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The Parking Lot Problem

Maria Arbatskaya
Emory University

Kaushik Mukhopadhaya
Emory University

Eric Rasmusen
The University of Tokyo / Indiana University

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Maria Arbatskaya, Kaushik Mukhopadhaya,
and Eric Rasmusen

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Abstract

If there is queueing for an underpriced good, the queueing can eat up the entire surplus, eliminating the social value of the good. An implication is that there is a discontinuity in social welfare between "enough" and "not enough" for certain goods such as parking spaces. This implies that if there is uncertainty in the number of demanders, the amount of the good should be set well in excess of the mean demand.

Keywords: parking lot, size, rent-seeking, timing, waiting line, queue.
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Maria Arbatskaya, Department of Economics, Emory University, Atlanta, GA 30322-2240; Phone: (404) 727 2770; Fax: (404) 727 4639; Email: marbats@emory.edu.

Kaushik Mukhopadhaya, Department of Economics, Emory University, Atlanta, GA 30322-2240; Office: (404) 727-6387, Fax: (404) 727 4639, Email: kmukhop@emory.edu.

Eric Rasmusen, Visiting Professor, CIRJE, University of Tokyo, Erasmuse@e.u-tokyo.ac.jp, and Professor of Business Economics and Public Policy and Subhedar Faculty Fellow, Indiana University, Kelley School of Business, BU 456, 1309 E 10th Street, Bloomington, Indiana, 47405-1701. Office: (812) 855-9219. Fax: 812-855-3354, Erasmuse@indiana.edu.
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School's Olin Center and the University of Tokyo's Center for International Research on the Japanese Economy for their hospitality while this paper was being written. Notes starting with xxx are notes to ourselves for future drafts. The most recent draft may be found at Http://Php.Indiana.edu/ erasmuse/papers/parking.pdf.

1. Introduction

People come close to getting killed over parking. In 1998, a Cal State student almost died over a parking space. In China, a 30 year- old man "was in critical condition after being stabbed with a knife by a 64-year-old man in a row over a parking space in Pei Street, Mongkok," Another news story starts "There were three elderly ladies – but only one handicapped parking space. Something had to give...." and continues predictably.[1] Parking rage is widespread and there are even services which help people cope with it.[2] Many drivers are forced to arrive at a parking lot well in advance of their preferred time to secure a spot. It is therefore surprising that planners claim that parking lots are operating "at peak efficiency" when their capacity utilization is 85–90%.[3] Shopping malls display an apparently useless excess of parking. One's natural response to their acres of parking lots is that although dying downtown retailing districts may have had too little parking, the malls have gone to a ridiculous extreme. After all, mall parking lots are almost never full.

It can nonetheless happen that more people want to park than there are spaces available – in the malls, the week before Christmas; in some universities, every day. the rent-seeking competition that ensues can dissipate most or all of

[1] Cal State student: April 2, 1998, http://www.aloha.net/~dyc/parking.html. China: *South China Morning Post*, 05/28/2000. Elderly ladies: Reuters News Service, 1998 xxx add cite.

[2] According to an article "Parking Rage" from *Parking Today*, March 1999, parking rage is on the rise. A writer for the *Chicago Sun Times* is cited saying that parking rage is "the result of too many people (cars) in too small a space." It is also attributed to drivers' expectations to find a garage full. Road rage is said to be one of the most popular contact sports in the United States (*Newsday*, 12/06/1999). Donald Anchorman, the author of a best-selling book *I Spit on Your Windshield,* is quoted as saying that road rage involves more people than baseball, basketball, football and hockey combined and that the only thing worse than road rage is parking rage. "People will fight to the death for their space. If they don't fight, it is the only time you will see grown men cry." Dr. Driving at *http://www.aloha.net/~dyc/* provides links to many acticles on parking rage.

[3] "It is important that the parking supply include a sufficient 'cushion' in excess of the necessary spaces to allow for the dynamics of vehicles moving in and out of parking stalls and to reduce the time to search for the last few available spaces. This cushion also allows for vacancies created by restricting lots to certain users, misparked vehicles, snow cover, and minor construction. Thus, a supply of parking operates at peak efficiency when occupancy is 85% to 90%. When occupancy exceeds this level, there may be delays and frustration in finding a space. The parking supply may be perceived as inadequate even though there are spaces available in the system." *1987 Regional Center Parking Study* at http://www.bts.gov/NTL/DOCS/rc.html

the rents from the parking lot, rendering it worse than useless. Thus, the planners are right: to avoid wasteful competition for parking spaces, parking lots should be surprisingly large. It can even be socially optimal to have parking lots less than 50% full on average. This can be done either by apparently wasteful expenditure on extra spaces or by apparently wasteful restriction on the number of people who are given permits to park in existing spaces.

Contrary to our conclusions, regulators insist on reducing standards for parking. Based on the finding that the average parking supply count exceeds demand count by 30%, it is recommended that "local jurisdictions consider reducing their parking requirements" to new and expanding developments and to allow property owners of existing work sites "to request reductions in parking supply." According to a Seattle study, the data indicate excess supply, which suggests that a reduction in parking supply is desirable. The authors propose alternative methods for reducing the supply. They admit, however, that *"A major policy-related issue is how much allowance to provide over the design-level demand in setting the size of a given parking facility."*[4]

Parking supply management is discussed in other studies, where it is proposed that a parking supply reduction will induce employees to look for other commuting options and this may promote travel effectiveness. The Cambridge City Council in England recently replaced the minimum standard on the number of spaces per new home that developers had to provide, 1.5, with the maximum standard of 1.5. The U.K. Government is changing planning guidelines to stop developers from building more than 1.5 parking spaces per dwelling. The effect of the policy is to have "ever more cars chasing ever fewer on-street parking spaces."[5]

We believe that strategic behavior of drivers, overlooked in the studies on parking, must be considered when deciding on the parking lot size. Having 10–15% of the spaces empty may well indicate too *small* a parking lot size, given uncertainty in the demand for parking. Our finding will arise from a strategic

[4] *1991 Parking Utilization Study* undertaken by the Research and Market Strategy Division of the Transit Department in the Municipality of Metropolitan Seattle. The supply counts include all types of parking (visitor, disabled, pool, reserved, general) excluding spillover and demand - all types of parking. The intricacies of optimal public regulation of parking–as opposed to the private problem of choosing lot size– are beyond the scope of the present article, depending as they do on the spillover from private lots onto public parking. We mention it here just to indicate that the importance of excess capacity is underappreciated.

[5] *The Sunday Telegraph,* United Kingdom, 05/16/1999.

asymmetry in the effects of over- and under-estimation of demand on social welfare. Extra parking spaces are costly, to be sure, in proportion to their number, but a small shortage in parking can result in a discontinuous and huge social loss.

To determine the optimal size of a facility with an unrestricted access, we construct a model of competition between drivers for parking spaces. The drivers face a trade-off between the disutility of arriving earlier than their preferred time and the increased probability of securing a space in the parking lot. Since the cost of arriving early is incurred regardless of whether a driver is successful in finding a parking space, the contest is a multi-unit all-pay auction. Due to the dynamic nature of the players' decisions on *when* to arrive at the lot, their strategies can be quite complex. Nevertheless, we are able to show that full rent-dissipation occurs in the equilibrium of the parking game whenever the size of the lot is too small.

The rest of the paper is organized as follows. Section 2 provides a brief discussion of the literature on the subject. In Section 3, we describe parking as competition between a known number of drivers under full observability of the parking lot. We derive the dynamic equilibrium strategies and show that the optimal size of the parking lot is equal to the number of drivers. Section 4 deals with the case of uncertainty in demand. When the size of the parking lot is set before the uncertainty about the number of drivers is resolved and the planners only know its distribution, they should build a sufficiently large parking lot. On average, a large proportion of the parking spots will be unoccupied for a parking lot of an optimal size. Section 5 offers discussion of the results and Section 6 concludes.

2. The Existing Literature

The problem of managing a transportation system has been analyzed mostly by urban economists and operations researchers. Queueing models with tolls, e.g. Naor (1969), assume an exogenous stationary customer arrival process and random service times. Customers benefit from the service, but have to incur a constant cost per unit of time from queueing. In an equilibrium, a consumer joins the queue if its length is below a threshold level. The last consumer in the queue is just indifferent between joining and staying out. The equilibrium outcome is not efficient due to the negative externality that a customer imposes on those arriving later. Nevertheless, rent dissipation through queueing is not full due to the randomness in the arrival and departure processes.

Arnott et al. (1993) compare alternative toll regimes in a model with a single traffic bottleneck and identical commuters incurring linear time inconvenience cost (the "schedule delay cost"). Richard Porter (1977), writes on the optimal size of underpriced facilities, and points out that congestion is better than queuing, but does not seem to note the discontinuity, or actually solve for the optimal size.

The strategic incentives of people to adjust their purchase schedules are analyzed by Robert Deacon and Jon Sonstelie (1991). Consumers choose the size of purchases to minimize the total cost of shopping for an underpriced good, which includes shopping and storage costs. The waiting time in a queue increases until the market clears. The authors note that consumers are no better off from a price ceiling while suppliers are worse off - there is a deadweight loss in rationing by waiting.[6]

Other economists have also looked at similar problems; notably, William Vickrey, more famous for his work on auction theory. In a "pure bottleneck", congestion in transportation facilities, queues form at a single route segment of fixed capacity. In Vickrey's pure bottleneck model (Vickrey, 1969), commuters have a preferred travel times through the bottleneck, distributed uniformly. For a sufficiently large capacity, no queue develops and the commuters arrive at their preferred times. For a smaller capacity, the latter is not possible, queue develops and some commuters arrive early and/or some arrive late. Each commuter faces a trade-off between the disutility of arriving at a less-preferred time and the cost of waiting in line. Vickrey (1969) notes a "sharp discontinuity" in the amount of delay at the level of capacity just sufficient to accommodate the traffic. Vickrey points out that the optimal investments in the capacity extension differ in the first-best and the second-best situations. In the first-best, with the presence of the optimal price structure (a toll fee that leads to the efficient use of the facility), the benefits of capacity extension are not as "capricious" as in the case when the access to the facility cannot be restricted by means of fees. In the second-best, "Expansion inadequate to take care of the entire traffic demand...may turn out to be hardly worthwhile..." In Vickrey's model, however, capacity extension reduces delays and is beneficial to travellers. We show that it may not have any positive effects to offset the costs of construction and be a pure waste.

[6] See also Deacon and Sonstelie (1985, 1989) and Deacon (1994).

Landsburg, in *The Armchair Economist*, has a story about an aquarium, showing how all gains from a facility with free access can be dissipated due to congestion. This means that building a new aquarium does not benefit anyone while it is costly to build. The argument of full rent dissipation holds when there are many identical potential customers, who are ready to jump in whenever there is a profitable opportunity. In a way, this result is similar to the zero-profit outcome in a contestable market: the potential competition puts a pressure on the competitors. At the margin, a person is indifferent between using a facility with unrestricted access and not using it.[7] However, Landsburg's focus is not on capacity.

In the theory of waiting lists proposed by Lindsay and Feigenbaum (1984), market clearing occurs due to depreciation of product value in time. Since delivery of a service in future can be less valuable than an immediate delivery, the potential consumers are discouraged to put their name on the list when the list is too long. By this argument the authors explain the persistence of long waiting lists for non-emergency in-patient care at Britain's National Health Service and fruitlessness of short-term measures aimed at a substantial reduction of the waiting list. See, however, the critique by Cullis and Jones (1986).

In our model, there are no waiting lists, congestion, or waiting in line to be served after the facility is full. The rent dissipation comes in the form of costly schedule adjustment by the travellers. In an effort to secure a parking spot, they come well in advance of their preferred time and dissipate all the rents whenever the number of drivers exceeds the number of parking spaces.

[7] In 1999 a Ticket Master in Windsor, Ontario, used an interesting approach to distributing tickets. To prevent the practice of overnight lineups and discourage scalping a random number line-up procedure was adopted. This approach assigns a random number to each ticket buyer present at the time the office is opening and the queue is formed accordingly. While the procedure may appear unfair to many ticket buyers, it eliminates incentives to arrive early camping over night or long hours of waiting in line by providing a "fair and equitable method that provides each customer with an opportunity to be first in line."

3. The Parking Game Under Full Observability and No Uncertainty

The Model

A set of players– the drivers– is denoted $I \equiv \{1, ..., N\}$. Assume that all drivers have a common preferred arrival time, $t = T$. We can think of drivers as workers who have to show up for work no later than time T. Each demands just one parking space. Let $S > 0$ be the size of the parking lot, $S \in Z_+$. The parking game arises when $S < N$, so players compete for a limited number of parking spaces by arriving earlier to secure a spot. The allocation mechanism which distributes parking spaces to first S drivers to arrive at the parking lot can be seen as a form of first-in first-out queuing discipline.[8]

We model the parking game in discrete time, with $t \in \{t_0, ..., T\}$. In each period t, a player decides whether to arrive at the parking lot instantly, given the observed history of other players' arrivals. Once a parking spot is taken by the player, the spot remains occupied and uncontested till the end of the game. Denote the time-t decision of player i to arrive at the parking lot as $d_i^t = 1$ and the decision to wait till the next period as $d_i^t = 0$. The decision set at time t is $D \equiv \{0, 1\}$ for any player who has not parked by time $t - 1$: player i chooses $d_i^t \in D$ if $d_i^{t-1} = 0$.

The history at time t can be summarized by the intensity of usage of the facility at time $t - 1$, called the capacity utilization, x_{t-1}.

Definition 1. The *capacity utilization* is the number of parking spots occupied at time t, $x_t = \min \left\{ \sum_{i=1}^{N} d_i^t, S \right\}$.

In the first best, all N players arrive at T, so $d_i^t = 0$ for $t < T$ and $d_i^T = 1$. S of them will get to park in the parking lot, so $x_t = 0$ for $t < T$ and $x_T = S$.

What matters to player i when deciding whether to rush for a parking spot is the current capacity utilization and the time of making the decision. Denote the

[8]On the first-in last-out queues, see a comment by Hassin (1985) and its discussion by Nalebuff (1989). Surprisingly, this alternative queue discipline leads to the socially optimal behavior of players. It is, however, rarely observed.

set of all possible time-t histories by H, $H = \{0, ..., S\}$ The set is time invariant and includes all levels of capacity utilization. The strategy of player $i \in I$ is a sequence of maps $\{d_i^t\}_{t=t_0}^T$ from the set of histories H to the decision set D, $d_i^t : H \rightarrow D$.

The payoff structure is that of an all-pay auction:

$$u_i(t_i) = \begin{cases} v - L(t_i) & \text{if} \quad t_i < t' \\ pv - L(t_i) & \text{if} \quad t_i = t' \\ -L(t_i) & \text{if} \quad t_i > t' \end{cases} \tag{1}$$

where $L(t_i) = \quad (T - t_i)w \quad \text{if} \quad t_i \leq T$ is the utility loss incurred by a driver who arrives at t_i and

$$p = \frac{S - \sum_{j \neq i} d_j^{t^{'}-1}}{\sum_{j \neq i}(d_j^{t^{'}} - d_j^{t^{'}-1}) + 1}$$

is the probability that player i wins one out of $(S - \sum_{j \neq i} d_j^{t^{'}-1})$ parking spots left at $t^{'}$, when competing with $\sum_{j \neq i}(d_j^{t^{'}} - d_j^{t^{'}-1})$ drivers arriving at $t^{'}$.

When a player observes at time t that the parking lot is full, the player waits till the preferred time and parks in a less convenient parking lot, obtaining payoff zero since $-L(T) = 0$. Let us define the *indifferent arrival time* as

$$t^* \equiv T - v/w. \tag{2}$$

A player arriving at this time will have a payoff of zero, since his payoff will be $v - w(T - (T - v/w))$. To avoid pesky but purely technical integer problems, assume t^* is an integer number.

Claim 1. *The following is a pure-strategy subgame- perfect Nash Equilibrium to the parking game ($N = 2, S = 1$).*

(i) Player 1 arrives at t^* if the rival has not arrived before t^*: $d_1^t = 0$ for $t < t^*$; if $x^{t^*-1} = 0$, then $d_1^{t^*} = 1$ while if $x^{t^*-1} = 1$, then $d_1^t = 0$ for $t \in [t^*, T)$ and $d_1^T = 1$.

(ii) Player 2 arrives at $t^* + 1$ if the rival has not arrived before $t^* + 1$: $d_2^t = 0$ for $t \leq t^*$; if $x^{t^*} = 0$, then $d_2^{t^*+1} = 1$ while if $x^{t^*} = 1$, then $d_2^t = 0$ for $t \in (t^*, T)$ and $d_2^T = 1$.

Proof. To prove that the listed strategies constitute a Nash Equilibrium we must show that there are no profitable deviations for any player, given the strategy of the opponent. Time t^* is such that player 1 is indifferent between arriving at t^* and not parking in the lot at all, $u_1(t^*) = v - L(t^*) = 0$. Arriving earlier than t^* (at $t < t^*$) yields the player a negative payoff, $v - L(t) < 0$. Arriving at $t^* + 1$, player 1 obtains $v/2 - L(t^*) < 0$. After $t^* + 1$ the parking lot is full and so player 1 would not arrive after $t^* + 1$. Similarly, for player 2 arriving earlier that t^* or at t^* yields negative payoffs. At t^* the parking lot is full and later arrival is not beneficial until T ■

The path of the strategy profile is $(d_1^t, d_2^t) = (0, 0)$ for $t < t^*$ and $(d_1^t, d_2^t) = (1, 0)$ for $t \geq t^*$.

Claim 2. *The following is a pure-strategy subgame- perfect Nash Equilibrium to the parking game* $(N \geq 2)$.

No player arrives before the *indifference arrival time*: $d_i^t = 0$ for $t < t^*$ and $i \in I$.

(i) Players 1 through n_1: if $x^{t^*-1} = 0$, then $d_i^{t^*} = 1$ while if $x^{t^*-1} = 0$, then $d_i^t = 0$ for $t \in [t^*, T)$ and $d_i^T = 1$; $i \in \{1, ..., n_1\}$; $n_1 = S - x^{t^*-1}$

(ii) Players $n_1 + 1$ through $S + n_2 + 1$: $d_i^{t^*} = 0$; if $x^{t^*} < S$, then $d_i^{t^*+1} = 1$ while if $x^{t^*} = S$, then $d_i^{t+1} = 0$ for $t \in [t^*, T)$ and $d_i^T = 1$; $i \in \{S + 1, ..., S + n_2 + 1\}$; $n_2 = S - x^{t^*}$

(iii) Other players: $d_i^t = 0$ for $t \in [t^*, T)$ and $d_i^T = 1$,

where $t^* = T - \frac{v}{w}$.

Proof. The proof is similar to that of Claim 1. Any time player i, $i \in \{1, ..., n_1\}$, deviates from the equilibrium strategy by not arriving at t^*, player $j \in \{S + 1, ..., S + n_2 + 1\}$ arrives at $t^* + 1$ and player i receives a payoff of zero at best. Arriving earlier than t^* never benefits a player. Given that the parking lot is full at t^*, player j, $j \in \{S + 1, ..., S + n_2 + 1\}$ does not arrive until T. ■

The path of the equilibrium strategy combination is $(d_1^t, ..., d_N^t) = (0, ..., 0)$ for $t < t^*$ and $(d_1^t, ..., d_S^t, d_{S+1}^t, ..., d_N^t) = (1, ..., 1, 0, ..., 0)$ for $t \geq t^*$. In the equilibrium, $n_1 = S$ and $n_2 = N - S$. In words: one group of S players arrive at time t^* and park in the lot, while a second group arrive at T and park elsewhere. Both groups receive the same payoff of zero.

Welfare and Parking Lot Size

The rent-seeking game between drivers is described and solved in the previous subsection for fixed parking lot size. We next characterize social welfare and study its dependence on the size of the lot.

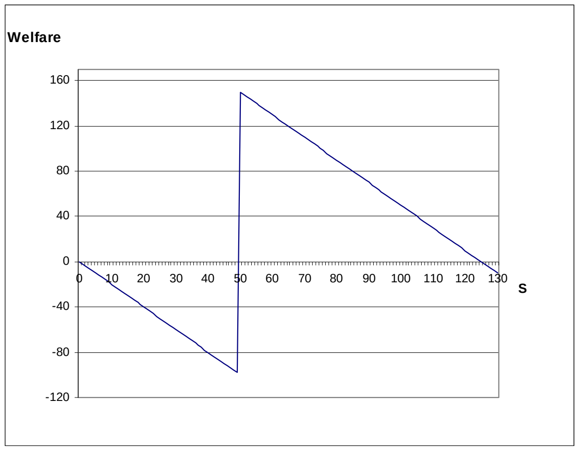
The drivers value each of the parking spaces in use at $v > 0$. Suppose the marginal cost of providing a parking space is constant, $c > 0$. The social welfare from a parking lot of size S is then

$$W(S) = \begin{cases} vN - cS - \sum_{i=1}^{N} L(t_i^*(N,S)) & \text{if} \quad S > N \\ (v-c)S - \sum_{i=1}^{N} L(t_i^*(N,S)) & \text{if} \quad S \leq N \end{cases} \tag{3}$$

We may decompose welfare into two parts: the *provision cost* cS and the *flow value*, which is either $vN - \sum_{i=1}^{N} L(t_i^*(N,S))$ or $vS - \sum_{i=1}^{N} L(t_i^*(N,S))$, depending on whether $S > N$ or not.

Unfortunately, in the competition for the limited number of parking spaces, drivers dissipate all the rents. In the equilibrium of Claim 2, they are just indifferent between arriving early enough to secure a spot and not parking in the parking lot at all. The full rent dissipation occurs each time the number of players exceeds the size of the lot. Therefore, to avoid wasteful rent-seeking activity, the parking lots have to be designed to accommodate all the people who need the parking. When the number of such people, N, is known with certainty, the parking lot should have N parking spaces. This result is summarized in Proposition 1 and illustrated by Figure 1.

Figure 1. The Welfare from a Parking Lot of size S when $N = 50$, $c = 2$, and $v = 5$



Proposition 1. *The optimal size of the parking lot under certainty equals the number of users, $S^* = N$. All smaller sizes have flow values of zero and negative welfare, with the minimum welfare being at $S = N - 1$. All greater sizes have flow values equal that for $S^* = N$, but increasingly high provision costs.*

Proof. If $S < N$, then in the SPNE all rents are dissipated and $W(S \mid S < N) = -cS$. If $S \geq N$, each player is guaranteed a parking space and arrives at the preferred time $t_i = T$. Therefore, $W(S \mid S \geq N) = vN - cS$. It follows that welfare is maximized at $S = N$ as long as it is socially beneficial to build a parking lot at all, i.e. when $v - c > 0$. ■

4. The Parking Game Under Full Observability and Uncertainty

To model the situation under uncertainty, we assume that the number of players is random. The planner has to decide on the size of the lot before the uncertainty about the demand is resolved. Maximization of expected welfare requires a much bigger parking lot than the expected value of potential demand, because the loss function in Figure 1 is asymmetric. In a stochastic model, there are usually many empty spaces in the lot. Planners nonetheless should not increase the number of parking permits.

Suppose the number of players is uncertain and is drawn from a known probability distribution $f(N)$. What is the optimal size of the parking lot? When there is competition for parking spots, $S < N$, the benefit from the parking lot is negative, $W = -cS$. When the size of the lot is large enough, $S \geq N$, the benefit to N players is $W = vN - cS$. Expected welfare, given risk neutrality, is

$$EW(S) = \Sigma_{N=0}^{S} f(N)(vN - cS) + \Sigma_{N=S+1}^{\overline{N}} f(N)(-cS)$$

or

$$EW(S) = v \sum_{N=0}^{S} Nf(N) - cS = vE(N \mid N \leq S) - cS \tag{4}$$

The size of the parking lot should be increased as long as the marginal net benefit is non-negative

$$MB \equiv EW(S) - EW(S-1) = vSf(S) - c \geq 0 \tag{5}$$

Since the drivers benefit from the Sth parking space only if $N = S$, the welfare increase equals the probability that there are exactly S drivers multiplied by the extra benefit from eliminating rent-seeking behavior, vS, net of the cost. The intuition is that it is more and more important to have a big parking lot as S gets bigger, because there are more people who could get benefit from it. At the same time, it could be less likely that larger parking lots are filled out. Whether the marginal benefit of a parking space is decreasing or increasing depends on the relative strengths of the two effects.

Inequality (5) can be rewritten as

$$Sf(S) \geq c/v \tag{6}$$

Expansion of the parking lot is welfare-improving if the probability that exactly S drivers compete for S parking spaces times the size of the parking lot exceeds the relative cost of building a parking space, $\gamma \equiv c/v$.

Example 1: Discrete Uniform Distribution

Consider a discrete uniform distribution on the support $\{0, ..., \overline{N}\}$ with p.d.f. $f(N) = 1/(\overline{N}+1)$. Notice that for larger capacity levels, the benefit from building an additional space in equation (5) increases. It is equally likely at any capacity that the parking demand will be just met, and at a larger capacity benefits accrue to more people. Hence, there is no interior solution. With a uniform distribution, the parking lot should be big enough to include all the people who might possibly want to park– if it should be built at all.

$$EW(S \mid S < \overline{N}) = \frac{v(S+1)}{2(\overline{N}+1)}S - cS$$

and $EW(S \mid S \geq \overline{N}) = v\overline{N}/2 - cS$. Comparing $EW(0) = 0$ and $EW(\overline{N}) = (v/2 - c)\overline{N}$ reveals that the parking lot of size large enough to accommodate all potential demanders should be constructed as long as $v/2 - c > 0$. On average, 50% of the parking spaces will be unclaimed since $E(N) = \overline{N}/2$.[9] Hence, a parking lot is desirable if the expected value of a spot, $v/2$, exceeds its cost, c.

A numerical example for $c = 1, v = 5, \overline{N} = 100$ shows welfare at different capacity levels. When 50 drivers ($E(N) = 50$) are arriving to the parking lot with certainty, the welfare is $W(S \mid S < 50) = -cS$ and $W(S \mid S \geq 50) = 50v - cS$.

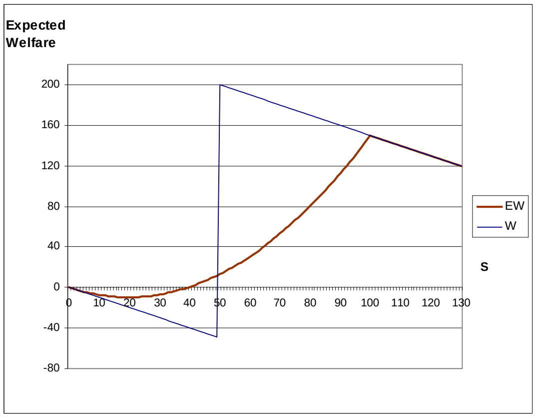
Table 1. Welfare and Parking Lot Size: Uniform Distribution

Number of Spaces, S	0	10	20	30	40	50	60	70	80	90	100
Expected Welfare, uncertainty	0	-7	-10	-7	1	13	31	53	80	113	150
Welfare, no uncertainty, N=50	0	-10	-20	-30	-40	200	190	180	170	160	150

Notes: Welfare is rounded to the nearest integer.

[9]For any probability distribution, $f(\cdot)$, such that the optimal capacity size is equal to the upper bound of the support of the distribution, capacity utilization is equal to the ratio of the expected number of drivers to the maximum number of drivers, $E(x_T) = E(N)/\overline{N}$.

Figure 2. Welfare and Parking Lot Size: Uniform Distribution



Even if $S = 40$, welfare is positive under certainty. This is a big difference from the case without uncertainty, where it would be near its minimum and very negative. The reason is that under uncertainty, even with very few parking spaces, it may happen that very few people need to park, and so there is no queuing and the parking spaces are valuable.

Example 2: Binomial Distribution

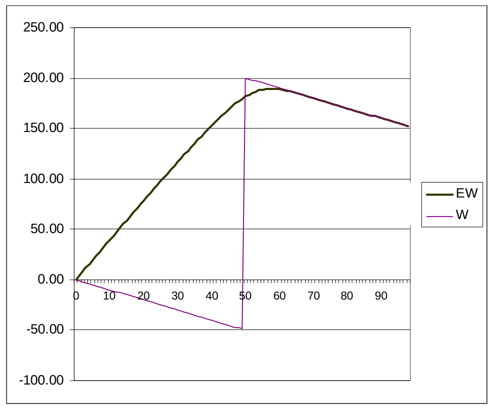
Consider a binomial distribution, which arises when drivers' needs for parking are independent random trials and suppose there is a fifty-fifty chance that each of 100 drivers will need parking. A numerical example for $c = 1, v = 5, \overline{N} = 100$, $p = 0.5$ shows welfare at different capacity levels.

Table 2. Welfare and Parking Lot Size: Binomial Distribution

Number of Spaces, S	0	10	20	30	40	50	60	70	80	90	100
Expected Welfare, uncertainty	0	39	78	117	153	182	189	180	170	160	150
Welfare, no uncertainty, N=50	0	-10	-20	-30	-40	200	190	180	170	160	150

Notes: Welfare is rounded to the nearest integer.

Figure 3. Welfare and Parking Lot Size: Binomial Distribution



In this example, the size of the parking lot should be $S = 58$. In contrast to the case of normal distribution, only about 8 out of 58 spaces are empty, on average. This corresponds to 86% utilization level. See Appendix A for an analysis of the parking problem when the number of drivers is large and uncertain. Appendix B provides further examples for discrete probability distributions.

5. Discussion

The model can be specified in two different ways, generating two different kinds of equilibrium, but full rent dissipation arises in either of them.

Observable Parking Lot

(a) *All drivers observe all arrival times, and can alter their arrival times instantly depending on what they observe.*

In equilibrium, S people show up at t^* and take the S parking spaces, where t^* is such that $w(T - t^*) = v$, and $N - S$ people show up at T. Each driver has a payoff of 0. The S early arrivers will not delay, because if one of them does, one of the $N - S$ late arrivers will alter his time of arrival to jump in ahead of him. Nobody has an incentive to unilaterally deviate by arriving earlier than the time assigned him by the equilibrium, because the earlier arrivers get a parking space anyway and the late arrivers would not get a parking space unless they deviated to a time earlier than t^*, in which case they would get negative payoffs.

Unobservable Parking Lot

(b) *Each driver chooses his time of arrival without observing what other drivers have chosen.*

In any mixed strategy equilibrium, drivers choose to arrive no earlier that time t^* and no later than time T.[10] Each driver has an expected payoff of zero and the total sum of all the losses incurred by drivers is equal to the total value of parking. The parking lot must still be built, however, so the social payoff is $-cS$. No driver can have a strategy with a positive payoff, or someone else would imitate that strategy by showing up slightly earlier.

If binding contracts could be made, the problem would be avoided. Everyone would arrive at T, S people would take the parking spaces, and the other $N-S$ would get side payments from the ones who park in the desirable lot. This, however, requires (a) communication to coordinate who parks, (b) low enough transaction costs for coordinating and making the payments, and (c) enforceability at low cost, so that people do not break their contract and arrive early or refuse to make the side payments later.

The rent-seeking behavior of drivers can be avoided by increasing the capacity level or by restricting the entry to the facility. Suppose there are S parking spaces and $N > S$ people willing to park. The access should be restricted to S people. If more people were given parking permits, the competition would not allow them to obtain benefit from parking. The effect of extra parking permits would be to reduce the benefit for the original permit holders to zero.

Note that it is assumed here that the drivers know in advance that N is small on a given day. If they believe that N is large, then they show up early, although there are plenty of spaces, and there is much more of a social loss.

[10]Holt and Sherman (1982) analyze the symmetric pure strategy equilibrium in a multi-prize waiting-line auction with heterogeneous players.

6. Conclusion

Our findings from the parking lot problem can be applied to any unregulated facility. If prices cannot serve as an allocation mechanism, the intertemporal variations in demand are reduced through the players' strategic behavior.

Consider the current electricity crisis in California. Since regulation is political, the good is underpriced. A peak in demand can cause a blackout of the whole system. Electricity, like parking, cannot be stored as other consumer products. The electricity facilities are able to supply much more than it takes on average to satisfy the demand.[11] A relatively low capacity utilization in this setting is similar to our finding for parking lots. The reasoning is similar as well: a slight excess demand can cause serious problems while an excess supply of the same size is less troublesome. Since there are millions of users, each user neglects his ability to cause an overload. The situation is somewhat more complex than in the parking lot problem because demand is not unitary– people choose the number of units to consume. Thus, when users are urged to and attempt to move their consumption from peak hours to off-peak, even if some users are persuaded, other less civic-minded users may increase their usage in response and leave the probability of overload unchanged.

Similarly, each year on New Year's Eve, it is nearly impossible to place an international call. Everybody is trying to reach a country at midnight (local time). Hence, some choose to call earlier to avoid the connection problems. This avoidance behavior is akin to queueing for a space in a parking lot before the lot is open.

To summarize, we believe that the strategic incentives of consumers are an essential element in planning capacity for an unpriced product– as important, or perhaps more important, than the obvious decision-theory problem of predicting uncertain demand and the obvious engineering problem of predicting capacity

[11] Currently, the reserve capacity in California is at the lowest level in three decades. The lagging supply and continuously increasing demand made it difficult for California to maintain the sufficient level of reserve to avoid emergency situations that it is currently suffering. For instance, operating at a "30 percent capacity reserve margin was the norm" in the past, but, the reserves have dipped to 15 to 20 percent capacity. The declaration of the stage-1 emergency when reserves drop below 7 pecent is designed to prevent the system failure. (Plants Sites & Parks 10/01/2000)".

cost. It can be very costly for society if the civil engineer knows no game theory.

Appendix A: Calculus for Large N

For larger N we can abstract from the integer problem and use of the calculus and re-write the expected welfare from the parking lot as

$$EW(S) = \int_0^S (vN - cS)f(N)dN + \int_S^N (-cS)f(N)dN$$

$$EW(S) = v\int_0^S Nf(N)dN - cS \tag{4*}$$

The optimal size of the parking lot is the solution to the first-order condition $\partial EW(S)/\partial S = vSf(S) - c = 0$, which can be re-written as

$$Sf(S) = c/v \tag{6*}$$

We can use (6*) to find the optimal level of S if the maximand is concave. Surprisingly, that seems unlikely. The second-order condition, $\partial^2 EW(S)/\partial S^2 < 0$, requires $Sf'(S) + f(S) < 0$. The probability distribution function has to be declining sufficiently fast: $f'(S) < -f(S)/S$.

Given that the second-order condition for welfare maximization is satisfied, the optimal size of the parking lot is negatively related to the relative cost of its construction (the result follows from the total differentiation of the first-order condition (6*) and the fact that $Sf'(S) < 0$ is necessary for the second-order condition to hold).

Example: Continuous p.d.f. $f(N) = N^{-\beta}$, $\beta \in (1, 2)$

The support for distribution $f(N) = N^{-\beta}$, $[a, b] = [1, (2-\beta)^{\frac{1}{1-\beta}}]$ is chosen to guarantee that $\int_a^b N^{-\beta}dN = 1$. The mean number of drivers in need of parking is $E(N) = \int_a^b Nf(N)dN = \int_a^b N^{1-\beta}dN$. Hence,

$$E(N) = \frac{(2-\beta)^{\frac{1}{1-\beta}+1} - 1}{2-\beta} \tag{7}$$

The first-order condition implies

$$S^* = \left(\frac{c}{v}\right)^{\frac{1}{1-\beta}} \tag{8}$$

(the second-order condition is satisfied.)

Let $E(x_T)$ be the mean number of parking spots taken. There are N drivers. If $N < S$, then all N drivers find parking; if $N \geq S$, then S out of N drivers find parking.

$$E(x_T) = \int_a^{S^*} Nf(N)dN + \int_{S^*}^{b} S^* f(N)dN = \int_a^{S^*} N^{1-\beta}dN + S^* \int_{S^*}^{b} N^{-\beta}dN$$

The mean period-T capacity utilization as a percentage of the lot size is, therefore,

$$E(x_T)/S^* = \frac{\gamma - \gamma^{\frac{1}{\beta-1}}(\beta-1) - (2-\beta)^2}{(2-\beta)(\beta-1)} \tag{9}$$

where $\gamma = c/v$. For example, when $\gamma = 0.5$ and $\beta = 1.5$, $S^* = \gamma^{\frac{1}{1-\beta}} = 4$ and $E(x_T) = \int_1^{(2-\beta)^\rho} N^{1/\rho}dk = 2$. On average, 50% of the parking spots will remain unoccupied. The 50%-utilized parking lot is socially optimal. The expected welfare for a parking lot of size S, relative to the value v of parking, is

$$EW(S)/v = \int_1^S N^{1-\beta}dN - \gamma S = \frac{S^{2-\beta} - 1}{2-\beta} - \gamma S \tag{10}$$

The normalized expected welfare for a parking lot of the optimal size is then

$$EW(S^*)/v = \frac{\gamma^{\frac{2-\beta}{1-\beta}}(\beta-1) - 1}{2-\beta} \tag{11}$$

To compare the results to those under no uncertainty, suppose that the distribution for N is degenerate, taking value $N = E(N)$ for sure, where $E(N)$ is specified in equation (7). Under no uncertainty, the optimal lot size is $E(N)$. The welfare from the parking lot is $W(S = E(N)) = (v - c)E(N)$.

Numerical examples can help us assess the likelihood and the extent of optimal capacity under-utilization and the welfare losses from sub-optimal capacity levels.

Table 3. Capacity Utilization and Normalized Expected Welfare

Parameter Values	$\gamma = 0.4$			$\gamma = 0.5$			$\gamma = 0.6$		
	$\beta = 1.6$	1.7	1.8	$\beta = 1.6$	1.7	1.8	$\beta = 1.6$	1.7	1.8
$E(N)$	2.11	2.25	2.48	2.11	2.25	2.48	2.11	2.25	2.48
S*, uncertainty	4.61	3.70	3.14	3.17	2.69	2.38	2.34	2.07	1.89
% Full, $E(x_T)/S^*$	45.71	57.59	65.95	62.92	71.41	77.28	76.63	82.18	85.97
[a]Welfare, $W(E(N))/v$	1.26	1.35	1.49	1.05	1.13	1.24	0.84	0.90	0.99
[b]Welfare $EW(S^{'})/v$	0.25	0.10	0.03	-0.12	-0.23	-0.26	-0.40	-0.43	-0.60
[c]Welfare $EW(S^{''})/v$	0.26	0.12	0.03	-0.12	-0.19	-0.24	-0.39	-0.43	-0.46
[d]Welfare, $EW(S^*)/v$	2.11	1.60	1.29	1.47	1.15	0.95	1.01	0.82	0.68

Notes: a) No uncertainty case, $S = E(N)$; b) Uncertainty case with $S' = trunc(S^*)$: the optimal size of the parking lot is truncated to an integer; c) Uncertainty case with $S'' = 0.99S^*$ - the optimal size of the parking lot is reduced by 1%; d) Uncertainty case with $S = S^*$.

Table 1 illustrates our assertion that a little bit too small parking lot may be worse than no parking lot at all. In case b), we truncate the optimal lot size to the nearest integer and in case c), the optimal size of the lot is reduced by 1%. These small changes greatly affects the normalized expected welfare.

Appendix B

Example: Uniform Distribution

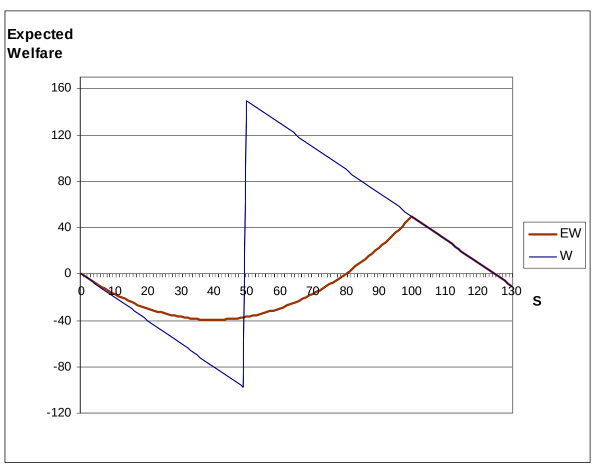
A numerical example for $c = 2, v = 5, \overline{N} = 100$ shows welfare at different capacity levels.

Table 4. Welfare and Parking Lot Size: Uniform Distribution

Number of Spaces, S	0	10	20	30	40	50	60	70	80	90	100
Expected Welfare, uncertainty	0	-17	-30	-37	-39	-37	-29	-17	0	23	50
Welfare, no uncertainty, N=50	0	-20	-40	-60	-80	150	130	110	90	70	50

Notes: The welfare is rounded to nearest integer.

Figure 4. Welfare and Parking Lot Size: Uniform Distribution



Notice that building a parking lot for 80 people out of 100 and not restricting an access to the facility is no better than building no parking lot at all. The welfare from a lot that can satisfy 80% of the demand is zero.

Example: Discrete p.d.f. $f(N) = p_N = 6/(\pi N)^2$, $N = \{1, 2, ...\}$

Assume that the probability that N drivers seek parking is $p_N = 6/(\pi N)^2$. It is easy to verify that $\sum_{N=1}^{\infty} p_N = 1$. Notice that with probability $\Pr(N = 1) = 6/\pi^2 \approx 61\%$ only one driver needs parking. If $\gamma = c/v = 0.5$, then from equation (5), the optimal capacity size is $S = 2$ since $Sf(S) = S\Pr(N = S) = 0.5$ for $S = 2$. The probability that there will be an empty space in a parking lot is $\Pr(N = 1) = 6/\pi^2 \approx 61\%$: more than 60% of the time only one driver arrives and the second parking space remains empty.

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