agent is normally distributed. Then  $\beta$  is simply the inverse of its variance. Thus, when the degree of uncertainty rises,  $\beta$  declines, and *vice versa*. In the limiting case, when  $\beta$  becomes zero, both  $\eta_1(x)$  and  $\eta_2(x)$ become 1/2. In this case, uncertainty is so great that economic decisions become equivalent to tossing a coin.

Now, the share of bulls, x changes stochastically, and so does GDP (recall equaion (3)). Specifically, it follows the jump Markov process with two transition rates (4) and (5). Denote the expected value of x by  $\phi$ :

(9) 
$$\phi = E(x)$$

Then  $\phi$  follows the following ordinary differential equation (note that  $\phi$  is not stochastic, and see Masanao Aoki (1996, 1998) for derivation of this equation):

(10) 
$$\dot{\phi} = (1-\phi)\eta_1(\phi) - \phi\eta_2(\phi)$$

The steady state of equation (10) is given by

(11) 
$$\frac{\eta_1(\phi)}{\eta_2(\phi)} = \frac{\phi}{1-\phi}$$

Thanks to equations (6) and (7), this equation is equivalent to

(12) 
$$2\beta g(\phi) = \log\left(\frac{\phi}{1-\phi}\right)$$

We observe that when there is little uncertainty, namely  $\beta$  is very large, equation (12) becomes equivalent to

$$(13) g(\phi) = 0$$

Thus, when there is little uncertainty (large  $\beta$ ), the expected value of x,  $\phi$  is equal to the zero of g(x) function, that is  $\overline{\phi}$  which satisfies

(14) 
$$g(\overline{\phi}) = 0$$

This  $\bar{\phi}$  is the unique stable equilibrium which satisfies  $g'(\phi) < 0$ , namely a critical point  $\bar{x}$  in Figure 1.

In this case, x changes stochastically, but spends most of time in the neighborhood of  $\bar{\phi}$ . Accordingly, GDP fluctuates stochastically but spends most of time in the neighborhood of

(15) 
$$Y = N[\bar{\phi}y^* + (1-\bar{\phi})y]$$

As we explained it above with respect to g(x) function, the standard comparative static analyses hold without any problem in this case. If policy makers find the current average level of Y too low, for example, then they can raise fiscal expenditures or lower the interest rate. These policies would shift g(x) function upward to the right as shown in Figure 2 (b). The expected value of Y would increase since in this case of low uncertainty (large  $\beta$ ), it is basically determined by the zero of g(x) function (equation (14)).

#### 2 Uncertainty and Policy Ineffectiveness

When the degree of uncertainty rises, however, the proposition that the stabilization policy framework of the mainstream textbooks applies does not hold. Most importantly, when the degree of uncertainty is high, the response of the economy to any policy action *necessarily* becomes small, or put another way, standard macroeconomic policies become less effective.

To explain this proposition, it is useful to introduce the *potential* function. It is given by

(16) 
$$U(x) = -2\int^x g(y)dy - \frac{1}{\beta}H(x)$$

The function g(y) and  $\beta$  are the same as the ones in equations (6) and (7), and H(x) is the Shannon entropy

(17) 
$$H(x) = -x \ln x - (1-x) \ln(1-x).$$

It would be necessary to explain H(x). Recall that each of N economic agents faces a binary choice of being either bull or bear. H(x) is nothing but the logarithm of binominal coefficient  ${}_{N}C_{n}$ , namely the number of cases where n out of N agents are bulls. Using the Stirling formular that  $\log N! \cong$  $N(\log N - 1)$ , we obtain

$$\log_N C_n = \log\left(\frac{N!}{(N-n)!n!}\right)$$
  
=  $N\left[-\left(\frac{n}{N}\right)\log\left(\frac{n}{N}\right) - \left(1 - \frac{n}{N}\right)\log\left(1 - \frac{n}{N}\right)\right]$   
=  $NH(x)$ 

The function H(x) expresses the combinatory aspect of our problem in which a large number of economic agents stochastically make binary choices. It is this combinatory aspect that the standard economic analysis entirely ignores, and yet that plays a crucial role in the analysis of any system, either physical or social, consisting of a large number of entities.

Let us keep this in mind, and go back to the analysis of the expected value of Y. The expected value of x,  $\phi$  which determines the expected value of Y, obeys ordinary differential equation (10). Now, it is easy to see that locally stable critical points of this dynamics given by equation (10) are local minima of the potential function (16):

(18) 
$$U'(\phi) = -2g(\phi) - \frac{1}{\beta}H'(\phi) = -2g(\phi) + \frac{1}{\beta}\log\left(\frac{\phi}{1-\phi}\right) = 0$$

When  $\beta$  is large (little uncertainty),  $U'(\phi) = 0$  is basically equivalent to  $g(\phi) = 0$ , and, therefore, the potential function has a unique minimum. As we explained it in the previous section, the standard textbook results hold.

When  $\beta$  is small, however, the expected value of x,  $\phi$  is not the zero of  $g(\phi)$ , but is determined by both  $g(\phi)$  and  $H'(\phi)/\beta$ . This should be clear from equation (18).

Suppose once again that g(x) function has a unique stable equilibrium as shown in Figure 1. And, for the sake of definiteness, consider the case where an "expansionary" policy such as lowering the real interest rate was taken. This is equivalent to an upward shift of g(x) function as shown in Figure 5 (b). Namely, we change g(x) in transition rates (6) and (7) to

$$g(x) + h(x)$$

where

$$h(x) > 0, \qquad h'(x) \cong 0.$$

With this change in g(x) function,  $\phi^*$  which satisfies equation (18) or  $U'(\phi^*) = 0$ , changes to  $\phi^* + \delta\phi$ . By definition,  $\phi^* + \delta\phi$  satisfies

(19) 
$$-2[g(\phi^* + \delta\phi) + h(\phi^* + \delta\phi)] + \frac{1}{\beta}\log(\frac{\phi^* + \delta\phi}{1 - \phi^* - \delta\phi}) = 0$$

This can be solved out to be

(20) 
$$\delta\phi = \frac{2h(\phi^*)}{\frac{1}{\beta}\left(\frac{1}{\phi^*(1-\phi^*)}\right) - 2g'(\phi^*)} > 0$$

Here we used the assumptions h'(x) = 0 (no particular bias in policy) and  $g'(\phi^*) < 0$  ( $\phi$  is a stable equilibrium).

Equation (20) shows how equilibrium  $\phi$ , which determines the expected value of Y, responds to a change in function g(x), here represented by  $h(\phi^*) > 0$ . It corresponds to the notion of multiplier in deterministic models. Equation (20), therefore, shows the effectiveness of macroeconomic policy.

Since we are considering an expansionary policy,  $\delta\phi$  is positive, that is  $\phi^*$  rises. However, the extent of an increase in  $\phi^*$  depends crucially on  $\beta$  or uncertainty. When uncertainty is negligible,  $\beta$  is so large that  $\delta\phi$  approaches its maximum value  $-h(\phi^*)/g'(\phi^*) > 0$ . On the other hand, as the degree of uncertainty rises ( $\beta$  declines),  $\delta\phi$  gets smaller and smaller approaching zero. This result is quite generic. When uncertainty rises, the effectiveness of macroeconomic policies which affect agents' economic incentives *necessarily* 

weakens. In the limit, the economy facing infinite uncertainty is trapped into a chaos in which no economic policy works or, in fact, no economic decision makes sense in that it is no different from thossing a coin.

#### **3** Concluding Remarks

The standard analysis in macroeconomics begins with micreconomic experiment on the assumption of the representative agent. Suppose, for example, that the authority cut the interest rate. The microeconomic theory tells us that for the representative household or firm, a lower interest rate raises the optimal level of investment. Translating this result to macroeconomic analysis, one conjectures that *ceteris paribus*, aggregate investment would increase. This kind of analysis, including the IS/LM anaysis, gives economists and policy makers a vigorous guidance so long as the degree of uncertainty facing the economy is limited.

However, when the degree of uncertainty becomes significant, we must depart from the representative agent assumption, and seriously take a simple fact that the macroeconomy consists of a large number of economic agents. In this case, stochastic approach is necessary; The combinatory aspect of the system plays a crucial role in the analysis of any system, either physical or social, consisting of a large number of entities. Though the standard economic analysis entirely ignores it, in this paper, we showed that it has, in fact, a very important implication for macroeconomics. Sprcifically, the effectiveness of policy necessarily weakens as the degree of uncertainty rises. One might call this probrem "uncertainty trap".

Once the economy is trapped into this "uncertainty trap," textbook macroeconomic policies including monetary policy, which correspond to a change in the g(x) function in the model, become ineffective. Here, let us take up Japan's long stagnation around 2000. Many economists argue that the BOJ facing the zero nominal interest rate bound can still lower the real interest by generating inflationary expectations (See Krugman (1998), Bernanke (2000), and Blanchard (2000), for example). In our model, it would change the g(x) function, and induce more economic agents to find a shift from "bear" to "bull" advantageous. When uncertainty is insignificant, and the minimum of the potential function is almost equivalent to the zero of the g(x) function, it certainly helps. This is a normal situation. However, when the combinatorial aspect cannot be ignored as the degree of uncertainty rises, policies which are effective in normal circumstances may not help.

Figure 3 shows the coefficient of variation (standerd deviation divided by mean) of the quarterly GDP growth (S.D. and mean are calculated for rolling 5 years or 20 quarters). For the sake of comparison, we also show it for the U.S. The figure shows that the coefficient of variation has risen extraordinarily in Japan during the 1990's, especially in the latter half, and suggests that the degree of uncertainty indeed appears to have risen.

Tobin (1975), in his "Keynesian models of recession and depression", suggests that "the system might be stable for small deviations from its equilibrium but unstable for large shocks." The same point was also made by Fisher (1933) long time ago. In our analysis, *uncertainty* plays the key role. When uncertainty is insignificant, the economy would fluctuate around the (unique) "natural" equilibrium, and policies are effective. However, when the degree of uncertainty rises above a critical level, the economy may be trapped, and policies *necessarily* become ineffective.

It is generally agreed that the performance of the postwar economy is better than that in the prewar period. Martin N. Baily (1978) argues that better safety nets provided by the government in the postwar period has contributed to this outcome. Our analysis suggests that uncertainty is indeed a very serious hindrance to the macroeconomy, and that once the economy faces mounting uncertainty, then the textbook remedies may not so readily work as we would wish.

### Appendix

This appendix offers microeconomic foundations for the transition rate  $\eta_1(x)$ , equations (6) and (7) in the model in Section 1. Namely, it explains how g(x) and  $\beta$  are obtained, and shows that  $\beta$  is a measure of uncertainty.

We offer two interpretations for our specifications of the function  $\eta$ . The first is based on approximate calculations of the perceived difference of the expected utilities, or advantages of one choice over the other. The second interpretation is based on discrete choice theory such as Anderson *et al.* (1993), or McFadden (1974).

#### (1) Representation of Relative Merits of Alternatives

Denote by  $V_1(x)$  the expected "return" from choice 1, given that fraction x has selected choice 1. For definiteness, think of the discounted present value of benefit stream based on the assumption that fraction x remain the same over some planning horizon. Define  $V_2(x)$  analogously. Let

$$\eta_1(x) = \Pr\{V_1(x) \ge V_2(x)\}.$$

We omit x from the arguments of V from now on.

Assume that the difference  $\Delta V = V_1 - V_2$  is approximately distributed as a normal random variable with mean g(x) and variance  $\sigma^2$ . We calculate the probability that the difference is nonnegative, namely choice 1 is preferred to choice 2

$$\eta_1(x) = \Pr{\{\Delta V \ge 0\}} = \frac{1}{2}[1 + erf(u)],$$

where the error function is defined by

$$erf(u) := \frac{2}{\sqrt{\pi}} \int_0^u e^{-y^2} dy,$$

with  $u = g(x)/(\sqrt{2}\sigma)$ . See Abramovitz and Stegun (1968) for example. Then, we follow Ingber (1982) to approximate the error function by

$$erf(u) \approx tanh(\kappa u),$$

with  $\kappa = 2/\sqrt{\pi}$ . This approximation is remarkably good and useful. For example for small |x|, we note that

$$erf(x) = \kappa(x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots),$$

and

$$tanh(\kappa x) = \kappa(x - \frac{x^3}{2.36} + \frac{x^5}{4.63} + \cdots)$$

By letting  $\beta$  to be  $\sqrt{2/\pi}\sigma^{-1}$ , we obtain the desired expression

$$\eta_1(x) = \Pr\{\Delta V \ge 0\} \approx X^{-1} \exp[\beta g(x)],$$

where  $X = \exp\{\beta g(x)\} + \exp\{-\beta g(x)\}.$ 

This offers one interpretation of  $\beta$  that appears in the transition rates. Large variances mean large uncertainty in the expected difference of the alternative choices. Such situations are represented by small values of  $\beta$ . Small variance means more precise knowledge about the difference in the values of two choices, represented by large values of  $\beta$ . This situation is represented by small  $\beta$ . Alternately put, we may interpret g(x) as the conditional mean of a measure that choice 1 is better than choice 2, conditional on the fraction x has decided on choice 1.<sup>1</sup>

# (2) Discrete Choice Theory and Extreme Value Distributions

Next, suppose that we calculate the probability that the discounted present value one,  $V_1$ , is higher than value two  $V_2$ , associated with alternative choices

<sup>&</sup>lt;sup>1</sup>Aoki (1996, Chap. 3, and 8) shows how  $\beta$  arises as a Lagrange multiplier to incorporate macrosignals as constraints. Parameter  $\beta$  is related to the elasticity of the number of microeconomic configurations with respect to macrosignals. Small values of  $\beta$  mean that the number of microeconomic configurations responds little when macroeconomic signals change. This is in accord with the interpretation that agents face large uncertainty in their choices. See Aoki (1996, p.216). Similar interpretation may be offered from the viewpoint of hazard function. See Aoki (2002, Section 6.2)

1 and 2 respectively. Suppose further that we represent some of the incompleteness and impreciseness of information or uncertainty of consequences surrounding the value calculation by adding random terms to the present values as

$$\hat{V}_1 = V_1 + \epsilon_1,$$

and

$$\hat{V}_2 = V_2 + \epsilon_2.$$

One interpretation is that these  $\epsilon$ s are noises to account for inevitable fluctuations in the present values. A second interpretation is to think of them as (additional) evidence to support a particular choice. Other interpretations are certainly possible. For example, McFadden (1973) speaks of common or community preference and individual deviations from the common norm in the context of utility maximization.

One quick assumption to obtain a Gibbs distribution expression in the case of two alternative choices is to assume that  $\epsilon = \epsilon_2 - \epsilon_1$  is distributed according to

$$\Pr(\epsilon \le x) = \frac{1}{1 + e^{-\beta x}},$$

for some positive  $\beta$ . With this distribution, a larger value of  $\epsilon$  supports more strongly the possibility that  $V_1 > V_2$ . Parameter  $\beta$  controls how much of changes in x translate into changes in probabilities. With a smaller value of  $\beta$ , a larger increase in x, that is, in "evidence" is needed to increase the probability that favors choice 1. The larger the value of  $\beta$  is, the smaller increase in x is needed to change the probability by a given amount.

With this distribution, then, it immediately follows that

$$P_1 = Pr(\hat{V}_1 \ge \hat{V}_2) = \frac{e^{\beta V_1}}{e^{\beta V_1} + e^{\beta V_2}} = \frac{e^{\beta g}}{e^{\beta g} + e^{-\beta g}},$$

with  $g = (V_1 - V_2)/2$ . We obtain also  $P_2 = 1 - P_1$ , of course.

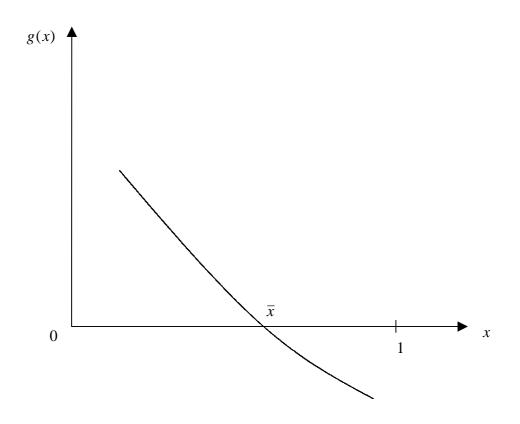
To reiterate, a smaller value of  $\beta$  implies a smaller difference of  $|P_1 - P_2|$ . Namely, with a larger the value of  $\beta$ , one of the alternatives tends to dominate.

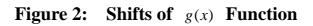
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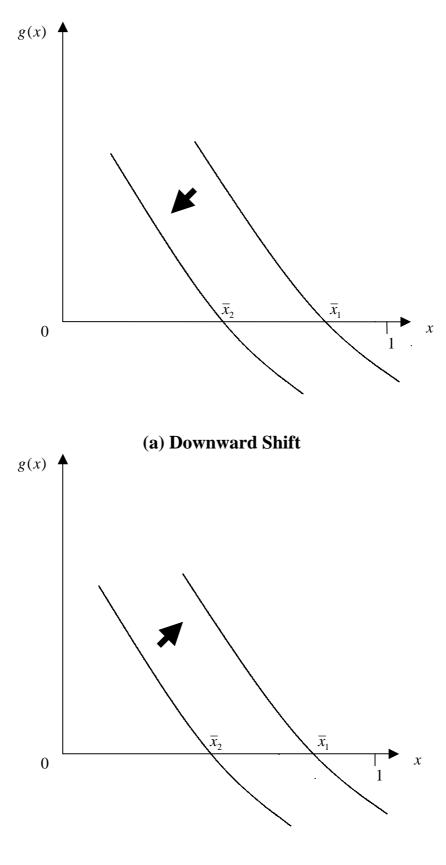
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**Figure 1:** g(x) **Function** 

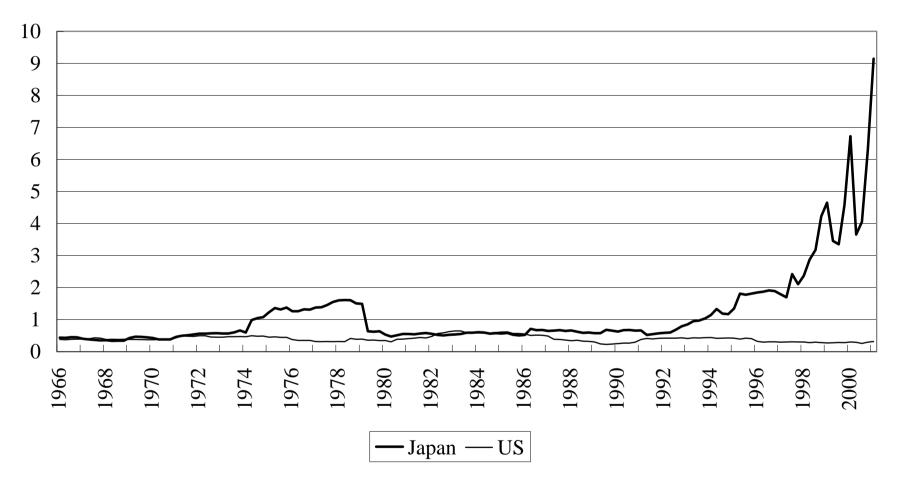






(b) Upward Shift

Figure 3: CV of Growth Rate of GDP for Japan and US



Note : CV = Standard deviation / Mean of quarterly GDP growth rates over the past 5years.