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Abstract

This paper presents a model of the economy with many sectors which face demand (quantity) constraints. Depending on the sign of the sectoral excess demand, the size of each sector either increases or decreases stochastically. We assume that reallocation of resources is not instantaneous, and, therefore, that sectoral differences in productivity always exist. We rely on the notion of holding time of continuous time Markov chains to select the sector which changes its size. We demonstrate that the total output fluctuates, and more importantly, that the level of the aggregate economic activity depends on the pattern of demand; The greater is demand for high productivity sectors, the higher is the expected value of GDP.

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1 Introduction

Fluctuations of aggregate economic activities or business cycles have long attracted economists' attention. A glance at traditional economic literature such as Haberler (1964) reveals that all kinds of theories had already been advanced by the end of the 1950s. New theories keep cropping up. The real business cycle (RBC) theory by Kydland and Prescott (1982) may arguably be the most influential current theory.

As typified by the RBC, economists often explain business cycle fluctuations as a direct outcome of the behavior of *individual agents*. This approach has been the standard in the mainstream economics in the last twenty years or so. The more strongly one wishes to interpret aggregate fluctuations as something 'rational' or 'optimal', the more likely one is led to this essentially microeconomic approach. The mission of this approach is to explain fluctiations as responses of the *representative agent* to changes in its economic environments. The consumer's intertemporal substitution, for example, is a device to achieve this goal.

There is a different approach to fluctuations, however. It starts with the fact that the economy consists of a large number of agents or sectors. In the real economy, perhaps, agents intertemporally maximize their respective objective functions subject to constraints. However, their economic environments keep changing due to idiosyncratic shocks that are all different. Plainly, an outcome of interactions of a large number of agents facing such incessant idiosyncratic shocks cannot be described by a response of the representative agent. It calls for a model of stochastic processes.[1] In a seminal work, Slutzky (1937) proposed such a stochastic approach. We

[1] The weak or strong laws of large numbers have been probably misapplied to justify deterministic macroeconomic models with representative agents. Deterministic models yes, but not representative agents. The hypotheses for the strong law of large numbers are not appropriate for some economic models. See Aoki and Shirai (2000) for an example of how deterministic model emerge from stochastic ones by letting the number of agents goes to infinity.

follow his lead in this paper to build a stochastic model of fluctuations and growth.

We use a simple model to accomplish two main objectives in this paper. First, we show that fluctuations of the aggregate economy arise as a natural outcome of *interactions of many agents/sectors*. Second, we demonstrate that the *level of the aggregate economic activity depends on the structure of demand*. In the standard neoclassical equilibrium where the marginal products of production factors such as labor are equal in all activities and sectors, demand determines only the composition of goods and services to be produced, but *not* the level of the *aggregate* economic activity.

Some recent works attempt to show that demand does affect the aggregate level of economic activities. One is externality associated with demand, which might produce multiple equilibria such as in Diamond (1982). Another is differences in productivity across sectors/activities. Murphy, Shleifer and Vishny (1989), and Matsuyama (1995), for example, emphasize the importance of increasing returns in order to demonstrate the role of demand in determining the level of the aggregate production. They, in effect, allow differences in productivity across sectors to produce multiple equilibria. In both approaches, demand plays an important role in the selection of equilibrium.

In this paper, we assume that productivities differ across sectors in the economy.[2] In the standard analysis, resources are assumed to be instantaneously reallocated so as to attain the equality of productivity across sectors. Here, we explicitly assume that reallocation of production factors takes time, and that differences in productivity across sectors persist.

Although studies of macroeconomy with demand (quantity) constraints are not new, dynamics in disequilibrium is not satisfactorily analyzed. Clower (1965) and Leijonhufvud (1968) pointed out that quantity adjustment might be actually more important than price adjustment in economic fluctuations.[3]

There are, in fact, some observations that quantities appear to fluctuate more swiftly than prices in the real economy. Figure 1 (a) and (b) show monthly rates of change (relative to the same month in the previous year) in output (the Index of Industrial Production) and price (the Wholesale Price Index) for the Japanese manufacturing industry as a whole and the automobile industry, respectively (January 1987-January 2000). A glance at these figures strikes us that output indeed fluctuates more than price. Figures 2 (a) and (b) show the frequency distributions for outputs and prices for the same data. We can easily reject the null hypothesis that variances for output and price are equal; the F statistics are 0.15 for the manufacturing industry, and 0.08 for the automobile industry while its critical value is 0.69.

For the U. S., Okun (1981, p.165) summarizes empirical evidences as follows:

[2]Yoshikawa (1995, 2000) taking the Japanese economy as an example, shows that productivity differences across secors actually persistently exist.

[3]The traditional Walrasian theory of general equilibrium is the egregious example of analysis of many agents. It focuses on price adjustment with the help of the non-existent auctioneer. More than forty years ago, Arrow (1959) pointed out that the notion of perfect competition was incompatible with disequilibrium, and that imperfect competition or demand constraints played a central role in disequilibrium.

"The empirical evidence for the United States suggests that cost-oriented pricing is the dominant mode of behavior. Econometrically, demand is found to have little, if any, influence on prices outside the auction market for raw materials."

The insight that quantity adjustment may be more important than price adjustment spawned a vast literature of the so -called 'non-Walrasian' or 'disequilibrium' analysis; see, for example, Bénassy (1975), Malinvand (1977) and Drèze (1991). Here, as Leijonhufvud (1968) puts it, quantitites determine quantities. This approach, however, suffers from its basically static nature of analysis because their purpose is primarily to show the existence of non-Walrasian quantity-constrained equilibrium. In contrast, our model abstracting from maximization of agents, focuses on the examination of dynamic behavior of the quantitiy-constrained economy.

This paper analyzes a very simple quantity adjsutment model composed of a large number of sectors or agents. We assume that sectors have different productivities. Resources are stochastically allocated to sectors in response to excess demand or supply; We assume the existence of underutilized production factors such as hours of work and work effort. Because of the linear structure, the level of aggregate production is inderterminate in equilibrium in a derministic model. However, we show that in our stochastic model, the total output fluctuates, and that the *average* level of aggregate production (or GDP) depends on the patterns of demand.[4] Specifically, the higher is the share of demand for high productivity sectors, the higher is the *average* GDP. These results are derived analytically for the two-sector economy. For economies with $K > 2$, they are shown by simulation.

In the first part of the paper we keep the number of sectors fixed. Then, we allow the number of sectors to grow stochastically. To that end we employ a scheme analogous to that of the Ewens sampling formula in introducing new sectors.

2 The Model

We propose a model of fluctuation and growth in which rate of utilization of a factor of production changes depending on demand. The assunmption is that there is always a room for a change depending on the economic environment, in a production factor such as hours of work or work intensity per hour. This assumption, in turn, presumes that allocation of resources takes time contrary to the standard assumption that tacitly presumes instantaneous adjustment.

[4] In this sense, the model may be thought of a particular kind of quantity adjustment model. Leijonhufvud (1974, 1993) described a Marshallian quantity adjustment model. He envisioned a representative firm which adjusts outputs to partially narrow the gap between the supply price and demand prices of the good produced by the firm. Since the demand price schedule is unknown to the firm, the market clearing price is substituted for it. Aoki analyzed his model in Aoki (1976, 193ff., 319 ff.) In this paper sectors are not the same, and sectors adopt stochastic response rules to gaps between the demand and supplies. Sectors are subject to aggregate externalities as we discuss in the text.

Suppose that there are K sectors in the economy. We keep K fixed for now. We will drop this assumption in section 6. Sectors adjust their outputs by varying the utilization rate of the factor of production in response to the excess demand for or supply of goods/services they produce. We model the state of this economy as a continous time Markov chain, also known as a jump Markov process.

We assume that sector i has productiviy coefficient, c_i, which is exogenously given and fixed. As we stated in Introduction, we assume that speed of changes in utilization of a production factor is much faster than that of equalization of productivities among sectors. Thus differences in productivity among sectors persist. Assume, for definiteness, that sectors are arranged in the decreasing order in productivity ($c_i > c_j$ if $i < j$). As we note it in Concluding Remarks, the fixed productivity coefficient is not a crucial assumption. We can allow variable productivity under concave production function to draw similar results. The crucial assumption is the existence of sectoral differences in productivity. It takes time for productivities to equalize among sectors. Meanwhile, utilization of production factor in *some* sector changes, and as we will see it shortly, the macroeconomic situation also changes.

Sector i employs $N_i(\mathrm{t})$ units of factor of production. It is a non-negative integer-valued random variable. We call its value as "size" of the sector. When $N_i(t) = n_i$, $i = 1, 2, \ldots, K$, the output of sector i is $c_i n_i$, and, therefore, the total output (GDP) of this economy is

$$Y(t) := \sum_{i=1}^{K} c_i n_i(t).$$

Demand for the output of sector i is denoted by $s_i Y(t)$, where $s_i > 0$ is the share of sector i, and $\sum_i s_i = 1$. The shares are also assumed to be exogeously given and fixed.[5]

We denote the excess demand for goods of sector i at time t by

$$f_i(t) := s_i Y(t) - c_i n_i(t),$$

$i = 1, 2, \ldots, K$. Denote the set of sectors with positive excess demand by

$$I_+ = \{i; f_i > 0\}$$

and similarly for the set of sectors with negative demand by[6]

$$I_- = \{j; f_j \leq 0\}.$$

To shorten notation, summations over these subsets are denoted as $\sum_+$ and $\sum_-$, respectively. Denote by n_+ the number of ns in the set I_+, that is, we write

$$n_+ := \sum_+ n_i,$$

[5]In the framework of the representative consumer, it corresponds to the Cobb-Douglas utility function.

[6]To be definite we include sectors with zero excess demands as well.

where the subscript + is a short-hand for the set I_+, and similarly

$$n_- := \sum_{-} n_j,$$

for the sum over the sectors with negative excess demand. Let $n = n_+ + n_-$. Sets I_+ and I_-, and n_+, n_- change over time.

Sectors with non-zero excess demand attempt to reduce the size of excess demand by adjusting their inputs, up or down, depending on the sign of the excess demand. This response pattern implies that firms increase their (positive) profits by raising the level of production when they faces excess demand, while they reduce their losses by lowering the production level when they face excess supply. The adjustment is *not* instantaneous, however. Also the same excess demand or supply does not necessarily bring about the same instantaneous response. For example, the same excess demand or supply may be taken to be either permanent or temporary. Firms would hold different initial levels of inventory stock. It is simply impossible for us to obtain enough information on how quickly sectors (firms) react to a given level of excess demand or supply. Here, a stochastic approach is necessary. The only assumption we make is that a sector in set I_+ raises the level of production whereas a sector in set I_- lowers the production level.

Specifically, the transitions in sizes of our model are

$$\Pr(N_i(t+h) = n_i + 1 | N_i(t) = n_i) = \gamma_i h + o(h) \qquad \text{for } i \in I_+,$$

and

$$\Pr(N_i(t+h) = n_i - 1 | N_i(t) = n_i) = \rho_i h + o(h) \qquad \text{for } i \in I_-,$$

The transition rates, γ and ρ of the jump Markov process are specified later. For simplicity, we normalize the size of jump to be one. This assumption is not essential, of course.

Holding Time

In a continuous time model, at each moment either one sector changes its production level or no sector does. This assumption is the same as the standard Poisson process, and is quite robust. The question is which sector, if any, adjusts its production level. We assume that the time it takes for sector i to adjust its size by one unit, up or down, T_i, is exponentially distributed,

$$\Pr(T_i > t) = \exp(-b_i t),$$

where b_i is either γ_i or ρ_i depending on the sign of the excess demand. This time is called *sojourn time* or *holding time* in the probability literature (See Lawler (1995)).

As we explained it earlier, the same excess demand or supply brings about a different reaction by sector (or firm) because each sector faces idyosyncratic economic environment or constraint. For the same reason, we do not know when the sector facing disequilibrium "reacts". It is stochastic.

The notion of "holding time" models the timing of stochastic reaction. We assume that the random variables Ts of the sectors with non-zero excess demand are independent.

The sector that adjusts first (call it sector a) is determined by the sector with the shortest holding time. Let T^* be the minimum of all the holding times of the sectors with non-zero excess demand. If sector a raises its production level, we have

$$\Pr(T_a = T^*) = \frac{\gamma_a}{\gamma_+ + \rho_-}, \qquad \text{for a} \in \mathrm{I}_+$$

where $\gamma_+ = \sum_+ \gamma_i$, and $\rho_- = \sum_- \rho_j$. If on the other hand, sector a cuts its production level (a $\in \mathrm{I}_-$), then the probability of the jump in sector a is given by

$$\rho_a/(\gamma_+ + \rho_-)$$

See Lawler (1995, 56) or Aoki (1996, Sec.4.2)

Transition Rates

We assume that γs and ρs depend on the total number of sizes and the current size of the sector that adjusts,

$$\gamma_i = \gamma_i(n_i, n),$$

and

$$\rho_j = \rho(n_j, n).$$

The idea is that a change which is significant enough to affect the economy as a whole, occurs more likely in large sectors/firms (sectors with large n_i) than in small sectors. Statistically speaking, this is an example of applying W. E. Johnson's sufficientness postulate; see Zabell (1982).[7]

There are several possibilities for the functional forms of γ and ρ. We specify initially the "birth rate", that is, the rate of size increase by

$$\gamma_i(n_i, n) = \frac{n_i}{n},$$

and that of the "death rate", namely, the rate of size decrease also by

$$\rho_i(n_i, n) = \frac{n_i}{n}.$$

Later in Section 5, we discuss the effects of modifying the entry rate to allow for new sectors.

When $\gamma_i = \rho_i$ for all i, time histories of n_i are those of fair coin tosses. We have K such coin tosses available at each jump. The sector that jumps is determined by the coin toss selected from these K coins.

[7] For specifications of entry and exit probabilities see Aoki (2002). See also Costantini and Garibaldi (1979, 1989), who give clear discussions on reasons for these specifications. As explained clearly by Zabell (1992), there is a long history of statisticians who have discussed this type of problems. There are good reasons for γ_i to depend only on n_i and n, and similarly for ρ_i. See Zabell for further references on the statistical reasons for this specification.

The Aggregate Output and Demand

When a sector changes its production level, it affects the aggregate output *and* the sectoral demand pattern. Specifically, after a change in the size of a sector, the total output of the economy or GDP changes to

$$Y(t+h) = Y(t) + sgn\{f_a(t)\}c_a, \tag{1}$$

where a is the sector that jumped first by the time $t+h$.[8]

Demand for each sector also changes. After the jump, sector a's excess demand changes to

$$f_a(t+h) = f_a(t) - c_a(1-s_a)sgn\{f_a(t)\}. \tag{2}$$

Other non-jumping sectors' excess demand changes to

$$f_i(t+h) = f_i(t) + sgn\{f_a(t)\}s_i c_a, \tag{3}$$

for $i \neq a$.

These equations show that an increase of size in one sector affects excess demand in *all* the sectors. When sector a raises its production level, GDP increases by c_a. Now sector a experiences an increase of its demand only by a fraction s_a of c_a, while all the other sectors experience increase of their demand by $s_i c_a$, $i \neq a$. Demand spillovers (Eq. (3)), a source of externality in this model, affect the behavior of the total output significantly. The index sets I_+ and I_- also change in general.

Defining $\Delta Y(t) := Y(t+h) - Y(t)$, and $\Delta f_i(t) := f_i(t+h) - f_i(t)$, we can rewrite (1) through (3) as

$$\Delta Y(t) = sgn\{f_a(t)\}c_a,$$

and

$$\Delta f_a(t) = -(1-s_a)\Delta Y(t),$$

and

$$\Delta f_i(t) = s_i \Delta Y(t),$$

for $i \neq a$.

Sizes of the Sectors and Aggregate Output in Equilibrium (Zero Excess Demand)

When excess demands of all sectors are zero, no sector changes its output, and, therefore, total output or GDP does not change, either. We call this state "equilibrium".

We solve K equations of zero excess demands $f_i = 0$, $i = 1, 2, \ldots, K$, to obtain the equilibrium sizes, denoted by superscript e, of the fractions of sector sizes, n_i^e/n^e for $i = 1, \ldots, K$, and also total output Y^e.

[8] For the sake of simplicity we may think of the skeleton Markov chain, in which the directions of jump are chosen appropriately but the holding times themselves are replaced by a fixed unit time interval. Limiting behavior of the original and the skeletal version are known to be the same under certain technical conditions, which hold for this example. See Cinlar (1975)

Define K dimensional column vectors $\mathbf{c} := (c_1, c_2, \ldots, c_K)'$, and $\mathbf{s} := (s_1, s_2, \ldots, s_K)'$. A diagonal $K \times K$ matrix $C := diag(c_1, c_2, \ldots, c_K)$ is introduced to simplify our discussion. Then the total output is $Y = \langle \mathbf{c}, \mathbf{n} \rangle$, and the set of zero excess demand conditions is expressed by $s_i Y = c_i n_i$, $i = 1, 2, \ldots, K$, which can be written compactly as

$$C\mathbf{n} = \mathbf{s}Y = \mathbf{s}\mathbf{c}'\mathbf{n},$$

or

$$\Phi\mathbf{n} = \mathbf{0}$$

with $\Phi = C - \mathbf{s}\mathbf{c}'$.

Noting that the shares sum to one, matrix Φ does not have full rank because $|\Phi| = |C|(1 - \mathbf{c}C^{-1}\mathbf{s}) = 0$. It has rank $K - 1$, and its null space has dimension one which is spanned by the solutions we give next.

The solution is

$$n^e = C^{-1}\mathbf{s}Y,$$

or

$$\frac{n_i^e}{n^e} = \frac{s_i/c_i}{\sum s_i/c_i}, i = 1, \ldots, K. \tag{4}$$

That is, the relative size of sectors in equlibrium n_i^e/n^e is uniquely determined by productivity coefficients c_i and demand shares s_i. The absolute levels of n_i^e and n^e are indeterminate, however. Multiply (4) by c_i and sum over i, and we obtain the relation between Y and n as

$$Y^e = \frac{n^e}{\sum_i s_i/c_i}. \tag{5}$$

Equation (5) shows that Y^e and n^e corresponds to each other. However, since n^e is indeterminate, so is Y^e. In the deterministic model, the equilibrium level of Y is indeterminate.

3 The Behaviour of the Economy out of Equilibrium or Sample Paths: Two Sector Model

We have seen that the level of total output or GDP is indeterminate in epuilibrium in a deterministic model. Our model is actually stochastic, however. The size of each sector n_i, and the total output Y stochastically change over time. In this section, we explore the behavior of the economy out of equilibrium, or of *sample paths* in a stochastic behaviour of the model. Despite its simplicity, the stochastic behavior of the model explained in section 2 turns out to be extremely richer. To gain insight, we analyse a simple two sector model, and later comment how the results of the two sector model may generalize.

Two Sector Model

In the two sector model, we have $s_2 = 1 - s_1$. This model is characterized then by two parameters s_1 and c_2/c_1. (If you wish, c_1 may be set to one with a suitable choice of unit to measure n_1.) Eq.(4) shows that $n_1^e/n_2^e = (s_1/c_1)/(s_2/c_2)$, that is, the sign of $s_2/s_1 - c_2/c_1$ determines the relative sizes of the two sectors in equilibrium. Hence it does matter in the details of stochastic evolution whether n_1^e is larger than n_2^e or not. We describe the model behavior assuming

$$\frac{s_2}{c_2} > \frac{s_1}{c_1}$$

that is, $n_2^e \geq n_1^e$. The other case may be examined by switching the subscripts. We supress time arguments.

Now, it is convenient to analyze the dynamics of this stochastic model, by namely *sample paths* by a diagram. The non-negative quadrant of the plane for n_1 and n_2, with the horizontal axis labelled by n_1, and the vertical by n_2, is divided into six regions, denoted by R_k, $k = 1, 2, \ldots, 6$ (See Figure 3). They are bounded by $n_i \geq 0$, $i = 1, 2$, and by five other straight lines denoted by $L_1, L_2, \ldots, L_5$, with a common slope

$$\beta := (s_2/c_2)/(s_1/c_1).$$

The slope is larger than one for our choice of the parameter values. The intercepts of the five lines are β, 1, 0, -1, and $-\beta$. Line 3 cuts the n_1 axis at 0, Line 4 at 1, and Line 5 at β, and so on.

In different regions, either the signs of the excess demands, or those after size changes in sector 1 or 2 are different, as detailed below. We note first that the two sector model is special in that $f_1 + f_2 = 0$ and, therefore, $f_1 f_2 < 0$. Further, denoting by $f_i^1(\pm)$, the value of the excess demand in sector i after a change of n_1 by ± 1, and similarly by $f_i^2(\pm)$, the excess demand in sector i after a change in n_2 by ± 1, we note that $f_1^1(\pm) + f_2^1(\pm) = 0$ and that $f_1^2(\pm) + f_2^2(\pm) = 0$.

We must note, for example, that only an increase in n_1 thus $f_i^1(+)$ is possible when $f_1 > 0$: No $f_i^1(-)$ is possible. Similarly in this case, because of $f_2 < 0$, $f_i^2(+)$ is not logically possible. For the same reason, when $f_1 < 0$, neither $f_i^1(+)$ nor $f_i^2(-)$ happens. Note that in R_1, R_2 and R_3 which are above L_3, we have $f_1 > 0$. Hence $f_2 < 0$ in these regions. States on L_3 are the equilibrium states analyzed in section 2.

The signs of excess demand and how the sign changes in each sector by a change in size in either sector 1 or 2 differ across regions. Take, for example, R_1. In this region, $n_2 > \beta n_1 + \beta$. Since $f_1 > 0$ in R_1, either n_1 increases or n_2 decreases. Now, after a change in n_1 by $+1$,

$$f_1^1(+) = s_1 c_2 [n_2 - \beta(n_1 + 1)] > 0,$$

above L_1. Similarly,

$$f_1^2(-) = s_1 c_2 [n_2 - \beta n_1 - 1] > 0.$$

In this way, we can determine the signs of f_i, $f_i^1(\pm)$ and $f_i^2(\pm)$. The results are summarized in Table 1. The five columns in Table 1 from the left to right show the signs of $f_1, f_1^1(+); f_1^1(-); f_1^2(+); f_1^2(-)$, respecrively. The logically impossible combinations are marked by $*$ in Table 1. Table 1 summarizes information necessary for understanding how the state stochastically evolves or sample paths.

In R_1, either n_1 increases or n_2 decreases. As long as the state remains in R_1 the state moves south-eastward, and consecutive increases in n_1 and/or decreases in n_2 will eventually bring the state near the boundary L_1. In this way, the state leaves R_1 to enter R_2 with probability one.

In R_2 and R_3, either n_1 increases or n_2 decreases. Now an increase of n_1 by 1 maps line L_1 onto line L_3, and L_3 onto L_5. This means that by an increase in n_1 region $R_2 \cup R_3$ is mapped onto $R_4 \cup R_5$. Conversely, a decrease of n_1 by 1 maps $R_4 \cup R_5$ back onto $R_2 \cup R_3$.

What happens when n_2 changes? A decrease of n_2 by 1 maps L_2 onto L_3, and L_3 into L_4. Conversely, an increase in n_2 by 1 maps L_3 onto L_2 and L_4 onto L_3. This means that by a decrease of n_2, R_3 is mapped into R_4, and by an increase in n_2, R_4 is mapped backed to R_3. It is a bit more complicated when n_2 decreases in R_2. With $\Delta n_2 = -1$, line L_1 is mapped into L_1' which lies above L_3. It lies below L_2 if $\beta < 2$, and above L_2 if $\beta > 2$. Parts of R_2 near L_1 may remain in R_2, and may take several reduction in n_2 to bring it into R_3. More precisely, for $k < \beta < k+1$ with a positive integer k, k consecutive reductions in n_2 or l consecutive decreases in n_2 followed by an increase in n_1 by 1, $0 \leq l \leq k-1$ will bring L_1 into R_3. Once state in R_3, the earlier argument applies.

Let P_+ be the probability $n_1 \to n_1 + 1$ and $P_- = 1 - P_+$ be the probability $n_2 \to n_2 - 1$ in region R_2 where $k < \beta < k+1$. Then the probability of the state in R_2 being mapped into $R_4 \cup R_5$ is

$$\begin{aligned} P_+ + P_-P_+ + P_-^2P_+ + \ldots + P_-^{k-1}P_+ + P_-^k \\ &= P_+(1 + P_- + \ldots + P_-^{k-1}) + P_-^k \\ &= P_+\left(\frac{1-P_-^k}{1-P_-}\right) + P_-^k = 1 \end{aligned}$$

i.e., the probability is one that R_2 is mapped into $R_4 \cup R_5$.

To summarize, whether n_i increases or decreases, states oscillate between regions $R_2 \cup R_3$ and $R_4 \cup R_5$ possibly in asymmetrical fashion. Call $R_2 \cup R_3 \cup R_4 \cup R_5$ 'near the equilibrium'. Then we have the following proposition.

Propositoion 1: Near the equilibrium on L_3, states oscillate (possibly asymmetrically) between $R_2 \cup R_3$ and $R_4 \cup R_5$. Sample path shows oscillation of (n_1, n_2), and so does the total output Y near the equilibrium.

This proposition shows that given differences in productivities and demand shares across sectors, aggregate fluctuations arise *endogenously* out of simple quantity adjustments.

General Case

In the previous simple two-sector model, excess demand is sufficiently large in magnitude in R_1 and R_6. In these regions the state of the economy is far removed from the equilibrium represented by Line L_3, and the excess demand sign patterns do not change by a change in a single sector.

In a general K-sector model, the same holds when the economy is sufficiently far from the equilibrium. Increases in the number of sectors, however, tend to decrease the transition rates for size increase in an individual sector with positive excess demand.

We next argue that the general model with $K > 2$ is similar to the two-sector model. Partition vectors into two subvectors with components in I_+ and I_- such as $\underline{n}$ into (n_+, n_-) and $\underline{f}$ into $(\underline{f}_+, \underline{f}_-)$. Let $c_\pm = diag(c_\pm)$ with $\underline{c}_+$ column vector of components c_i, $i \in I_+$ and so on. Then the equilibrium states are

$$\begin{aligned} \underline{n}^e_\pm &= c_\pm s_\pm Y \\ Y &= \underline{c}'_+ \underline{n}_+ + \underline{c}'_- \underline{n}_- . \end{aligned}$$

Denote by $\underline{n}_+ + \underline{e}_a$, $a \in I_+$, an increase in n_a by one (a for active). After this change, we observe that

$$\begin{aligned} f'_-(+) &= f_-(\underline{n}_+ + \underline{e}_a) \\ &= f_-(n_+) + c_a \underline{s}_- \end{aligned}$$

and

$$\begin{aligned} \underline{f}'_+(+) &= \underline{f}_+(\underline{n}_+ + \underline{e}_a) \\ &= \underline{s}_+(\underline{c}'_+ \underline{n}_+ + \underline{c}'_- \underline{n}_- + c_a) - c_+ \underline{n}_+ - c_+ e_a \\ &= \underline{f}_+(\underline{n}_+) + \underline{s}_+ c_a - c_a \underline{e}_a , \end{aligned}$$

i.e.,

$$\begin{aligned} f'_a &= f_a - c_a(1 - s_a) \\ f'_b &= f_b + c_a s_b \qquad b \neq a, \quad b \in I_+ \end{aligned}$$

Note that $c_a \underline{s}_-$, $c_a(1 - s_a)$ and $c_a s_b$ are all positive.

Similar analysis is conducted for $\underline{n}_- - \underline{e}_a$. These changes in the excess demands are basically the same as those of a two sector model with the only complication that different sectors become active and the sets I_+ and I_- change in general. However K being finite, we can think of the $\begin{pmatrix} K \\ 2 \end{pmatrix}$ projections of the trajectory in the K-dimensional Euclidian space, and conclude that the trajectory in the K dimensional space near the equilibrium hyperplane undergo possibly asymmetrical movements in $\underline{n}$, analogues to those described in Proposition 1.

4 Stationary Probability Distribution for the Two Sector Model

In the previous section, we have seen that sample paths generally oscillate. Next, we consider the stationary probability distribution for the total output or GDP in the two sector model. Our goal is to show that the expected value of GDP depends on the demand pattern. Dynamics can be analyzed by the Chapman-Kolmogorov (master) equation. We derive the stationary probability distribution near the equilibrium represented by L_3. Devote the stationary probability by $\pi(\cdot)$, e. g., at b by $\pi(b)$.

Take the initial state b which is on or just below L_3, namely in or near equilibrium. Let $n(b) = (n_1(b), n_2(b))$ be the state, and define two adjacent positions e and c by $n_1(e) = n_1(b) + 1$, $n_2(e) = n_2(b) + 1$, $n_1(c) = n_1(b) - 1$, and $n_2(c) = n_2(b) - 1$. Denote the stationary probability by $\pi(\cdot)$.

By the detailed balance conditions between states e and b, and those between b and c, we derive the relations for the stationary probabilities

$$\frac{\pi(e)}{\pi(b)} = \frac{n_1(b)}{n_1(b)+1}\frac{n_2(b)}{n_2(b)+1}\frac{n+2}{n},$$

where $n := n_1(b) + n_2(b)$, and

$$\frac{\pi(c)}{\pi(b)} = \frac{n_1(b)}{n_1(b)-1}\frac{n_2(b)}{n_2(b)-1}\frac{n-2}{n}.$$

By repeating the process of expressing the ratios of probabilities, we obtain

$$\frac{\pi(b+(k,k))}{\pi(b)} = (\frac{n_1(b)}{n_1(b)+k})^2\frac{n+2k}{n},$$

for $k = 1, 2, \ldots$, where we use $n_2 = \beta n_1$ on or near L_3.

Similarly

$$\frac{\pi(b-(l,l))}{\pi(b)} = (\frac{n_1(b)}{n_1(b)-l})^2\frac{n-2l}{n},$$

for $l = 1, 2, \ldots, \bar{l}-1$, where $\bar{l}$ is the largest positive integer such that $n - 2\bar{l} \geq 0$. Without loss of generality, we treat it as an integer.

Noting that $n = (1+\beta)n_1$, we write these ratios as

$$\frac{\pi(b+(k,k))}{\pi(b)} = \gamma^{-\mu k},$$

and

$$\frac{\pi(b-(l,l))}{\pi(b)} = \gamma^{\mu l},$$

with $\gamma = \exp(2/n_1(b))$, and $\mu = \beta/(1+\beta)$. From now on we write b for $n_1(b)$ since there is no ambiguity.

The stationary distribution is then

$$\pi(b+k) = A\gamma^{-\mu k},$$

for $k = 1, 2, \ldots$, and

$$\pi(b-l) = A\gamma^{\mu l},$$

for $l = 1, 2, \ldots, \bar{l} - 1$, where A is the normalizing constant $A^{-1} = \sum_0^{\bar{l}-1} \gamma^{\mu l} + \sum_{k \geq 1} \gamma^{-\mu k} = \gamma^{\mu \bar{l}} / [\gamma^{\mu} - 1]$.

Now, the expected value of the total output in the economy or GDP is $E(Y) = (c_1 + c_2\beta)E(n_1)$ where

$$E(n_1) = A[\sum_{k \geq 1} (b + k)\gamma^{-\mu k} + \sum_0^{\bar{l}-1} (b - l)\gamma^{\mu l}].$$

We can calculate this sum by means of the generating function,

$$E(n_1) = b + AG'(1),$$

with

$$G(z) = \sum_k (\gamma^{-\mu} z)^k - \sum_l (\gamma^{\mu} z)^l,$$

where we drop the obvious upper limits of summation for simpler notation. Note that

$$G(z) = \frac{z}{\gamma^{\mu} - z} - \frac{\gamma^{\mu} z - (\gamma^{\mu} z)^{\bar{l}}}{1 - (\gamma^{\mu} z)^{\bar{l}}}.$$

Substituting $(1 + \beta)b/2$ for $\bar{l}$, we obtain, after some algebra

$$E(n_1) = \frac{1}{1 - \gamma^{-\mu}} + b - \bar{l} = b - \bar{l} - \frac{b}{2}(\beta - 1).$$

For the value of β=1, this is clearly positive ($b - \bar{l} > 0$). Then by continuity of $E(n_1)$ with respect to β, it is positive with β close to 1. With $\beta > 1$, it is positive if $1/(1 - \gamma^{-\mu}) > (b/2)(\beta - 1)$, which is satisfied for $\beta > \beta*$ for some $\beta*$. We assume that this condition is satisfied.

We note

$$\frac{dE(y)}{ds} = \frac{dE(Y)}{d\beta}\frac{d\beta}{ds}.$$

Since we have $d\beta/ds = -(c_1/c_2)(1/s^2) \leq 0$, $E(Y)$ increases with a small increase in s if and only if $dE(Y)/d\beta < 0$. Now recall

$$E(Y) = (c_1 + c_2\beta)E(n_1).$$

and we obtain

$$\frac{dE(Y)}{d\beta} = [-H(\beta)c_1 - G(\beta)c_2](\gamma^{\mu} - 1)^{-2}.$$

Here

$$H(\beta) = \frac{2}{b}\frac{1 + \beta^2}{(1 + \beta)^2} + o(1/b) > 0$$

and

$$G(\beta) = \frac{2}{b}\frac{2\beta^2(\beta - 1)}{(1 + \beta)^2} + o(1/b) > 0.$$

Therefore, we have indeed

$$\frac{dE(Y)}{d\beta} \leq 0,$$

for all $\beta \geq 1$. Thus we have shown $dE(Y)/ds < 0$.

Obviously, $E(Y)$ depends on the initial state b. Given the initial state b, we have obtained the following proposition.

Proposition 2: Given the initial state, the expected value of Y increases as the share of demand for sector 1 (high productivity sector) increases in the range of $\beta > 1$.

Analogous proposition may be established for the range $\beta < 1$ in similar manner.

Proposition 2 means that the expected value of aggregate economic activity depends on the pattern of demand. Specifically, the higher is the share of demand for high productivity sector, the higher is the expected value of aggregate economic activity. Note that the level of aggregate economic activity is indeterminate in equilibrium of a corresponding deterministic model. In a stochastic model, the higher the share of demand for high productivity sector is, the more likely the sector faces excess demand and raises its production level. Therefore, the higher is the share of demand for high productivity sector, the greater would be externality generated by an increase in the size of a sector. In this way we obtain Proposition 2. This proposition provides a new perspective to the principle of effective demand.

5 Emergence of New Sectors

So far, we have assumed that the number of sectors is given. Before we proceed to our simulation analysis for the case of $K > 2$, we introduce the possibility of emergence of new sectors. Namely, K is no longer a given constant, but a stochastic variable. We assume that a new sector emerges as a branch off from a sector with excess demand. There are now two possible changes for sectors with positive excess demand. (i) Just as in our analysis above, one of the existing sectors with positive excess demand increases its size by one with probability $(\alpha + n_j)/(K_+\alpha + n_+)$, where K_+ denotes the number of sectors with positive excess demand, and n_+ is the total size of such sectors. (ii) A new sector emerges with rate proportional to $(K_+ - 1)\alpha/(K_+\alpha + n_+)$. These transition rates approach $n_j/(\theta + n_+)$ and $\theta/(\theta + n_+)$, respectively as parameter α goes to zero while $K_+\alpha$ approaches a positive value θ.

Schmookler (1966) by analyzing the U. S. patent data, demonstrates that invention and technical progress are very strongly conditioned by demand prospectus. Given his findings, we can resonably assume that a new sector emerges as a branch off from a sector with excess demand. In the limit of letting α go to zero, and assuming that $K_+\alpha$ approaches a common positive value for the sake of simplicity, we have a model in which either one of the existing sectors with positive excess demand increases size by one, or a new sector emerges.[9] That is, (1) is now modified to read that the conditional

[9] We could assume that $K_+\alpha$ converges to θ_+ which may change each epoch. This would lead to a slight modification of the Ewens sampling formula. See Aoki (2002, Sec 8.6)

change in $Y(t+h)$ given $Y(t)$ consist of two terms; the one conditional on the event of the new sector appearing, which occurs with probability $\theta/(\theta+n_+)$, and the second one conditional on the event that no new sector appears.

We assume that a new sector when it emerges inherits characteristics, namely c an s, of one of the existing sectors with equal probability. That is, if there are K sectors, then with probability $1/K$, the value of c and s of the randomly selected sector is inherited. The shares of demand s are then renormalized so that they sum to one, including the newly born sector. This assumption is merely for convenience. Other schemes may also be tried.

To study how n or equivanently Y behaves, we introduce another continuous time Markov chain for n by the transition rates

$$w(n, n+1) = \lambda,$$

and

$$w(n, n-1) = \nu n.$$

Then, the stationary distribution of n is the Poisson distribution $\pi(n) = e^{-\mu}\mu^n/n!$, with $\mu = \lambda/\nu$. [10]

6 Simulation Runs for Multi Sector Model

We have seen in sections 3 and 4 that our simple two sector model is such that (i) sample path exhibits cycles or oscillations near the equilibrium, and (ii) the greater is the share of demand for high productivity sector, the higher is the expected value of aggregate economic activity. It is extremely difficult, however, to analyze the multi-sector model explicitly. In this section, we check the robustness of the two propositions which we derived for the two-sector model by simulation.

We keep the total number of sectors at $K = 10$. Our discussion in Sec 5 indicates that there is not much loss of generality in keeping the value of K fixed for small value of θ, which ranges from 0.2 to 0.6 in our experiments. We also keep fixed the order of the productivities from $c_1 = 1$ to $c_K = 1/K$ at equal interval. We start the the simulation runs with the initial condition $n_i = 10$ for all sectors, $i = 1, 2, \ldots, 10$.

We vary the demand patterns for the outputs of the sectors as follows. We try five patterns, Pi, $i = 1, \ldots, 5$:

Pattern $P1$ has $\mathbf{s} = (5,5,4,4,3,1,1,1,1,1)/26$;
Pattern $P2$ has $\mathbf{s} = (5,3,2,1,1,1,1,1,1,1)/17$;
Pattern $P3$ has $\mathbf{s} = (2,2,2,2,2,1,1,1,1,1)/15$;
Pattern $P4$ has $\mathbf{s} = (1,1,1,3,3,3,3,1,1,1)/18$;
Pattern $P5$ has $\mathbf{s} = (2,2,2,1,1,1,1,1,1,1)/13$.

The sum of the shares of the top five sectors are 0.8, 0.7, .66, .5, and .61 respectively. By lumping the top five sectors and the bottom five sectors in this way, we might interpret these five patterns as corresponding to the two sector model we analyzed with different demand parameters.

[10] Actually, the rates are functions of n in the jump Markov process. Since we are using the skeletal chain, this point does not matter in the simulations.

Figures 4 through 7 show the results.[11] As suggested by Proposition 1, they all show oscillations of Y. Proposition 2 in section 4 shows that the greater is the share demand for more productive sectors, the higher the expected value of total output Y is. Therefore, it suggests that in our simulation, the avarage level of Y is highest for P_1, and lowest for P_4 with P_2 and P_3 in between. This is exactly what we observe in Figure 4. By putting larger demand shares at higher productivity sectors, the averge output shifts up.[12]

We also checked some histograms of the number of size of sector 1 and sector 10 increasing or decreasing during the Monte Carlo runs. They show, that the sectors grow when demand shares are high, and decay when demand shares are small.

Fig. 4 shows outputs, averaged over 200 Monte Carlo runs, with demand shares $P1$ through $P5$, each for the case of $\theta = .6$. Fig. 5 is the plot of $P2$ outputs averaged over 200 runs with $\theta = .2$. Fig. 6 shows 1000 time periods of outputs averaged over 400 runs, with $\theta = 0.6$. Fig. 7 shows per unit output Y/n averaged over 200 runs. The equilibrium value is $y^e/n^e = .4196$. This value is independent of θ values. Fig. 8 shows the outputs for $\theta = .1$, and $\theta = 1$ with P3 demand share pattern. These two figures are included to give some feel for the effects of the magnitude of θ on the outputs. As is pointed out by Feller (1968, chap. 3), the random walks generated by fair coin tosses show many counter intuitive behavior. The numbers of periods and runs are not large enough to draw any precise conclusions. These simulation experiements serve to show the existence of equilibrium cycles even in this extremely simple quantity adjustment model.

As an example, suppose that $\mathbf{s} = (2,2,2,2,2,1,1,1,1,1)/15$. This is one of the patterns of demands we simulate (pattern 3 above). We take $c_j = (K - j + 1)/K$, $j = 1, \ldots 10$, with $K = 10$. Then, the model has $\sum_i s_i/c_i = 2.38$. The equilibrium sizes are: $n_1^e/n^e = .056$, $n_2^e/n^e = .06$, $n_3^e/n^e = .07$, $n_4^e/n^e = .08$, $n_5^e/n^e = .09$, $n_6^e/n^e = .056$, $n_7^e/n^e = .14$, $n_8^e/n^e = .09$, $n_9^e/n^e = .14$, and $n_{10}^e/n^e = .28$.

The simulation results for this case with 450 periods, averaged over 200 runs, show that the total output cycles in the range of 99.5 to 100.3. Therefore, the average is about 99. Taking this value as a proxy for n^e, we obtain $n_1^e = 5.6, n_2^e = 6, n_3^e = 7, \cdots n_7^e = 14, n_8^e = 9, n_9^e = 14, n_{10}^e = 28$. These numbers agree with the numbers from the simulation very well. They are approximately $\mathbf{n}^e = (5,6,7,8,9,5,14,9,14,28)$.

The reason for less efficient sectors to become larger than the initial starting values is that they must grow to meet the demands, while the efficient sectors shrink in size, because being efficient the initial sizes are too large for the demands.

The theoretical value of Y^e/n^e is about 0.42. The corresponding sim-

[11] All patterns are run 200 times for 500 periods with $\theta = .6$ except as we note below. Pattern $P2$ has also been run with $\theta = .2$. Pattern $P5$ is also run for 1000 periods 400 times. We skip the first 150 periods to avoid transient responses in the figure 5.

[12] Because standard deviation of outputs are still large due to small numbers of runs, effects of different demand patterns on the statistical features of cycles are not so clear cut. Peak-to-peak swings are about 2 per cent of the mean levels of outputs.

ulation number is also about 0.42. Here we also note that the maximum excursion $(Y_{max} - Y_{min})/Y_{mean}$ is about 2.5 per cent of the mean output.

The four panels of Fig. 9 show how the number of sectors increases, together with the total number of sizes, total output, and output per unit size, for the demand pattern $P3$ with $\theta = .3$. As the value of θ is increased, the number of new sectors increases more quickly. For samll values such as $\theta = 0.01$, new sectors come in much more slowly.

7 Concluding Remarks

This paper demonstrated that sectoral adjustments and reallocations of resources generated fluctuations of aggreagate economic activity, and that the average level of aggregate output depended on the sectoral demand pattern. By a careful examination of the U. S. job creation/destruction data, Davis *et al.* (1996), were led to the conclusion that a kind of approach we took in this paper was, in fact, important;

> Prevailing interpretations of business cycles stress the role of aggregate shocks and downplay the connection between cycles and the restructuring of industries and jobs. Several aspects of gross job flow dynamics do not fit comfortably with prevailing views. Rather, the empirical evidence points to the need for a richer view of business cycles that highlights their connection with the restructuring process (Davis *et al.* [1996; p. 83]).

Economists routinely assume that resources are instantaneously reallocated so as to make productivities in all the sectors equal. Instead of this standard assumption, we assume that allocation of resources takes time. This process is fundamentally stochastic. The model rests on the idea of *holding time* which determines the probability of the sector which actually increases/decreases the level of output in response to excess demand or supply. The model solves the conceptual problem of which agent moves first. In the usual agent-based simulation models, agents are random with uniform probability.

Using a simple model, we show how fluctuations of aggregate enconomic activity *endogenously* arises, and that the greater demand for high productivity sector is, the higher the *expected* level of GDP is. This result provides a new perspective to *the principle of effective demand.* Since the model is extremely simple, we believe that the results are generic.

Some final comments. In this paper we have taken the 'birth' and 'death' probabilities to depend on the sizes of the sectors. An alternative specification may be that these probabilities depend on the sizes of excess demand themselves. This possibility is definitely worth pursuing.

We note that changing the production technology from linear as in equation (1) to concave such as $c_i n_i^{\gamma}$, with $0 < \gamma \leq 1$, does not significantly change the basic results; Specifically, the patterns of the sign changes of excess demands in response to changes in n_i in the two sector model do not change if we replace β by $\beta^{1/\gamma}$. In this case, the inequality $n_2 > \beta(n_1 + 1)$ is replaced with $(n_2)^{\gamma} > \beta(n_1 + 1)^{\gamma}$, that is with $n_2 > (\beta)^{1/\gamma})(n_1 + 1)$, for example. The regions R_1 through R_6 are analogously defined by lines L_1 through L_5 with slope $(\beta)^{1/\gamma}$. Arguments leading to the derivation of the stationary distribution go through with β replaced by $\beta^{1/\gamma}$. Since the Proposition 2 in section 4 holds for all values of β, it also holds for economies with $c_i n_i^\gamma$, $i = 1, 2, \dots, K$.

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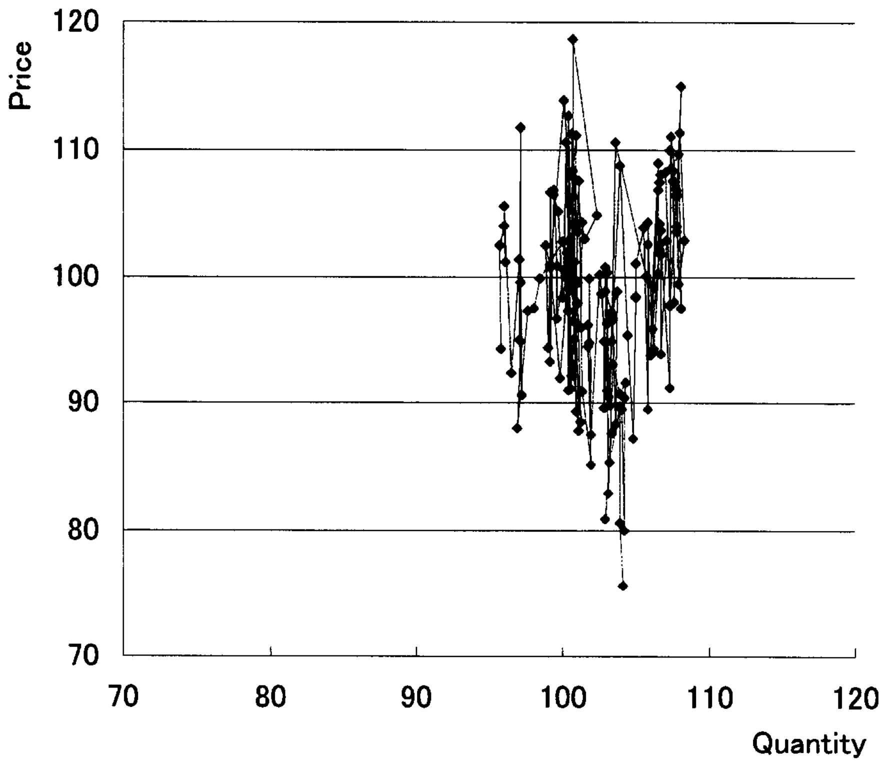
Table 1: Dynamics of Two-Sector Model

Region	f_1	$f_1^1(+)$	$f_1^1(-)$	$f_1^2(+)$	$f_1^2(-)$
R_1	+	+	*	*	+
R_2	+	–	*	*	+
R_3	+	–	*	*	–
R_4	–	*	+	+	*
R_5	–	*	+	+	*
R_6	–	*	–	–	*

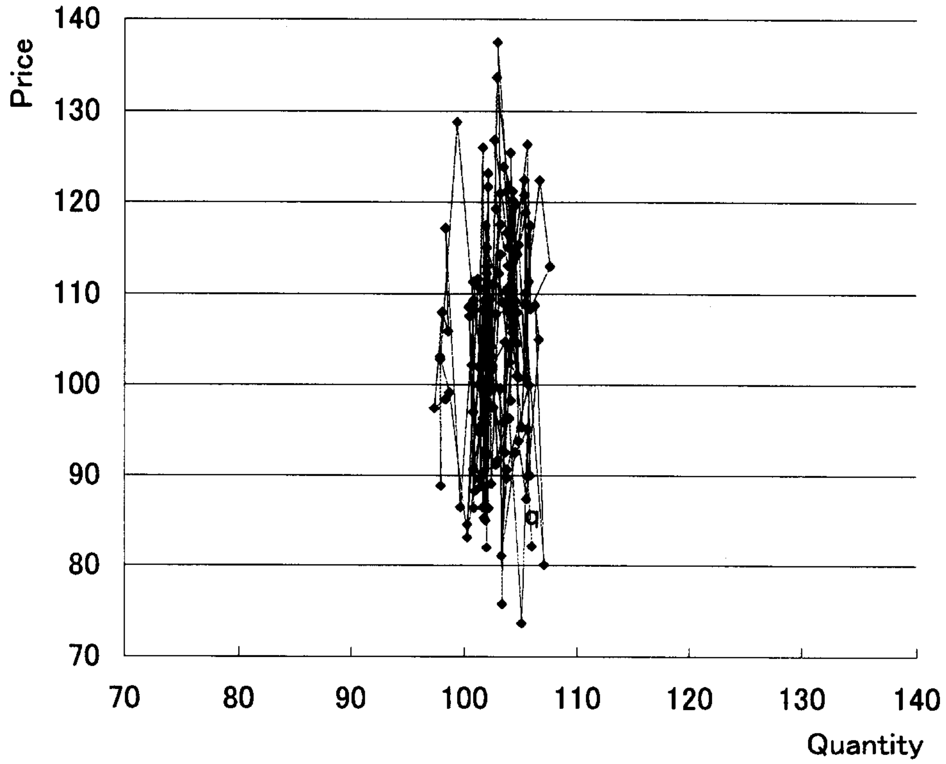
Note: See the text and Figure 1 for regions R_1 though R_6. The table shows the signs of f_1, $f_1^1(+)$, and so on. The symbol "$*$" means "no entry", that is logically impossible combinations.

Figure 1(a) : Price and quantity: Manufacturing industry
1987(Jan.)-2000(Jan.)



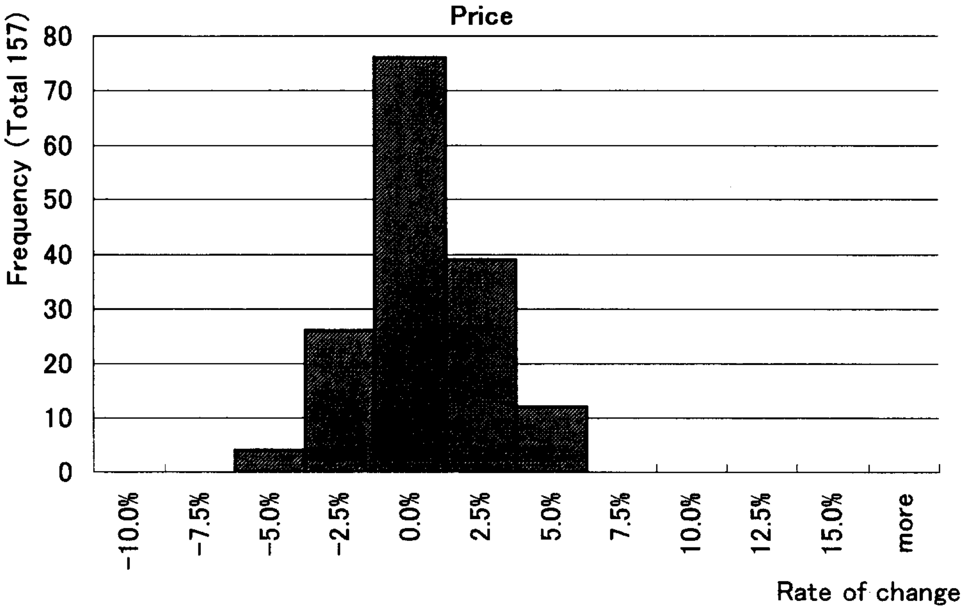
Source: Price is *wholesale price index* (WPI)(1995=100);
quantity is the *index of industrial production* (IIP)(1995=100).

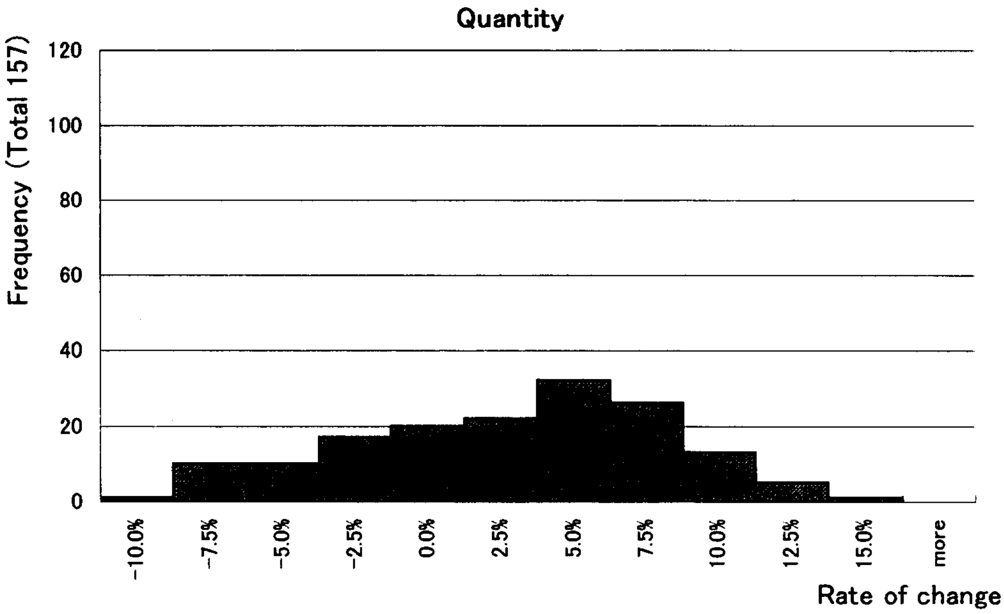
Figure 1(b) : Price and quantity: Transport machinery
1987(Jan.)-2000(Jan.)



Source: Price is *wholesale price index* (WPI)(1995=100);
quantity is the *index of industrial production* (IIP)(1995=100).

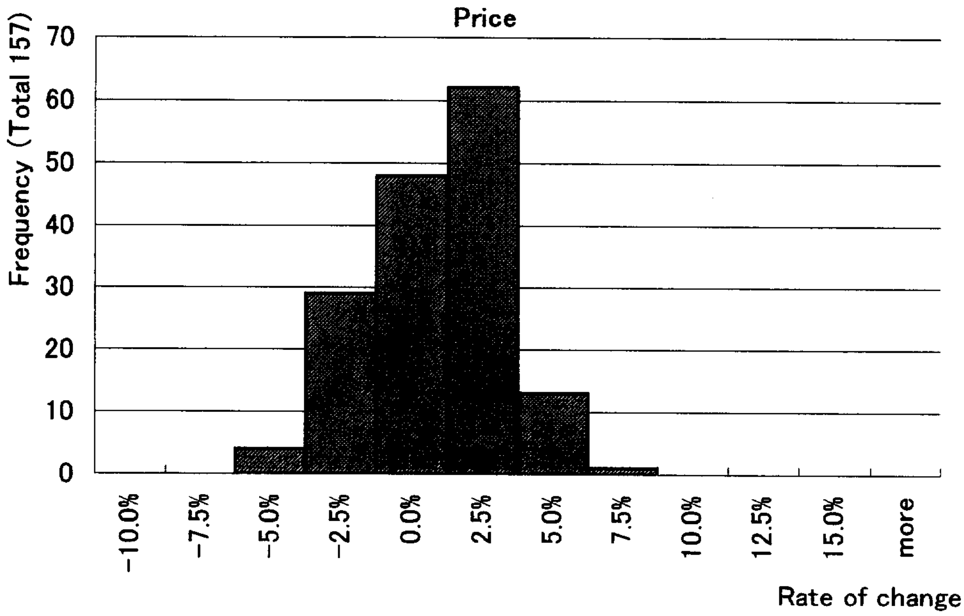
Figure 2 (a) : Frequency distributions of rate of changes in price and quantity: Manufacturing industry 1987(Jan.)–2000(Jan.)

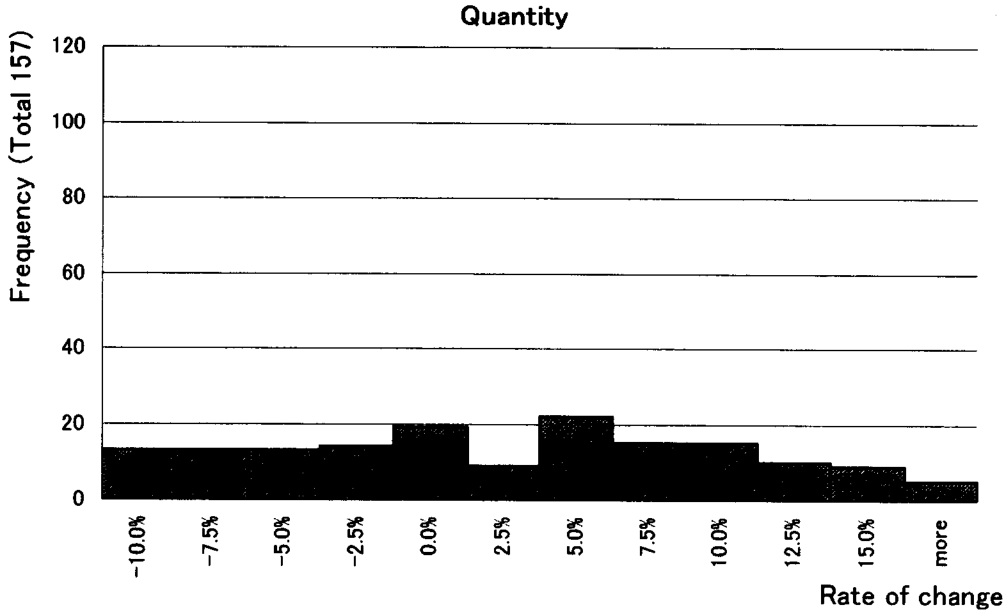




Source: Price is wholesale price index (WPI)(1995=100);
quantity is the index of industrial production (IIP)(1995=100).
Rate of change is to the same month in preceding year.

Figure 2 (b) : Frequency distributions of rate of changes in price and quantity: Transport machinery 1987(Jan.)-2000(Jan.)





Source: Price is wholesale price index (WPI)(1995=100); quantity is the index of industrial production (IIP)(1995=100). Rate of change is to the same month in preceding year.

$\beta > 1$

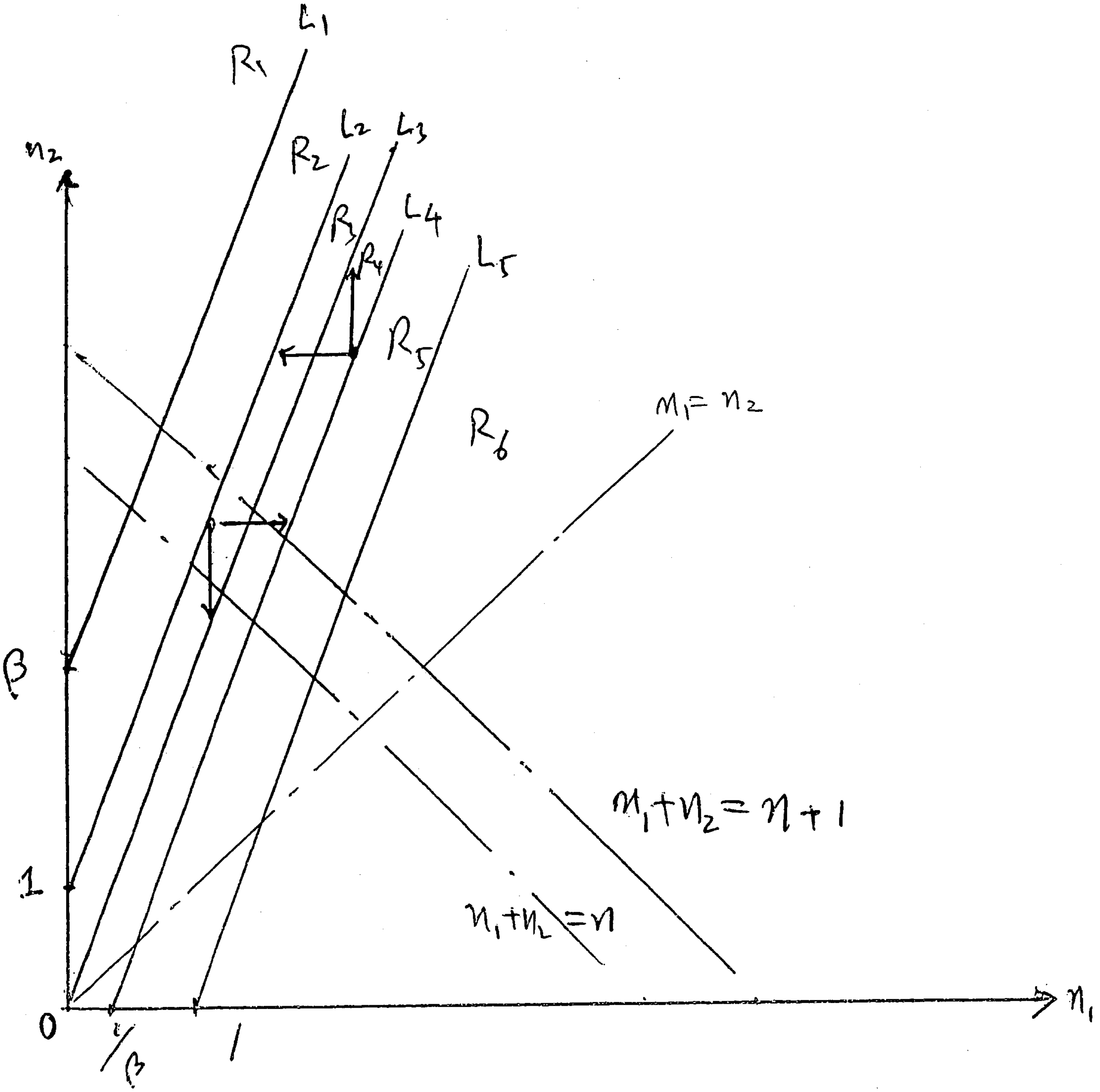
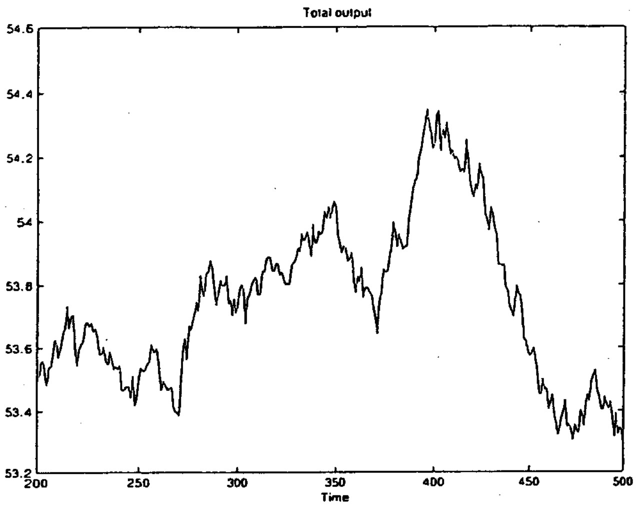
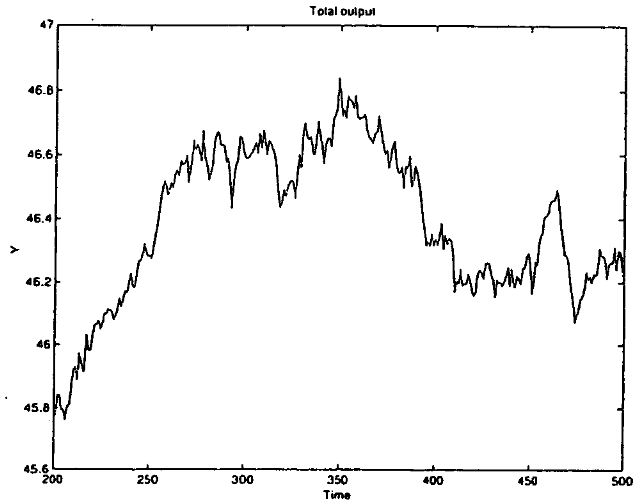


Figure 3

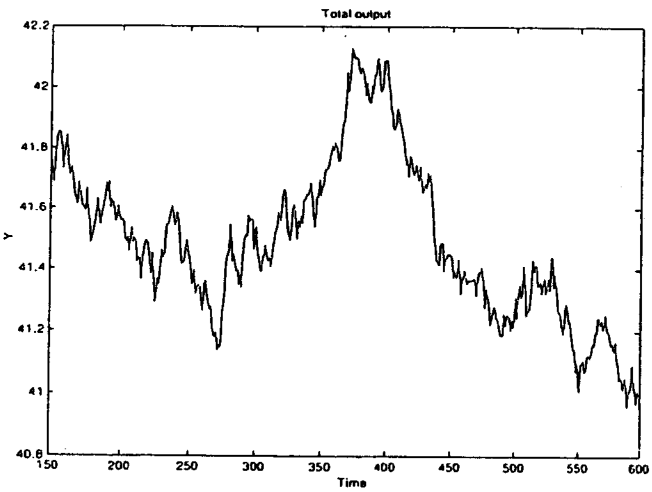
Figure 4



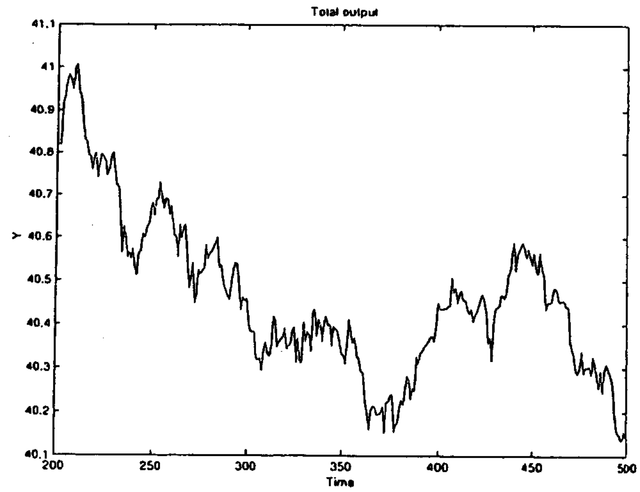
P1



P2



P3



P4

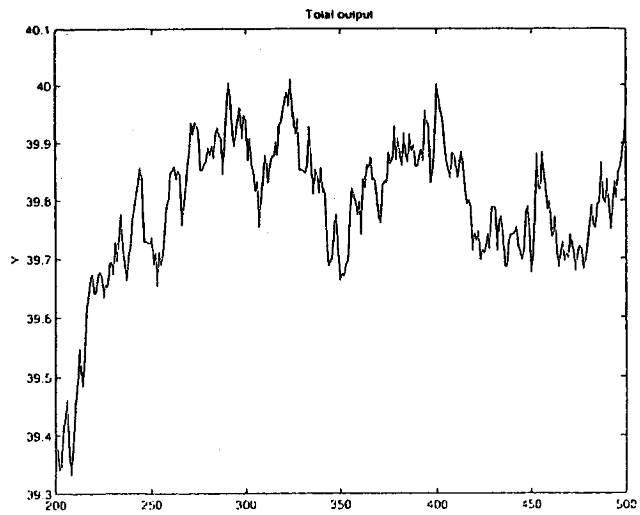
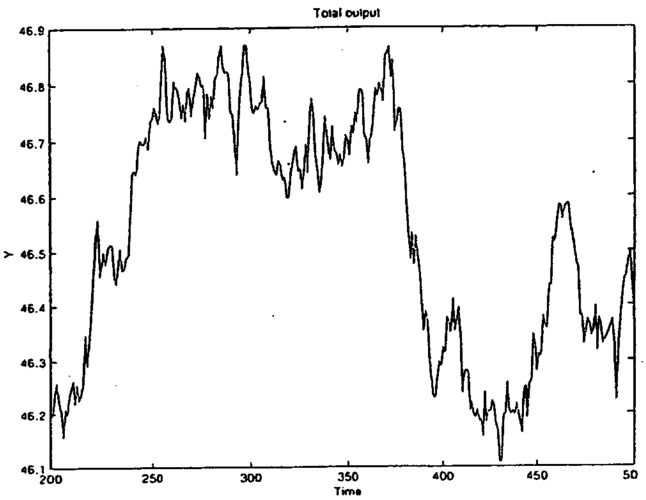
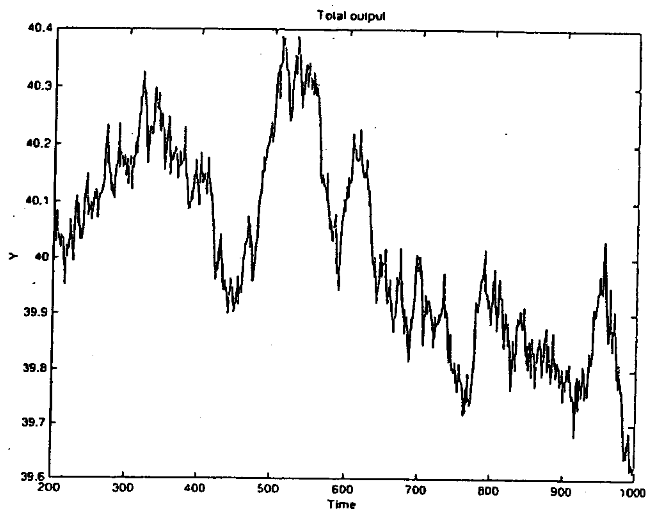


Figure 5



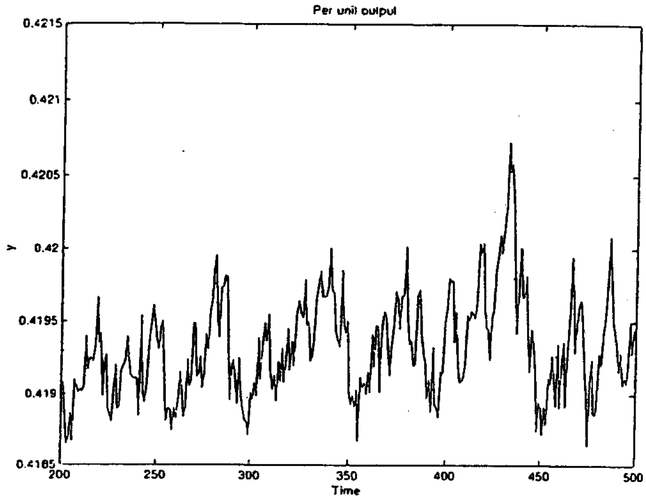
P2 , θ = .2

Figure 6

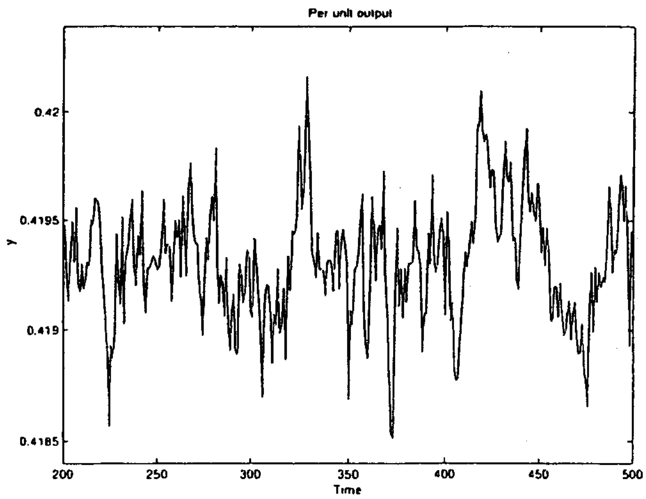


P5 θ = .6

Figure 7

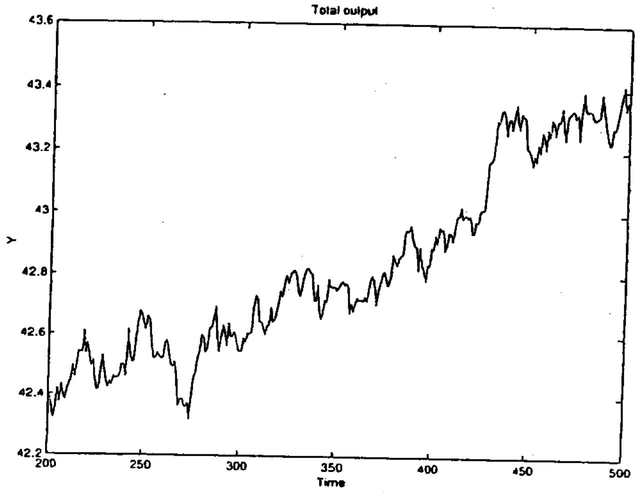


P3
θ = .1



P3
θ = 1

Figure 8



P3 D=.11

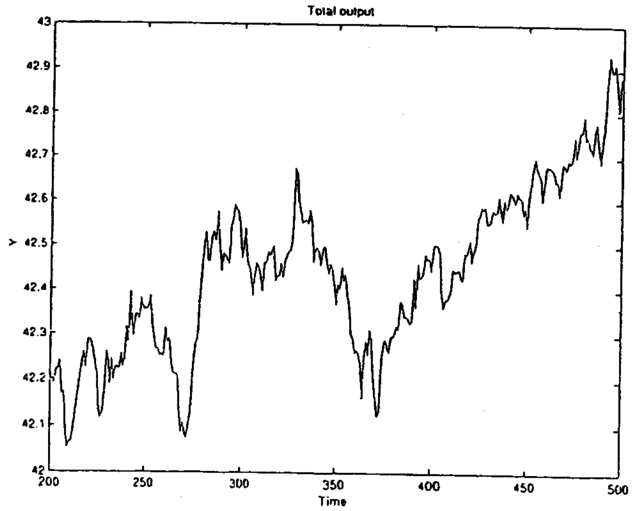
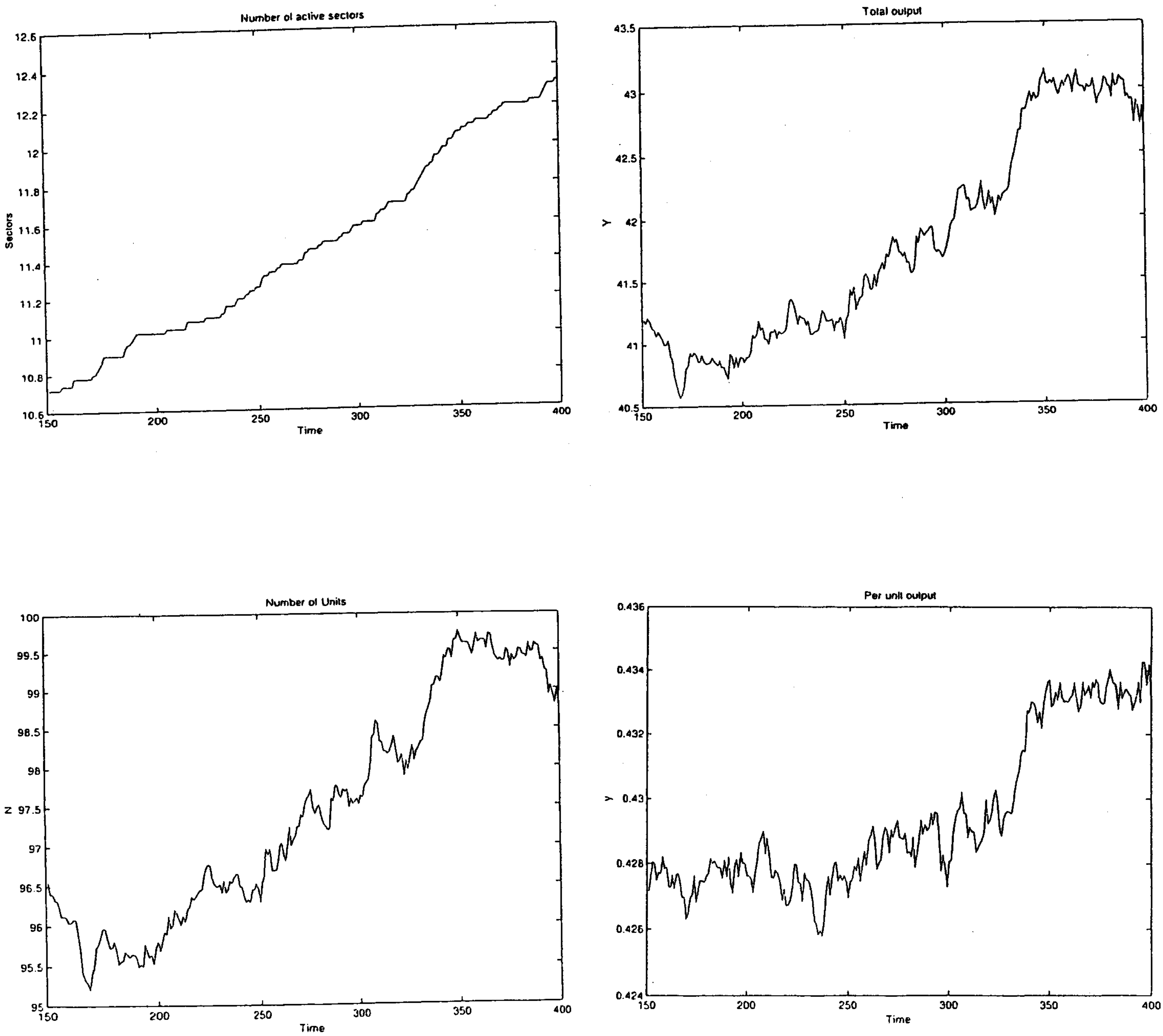


Figure 9



Number of active sectors
Sectors
Time
Total output
Y
Time
Number of Units
N
Time
Per unit output
y
Time