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Abstract

This paper extends Diamond (1965)'s one-sector neoclassical growth model with two-period lived, overlapping generations, by allowing the agents to make the labor force participation decision in their second period, in the spirit of Feldstein (1974). If the agents earn a high wage income when young, they choose to retire when old. This reduces the labor supply (through a lower participation rate of the elderly) and stimulates capital accumulation (through saving for retirement). The resulting high capital-labor ratio leads to a higher wage income for the next generation. If the agents earn a low wage income when young, they continue to work when old and save little, which implies a low capital-labor ratio and a low wage income for the next generation. Due to such positive feedback mechanisms, the endogeneity of retirement magnifies the persistence of growth dynamics, thereby slowing down a convergence to the steady state, and even generating multiple steady states for empirically plausible parameter values.

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1. Introduction

The labor force participation rate of the elderly declines with economic growth, both in time series and in cross sections of countries.[1] Although social security systems undoubtedly had major impacts on the retirement behavior, the observed patterns are remarkably similar across countries, suggesting that the trend for early retirement is as a consequence of the rising income.[2] Early retirement in turn affects the growth process of the economy through its effects on the labor supply and the saving behavior.

To study such interactive processes between induced retirement and economic growth, this paper develops a variant of Diamond's (1965) one-sector neoclassical growth model of an overlapping generations economy, in which each generation lives for two periods. The original Diamond model assumes that the agents work in the first period (when they are young) and retire in the second (when they are old). The present paper endogenizes the labor force participation decision by the elderly, in the spirit of Feldstein (1974), while maintaining the original assumption that the young always work. In order to generate the patterns observed both in time series and in cross sections of countries, it is assumed that the income effect of higher wage, which encourages retirement, dominates the price effect, which discourages retirement.[3]

It turns out that endogenizing the labor force participation of the elderly has significant effects on the mechanics of economic growth. If the agents earned a high wage income when young, they choose to retire when old. This leads to a higher capital-labor ratio by reducing the labor supply (through a lower labor force participation rate of the elderly) and by stimulating capital accumulation (through a higher saving for retirement). The high capital-labor ratio in turn leads to a high wage income for the next generation. If the agents earned a low wage income when young, on the other hand, they continue to

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[1] See, for example, Fuchs (1983), Diamond & Gruber (1999) and Gruber & Wise (1999).

[2] See, for example, Costa (1998), who found, using the historical US data prior to the introduction of the Social Security system, that the rising income is a major factor responsible for the decline in the elderly's labor force participation rate.

[3] It is well known that, within each cohort, the individuals with lower income tend to retire early. The assumption that the income effect dominates the price effect is not inconsistent with this well-known correlation, which can easily be explained once we take into account the heterogeneity of the agents in health and in education. For example, the agents in poor health are more likely to earn lower income and to retire

work when old, which increases the labor supply. Furthermore, expecting to earn some wage income when old, they save little. Therefore, the economy has a low capital-labor ratio and a low wage income for the next generation. Due to such positive feedback mechanisms, the endogeneity of retirement magnifies the persistence of growth dynamics, thereby slowing down a convergence to the steady state, and even leading to multiple steady states for empirically plausible parameter values.

The result that endogenous retirement magnifies the persistence of growth dynamics suggests that the mechanism discussed in this paper may offer an explanation for the slow convergence puzzle, identified by Barro & Sala-i-Martin (1992). The present mechanism, however, should not be viewed as an alternative to the other persistence mechanisms previously proposed in the literature, such as endogenous total factor productivity. It should be viewed instead as complementary, because endogenizing retirement also amplifies persistence caused by these mechanisms. The possibility of multiple steady states suggests that there are two-way causalities between early retirement and economic development.

Some readers may think that the labor supply effect cannot be important because the elderly accounts for a small share of the labor supply, even when their labor force participation rate is high. Feldstein (1974, p.924), for example, expressed such a view. To respond to such a skepticism within the two-period overlapping generations framework, the model assumes that the old agent's effective labor supply be a fraction of the young agent's, given by a parameter, θ. It turns out that the magnification effect of endogenous retirement is independent of θ.[4] The reason why the main result does not have to depend on the elderly's share in the labor supply is that we are dealing with the effect of *endogenous* retirement on growth dynamics. The relevant question is not only how much early retirement causes an increase in the wage rate, but also how much an increase in the wage rate induces early retirement.

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early. Or the less educated are more likely to be in physically demanding occupations and retire early. Section 6 suggests a way of extending the present model to generate this correlation without changing the main results.

Many studies have previously explored a variety of extensions to Diamond's overlapping generations model: see Azariadis (1993, Part II) for a survey. None seems to have endogenized the retirement decision. Within this literature, the closest is Reichlin (1986), who introduced variable labor supply by the young and demonstrated the possibility of endogenous cycles. He assumed, as Diamond, that the old does not work.

Closest to the present paper in spirit is Feldstein (1974), who discussed the possibility that social security may end up increasing the aggregate saving through induced retirement, instead of reducing it as the standard analysis might suggest. Feldstein made, however, neither attempt to analyze full general equilibrium effects of such an induced retirement nor to explore its growth implications.[5]

In an important study on child labor, Kasu and Van (1998) explored the positive feedback mechanism that resembles the labor supply effect in the present model. If the parents earn lower wages, they are forced to send their children to work. The resulting increase in the labor supply reduces the wage income of the parents. If the parents earn high wages, on the other hand, they can afford to withdraw their children from the labor market, which in turn leads to the high wage income for the parents. If all the parents withdraw their children from the labor market, the economy could jump from a low-wage equilibrium with child labor to a high-wage equilibrium without child labor, at the expense of the employers. The analogy should be clear. The adult (child) labor in their model corresponds to the

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[4] Needless to say, the independence is due to the particular functional forms assumed in the paper. However, this means that the effect of an increase in the elderly's share in labor supply on the magnification effect is ambiguous in general, which should be sufficient to refute the argument that the elder's share in labor supply must be large enough to have significant effects.

[5] There is an extensive literature in labor economics and public finance, which analyzes the elderly's labor force participation decision. The main objective of this literature is an empirical assessment of the impact of social security systems and pension plans on retirement behavior, instead of understanding growth implications of retirement behavior. In an attempt to explain how social security provisions affect the timing of retirement, this literature utilizes increasingly sophisticated models of retirement behavior, including those of stochastic dynamic programming. On the other hand, the present model keeps retirement behavior as simple as possible in order to maintain the tractability of general equilibrium analysis. For example, it is assumed that the agents live only for two periods, that the labor supply when young is exogenous, that there is no uncertainty, and that retirement is a zero-one decision, etc. One may hence be surprised to find that the model of retirement behavior developed below is *not* a special case of those developed in this literature. The reason is that the present model is designed to evaluate the effect of retirement on saving through a change in the time profile of labor income, not through a change in the intertemporal preferences over consumption. This requires that the preferences for retirement be weakly separable from the intertemporal preferences for consumption. (Interestingly, Feldstein (1974, Fig. 1) made this assumption implicitly in his graphic analysis.) It turns out that this condition, when the retirement decision is endogenized, the preferences cannot be intertemporally separable. The recent literature on retirement behavior, on the other hand, imposes the intertemporal separability of preferences so as to make the standard tool of dynamic optimization readily applicable to the problem. It should also be noted that the existing empirical studies impose functional forms that rule out the possibility of nonhomothetic preferences, which are too restrictive for the present analysis. This, too, is a reflection of the difference in objective. To evaluate the incentive

young's (old's) labor supply in the present model. There are some significant differences, however. Their model is static and the feedback mechanism operates contemporaneously to generate multiple equilibria. On the other hand, the feedback mechanism in the present model operates intertemporally, from one generation to the next. This generates multiple steady states, but the equilibrium path is unique. A transition from a low wage steady state without retirement to a high wage steady state with retirement, while it improves the steady state welfare of the economy, can be achieved only at the expense of the current generation. Note also that the Kasu and Van model does not have anything that corresponds to the feedback mechanism operating through the saving effect in the present model.

Before proceeding, it is worth clarifying the measure of development adopted in this paper. The standard measure of development, per capita income, is highly misleading, when comparing countries that differ significantly in their labor force participation rates. For example, per capita income in Japan may be higher than those in some European countries, only because the labor force participation rate in Japan is much higher. It is possible that, because of higher output per worker, many people in these European countries can afford to retire early, which make their per capita incomes lower than Japan's. In this case, the output per worker is a better measure of development. The ultimate measure of development, of course, should be the standard of living. In the model developed below, the lifetime utility of the agent is higher if and only if the wage rate, which moves together with the capital/labor ratio and the output per worker, is higher. On the other hand, higher per capita income does not necessarily imply the higher lifetime utility. It is for this reason that the wage rate (or the capital/labor ratio or the output per worker), but not per capita income, is used as the measure of development.

The rest of the paper is organized as follows. Section 2 sets up the framework. As a warm-up exercise, Section 3 considers the special case, in which the agent cares only consumption and leisure when old. This implies that the young worker saves all the wage income. By making saving decision

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effects of social security provisions on the timing of retirement, the income effect may not be important. For the present study, which is concerned with growth implications, it is essential.

independent of the retirement decision, this helps to focus on the labor supply effect, without worrying about some complications that arise from the joint saving/retirement decision. The labor supply effect alone can generate persistence large enough to generate multiple steady states, but only when the share of capital is implausibly large. Section 4 introduces the retirement motive for saving, and shows that a combination of the labor supply effect and the saving effect generates such a large persistence that multiple steady states are possible for empirically plausible parameter values. Section 5 introduces a form of heterogeneity among agents. Section 6 suggests some directions for future research.

2. The Framework

Time is discrete and extends to infinity. There is a single final good, the numeraire, which can either be consumed or invested. It is produced competitively by a standard constant-return-to-scale technology, $Y_t = F(K_t, L_t)$. Let $k_t \equiv K_t/L_t$ denote the capital-labor ratio, and $f(k_t) \equiv F(k_t, 1)$ denote the production function in its intensive form, which is increasing and concave in k_t. The factor markets are competitive, and both capital and labor earn their marginal values, as follows.

(1) $$r_t = R(k_t) \equiv f'(k_t).$$

(2) $$w_t = W(k_t) \equiv f(k_t) - k_t f'(k_t).$$

The economy is populated by overlapping generations of the equal size, normalized to be one. Each generation lives for two periods. Aside from the fact that they may live in different periods, the agents are homogenous (until section 5, where one way of dealing with heterogeneity is discussed.). The young in period t−1 supplies one unit of labor inelastically, and earns wage income, w_{t-1}. The agent may consume some of the wage income, c^y_{t-1}, and save the rest, $s_t = w_{t-1} - c^y_{t-1}$, in capital. When the agent becomes old in period t, s/he earns capital income, $r_t s_t$. In addition, the agent may supplement the capital income by continuing to work and earning wage income, equal to $w_t\theta$, where θ is the effective unit of labor supply by the old. Alternatively, the agent may retire. The old agent's labor force participation is a zero-one decision. That is, $e_t = 0$ if s/he retires, $e_t = 1$ if s/he works. The agent's old consumption is equal to $c^o{}_t = r_t s_t + w_t \theta e_t$. The agent's choice can thus be described as the solution to the following maximization problem: given w_{t-1}, choose $c^y{}_{t-1} \geq 0$, $c^o{}_t \geq 0$, and $e_t \in \{0,1\}$ to maximize $U(c^y{}_{t-1},\ c^o{}_t\ ,\ e_t\ )$ subject to $c^o{}_t = r_t\, s_t + w_t\, \theta\, e_t = r_t(\, w_{t-1} - c^y{}_{t-1}) + w_t\, \theta\, e_t$. The parameter, θ, is introduced to allow for the possibility that the elderly accounts for only a small share of the labor supply, even if their labor force participation rate is high. It turns out that the main results obtained below are independent of θ. Endogenizing the young's labor force participation decision would be straightforward (and perhaps uninteresting), and it is hence omitted here to simplify the notation.[6]

In equilibrium, the agent may be indifferent between working and retiring when old, so that, in spite of the homogeneity, the old generation's labor force participation rate, x_t , may take a value between zero and one, and the labor supply in period t is $L_t = (1+\theta x_t)$. The supply of capital in period t is equal to the total saving made by the young in period t−1. An agent's saving in period t−1 generally depends on whether s/he retires or not in period t. If we indicate the dependence by $s_t(e_t)$, then the gross saving by the young generation in period t−1, and hence the capital stock available in period t is given by $K_t = s_t(0)(1-x_t)+s_t(1)x_t$. The capital-labor ratio is therefore $k_t = K_t/L_t = \{s_t(0)(1-x_t)+s_t(1)x_t\}/(1+\theta x_t)$.

Note that this model would be identical to Diamond's original model, if the agent is forced to retire, $e_t = 0$, and hence the labor force participation rate by the old is exogenously equal to zero ($x_t = 0$).

3. Consumption/Retirement Trade-off: the Labor Supply Effect.

[6] It is straightforward, because, in order to keep the results intact, it would be sufficient to assume that the value of leisure when young is relatively smaller than the value of leisure when old. It would be perhaps uninteresting because, in reality, it is impractical for someone to stay out of the labor market during the prime working age (ages 25 to 50) and then join the labor market when old (after 50). First of all, it would be almost impossible for a fifty-year old man who never had a full- time job to find a decent job. Second, there is the borrowing constraint. It would be almost impossible to finance consumption during the prime working age (ages 25 to 50) by borrowing against the wage earning after the age of fifty. Most people need to start working long before (say, by the age of thirty) to make living. Although these factors are not explicitly modeled here, they should provide enough justifications for the assumption that the young always work.

As a warm-up exercise, let us consider a special case, where the utility function does not depend on c^{y}_{t-1}. That is, the agent cares only consumption and leisure when old. Then, the agent saves all the wage income, $s_t = w_{t-1}$, independent of the retirement decision, and hence the total supply of capital in period t is simply $K_t = w_{t-1}$. This simplification has two advantages. It allows us to focus on the retirement/consumption trade-off faced by the old. It also helps us to see how the retirement decision by the current old generation will affect all the future generations through the labor market, without the complication that arises from the joint saving-retirement decision.

The preferences of the old agent in period t are now described by $U(c^{o}_{t}, e_t)$. It is increasing in c^{o}_{t} and $U(c^{o}_{t},1) < U(c^{o}_{t}, 0)$ for all $c^{o}_{t} > 0$. Define a positive-valued function, ϕ, by

(3) $$U(c^{o}_{t} + \phi(c^{o}_{t}), 1) \equiv U(c^{o}_{t}, 0),$$

which may be interpreted as the value of leisure when old.

An agent, who has earned w_{t-1} and saved $K_t = w_{t-1}$ when young, receives $r_tK_t = r_tw_{t-1}$. If s/he retires, the utility level is $U(r_tK_t, 0) = U(r_tK_t + \phi(r_tK_t), 1)$. On the other hand, if s/he continues to work, it is $U(r_tK_t + w_t\theta, 1)$. Therefore, an old agent in period t chooses to work if $\phi(r_tw_{t-1}) < w_t\theta$; s/he chooses to retire if $\phi(r_tw_{t-1}) > w_t\theta$; and s/he is indifferent $\phi(r_t w_{t-1}) = w_t\theta$.

Given $w_{t-1} = K_t$, the equilibrium in period t is given by eqs. (1) and (2), and

(4) $$k_t = K_t/(1+\theta x_t) = w_{t-1}/(1+\theta x_t),$$

(5) $$x_t \begin{cases} = 1 & \text{if} \quad \phi(r_t w_{t-1}) < w_t\theta \ , \\ \in [0,1] & \text{if} \quad \phi(r_t w_{t-1}) = w_t\theta \ , \\ = 0 & \text{if} \quad \phi(r_t w_{t-1}) > w_t\theta \ . \end{cases}$$

These conditions jointly determine the mapping from w_{t-1} to w_t, $w_t = \Psi(w_{t-1})$, which can be applied iteratively to solve for the equilibrium trajectory of the economy, for any initial condition, $w_0 = K_1$.

3.A. Exogenous Retirement

Before proceeding, let us first consider the case, where the old generation's labor force participation rate is given exogenously, $x_t \in [0,1]$, for all t. That is, a fraction, $1-x_t$, of the old generation in period t retires, and the rest stays in the labor force. The Diamond overlapping generations model is a special case, where $x_t = 0$ for all t. It is well known (see, for example, Azariadis 1993, Part II) that the dynamics in the Diamond model may be complicated unless additional restrictions are imposed on the production function. Since the goal here is to provide a benchmark for the case of endogenous retirement, we impose the restriction so that, without the endogeneity of retirement, the dynamics is "well-behaved."

More specifically, it is assumed that the production function is a Cobb-Douglas, $f(k) = Ak^{\alpha}$, where $\alpha \in (0,1)$ is the capital share. Then, (2) becomes $w_t = (1-\alpha)A(k_t)^{\alpha}$. From (4) and $w_{t-1} = K_t$, the dynamics is described as

$$(6) \qquad w_t = (1-\alpha)A(k_t)^{\alpha} = \frac{(1-\alpha)}{(1+\theta x_t)^{\alpha}} A(w_{t-1})^{\alpha}$$

Figure 1 illustrates the dynamical system, (6), under the assumption that the labor force participation rate is constant over time, $x_t = x$. For any x, the mapping is globally concave and the dynamics has a unique steady state. A higher x shifts down the mapping, reducing the wage rate and capital stock in the steady state. The parameter that governs the persistence of the dynamics is equal to the capital share, α, independent of x.

3.B. Endogenous Retirement

We are now ready to examine the effect of endogenous retirement. The dynamics is now described by (5) as well as (6), with $w_t = (1-\alpha)A(k_t)^{\alpha}$ and $r_t = \alpha A(k_t)^{\alpha-1}$. In order to obtain a closed-form solution for the mapping, let us consider $\phi(c^o) = \Lambda(c^o)^{\lambda}$; $\Lambda > 0$ and $\lambda \in (1, \infty)$. The assumption, $\lambda > 1$, ensures that, as the economy develops, the income effect of a rising wage income, which encourages retirement, dominates its price effect, which discourages retirement, as the empirical evidence suggests. Some algebra yields

$$(7)\qquad w_t = \Psi(w_{t-1}) \equiv \begin{cases} \dfrac{(1-\alpha)}{(1+\theta)^{\alpha}} A(w_{t-1})^{\alpha} & \text{if} \quad w_{t-1} \in (0, w^{-}], \\[2ex] (1-\alpha)\Omega^{(\mu-1)}\left[A(w_{t-1})^{\alpha}\right]^{\mu} & \text{if} \quad w_{t-1} \in (w^{-}, w^{+}), \\[2ex] (1-\alpha)A(w_{t-1})^{\alpha} & \text{if} \quad w_{t-1} \in [w^{+}, \infty), \end{cases}$$

where

$$(8)\qquad \mu \equiv \frac{\lambda}{\alpha + \lambda(1-\alpha)} > 1,$$

$$(9)\qquad \Omega \equiv \left(\frac{\Lambda \alpha^{\lambda}}{\theta(1-\alpha)}\right)^{1/(\lambda-1)}$$

and

$$(10)\qquad w^{-} \equiv \frac{(1+\theta)^{1/(1-\mu)}}{(A\Omega)^{1/\alpha}} < w^{+} \equiv \frac{1}{(A\Omega)^{1/\alpha}}.$$

The choice of parameterization in (7)-(10) was made so that the reader can see how the map, (7), depends on A and Λ, by inspection. The map is also illustrated in Figures 2a and 2b, which assume that the map intersects with the 45° line in the interval, (w^{-}, w^{+}). (It is easy to find a set of parameter values that ensures the existence of such an intersection by adjusting, say, Λ.)

If $w_{t-1} \in (0, w^{-}]$, the old generation, having earned and saved little when young, does not retire: the labor force participation rate is $x_t = 1$. If $w_{t-1} \in [w^{+}, \infty)$, the old generation, having earned and saved enough when young, chooses to retire: $x_t = 0$. Thus, the dynamics of the economy in both the lower and higher ranges is similar to the case of an exogenous labor force participation rate. In particular, the persistence parameter is equal to α.

In the middle range, (w^-, w^+), the labor force participation rate changes with w_{t-1}. Some algebra shows that the capital-labor ratio and the labor force participation rate in this range change with $K_t = w_{t-1}$, as follows.

$$(11) \qquad k_t = \left(A\Omega\right)^{\mu-1/\alpha}\left(w_{t-1}\right)^{\mu} = \left(w^{+}\right)^{1-\mu}\left(w_{t-1}\right)^{\mu} \quad ,$$

$$(12) \qquad x_t = \frac{1}{\theta}\left[\left(\left(A\Omega\right)^{1/\alpha} w_{t-1}\right)^{1-\mu} - 1\right] = \frac{\left(w_{t-1}\right)^{1-\mu} - \left(w^{+}\right)^{1-\mu}}{\left(w^{-}\right)^{1-\mu} - \left(w^{+}\right)^{1-\mu}} \quad .$$

An increase in the wage rate leads to a decline in the participation rate, and to a more-than-proportionate increase in the capital-labor ratio.

The homogeneity of the agents makes it simple to derive the map in the middle range, where $x_t \in (0,1)$. It is given by the condition that the agents are indifferent between working and retiring when old. A large Λ (a higher value of retirement), by increasing Ω, reduces the equilibrium labor force participation rate and the capital-labor ratio, as seen in (11) and (12). Therefore, it shifts the map, Ψ, upwards in the middle range, while it has no effect on the map in both the low and high ranges. The middle range itself moves to the left in response.

An exogenous increase in A, the total factor productivity, shifts the map upward everywhere. However, the shift is bigger in the middle range, and the range itself moves to the left, because the old generation earns more capital income out of saving, which reduces the labor force participation rate and the capital-labor ratio, as seen in (11) and (12).

3.C. Persistence

Note that, in the middle range, the elasticity of the map is equal to $\alpha\mu > \alpha$, higher than the case with a constant x. In other words, the persistence of the dynamics is magnified by the factor to $\mu > 1$. Eq. (8) shows that the magnification factor μ is increasing in λ. When the value of retirement rises sharply with economic growth, a higher wage rate would be needed to keep the old generation in the labor force. Note also that the magnification factor μ is independent of θ.

This feature of endogenous retirement, the magnification of persistence, has important implications for the growth process. If $\alpha\mu < 1$, the case depicted in Figure 2a, the economy still converges to the unique steady state. However, the speed of convergence is slower, as the economy traverses through the middle range. If $\alpha\mu = 1$, the map generates a unit-root dynamics in the middle range. If $\alpha\mu > 1$, the map becomes convex in the middle, so that growth accelerates. Furthermore, there may exist multiple steady states, two stable and one unstable, as depicted in Figure 2b. In the lower stable steady state, the wage is low and people do not retire. In the higher stable steady state, the wage is high and people retire. Two otherwise identical economies, if their initial positions are separated by the unstable steady state, converge to different steady states.

When an economy is trapped in the lower of the two stable steady states, a variety of the government policies can be used to lift the economy out of the trap and to move toward the higher stable steady state. This can be done either by forcing the elderly to retire or by subsidizing the elderly to retire. (Indeed, the subsidy does not need to be conditioned on retirement; even a simple, unconditional transfer to the elderly can induce the elderly to retire because retirement is a normal good.) It should be pointed out, however, that the above mentioned policies are not Pareto-improving. These policies must reduce the welfare of either the current old generation (if they are forced to retire) or that of the future generations (if they are imposed taxes to finance the subsidies). The reason is simple. The model has neither externality (because all the intertemporal linkages operate through markets) nor dynamic inefficiencies (because the steady state interest rate is positive, and hence higher than the steady state growth rate of the economy, which is equal to zero). Hence, the equilibrium allocation of the economy described above, even in the case where the economy is trapped in the lower stable steady state, is Pareto-efficient, and hence cannot be Pareto improved by means of taxes, subsidies, transfers, and other standard corrective policy measures.

How big is the magnification effect of endogenous retirement? As it stands, the model does not generate quantitatively significant effects. Even when λ is taken arbitrarily large, $\alpha\mu$ cannot be greater than $\alpha/(1-\alpha)$, which is equal to 0.5 for $\alpha = 1/3$ and to 2/3 for $\alpha = 0.4$. For the map to become convex in the middle and to have multiple steady states, it is necessary to have $\alpha > 1/2$.

This merely suggests, however, that the labor supply effect of endogenous retirement *alone* cannot explain a quantitatively large persistence. The next section will introduce a retirement motive for saving. It will be shown that, when the labor supply and saving effects are both operative, endogenous retirement can explain a quantitatively large persistence for an empirically plausible value of α.

Alternatively, endogenous retirement can be combined with other mechanisms for persistence. It is worth noting that endogenous retirement not only supplements other mechanisms for persistence, but also enhances their power of generating large persistence. As an illustration, let us suppose that the total factor productivity, A, is now endogenous, and evolves according to

(13) $$A_t = A_0(K_t)^{\gamma}.$$

The idea is that the level of aggregate capital stock can also be viewed as a proxy for knowledge capital. Through knowledge spillover, capital stock affects the total factor productivity of the economy, and γ measures the externality effect of knowledge spillover. Since this effect is purely external, the agent does not take it into account when making decisions. Aggregate externality of this kind has been suggested by Romer (1986) and other studies in the endogenous growth literature as a way of generating persistence in dynamics.

By inserting (13) into (6) and by recalling $K_t = w_{t-1}$, it can be shown that the dynamics would follow

(14) $$\ln\left(w_t\right) = \text{const.} + \left(\alpha + \gamma\right)\ln\left(w_{t-1}\right),$$

when the old generation's labor force participation rate is exogenous and constant. To generate accelerating growth and the possibility of multiple steady states, γ has to be much larger than α for any plausible value of α. For example, $\alpha = 1/3$ implies that γ must be more than twice as large as α. Even with $\alpha = 0.4$, γ must be more than 50% larger than α.

With endogenous retirement, on the other hand, the dynamics follows, from (7) and (13),

(15) $$\ln\left(w_t\right) = \text{const.} + \mu\left(\alpha + \gamma\right)\ln\left(w_{t-1}\right) = \text{const.} + \frac{\lambda\left(\alpha+\gamma\right)}{\alpha+\lambda(1-\alpha)}\ln\left(w_{t-1}\right),$$

in the middle range. Note that endogenous retirement not only generates persistence in addition to the externality effect, but also enhances the externality effect. Hence, multiple steady states are possible for a plausible value of α, and much smaller γ. (For example, for $\alpha = 1/3$, $\gamma > 1/3$ would suffice for a sufficiently large λ. For $\alpha = 0.4$, $\gamma > 0.2$ would suffice.)

4. Introducing a Joint Saving-Retirement Decision

Let us now introduce the retirement motive for saving. As stated before, the problem of the agent, who becomes old in period t, can be described as: given w_{t-1}, choose $c^y_{t-1} \geq 0$, $c^o_t \geq 0$, and $e_t \in \{0,1\}$ to maximize $U(c^y_{t-1}, c^o_t, e_t)$ subject to $c^o_t = r_t s_t + w_t\,\theta\, e_t = r_t(w_{t-1} - c^y_{t-1}) + w_t\,\theta\, e_t$. For the rest of the analysis, two strong restrictions will be imposed on the utility function.

First, c^y_{t-1} and c^o_t are assumed to be weakly separable from e_t, so that the intertemporal preferences for consumption is independent of the retirement decision. Without such a restriction, the effect of retirement decision on saving can be arbitrary. The result can change according to how the marginal rate of substitution between consumption in two periods depends on e_t. Retirement may reduce marginal utility of some consumption items, such as business suits, while raising that of other items, such as books. Such an introspection, however, may not be useful for the level of aggregation that we are dealing with. Although retirement may increase marginal utility of books, reading books may help the retired person's need for other items, thereby reducing marginal utility of consumption in general. The assumption of weak separability, while restrictive, seems to offer a most natural benchmark.[7] Note that the weak separability assumption does not eliminate the retirement motive for saving, because the retirement decision affects the time-profile of labor income. It simply means that the retirement cannot affect saving by changing the intertemporal preferences over consumption.

Second, in order to focus on the magnification effect of endogenous retirement, one needs to impose the restriction on the preferences in such a way that, when the labor force participation rate is constant, $x_t = x$, the persistence parameter is equal to α, independent of x. This is satisfied if and only if intertemporal preferences are Cobb-Douglas.

These two restrictions jointly imply that the utility function can be written in the form,

$$U(c^y_{t-1},\ c^o_t,\ e_t) \equiv U(z_t, e_t), \quad \text{with } z_t \equiv Z(c^y_{t-1},\ c^o_t) = \left(\frac{c^y_{t-1}}{1-\beta}\right)^{1-\beta}\left(\frac{c^o_t}{\beta}\right)^{\beta},$$

where $\beta \in (0,1]$ is a constant, independent of e_t. Note that the preferences here are not intertemporally separable.

Given the preference assumed, the utility maximization yields,

[7] At least, the results under this assumption should be considered "neutral." Interestingly enough, the assumption of weak separability between the retirement decision and two period consumptions was made implicitly in Feldstein's (1974) graphic analysis, when he drew the indifference curves defined over the space, (C_1, C_2) in his notation, independent of the agent's retirement behavior.

(16) $s_t = w_{t-1} - c^y_{t-1} = \beta w_{t-1} - (1-\beta)\left(\dfrac{w_t\theta}{r_t}\right)e_t$.

Eq. (16) shows that the agent's saving is contingent on the retirement decision. Those who retire save more than those who don't. The difference, "retirement motive for saving," would be larger if β is smaller. Eq. (16) also shows the positive correlation between the asset holding of the agents at the beginning of their second period and their retirement decision. This should not be interpreted that the wealthy retires early, because the wealth and the retirement are jointly determined in this model.

The value of z_t also depends on the retirement decision, as follows.

(17) $z_t = \left[w_{t-1} + \left(\dfrac{w_t\theta}{r_t}\right)e_t\right]\left(r_t\right)^{\beta}$,

hence, the opportunity cost of retirement is equal to $w_t\theta(r_t)^{1-\beta}$. Therefore, the retirement decision is given by

$$
(18)\qquad e_t \quad \begin{cases} = 1 & \text{if} \quad \phi\left((r_t)^{\beta} w_{t-1}\right) < w_t\theta(r_t)^{1-\beta}\ , \\ \in \{0,1\} & \text{if} \quad \phi\left((r_t)^{\beta} w_{t-1}\right) = w_t\theta(r_t)^{1-\beta}\ , \\ = 0 & \text{if} \quad \phi\left((r_t)^{\beta} w_{t-1}\right) > w_t\theta(r_t)^{1-\beta}\ , \end{cases}
$$

where ϕ, the value-of-retirement function, is defined in a manner similar to (3), as follows. $U(z_t+\phi(z_t), 1) \equiv U(z_t, 0)$.

The aggregate saving by the young in period t−1, which is equal to K_t, can be obtained from (16) by aggregating across all the agents, as follows.

(19) $K_t = \beta w_{t-1} - (1-\beta)\left(\dfrac{w_t\theta}{r_t}\right)x_t$

where x_t is the labor force participation rate when old. Since the labor supply is $L_t = (1+\theta x_t)$, this implies that

$$(20) \qquad k_t = \frac{\beta w_{t-1} - (1-\beta)\theta(w_t / r_t)x_t}{1+\theta x_t}.$$

Note that the retirement decision by the old affects the capital-labor ratio through two different routes. One is the labor supply effect, which appears in the denominator of (20). The other is the saving effect, captured in the numerator. Both effects work in the same direction. A higher labor force participation rate by the elderly thus implies a lower capital-labor ratio, hence a lower wage for the next generation.

The labor force participation rate satisfies in equilibrium,

$$(21) \qquad x_t \begin{cases} = 1 & \text{if} \quad \phi\left((r_t)^{\beta} w_{t-1}\right) < w_t \theta (r_t)^{1-\beta}, \\ \in [0,1] & \text{if} \quad \phi\left((r_t)^{\beta} w_{t-1}\right) = w_t \theta (r_t)^{1-\beta}, \\ = 0 & \text{if} \quad \phi\left((r_t)^{\beta} w_{t-1}\right) > w_t \theta (r_t)^{1-\beta}. \end{cases}$$

Given w_{t-1} the equilibrium in period t is given by eqs. (1), (2), (20) and (21).

4.A. Exogenous Retirement

Before proceeding, let us take a brief look at the case of an exogenous retirement. Under the assumption of the Cobb-Douglas production function, (20) implies that the wage rate in period t is given by,

$$(22) \qquad w_t = (1-\alpha)A(k_t)^{\alpha} = (1-\alpha)\left[\frac{\beta}{1+\{\theta+(1-\beta)\theta(1-\alpha)/\alpha\}x_t}\right]^{\alpha} A\left(w_{t-1}\right)^{\alpha}$$

When $x_t = x$, the map is globally concave, and the economy converges to the unique steady state, as depicted in Figure 1. Note also that the persistence of the dynamics is equal to α, independent of x.

4.B. Endogenous Retirement

Let us now look at the case, where the labor force participation rate is endogenous. Again, assuming that $f(k) = A(k)^{\alpha}$, $\alpha \in (0,1)$, and $\phi(z) = \Lambda(z)^{\lambda}$; $\Lambda > 0$, $\lambda \in (1, \infty)$, eqs. (21) and (22) imply

$$(23)\quad w_t = \Psi(w_{t-1}) \equiv \begin{cases} (1-\alpha)(\beta^*)^{\alpha} A(w_{t-1})^{\alpha} & \text{if} \quad w_{t-1} \in (0, w^-], \\ (1-\alpha)\left[\Omega^{\alpha} A^{\beta-1}\right]^{\frac{\mu-1}{1-\beta(1-\alpha)}}\left[A(w_{t-1})^{\alpha}\right]^{\mu} & \text{if} \quad w_{t-1} \in (w^-, w^+), \\ (1-\alpha)\beta^{\alpha} A(w_{t-1})^{\alpha} & \text{if} \quad w_{t-1} \in [w^+, \infty), \end{cases}$$

where

$$(24)\qquad \mu \equiv \frac{\lambda}{\alpha + (\lambda\beta + 1 - \beta)(1-\alpha)} > 1,$$

$$(25)\qquad \beta^* \equiv \frac{\beta}{1 + \theta + (1-\beta)\theta(1-\alpha)/\alpha} < \beta$$

$$(26)\qquad \Omega \equiv \left(\frac{\Lambda\alpha^{\lambda\beta+(1-\beta)}}{\theta(1-\alpha)}\right)^{1/(\lambda-1)}.$$

$$(27)\qquad w^- \equiv \frac{(\beta^*)^{1/\mu-1}}{\left(A^{\beta}\Omega\right)^{1/1-\beta(1-\alpha)}} < w^+ \equiv \frac{\beta^{1/\mu-1}}{\left(A^{\beta}\Omega\right)^{1/1-\beta(1-\alpha)}}\quad.$$

It is easy to verify that (23) is an extension of (7), by letting $\beta = 1$. This extension keeps the qualitative features of (7). In particular, there is a middle range, in which the labor force participation rate changes, and the persistence parameter is greater than α. Here, the magnification factor is given by (24). In this range, the capital-labor ratio and the labor force participation rate change as follows.

$$(28)\qquad k_t = \left(A^{\beta}\Omega\right)^{\mu-1/1-\beta(1-\alpha)}\left(w_{t-1}\right)^{\mu} = \beta\left(w^+\right)^{1-\mu}\left(w_{t-1}\right)^{\mu}\quad,$$

$$(29)\qquad x_t = \frac{\alpha}{\alpha\theta + (1-\alpha)(1-\beta)\theta}\left[\beta\left(\left(A^{\beta}\Omega\right)^{1/_{1-\beta(1-\alpha)}} w_{t-1}\right)^{1-\mu} - 1\right] = \frac{\left(w_{t-1}\right)^{1-\mu} - \left(w^{+}\right)^{1-\mu}}{\left(w^{-}\right)^{1-\mu} - \left(w^{+}\right)^{1-\mu}} \ .$$

It can also easily be seen that the effects of A and Λ are similar as before.

The effect of a change in β, on the other hand, is difficult to evaluate; as seen in (23)-(27), β appears everywhere. Nevertheless, what interests us most is how the magnification effect of endogenous retirement changes with the introduction of retirement motive for saving. Eq. (24) shows that a smaller β makes μ larger. For example, let $\beta = 1/2$, so that the agent puts equal weight on each period. Then, for a sufficiently large λ, the map becomes convex in the middle, for any $\alpha > 1/3$. If $\beta = 1/3$, which implies the agent's discount rate is about 2.2% per year, if the period length is 30 years, then for $\alpha = 1/3$, $\lambda > 1.4$ is enough for the convexity. Note, again, that μ is independent of θ.

5. Heterogeneity of Agents

In the model presented above, the homogeneity of the agents and the zero-one nature of the retirement decision ensured that the price elasticity of the labor supply by the old generation (as a group) is infinite. This implies that, if the old generation's labor force participation rate is between zero and one, the agents must be indifferent whether to retire, which greatly simplified the analysis. This supply-side condition alone determines the map, $w_t = \Psi(w_{t-1})$, in the middle range. The demand factors for labor need to be evoked only to pin down the value of x, the elderly's labor force participation rate. Generally speaking, the presence of heterogeneity or allowing for a partial retirement would make the analysis more difficult because both the supply and demand sides would then need to be taken into account to derive the map. This section discusses one relatively simple way of making the price elasticitity of the elderly's labor supply finite without losing the tractability of the model, by introducing a form of heterogeneity into the analysis.

Suppose now the agents differ only in their reservation wage function, $\phi(z)$, which has the following simple form: $\phi(z) = 0$ if $z < \eta$, and $\phi(z) = M$ if $z \geq \eta$, where M is a sufficiently large, but finite number. The agents differ in their values of η, and let $G(\eta)$ be the distribution function. Then, from (18), $e_t = 1$ if and only if $(r_t)^{\beta} w_{t-1} < \eta$, and hence the labor force participation rate is

$$(30) \qquad x_t = 1 - G\left((r_t)^{\beta} w_{t-1}\right) = 1 - G\left(\left(\alpha^{\alpha}(1-\alpha)^{1-\alpha} A\right)^{\frac{\beta}{\alpha}} (w_t)^{\frac{\beta(\alpha-1)}{\alpha}} w_{t-1}\right).$$

The equilibrium dynamics is now determined jointly by (22) and (30). Clearly, the case of an exogenous retirement is a special case, where G is constant. In this case, the persistence parameter is α. When G is degenerate and has a mass on a single point, eq. (30) shows that $x_t \in (0,1)$ requires that $(w_t)^{\beta(\alpha-1)}(w_{t-1})^{\alpha}$ must be constant. This implies that the persistence parameter is equal to $\alpha/\{\beta(1-\alpha)\}$. (Note that the magnification factor in this case is $1/\{\beta(1-\alpha)\}$, which coincides with (24) with $\lambda = +\infty$.)

Except these two extreme cases, one cannot solve for the map in a closed form (unless G is allowed to depend on parameters, such as α, β, and θ). Nevertheless, some qualitative effects of endogenous retirement can be seen from (22) and (30), which jointly imply the dynamics in the follow form:

$$(31) \qquad w_t = H\left((w_t)^{\beta(\alpha-1)}(w_{t-1})^{\alpha}\right)\left(w_{t-1}\right)^{\alpha},$$

where H is a function, which is increasing if and only if G is increasing. A total differentiation of (31) yields

$$(32) \qquad \frac{d\log(w_t)}{d\log(w_{t-1})} = \alpha\left[\frac{\alpha+\xi}{\alpha+\beta(1-\alpha)\xi}\right],$$

where ξ is the elasticity of H. Since $\beta(1-\alpha) < 1$, the magnification factor, given in the bracket, is greater than one, whenever $\xi > 0$. Hence, in any range where G is increasing, the map is steeper and the persistence parameter is bigger than the case of an exogenous retirement, as illustrated in Figure 3. A large heterogeneity (i.e., a smaller ξ) makes the magnification effect smaller, but expands the range in which the magnification effect operates.

6. Concluding Remarks

This paper endogenized the retirement decision in Diamond's overlapping generations model and studied the interdependence between the labor force participation by the elderly and economic growth. If the agents earned a high wage income when young, they retire when old. This decision leads to a lower labor supply (through a low participation rate of the elderly), and to a high capital accumulation (through saving for retirement). The resulting high capital-labor ratio implies a high wage income for the next generation. When the agents earned a low wage income, they continue to work and save little, which implies a low capital-labor ratio and a low wage income for the next generation. Due to such positive feedback mechanisms, the endogeneity of retirement magnifies the persistence of growth dynamics, thereby slowing down a convergence to the steady state, and even generating multiple steady states for empirically plausible parameter values.

Obviously, there are many ways in which the model can be extended. Only a few will be suggested below. First, while section 5 discusses the case where the agents differ in their reservation wages, differences in the asset holding and in earning capacity might be more important as a source of heterogeneity that affects the retirement decision. Earning capacity differences can be modeled by letting different agents endowed with different effective units of labor. Such an extension would generate some interesting predictions. For example, if the ratio of the effective units endowed when young and when old is the same across agents, those with higher ability would tend to retire than those with less ability. On the other hand, if those with lower earning when young happen to be those whose earning capacity depreciate faster as they age,--such an assumption may be a reasonable way of capturing the situation that the job held by unskilled may be more physically demanding--, then this result can be reverse, and the poor may retire early, the prediction consistent with the evidence that the better educated tends to retire later (see, for example, Fuchs 1983).

Second, the model can be extended to allow stochastic shocks. For example, suppose that the total factor productivity, A, is subject to i.i.d. shocks. If there is a negative shock in period t, the old generation may find themselves short of the retirement income, and be forced to work. The negative shock would thus reduce the young generation's wage income in period t not only through the direct effect of lower productivity but also through the indirect effect of higher labor force participation by the old generation. The lower wage income when young in turn force this generation to work in period t+1, which depresses the next generation's wage income. Thus, a temporary technology shock propagates across periods. Or, the population growth may be subject to shocks. A baby boom generation might have to work longer, which affects the next generation. Or, the impacts of a large inflow of migration in one period may affect not only the current generation, but also the future generations. Or, when wars, plagues and other shocks that decimate one generation could boost the wage income for the next generation so much that the economy may escape from the lower steady state, and all the future generations may enjoy higher standard of living. (For example, some historians suggest that Black Death set up the stage for the future European Miracle, by boosting the wage income.) It should be pointed out that the present model has neither externalities nor dynamic inefficiencies. The above argument merely suggests that the loss in one generation is accompanied by the gains in the future generations, and does not imply any Pareto improvement.

Third, the mechanism identified in the present paper, macroeconomic impacts of retirement decision, should be an important factor when evaluating social security systems, which is indeed the original motivation of Feldstein (1974). To make a compelling numerical assessment of the impacts of such induced retirement, however, it would be necessary to extend the model to allow the agents to live many periods and to decide how soon they would retire. It is hoped that the present model would be a useful first step for this purpose.

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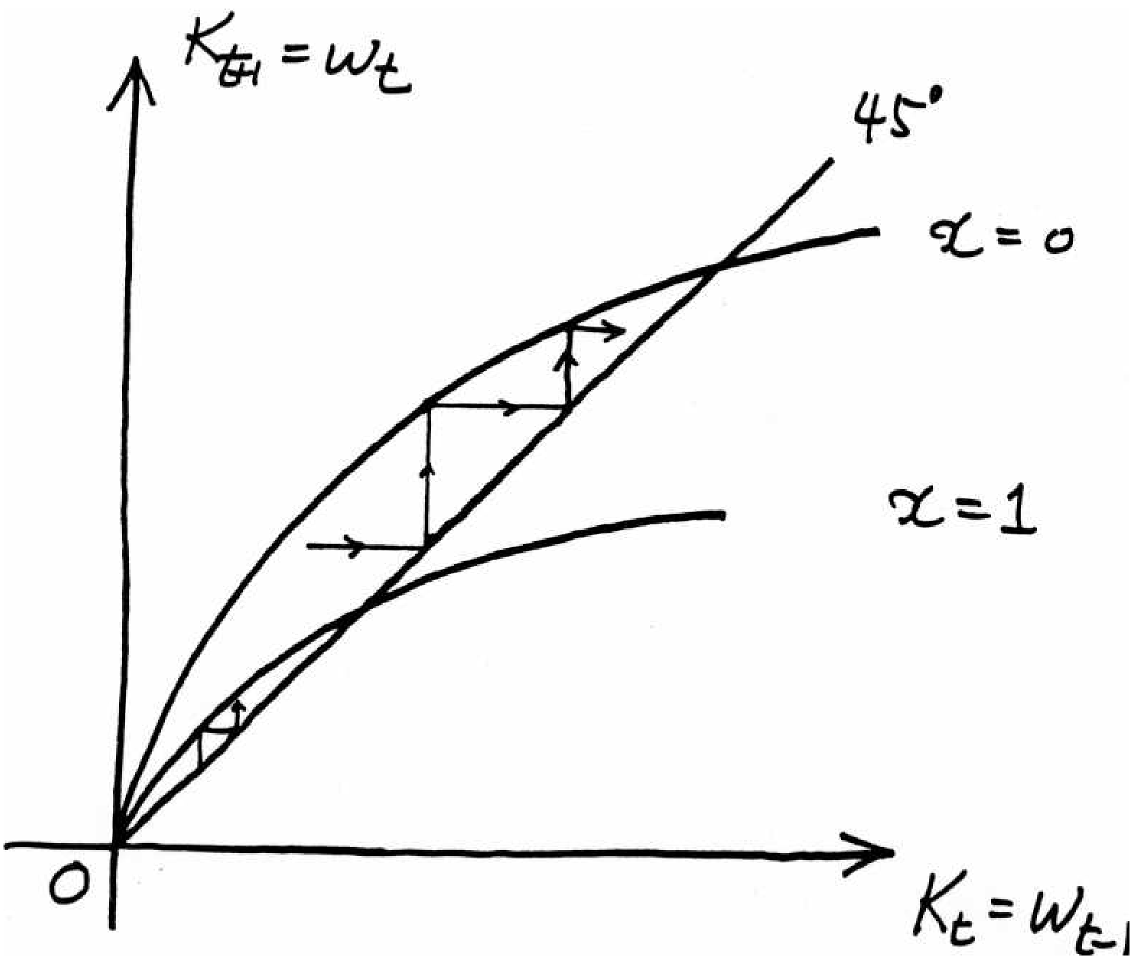

Fig. 1.

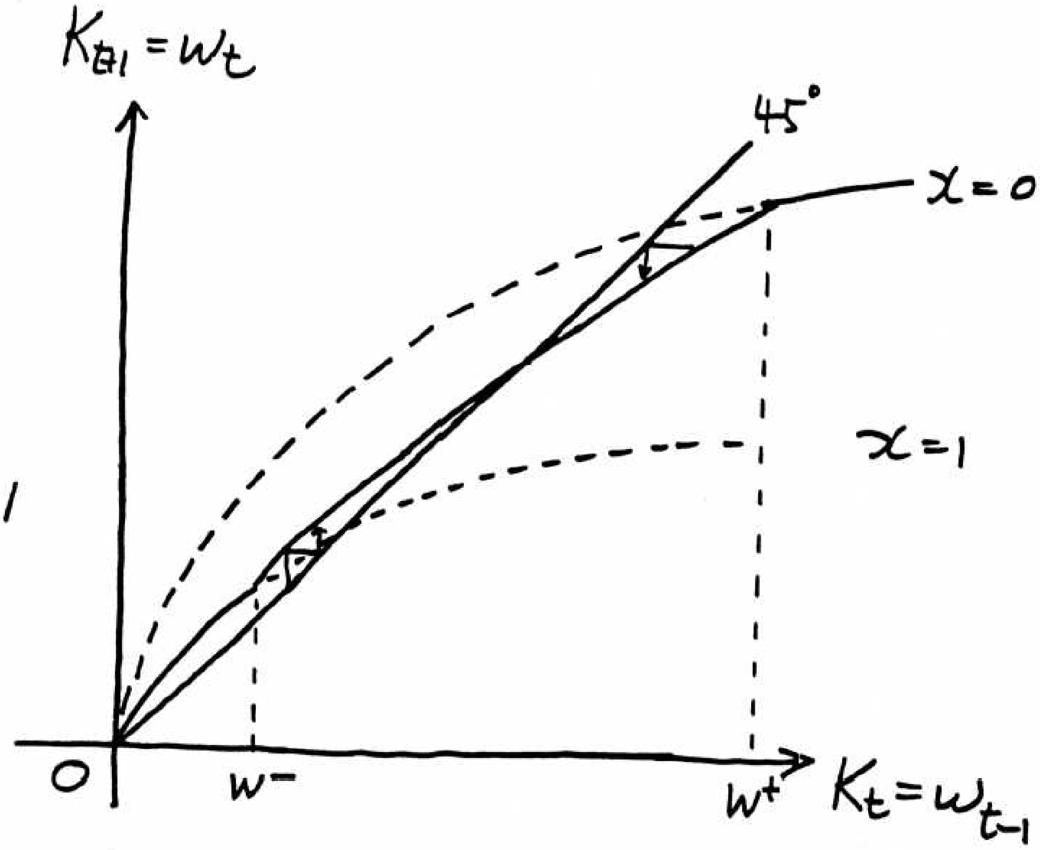

Fig 2a.
$\alpha\mu < 1$

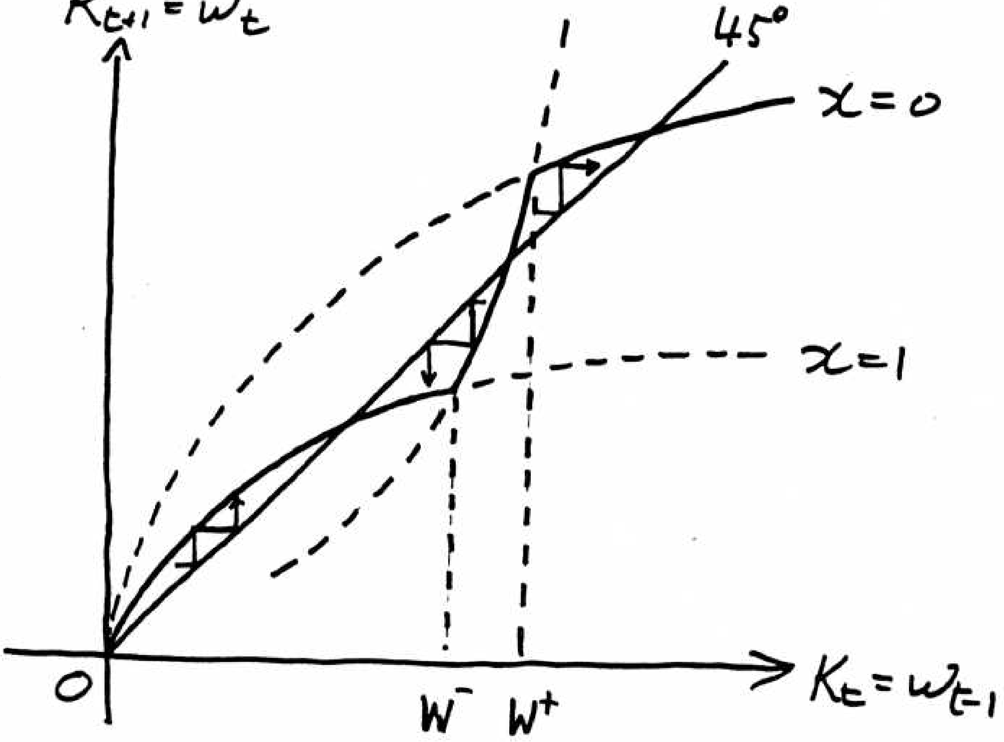

Fig 2b.
$\alpha\mu > 1$

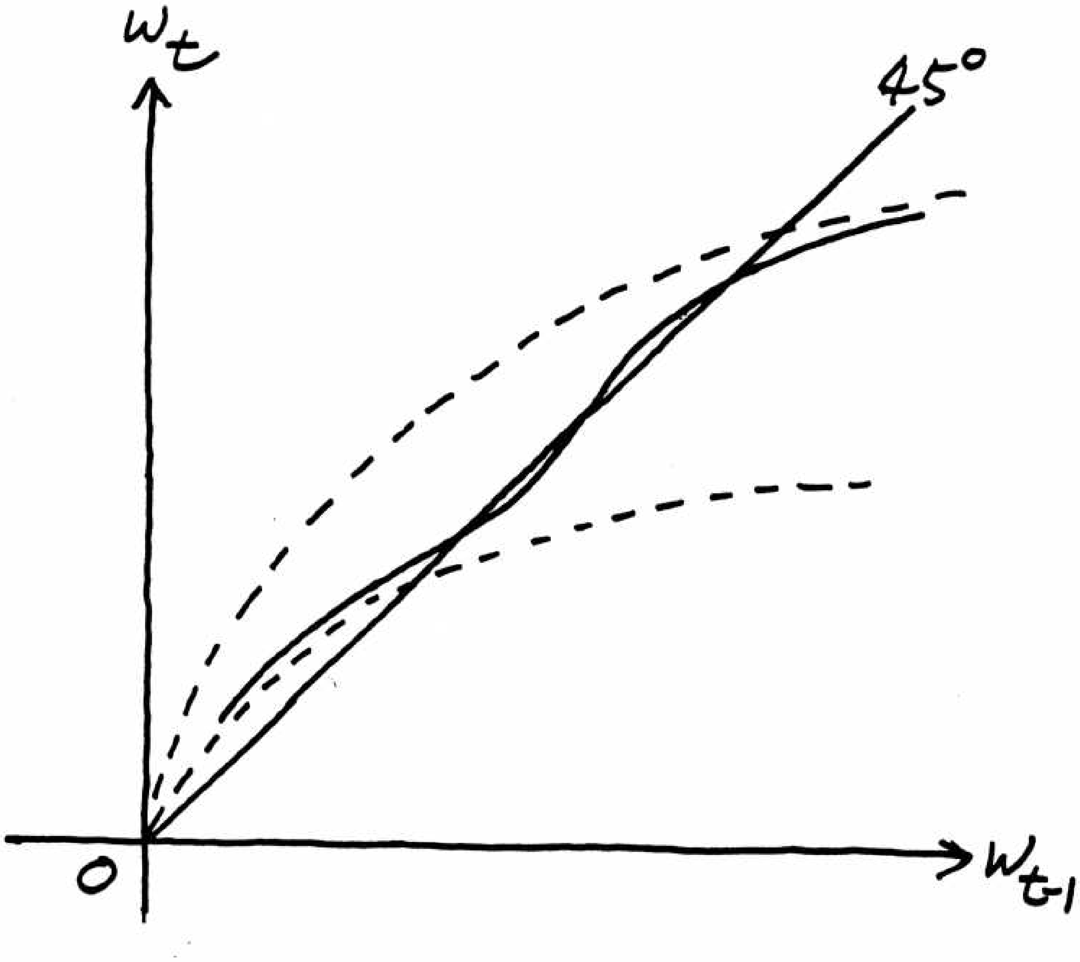

Fig. 3