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Randomization, Communication and Efficiency in Repeated Games with Imperfect Public Monitoring

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Randomization, Communication and Efficiency in Repeated Games with Imperfect Public Monitoring[†]

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[†] The original paper was entitled, "Check Your Partners' Behavior by Randomization" (Kandori (1999)). In particular, the present paper is mainly based on Section 4 of the original paper, while Section 3 (on private equilibria) is incorporated in Kandori and Obara (2000).

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Abstract

The present paper shows that the Folk Theorem under imperfect (public) information (Fudenberg, Levine and Maskin (1994)) can be obtained under much weaker set of assumptions, if we allow communication among players. Our results in particular show that *for generic symmetric games with at least four players, we can drop the FLM condition on the number of actions and signals altogether and prove the folk theorem under the same condition as in the perfect monitoring case*.

1. Introduction

The present paper shows that the Folk Theorem under imperfect (public) information (Fudenberg, Levine and Maskin (1994), FLM hereafter) can be obtained under much weaker set of assumptions, if we allow communication among players. The FLM Folk Theorem considers long-term relationships, where players can publicly observe some signal, whose probability distribution is affected by their unobservable actions. The FLM Folk Theorem shows that in such a situation, any feasible and individually rational outcomes can be sustained *when the signal takes on sufficiently many values relative to the number of available actions*. In particular, in generic symmetric games with K actions for each player and L different values for the signal, the FLM Folk Theorem obtains when $2K-1 \leq L$.

While this condition covers a wide range of applications with "rich" signal spaces, it fails when only coarse information is available. For example, the FLM Folk Theorem fails when the *outcome (of the signal) is either success or failure* (L=2). What is perhaps more troubling, however, is the fact that we may not have a precise idea on the number of possible signal outcomes and actions in any given application. Therefore, it would be valuable to know if the Folk Theorem also holds for the case when the number of possible outcomes is rather small. In the present paper, we show that the Folk Theorem also holds for the case with small signal space, once we let players communicate.

In particular, we show that *for generic symmetric games with at least four players, we can drop the FLM condition on the number of actions and signals altogether and prove the folk theorem under the same condition as in the perfect monitoring case*. We mentioned the symmetric case as it provides a clean sufficient condition, but the symmetry is not essential; our condition for the Folk Theorem, (3), can also be satisfied for a wide class of (asymmetric) games, even when the signal is binary (success or failure) so that the available information is minimal.

Our equilibrium has the following features. At the end of each period, each player announces which action he has just taken. Players can tell a lie but we will construct equilibria where they voluntarily reveal the true actions[1]. Thus the players' future payoffs can depend both on the signal and actions, and a rich class of information can be generated when players *randomize.* The basic logic is similar to the random sampling technique for quality control. A factory manager randomly picks up one of the products assembled by workers and examines its quality. Even though the outcome of the test is only binary (pass or failure), it can discipline a large number of workers. Note well that, to provide incentives for the workers, their compensation should depend

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[1] Communication has been introduced in repeated games with imperfect *private* monitoring, where each player privately receive a different signal about the past actions (Ben-Porath and Kahneman (1996), Compte (1998) and Kandori and Matsushima (1998)). The main role of communication in those works, which mainly look at pure strategies, is to coordinate the players' actions, rather than to generate more information.

not only on the test result (the value of the *signal*) but also on whose product was examined (the *action* of the manager). Similarly in repeated games, if players randomize over different actions and the future payoffs can depend *both* on the realized signal *and* actions, rich information can be generated to discipline the players[2].

A similar idea can be embodied in games without communication. Even without communication, it is possible to construct an equilibrium where players randomize over different actions and the future payoffs depend *both* on the realized signal *and* actions. Such an equilibrium is called a *private equilibrium*, which is ignored by the majority of the existing work, including FLM and Abreu, Pearce and Stacchetti (1990). A companion paper by Kandori and Obara (2000) explores this issue and shows that such an equilibrium can provide significant welfare improvements.

2. The Model

A group of players i=1,2, …, N repeatedly play a stage game over an infinite time horizon, t=0,1,…. The stage game payoff function for player i is given by $u_i(a_i, \omega)$, where $a_i \in A_i$ is player i's action and $\omega \in \Omega$ is publicly observable signal. Each player's action is not observable to other players. Given an action profile $a \in A = A_1 \times \cdots \times A_N$, the probability of ω is denoted by $p(\omega|a)$. (We assume that Ω and A are finite sets). Player i's *expected* stage game payoff is defined by $g_i(a) = \sum_{\omega \in \Omega} u_i(a_i, \omega) p(\omega|a)$. The *average payoff* of player i in the repeated game is given by

$(1-\delta)\sum_{t=0}^{\infty} \delta^t g_i(a(t))$, where a(t) refers to the action profile in period t and $\delta \in (0, 1)$ is the

discount factor. We denote a mixed strategy profile by $\alpha \in \Delta = \Delta_1 \times \cdots \times \Delta_N$, and abuse the notation to represent the corresponding expected payoff and signal distribution by $g_i(\alpha)$ and $p(\omega|\alpha)$. Finally, we define the stage game payoff profile by $g=(g_1,\ldots,g_N)$.

Let us now introduce communication in the model. In each period, each player i takes an action, observes ω, and then[3] (simultaneously with other players) announces a message m_i. As we examine the case where the players reveal their action, we assume $m_i \in A_i$. The players can tell a lie but we will construct equilibria where they voluntarily tell the truth, as we will see in the next section.

3. The Folk Theorem Conditions under Communication

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[2] More detailed explanation can be found in Section 2 (see the discussion following Table 2).

[3] Alternatively, we can assume that the players communicate *before* the realization of ω. Our results in Section 4 and 5 show that the players truthfully reveal their actions after each realization of ω. Then, under the same arrangements the players have incentives for truth-telling before observing ω. Hence our results hold for either specification.

Our equilibrium has the property that each player's future payoff is independent of what he announces. (Therefore, he has a *weak* incentive to tell the truth[4].) At the same time, we need to avoid punishing players simultaneously, as it entails welfare loss (see the explanation below). To put these ideas together, we employ the following construction; for any given pair of players, say 1 and 2, we *transfer* some of 1's future payoff to 2, when 1's cheating is suspected (and *vice versa*). Compared to the case where 1 and 2 are punished simultaneously, the transfer entails negligible welfare loss. This is one of the essential ideas employed by the FLM folk theorem. The transfer between 1 and 2 is based partly on the publicly observable signal ω, as in FLM, but partly on the announced actions of other[5] players ($i=3,4,\ldots,N$), which is the key to the improved efficiency obtained in the present paper. The transfer scheme to provide the right incentives can be constructed whenever player 1's actions are *statistically distinguished* from player 2's, so that we can tell which player is suspected of deviation (the precise conditions will be presented shortly). Note also that it needs *at least three players*, which will be assumed throughout the paper.

First let us introduce some notation. For a given pair of players i and j, we denote their opponents' action profile by $a_{-ij} \in A_{-ij} \equiv \prod_{k\neq i,j} A_k$, and their mixed action profile $\alpha_{-ij}\in\Delta_{-ij}$ is defined similarly. Let R_i be the matrix whose rows and columns are indexed by a_i and (ω,a_{-ij}) respectively, with its $(a_i,(\omega,a_{-ij}))$-element given by

$$\Pr(\omega,a_{-ij}|a_i) = \sum_{a_j} p(\omega|a)\prod_{k\neq i}\alpha_k(a_k).$$

Roughly speaking, R_i summarizes the observability of various actions of player i. *Each row in* R_i *represents the distribution of* (ω, a_{-ij}) *given each action of player i*, and recall that (ω, a_{-ij}) is the relevant information to determine the transfer between i and j. As this matrix depends on who else is in the given pair (i.e., player j) and other players' mixed action profile α_{-i}, we denote $R_i= R_i(j,\alpha_{-i})$. Now define $Q_{ij}(\alpha)$ by "stacking" R_i on R_j;

$$Q_{ij}(\alpha) = \begin{pmatrix} R_i(j,\alpha_{-i}) \\ R_j(i,\alpha_{-j}) \end{pmatrix}.$$

If $Q_{ij}(\alpha)$ has the maximum row rank, we can *statistically distinguish* what i does from what j does,

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[4] It is possible to provide a strict incentive for truth telling by modifying our equilibrium construction. Kandori and Matsushima (1998) show that when players privately observe random variables that are *not independent*, we can provide strict incentive to tell the truth. Hence it is not obvious that mixed actions, which are totally independent, can also be revealed with strict incentives. The crux of the matter is that, *conditional on* ω, the mixed actions are in fact *not* independent. The details can be found in the discussion paper version of the present paper (Kandori (1999)).

[5] This induces the truth-telling: players $i=2,3,\ldots,N$ has an (weak) incentive to tell the truth, as their future payoffs are not affected by what they say.

by looking at (ω,a_{-ij}), and it is possible to construct the efficient punishment (transfer) discussed above. This is basically the *pairwise full rank condition* of FLM, *except* that we utilize *both* ω *and* a_{-ij} to police player i and j, while FLM utilize only ω. The crux of the matter here is that *letting players randomize and supplementing the signal* ω *with realized actions* a_{-ij} *provide more information*.

The crude intuition for this can be obtained by the random sampling analogy discussed in the introduction, but let us provide more detailed explanation here. Suppose we have a joint production problem with three players, where each player's (unobservable) effort level is either H (high) or L (low). The observable signal ω takes on two values, S (success) or F (failure). Assume now a (possibly mixed) action profile α is being played. Without communication, we must determine the transfer between player 1 and 2 solely on the basis of ω. This is the traditional approach taken by FLM. The probability distribution of ω is given by the following matrix[6] (the counterpart of our matrix $Q_{ij}(\alpha)$):

	S	F
a_1=H		
a_1=L		
a_2=H		
a_2=L		

Table 1

For example, the first row represents the distribution of ω, when a_1=H, given that other players are mixing according to α_{-1}. To distinguish 1 and 2's behavior, we need that those rows are "distinct" in the sense of linear independence. As there is one linear constraint[7] for the rows, the distinguishability holds when the matrix has the maximum possible row rank 3. This is the *FLM pairwise full rank condition*. This condition, however, cannot possibly be satisfied in this example, as the matrix has only two columns. In other words, the binary signal ω=S,F cannot possibly distinguish two players' actions.

Suppose, in contrast, that we let players randomize and introduce communication, to use both ω and a_3 (the truthfully announced action by player 3) to determine the transfer between player 1 and 2. The distribution of (ω, a_3) changes according to 1 and 2's actions, and it is summarized by our matrix $Q_{12}(\alpha)$:

[6] The entries are left blank, as they are not essential for the argument here.

[7] $\alpha_1(H)\times$(the first row)+$\alpha_1(L)\times$(the second row) represents the distribution of ω under α, which is also equal to $\alpha_2(H)\times$(the third row)+$\alpha_2(L)\times$(the fourth row).

	a_3=H		a_3=L	
	S	F	S	F
a_1=H				
a_1=L				
a_2=H				
a_2=L				

Table 2

Now that we have more columns[8], it is possible that the matrix has the maximum row rank 3 (for a generic choice of entries[9]). In other words, with the richer information (ω, a_3), we can now distinguish player 1 and 2's actions. To understand why this is the case, following consideration may help. When player 1 reduces his effort, it may always increases the chance of failure (F), but *the magnitude of this effect depends on (among other things) player 3's effort level*. For example, if there is a strong complementarity between 1 and 3's efforts, the reduction of 1's effort sharply increases the chance of failure, when 3 is working hard (while the effect may be minute when 3's effort is low, as 1's effort is rather useless without the help of 3). When the complementarity is not so strong between 3 and 2's efforts, in contrast, the reduction of 2's effort has somewhat different effects on the relative likelihood of (ω, a_3). Hence we may statistically distinguish 1 and 2's effort levels by looking at (ω, a_3).

This is why letting 3 randomize and reveal his action through communication may be useful. *Although 3's action itself does not contain any information, the informativeness of the signal ω (concerning 1 and 2's actions) does in general depends on 3's action.*

Let us now generalize the observations obtained in the above example. Let $q_{ij}(a_k)$ (k=i,j) and $q_{ij}(\omega,a_{-ij})$ be the a_k-th row and the (ω,a_{-ij})-th column of matrix $Q_{ij}(\alpha)$. The simple case presented in Table 2 would be useful to understand the (unavoidably) heavy notation. The number of the columns is $|\Omega|\times|A_{-ij}|$, but as in Table 2 (see footnote 8), the identity $\sum_\omega \Pr(\omega,a_{-ij}|a_k)=\Pr(a_{-ij})=\alpha_{-ij}(a_{-ij})$ for k=i,j imposes some linear dependence among them. Namely, we have

[8] Note, however, that there is one linear dependence for the columns, as the first two columns add up to $\alpha_3(H)(1,1,1,1)'$, which is proportional to the sum of the last two columns $\alpha_3(L)(1,1,1,1)'$.

[9] It is not completely obvious that we can suitably perturb the entries by slightly changing the information structure $p(\omega|a)$, as they also depend on the mixed strategy profile under consideration. Note that we have to consider various mixed strategy profiles to obtain the Folk Theorem. Lemma 2 formally addresses this issue.

$$\sum_{\omega} q_{ij}(\omega, \alpha_{-ij}) = \alpha_{-ij}(\alpha_{-ij}) \begin{pmatrix} 1 \\ . \\ . \\ . \\ 1 \end{pmatrix}$$

for each a_{-ij}, and this implies that we have $|A_{-ij}|-1$ linear constraints for the columns[10]. (Note that those constraints do not exist in FLM (see Table 1)). Therefore, at most $(|\Omega| \times |A_{-ij}|) - (|A_{-ij}|-1) = (|\Omega|-1)|A_{-ij}|+1 =$

$$(|\Omega|-1)\prod_{k \neq i,j} A_k + 1 \tag{1}$$

columns can be independent. The number of rows are $|A_i|+|A_j|$, but as in the above example (see footnote 7), there is one linear constraint among them. Namely, we have

$$\sum_{a_i} q_{ij}(a_i)\alpha_i(a_i) = \sum_{a_j} q_{ij}(a_j)\alpha_j(a_j)$$

as both sides are equal to the probability distribution of (ω, a_{-ij}) under mixed strategy profile α. Hence at most

$$|A_i|+|A_j|-1 \tag{2}$$

rows can be independent. (Note that the same constraint also appears in FLM.) Therefore, a necessary condition for $Q_{ij}(\alpha)$ to have the maximum row rank is (1) ≥ (2), or

$$(|\Omega|-1)\prod_{k \neq i,j} |A_k| \geq |A_i|+|A_j|-2. \quad \text{for each } i \neq j \tag{3}$$

Once this is satisfied, matrix $Q_{ij}(\alpha)$ has the maximum row rank for a generic choice of its elements. However, it is not completely obvious that we can freely perturb each element by changing the

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[10] Fix any a^0_{-ij}, we have

$$\frac{1}{\alpha_{-ij}(\alpha_{-ij})}\sum_{\omega} q_{ij}(\omega, \alpha_{-ij}) = \frac{1}{\alpha_{-ij}(\alpha^0_{-ij})}\sum_{\omega} q_{ij}(\omega, \alpha^0_{-ij})(=(1,...,1)') \quad \text{for all } a_{-ij} \neq a^0_{-ij}.$$

underling information structure p(ω|a) (see footnote 9). Lemma 2 in Appendix shows that this is not a problem. This leads us to the following folk theorem. Let $\underline{v}_i$ be player i's minimax value $\underline{v}_i = \max_{a_i} \min_{\alpha_{-i}} g_i(a_i, \alpha_{-i})$. Define the set of feasible and individually rational payoff set by

V*={v∈cog(A)|∀i $v_i \geq \underline{v}_i$ }, where cog(A) is the convex hull of pure strategy stage payoffs.

Proposition (Folk Theorem with Communication). *Suppose N≥3 and V* has non-empty interior. Under (3), for a generic choice of the signal distribution, any feasible and individually rational payoff profile can be approximately achieved by a sequential equilibrium with communication, if the discount factor is sufficiently close to unity:*

A sketch of proof is presented in Appendix, and the full proof can be found in the discussion paper version Kandori (1999).

Condition (3), $(|\Omega|-1)\prod_{k\neq i,j}|A_k| \geq |A_i|+|A_j|-2$, is *always weaker than the FLM condition*,

$$|\Omega| \geq |A_i|+|A_j|-1,$$

whenever N≥3. Recall that our equilibrium with communication works only when N≥3, and we are assuming $|A_k|>1$ (otherwise player k has nothing to choose). Let us now examine when (3) is likely to be satisfied. Note that (3) is most difficult to hold when $|A_i|$ and $|A_j|$ are large and $|A_k|$ is small (k≠i,j). However, even in such a strongly asymmetirc case, (3) can be satisfied if there are many players. As we may assume that |Ω|>1 (otherwise no information is available) and each player has something to choose, the left side of (3) is at least 2^{N-2}. This quickly becomes a large number as N increases, so we conclude that *our Folk Theorem is likely to hold when there are many players*. If players have similar numbers of actions, in contrast, our Folk Theorem quite generally holds. A clean statement is possible when players have the same number of available actions, such as in (the finite version of) the Green-Porter oligopoly model (1984), where A_i = {0,1,2,…,H} is the set of possible outputs. Let K=H+1 be the number of available actions. Then, condition (3) reduces to $(|\Omega|-1)K^{N-2} \geq 2K-2$, and this is always satisfied[11] when N≥4. In other words, *we can drop the FLM conditions altogether* for generic games in this class.

Corollary 1. *Consider a game where the players have an equal number of actions, and suppose N≥4 and V* has non-empty interior. Then, for a generic choice of the signal distribution, the Folk*

[11] As we exclude the case where there is no information, we have |Ω|≥2 . Then N≥4 implies $(|\Omega|-1)K^{N-2} - (2K-2) \geq K^2 - 2K + 2 \geq (K-1)^2+1 \geq 0$.

Theorem under imperfect public information holds with communication.

A similar result also holds for symmetric games. As some restrictions are imposed by the symmetry of $p(\omega|a)$, we have to check if the generic full rank properties (Lemma 2 in Appendix) also hold for symmetric games. The answer turns out to be positive, and we obtain the following result (see the discussion paper version Kandori (1999) for the proof).

Corollary 2. *Consider symmetric games, and suppose $N \geq 4$ and V^* has non-empty interior. Then, for a generic choice of the (symmetric) signal distribution, the Folk Theorem under imperfect public information holds with communication.*

Remark: As d'Aspremont and Gerard-Varet (1998) note, the contract problem in Appendix with $\lambda=(1,\ldots,1)$ can be interpreted as the *static contract (moral hazard) problem in joint production* with risk neutral players. Our proof shows that efficiency can generically be achieved with communication in such a problem, under a quite weak condition (3). In particular, players in a joint production problem may well have the same number of available actions, and our Corollaries show that we can generically achieve efficiency with at least four players, when the compensation scheme depends not only on the outcome ω but also on verifiable messages of players.

Appendix

Here we present the sketch of proof of our Folk Theorem (Proposition). This is done by the Fudenberg and Levine (1994) (FL hereafter) algorithm, which allows us to determine the limit ($\delta \to 1$) equilibrium payoff set in the repeated game by means of a set of *associated static contract problems*. Let us present the modified version of the FL algorithm to fit our model with communication. Let $m_i = c_i(a_i, \omega)$ be player i's communication strategy in the stage game (i.e., player i announces that $m_i = c_i(a_i, \omega) \in A_i$ is the action he has taken, when he actually chose a_i and the realized signal is ω). Let $x(\omega, m) = (x_1(\omega, m), \ldots, x_N(\omega, m))$ represents the "transfer" to each player, given ω and announced action profile m. For a given welfare weight $\lambda = (\lambda_1, \ldots, \lambda_N) \neq 0$, we define the "maximized welfare" (or, following FL, maximum "score") $k(\lambda)$ by

$$\sup_{\alpha, c, x} \lambda \cdot (g(\alpha) + \mathrm{E}[x \mid \alpha, c])$$

$$\text{s.t.} \quad \text{(IC)}\ \forall i \forall (a'_i, c'_i)\ g_i(\alpha) + \mathrm{E}[x_i \mid \alpha, c] \geq g_i(a'_i, \alpha_{-i}) + \mathrm{E}[x_i \mid a'_i, \alpha_{-i}, c_i', c_{-i}]$$
$$\text{(B)}\ \forall \omega \forall m \quad \lambda \cdot x(\omega, m) \leq 0,$$

where $\mathrm{E}[x|\alpha,c]$ is the expected value of $x(\omega,m)$ under (α,c). This could be interpreted as a static contract problem, where the social planner tires to maximize the social welfare by means of side payment scheme x. This is closely related to the incentive constraints for the repeated game, where x corresponds to the variation of continuation payoffs in the repeated game[12]. This is a rough intuition why the above programming problem is useful in calculating the equilibrium payoff set in the repeated game. Define half space $D(\lambda) = \{v \in \Re^N | \lambda v \leq k(\lambda)\}$ and let $V^c(\delta)$ be the set of perfect semipublic[13] equilibrium payoffs in the game with communication under discount factor δ. Then we have the following[14].

[12] Condition (B), which may be interpreted as the budget constraint in terms of the contract problem interpretation, stipulates that the payoff variations cannot go beyond the boundary of the equilibrium payoff set, whose tangent vector is given by λ.

[13] A perfect semipublic equilibrium is a sequential equilibrium where each player's action at time t depends only on the realization of past signals $\omega(s)$ and messages $m(s)$, $s<t$.

[14] This is the modification of FL algorithm by means of communication, obtained by Compte (1998) and Kandori and Matsushima (1998).

Lemma 1 (FL Algorithm with Communication).

$$\lim_{\delta\to 1} V^c(\delta) \supset \bigcap_{\lambda\neq 0} D(\lambda),$$

if the right hand side has non-empty interior[15]*.*

The proof of Proposition is obtained by showing that the maximum possible welfare can be attained for each $\lambda\neq 0$ in the above programming problem. Lemma 2 below, which corresponds to the pairwise and individual full rank conditions of FLM, plays a key role in designing incentive compatible side payment schemes which entail no welfare loss ($\lambda\bullet E[x|\alpha,c]=0$). An informal description of such transfer schemes is given at the beginning of Section 3, and the formal construction can be found in the discussion paper version (Kandori (1999)).

Lemma 2 (Generic Full Rank Conditions with Communication). *Under (3), the following statements are true for a generic choice of signal distribution* $p(\omega|a)$*.*

(i) For any $\varepsilon>0$ *and any pure strategy profile a, there is a mixed strategy profile* α *such that* $||g(a)-g(\alpha)||<\varepsilon$ *and* $Q_{ij}(\alpha)$ *has the maximum row rank for any pair i, j.*

(ii) Take any $\varepsilon>0$ *and any profile* (a_i, α_{-i}) *where* a_i *is a best response against* α_{-i}*. Then there is a profile* (a'_i, α'_{-i}) *such that* $||g_i(a_i, \alpha_{-i})- g_i(a'_i, \alpha'_{-i})||<\varepsilon$, a'_i *is a best response against* α'_{-i}*, and* $R_j(i, (a'_i, \alpha'_{-ij}))$ *has full row rank for each* $j\neq i$*.*

Remark: (i) and (ii) are generic versions of the *pairwise and individual full rank conditions* of FLM, suitably modified to fit our model with communication. The individual full rank condition (ii) is necessary to support the point that gives player i his maximum payoff.

Proof. (i) For each pair i, j, fix a profile $\alpha(i,j)=(a^0_i,a^0_j,\alpha^0_{-ij})$, where α^0_{-ij} is completely mixed[16]. Then, each element of matrix $Q_{ij}(\alpha(i,j))$ is $p(\omega|a)$ times a positive constant (i.e., the probability of a_{-ij} under α^0_{-ij}). Since (3) guarantees that the number of potentially linearly independent columns exceeds the number of potentially linearly independent rows, it is clearly possible that $Q_{ij}(\alpha(i,j))$ has the maximum row rank for *a particular choice* of the information structure of the game (we denote it by $p^*(\omega|a)$). Take any $(|A_i|+|A_j|-1)\times(|A_i|+|A_j|-1)$ submatrix of $Q_{ij}(\alpha(i,j))$ that is nonsingular when $p(\omega|a)= p^*(\omega|a)$, and denote its determinant by D_{ij}. Since D_{ij} is a polynomial in $p(\omega|a)$ and nonzero for $p^*(\omega|a)$, we conclude that it is nonzero on an open and dense set, say $P(i,j)$, in the space of all

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[15] If we used mixed communication strategies in the definition of $k^c(\lambda)$, we would have equality rather than set inclusion $\supset$. For our purpose this weaker version suffices.

[16] The first two elements in $\alpha(i,j)$, a^0_i, and a^0_j, are interpreted as the degenerate mixed strategies

possible signal distributions. In other words, $Q_{ij}(\alpha(i,j))$ has the maximum row rank under (3) in an open and dense set, say $P(i,j)$, in the space of all possible signal distributions $p(\omega|a)$. Thus in an open and dense set $\bigcap_{i \neq j} P(i,j)$, for each pair i,j, $Q_{ij}(\alpha(i,j))$ has the maximum row rank. Now fix a $p(\omega|a)$ in this generic set. Since D_{ij} is also a polynomial in α and nonzero for $\alpha=\alpha(i,j)$, we conclude that it is nonzero on an open and dense set, say $\Delta(i,j)$, in the space of mixed strategy profiles. Thus, for a generic choice of the signal distribution, $Q_{ij}(\alpha)$ has the maximum row rank for each i, j on an open and dense set $\bigcap_{i \neq j} \Delta(i,j)$. Claim (i) then follows from the continuity of the stage payoff function g.

(ii) By a similar argument as above, we can show that for a generic choice of the signal distribution, there is an open and dense set Δ' such that for any pair i,j and any a''_i, matrix $R_j(i, (a''_i, \alpha''_{-ij}))$ has full row rank if $\alpha'' \in \Delta'$. Choose $\alpha' \in \Delta'$ so that α'_{-i} is sufficiently close to the given profile α_{-i}. Then, we have $\|g_i(a_i, \alpha_{-i})- g_i(a'_i, \alpha'_{-i})\|<\varepsilon$ by Berge's Theorem (on the continuity of maximized value), and $R_j(i, (a'_i, \alpha'_{-ij}))$ has full row rank for each $j \neq i$. □

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which play a^0_i, and a^0_j with probability one.

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