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A Model of Keynesian under Knightian Uncertainty

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Abstract

The purpose of this paper is to explore a source of nominal price rigidity and non-neutrality of money in a model of monopolistic competition under Knightian uncertainty. The decision-making theory in the analysis is that of expected utility under a nonadditive probability measure, that is, the Choquet expected utility model of preference. We apply this decision theory to a model of monopolistic competition without fixed cost of price adjustment. We show that when aversion to Knightian uncertainty exists, nominal price becomes rigid in a model of monopolistic competition. The model therefore has a Keynesian feature that nominal disturbances, particularly anticipated changes of money supply, have real effects on aggregate output fluctuations. The feature holds even if aversion to Knightian uncertainty is very small.

Keywords: Uncertainty, Choquet expected Utility, Price rigidity, Neutrality of money
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1. Introduction

During the past decades, the most standard theories in macroeconomics under uncertainty have been based on Savage's axioms. Extending von Neuman and Morgenstern (1944), Savage (1954) devised an axiomatic treatment of decision-making in which actions could be justified as maximizing expected utility with the assignment of subjective probabilities. Rational expectations theory applies this Savage's approach by imposing consistency between agents' subjective probabilities and the probabilities emerging from the economic model containing those agents. The consistency of models imposed by rational expectations has profound implications about the design and impact on macroeconomic policy-making.

Knight (1921), however, emphasized the distinction between quantifiable "risks" and unknown "uncertainty" in economic decision-making. In particular, a large number of empirical studies inspired by the Ellsberg paradox (Ellsberg (1961)) have challenged the appropriateness of the full array of Savage's axioms.[1] The studies suggest that there exists a reasonable room to rethink the foundations of rational expectations in macroeconomics. Recent studies thus made several attempts to incorporate Knightian uncertainty in economic decision-making. Studies by Gilboa (1987), Gilboa and Schmeidler (1989), and Schmeidler (1989) are pioneering attempts to formulate Knightian uncertainty by a new axiomatic treatment.[2] Based on Gilboa-Schmeidler's axioms, studies such as Epstein and Wang (1994), Aizenman (1997), and Hansen and Sargent (2000) incorporate Knightian uncertainty in macroeconomic models.

The purpose of this paper is to explore a source of nominal price rigidity and non-neutrality of money in a model of monopolistic competition under Knightian uncertainty. We investigate under what circumstances nominal price becomes rigid when the monopolistic firms follow a decision-making theory under Knightian uncertainty. The decision-making theory we use in the following analysis is that of expected utility under a nonadditive probability measure, that is, the Choquet expected utility model of preference, developed by Gilboa (1987) and Schmeidler (1989).[3] We apply this decision theory to a model of monopolistic competition without fixed cost of price adjustment. In the absence of Knightian uncertainty, rational expectations theory suggests that monetary policy would have no

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[1] See Camerer and Weber (1992) for their survey.
[2] Another early attempt is Bewley (1986).
[3] The Gilboa-Schmeidler's decision-making theory was elaborated by several authors such as Ozaki and Streufert (1996), Epstein and Zhang (1999), and Ozaki (2000).

systematic effect on real output even if there exists informational imperfection (see Lucas (1972, 1973), Sargent and Wallace (1975)). The following model of monopolistic competition, however, shows that when aversion to Knightian uncertainty exists, nominal price can be rigid for some measurable range. The model therefore has a Keynesian feature that nominal disturbances, particularly anticipated changes of money supply, have real effects on aggregate output fluctuations. The feature holds even if aversion to Knightian uncertainty is very small.

In the previous literature, Dow and Werlang (1992a,b) and Epstein and Wang (1994) seek to distinguish between “risks” and “uncertainty” and provide models of asset price indeterminacy in static and dynamic frameworks respectively. Our model of Keynesian has a common feature with these models in that aversion to Knightian uncertainty is a primary factor for the existence of multiple equilibrium prices. However, while the previous studies analyzed financial market instability, a main interest in our analysis is price rigidity in monopolistic goods markets. In particular, our model sheds light on traditional Keynesian questions, that is, what is a source of nominal price rigidity and what makes money non-neutral.

In previous empirical studies, there is abundant evidence for the phenomenon of price sluggishness (see, for example, Okun (1981) and Blinder (1991)). Traditional Keynesian theories thus stressed the role of nominal price rigidity to explain why nominal disturbances are a primary source of aggregate fluctuations. “New Keynesian economies” explains nominal price rigidities by assuming exogenous fixed costs of price adjustment, that is, menu costs. In particular, Mankiw (1985) and Akerlof and Yellen (1985) show that even small price adjustment costs (“menu costs”) can produce large social costs of nominal rigidities (see also Blanchard and Kiyotaki (1987) and Ball and Romer (1989)).

Except introducing multiple priors and ruling out exogenous menu costs, our model of Keynesian follows a standard New Keynesian model. One can interpret our model as differing from standard models of monopolistic competition only by replacing Savage’s sure-thing principle by the Gilboa-Schmeidler set of axioms. Our model thus has several similarities with menu cost models. For example, even small aversion to Knightian uncertainty can produce large social costs of nominal rigidities in our model. The result reminds us a key feature of menu cost models that small “menu costs” can cause large business cycles. In addition, as in menu cost models, output-inflation trade-off tends to exist in our model only when increases of money supply are moderate. An empirical result by Ball, Mankiw, and Romer (1988) is therefore consistent not only with menu cost models but also with our model under Knightian uncertainty.

However, since there exists no exogenous transaction cost, our model can derive several new implications that menu cost models may not provide. First, our model can explain why both small and large price changes occur for most commodities and transaction types.[4] In menu cost models, frequent and irregular small price changes rarely happen because they increase menu costs. In contrast, depending on the initial price level and aversion to uncertainty, both frequent small price changes and infrequent large price changes are likely to happen in our model. Our model thus can explain the existence of frequent small price changes after a long period of nominal price rigidity.

Secondly, in explaining the nominal price rigidity, we do not impose an asymmetric assumption that firms have a cost of price adjustment but no cost of quantity adjustment. In menu cost models, the implicit assumption is crucial because output rigidity may arise when a cost of quantity adjustment exists. In contrast, without any transaction costs, our model can derive rigid nominal price and flexible real output as a natural consequence of the price setting behavior of monopolistic firms.

Thirdly, our model may explain why nominal price rigidity can cause serious losses of real output under some pessimistic circumstances. Exogenous menu costs are, if any, very small. The derived first-order losses of output in menu cost models are thus generally moderate and are not sufficient to explain serious losses of output under depressions. In contrast, Knightian uncertainty aversion may show substantial variations because it is a consequence of psychological multiple-priors. Some pessimistic circumstances may therefore make the uncertainty aversion very large. If this is the case, pessimistic "animal spirits" may cause serious losses of output not only through causing serious decline of aggregate demand but also through causing persistent nominal price rigidity.

In the sense that demand uncertainty causes price rigidity, our model of Keynesian has several common features with previous studies on kinked demand curves (Negishi (1979), Woglom (1982), Nishimura (1992)). In particular, since the price-setting firm knows the quantity demanded at the status quo price but has imperfect information otherwise, a source of nominal price rigidity in our model is similar to that in Dreze (1979) and Weinrich (1997). However, these two studies were based on partial equilibrium models and did not make the distinction between quantifiable "risks" and unknown "uncertainty". Thus, without imposing

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[4] The evidence is reported by the authors such as Carlton (1986), Cecchetti (1986), and Kashyap (1995) who made empirical studies of price adjustment by individual firms.

restrictive assumptions, their models did not have nominal price rigidity.[5] Our general equilibrium model under Knightian uncertainty may provide some theoretical background for their restrictive assumptions.

The paper proceeds as follows. Section 2 sets up our basic model and section 3 explains its information structure. After formulating the expectations under Knightian uncertainty in section 4, section 5 shows the existence of multiple equilibrium prices in our model. Section 6 elaborates section 5 by two special cases. Section 7 investigates the effects of anticipated money supply. Section 8 shows that small menu costs can produce large business cycles. Section 9 discusses implications of our model and section 10 summarizes our main results.

2. The Basic Model

In the following analysis, we consider the economy that comprises a labor market, a money market, and a continuum of product markets. The labor market and the money market are competitive. Each product market, however, has a monopolist that produces a differentiated product. The economy is composed of consumers and firms, both of which are distributed along the unit intervals. All consumers are identical, so that their behavior is represented by a single representative consumer.

In each period, our representative consumer has the following utility function:

$$(1) \quad U = \frac{1}{1-\alpha}\int_0^1 q_i^{1-\alpha}di \;+ \gamma log(M^d/P) - \rho L,$$

where U is utility, q_i is the quantity of product i he (or she) consumes, α is the reciprocal of the elasticity of substitution between the different products ($0 < \alpha < 1$), M^d is his (or her) money demand, P is the general price level, L is his (or her) labor supply, and γ is a money demand parameter ($\gamma > 0$).

[5] To derive price rigidity, Dreze assumed the "truncated minimax" decision criterion which calls for maximizing expected value of profit minus a multiple of profit's standard deviation. In contrast, Weinrich considered an expected utility maximization but assumed that the variance of the firm's subjective distribution over quantities demanded as a function of price displays a kink at the status quo.

Define P_i as the nominal price of the good produced by firm i and W by the nominal wage.[6] Then, in each period, given W and P_i for all i, our consumer maximizes U subject to the following budget constraint:

$$(2) \quad \int_0^1 P_i q_i \, di + M^d = WL + M_0 + Profits,$$

where M_0 is the amount of money at the beginning of the period.

With this particular utility function, the first-order conditions give product demand and money demand in each period as follows.

$$(3) \quad q_i^{-\alpha} = \rho(P_i/W),$$

$$(4) \quad \gamma/M^d = \rho/W.$$

Since all consumers are identical, equilibrium in the money market is simply money supply M equaling money demand M^d, that is, $M^d = M$. Thus, from (3) and (4), we obtain:

$$(5) \quad q_i = \{(1/\rho)(W/P_i)\}^{1/\alpha},$$
$$= \{(M/\gamma)(1/P_i)\}^{1/\alpha},$$

$$(6) \quad W = \rho(M/\gamma).$$

Equation (5) is the inverse demand function faced by the firm. It shows that the product demand is a function of real wage W/P_i. The price elasticity of demand is $1/\alpha$. Equation (6) determines the equilibrium nominal wage in the economy.[7] Since the labor market is competitive, all firms face the same nominal wage W. It shows that the nominal wage W is proportional to nominal money supply M.

Given the inverse demand function (5), each monopolist firm sets its price level so as to maximize its profit in each period. For analytical simplicity, we assume that each firm has the following linear production function.

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[6] The general price level is the geometric average of all P_i. That is, $P = \exp\left(\int_0^1 \log P_i di\right)$.

[7] Coupled with the equilibrium both in the product markets and in the money market, the equilibrium in the labor market is guaranteed by Walras's Law,

(7) $q_i = (1/z)L_i$,

where L_i is labor input of firm i, and z is the inverse of its productivity. By using (5) and (7), the implied profit function is thus written as:

(8) $\Pi(P_i) = P_i q_i - WL_i$,

$= (M/\gamma)^{1/\alpha}(P_i^{1-1/\alpha} - zW P_i^{-1/\alpha})$.

Without uncertainty, $\partial\Pi(P_i)/\partial P_i = 0$ leads to the profit maximizing price level as follows.

(9) $P_i = zW/(1-\alpha)$.

Assuming no transaction costs such as menu costs, equations (3), (6), and (9) lead to

(10) $q_i = q_n \equiv [(1-\alpha)/(z\rho)]^{1/\alpha}$,

(11) $P_i = z\rho(M/\gamma)/(1-\alpha)$.

Because all firms are identical, equations (10) and (11) hold for all i. Equations (10) and (11) thus imply that the equilibrium price level is proportional to the nominal money supply, while the equilibrium output is equal to its "natural rate" q_n which is independent of the nominal money supply. In other words, when there exists no uncertainty, the neutrality of money holds in our model.

3. Information Structure

In the following model, we consider the economy where consumer's preference parameters, α, γ, and ρ, are uncertain for the firms. At the beginning of each period, each monopolistic firm observes its productivity $1/z$, nominal wage W, and new nominal money supply M. It also observes how much quantity is demanded when its selling price is the same as that in the last period. However, the firm can observe neither the consumer's preference parameters nor the product demand function directly. The price-setting firm therefore has imperfect information of the total quantity demanded in the present period unless it keeps the equilibrium price constant. In contrast, each consumer observes his (or her) preference parameters, nominal wage *W*, and the price of each product P_i in each period. Each consumer thus decides his (or her) functional forms of product demands, money demand, and labor supply without uncertainty.

Define P_0 as the selling price in the last period and assume that P_0 is common for all firms. If we define q_0 as the total quantity demanded at $P_i = P_0$ in the present period, then equations (3) and (5) imply that for all firm *i*,

$$(12) \quad q_0 = \{(W/\rho)(1/P_0)\}^{1/\alpha}, \\ \quad = \{(M/\gamma)(1/P_0)\}^{1/\alpha}.$$

From (6) and (12), we can see that the observations of *W, M,* and q_0 partially reveal the information of preference parameters, α, γ, and ρ to the firm. However, except for special cases, the firm cannot identify the exact values of α, γ, and ρ by the observations. Thus, before a new selling price is announced, the firm generally has imperfect information on the consumer's demand function under the new preference parameters.

Given the above information structure, each monopolistic firm maximizes its "expected" profits. In the standard theory of rational expectations, the monopolistic firm sets its price level so as to maximize its expected profits, $E\Pi_i$ based on Savage's axioms. Under such a standard expected profit-maximization, the price level is not sticky unless there exist costs of changing prices such as menu costs. In addition, the money is neutral in a sense that only unanticipated money supply can affect real output.

However, when the firm has aversion to Knightian uncertainty, it becomes important to distinguish between quantifiable "risks" and unknown "uncertainty." The standard expected profit-maximization is thus not an appropriate expected profit-maximization when the firm has multiple priors under Knightian uncertainty. Based on previous contributions, the following analysis investigates the firm's expected profit-maximization under Knightian uncertainty by using the concept of a probability capacity and Choquet integral.

4. The Expectation under Knightian Uncertainty

Let *S* be a finite set of states of nature, and let $\Gamma(S)$ denote the set of all subsets of *S*. Then, a <u>probability capacity</u> is defined as a function $\theta : \Gamma(S) \rightarrow [0, 1]$ which satisfies $\theta(\phi) = 0$, $\theta(S) =$

1, and $A \subseteq B \Rightarrow \theta(A) \leq \theta(B)$ for all $A, B \subseteq S$. A probability capacity is called <u>convex</u> if $\theta(A \cup B) + \theta(A \cap B) \geq \theta(A) + \theta(B)$ for all $A, B \subseteq S$. [8]

Now, suppose that θ is a convex probability capacity. Let $u(\alpha, X)$: S x $\Re \rightarrow \Re$ be a function such that $u(\cdot, X)$ is Borel-measurable for all $X \in \Re$ and that $u(\alpha, \cdot)$ is a differentiable concave function for all $\alpha \in S$. Then, under Knightian uncertainty, the expected value of a random variable $u(\alpha, X)$ is defined by the following Choquet integral:

$$(13) \quad E_Q\, u(\alpha, X) \equiv \int u(\alpha, X)\theta(d\alpha),$$

$$= \int_0^{+\infty} \theta(\{\alpha |\ u(\alpha, X) \geq y\})dy + \int_{-\infty}^{0} \left[\theta(\{\alpha |\ u(\alpha, X) \geq y\}) - 1\right]dy,$$

whenever these integrals exist in the improper Riemann sense and are finite. In particular, if $u(\alpha, X)$: S x $\Re \rightarrow \Re_+$ is a function such that $u(\alpha_1, \cdot) \geq u(\alpha_2, \cdot) \geq \cdots \geq u(\alpha_n, \cdot) \geq 0$, where n is the number of outcomes of α, it holds that

$$(14) \quad E_Q\, u(\alpha, X) = \sum_{i=1}^{n-1} [u(\alpha_i, \cdot) - u(\alpha_{i+1}, \cdot)]\, \theta(\cup_{j=1}^{i} A_j) \ + u(\alpha_n, \cdot).$$

Under the above definition, the following properties of the integral are proved in the previous studies for random variables $u(\alpha, X)$ and $v(\alpha, X)$.

Property (i): $u(\alpha, X) \geq v(\alpha, X) \Rightarrow E_Q\, u(\alpha, X) \geq E_Q\, v(\alpha, X)$.

Property (ii): $E_Q\, [u(\alpha, X) + v(\alpha, X)] \geq E_Q\, u(\alpha, X) + E_Q\, v(\alpha, X)$.

Property (iii): $-E_Q\, [-u(\alpha, X)] \geq E_Q\, u(\alpha, X)$.

Property (iv): $\forall a \geq 0$ and $b \in \Re$,, $E_Q\, [a\, u(\alpha, X) + b] = aE_Q\, u(\alpha, X) + b$.

Property (v): If $u(\alpha, X)$ is strictly concave in X, $E_Q\, u(\alpha, X)$ is strictly concave in X.

In general, even if $u(\alpha, X)$ is differentiable in X, $E_Q\, u(\alpha, X)$ may not be not differentiable in X. However, since $u(\alpha, X)$ is concave and differentiable in X, Proposition in Aubin (1979, p.116) leads to the following lemma:

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[8] It is additive, and therefore a probability measure if the inequality is always satisfied as an equality.

Lemma: Suppose that for all $\alpha \in S$, $u(\alpha, X)$ is independent of α when $X = X_0$. Then, it holds that

$$\partial E_Q u(\alpha, X)/\partial X\Big|_{X=X_0-0} = -E_Q(-\partial u(\alpha, X)/\partial X\Big|_{X=X_0}),$$

$$\partial E_Q u(\alpha, X)/\partial X\Big|_{X=X_0+0} = E_Q(\partial u(\alpha, X)/\partial X\Big|_{X=X_0}).$$

Proof. See Appendix.

5. The Equilibrium under Knightian Uncertainty

The purpose of this section is to characterize the equilibrium price in our model of monopolistic competition when the firm has aversion to Knightian uncertainty. From equations (8) and (12), with some algebraic rearrangements, we obtain

(15) $\Pi(P_i) = (P_i/P_0)^{-1/\alpha}(P_i - zW)q_0$.

When the firm has aversion to Knightian uncertainty, the firm maximizes $E_Q\Pi(P_i)$ based on its information set where E_Q is the expectation operator formulated in the last section. Thus, by using the properties in the last section, we can prove the following proposition.

Proposition 1: Suppose that

(16) $-1/E_Q(-1/\alpha) \leq 1 - zW/P_0 \leq 1/E_Q(1/\alpha)$.

Then, $E_Q\Pi(P_i)$ is maximized at $P_i = P_0$, where P_0 is the price level in the last period.

Proof: From lemma in the last section, it holds that

$$\partial E_Q\Pi(P_i)/\partial P\Big|_{P_i=P_0-0} = -E_Q\left[-\partial\Pi(P_i)/\partial P\Big|_{P_i=P_0-0}\right] = -E_Q\{(1/\alpha)(1-zW/P_0)-1\}q_0.$$

Since $-E_Q\{(1/\alpha)(1-zW/P_0)-1\} = -(1-zW/P_0)E_Q(1/\alpha)+1 \geq 0$ under Property (iv), this implies that when (16) holds, $\partial E_Q\Pi(P_i)/\partial P\Big|_{P_i=P_0-0} \geq 0$. Similarly, the lemma implies that

$$\partial E_Q \Pi(P_i)/\partial P\Big|_{P_i=P_0+0} \;=\; E_Q\Big[\partial \Pi(P_i)/\partial P\Big|_{P_i=P_0-0}\Big] \;=\{(1\text{-}zW/P_0)E_Q(\text{-}1/\alpha)+1\}q_0.$$

Thus, noting that $E_Q(-1/\alpha) < 0$, $\partial E_Q \Pi(P_i)/\partial P\big|_{P_i=P_0+0} \leq 0$ when (16) holds. Since $E_Q\Pi(P_i)$ is strictly concave in P_i under Property (v), this proves the proposition. [Q.E.D.]

The above proposition implies that as long as the condition (16) holds, the monopolistic firm will not change its price level even if the parameters of the consumer's preference and the firm's productivity vary. When the expected maximization is based on Savage's axioms, we never observe such price rigidity because $-E(-1/\alpha) = E(1/\alpha)$. However, when expectations are based on the Choquet integral (13), Property (iii) implies that $-E_Q(-1/\alpha) \geq E_Q(1/\alpha)$. Thus, there can exist a measurable range of price level for which exogenous shocks on parameters do not change the monopolist's price level under Knightian uncertainty. The intuition behind the result is that the profit is uncertain for the firm unless $P_i = P_0$. Thus, even if P_0 is not the profit-maximizing price level in the absence of uncertainty, the firm tends to set its equilibrium price at P_0 under Knightian uncertainty.

In previous literature, Dow and Werlang (1992a) define "uncertainty aversion" of a convex probability capacity θ at event A by $c(\theta, A) = 1\text{-}\theta(A)\text{-}\theta(A^c)$, where A^c is the set of elements not in A. Under the definition, they show that $-E_Q(-u) - E_Q(u)$ becomes larger as "uncertainty aversion" is larger for a random variable u. If we follow their definition of "uncertainty aversion", the above proposition implies that the range of the price rigidity is larger as the "uncertainty aversion" is larger.

6. Two Special Cases

(i) The Case of Two States of Nature

The decision theory under Knightian uncertainty can be formulated in a tractable form for the case of two states of nature. For analytical simplicity, we assume that $\gamma = \rho = M/P_0$ for all states. Under the assumption, (6) and (12) imply that $W = M$ and $q_0 = 1$. Thus, the observations of W, M, and q_0 reveal no additional information of α to the firm. The firm, however, knows that the parameter α takes a distinct value for each state.

Suppose that $\alpha = \alpha_1$ in state 1 and $\alpha = \alpha_2$ in state 2 where $\alpha_1 < \alpha_2$. Let $S = \{\alpha_1, \alpha_2\}$, and assume that $\theta(\{\alpha_1\}) = \theta(\{\alpha_2\}) = \mu \leq 1/2$, $\theta(\phi) = 0$, and $\theta(S) = 1$, where θ is a convex probability capacity. It is easy to see that θ is a probability measure when $\mu = 1/2$. One can thus interpret that there exists Knightian uncertainty if and only if $0 \leq \mu < 1/2$. In particular, if we follow the definition of "uncertainty aversion" by Dow and Werlang, we can see that uncertainty aversion is larger when μ is smaller.

Since $\alpha_1 < \alpha_2$, (14) leads to

$$\text{(17a)} \quad E_Q(1/\alpha) = \mu(1/\alpha_1 - 1/\alpha_2) + (1/\alpha_2), \\ = \mu(1/\alpha_1) + (1-\mu)(1/\alpha_2).$$

$$\text{(17b)} \quad E_Q(-1/\alpha) = E_Q(-1/\alpha + 1/\alpha_1) - 1/\alpha_1, \\ = \mu(-1/\alpha_2 + 1/\alpha_1) - (1/\alpha_2), \\ = (1-\mu)(-1/\alpha_1) + \mu(-1/\alpha_2).$$

The condition (16) is thus written as

$$\text{(18)} \quad 1/[(1-\mu)(1/\alpha_1) + \mu(1/\alpha_2)] \leq 1 - zW/P_0 \leq 1/[\mu(1/\alpha_1) + (1-\mu)(1/\alpha_2)].$$

From proposition 1, we can see that the equilibrium price P_i is rigid at P_0 when (18) holds. The condition (18) is never satisfied when $\mu = 1/2$, that is, when there exists no Knightian uncertainty. However, to the extent that $\mu < 1/2$, some algebraic arrangements verify that the range of the price rigidity is large when α_2 is large or when μ is small.

In general, $E_Q \Pi(P_i)$ is equal to

$$\text{(19a)} \quad \Pi^L(P_i) \equiv [\mu(P_i / P_0)^{-1/\alpha_1} + (1-\mu)(P_i / P_0)^{-1/\alpha_2}](P_i - zW)q_0 \quad \text{when } P_i \leq P_0,$$

$$\text{(19b)} \quad \Pi^U(P_i) \equiv [(1-\mu)(P_i / P_0)^{-1/\alpha_1} + \mu(P_i / P_0)^{-1/\alpha_2}](P_i - zW)q_0 \quad \text{when } P_i > P_0.$$

Therefore, the equilibrium price is

$$P_i = P_0 \quad \text{when } 1/[\mu(1/\alpha_1) + (1-\mu)(1/\alpha_2)] < 1 - zW/P_0 < 1/[(1-\mu)(1/\alpha_1) + \mu(1/\alpha_2)],$$

$$P_i = p^L \quad \text{when } 1/[(1-\mu)(1/\alpha_1) + \mu(1/\alpha_2)] < 1 - zW/P_0,$$

$$P_i = p^U \quad \text{when } 1/[\mu(1/\alpha_1) + (1-\mu)(1/\alpha_2)] > 1 - zW/P_0.,$$

where p^L and p^U are P_i's such that $\partial\Pi^L(P_i)/\partial P_i = 0$ and $\partial\Pi^U(P_i)/\partial P_i = 0$ respectively. Since a productivity shock, $1/z$, changes both p^L and p^U, the result indicates that the equilibrium price is flexible unless the condition (18) holds.

(ii) The Case of Maxi-min Rule under Complete Ignorance

The maxi-min rule under complete ignorance proposed by Wald (1950) is a special case of the decision theory under Knightian uncertainty. Under the maxi-min rule, a person with extreme uncertainty aversion who is completely uninformed maximizes the payoff of the worst possible outcome. We assume that the parameters α, γ, and ρ have the upper and lower values, that is, $\alpha \in [\alpha^*,\alpha^{**}]$, $\gamma \in [\gamma^*, \gamma^{**}]$, and $\rho \in [\rho^*, \rho^{**}]$. For analytical simplicity, we also assume that γ is equal to ρ.

Given the assumptions, the firm extracts the upper and lower values of α observing q_0. Define α^L and α^U respectively as the extracted upper and lower values of α from q_0. Then, from (12), it holds that

(20a) $\alpha^L = \max\{\alpha^*, [\log(W/P_0)-\log(\rho^U)]/\log q_0\}$,

(20b) $\alpha^U = \min\{\alpha^{**}, [\log(W/P_0)-\log(\rho^L)]/\log q_0\}$.

In general, $E_Q(-1/\alpha) = -1/\alpha^L$ and $E_Q(1/\alpha) = 1/\alpha^U$ under the maxi-min rule. Proposition 1 therefore implies that if

(21) $\alpha^L \leq 1 - zW/P_0 \leq \alpha^U$,

the equilibrium price P_i is rigid at P_0 under the maxi-min rule.[9] The range of the price rigidity is larger as the degree of uncertainty on α, i.e., $\alpha^U - \alpha^L$, is larger.

In general, since $(P_i/P_0)^{-1/\alpha}$ is decreasing in α when $P_i \leq P_0$ and increasing in α when $P_i > P_0$, (12) implies that Max Min Π is equivalent to maximize

$$\Pi^*(P_i) \equiv \left(P_i / P_0\right)^{-1/\alpha^U} (P_i - zW)q_0 \quad \text{when } P_i \leq P_0,$$
$$\equiv \left(P_i / P_0\right)^{-1/\alpha^L} (P_i - zW)q_0 \quad \text{when } P_i > P_0,$$

The equilibrium price is thus

[9] When $\alpha_1 = \alpha^L$ and $\alpha_2 = \alpha^U$, this corresponds to the case that $\mu = 0$ in (18) in the case of two states of nature.

$P_i = P_0$ when $\alpha^L \leq 1 - zW/P_0 \leq \alpha^U$,

$P_i = zW/(1-\alpha^U)$ when $\alpha^U < 1 - zW/P_0$,

$P_i = zW/(1-\alpha^L)$ when $\alpha^L > 1 - zW/P_0$.

The result indicates that exogenous shocks will not affect the equilibrium price if and only if $\alpha^L \leq 1 - zW/P_0 \leq \alpha^U$. When this inequality does not hold, the monopolistic firm changes its price level when there exist exogenous shocks in the economy.

7. The Effects of Anticipated Money Supply

In Keynesian models, nominal price rigidity is a primary source for nominal disturbances to have real effects. In those models, an increase in nominal money supply can raise real output and improve social welfare systematically. To the extent that nominal prices are rigid, our model thus has a similar feature with Keynesian models.

Suppose that the condition (16) holds and that $P_i = P_0$ for all i. Then, from (12), the output level of each firm is equal to $q_0 = \{(M/\gamma)(1/P_0)\}^{1/\alpha}$. Since $\partial q_0/\partial M > 0$, this implies that an increase in anticipated money supply can increase real output level in the economy when (16) holds. In addition, since

$$(22) \quad U = \frac{1}{1-\alpha} q_0^{1-\alpha} + \gamma log(M/P_0) - \rho z\, q_0,$$

it holds that $\partial U/\partial M > 0$ when nominal prices are rigid. Thus, an increase in the anticipated money supply can increase social welfare systematically when (16) holds

However, because of (6), an increase in M raises W proportionally. A change in M may also affect $E_Q(1/\alpha)$ and $E_Q(-1/\alpha)$ under imperfect information. Thus, even if (16) holds for the initial level of money supply, a large change in M may violate the condition (16). This implies that a large increase in money supply tends to increase the equilibrium price level and may not have a desirable impact on real output even under Knightian uncertainty.

For example, consider the case of the maxi-min rule under complete ignorance discussed in the last section. In this special case, the equilibrium price is rigid if and only if the condition (21) holds, that is, $\alpha^L \leq 1 - zW/P_0 \leq \alpha^U$. However, even if α^L is given, (6) implies that a large increase in M may violate the condition that $\alpha^L \leq 1 - zW/P_0$ because W arises proportionally as

M increases. Since $P_i = zW/(1-\alpha^L)$ when $\alpha^L > 1 - zW/P_0$, this indicates that given α^L, a large increase in money supply increases the price level proportionally and becomes neutral in affecting real output and social welfare. Moreover, since $[\log(W/P_0)-\log(\rho^U)]/\log q_0 = \alpha[\log(W/P_0)-\log(\rho^U)]/[\log(W/P_0)-\log(\rho)]$, equations (6) and (20a) imply that an increase in M raises α^L when $\alpha^* < [\log(W/P_0)-\log(\rho^U)]/\log q_0$. Since an increase in M always raises W, this indicates that an increase in M tends to violate the condition (21) more easily when $\alpha^* < [\log(W/P_0)-\log(\rho^U)]/\log q_0$. Therefore, recalling that $P_i = zW/(1-\alpha^L)$ when $\alpha^L > 1 - zW/P_0$, a large increase in M can raise the price level more than proportionally when α^L arises. In this extreme case, we can verify that a large increase in money supply reduces real output and deteriorates social welfare.

In previous literature, Ball, Mankiw, and Romer (1988) show that there exists an output-inflation trade-off only when average inflation rates are low. Since only a moderate increase in the money supply can increase real output and social welfare in our model, the empirical result is consistent not only with menu cost models but also with our model under Knightian uncertainty. However, since a large increase in M can have a negative effect on real output and social welfare, the role of active monetary policy in model may be more limited than that in menu cost models.

8. Small Knightian Uncertainty and Large Business Cycles

Because of a large number of empirical studies inspired by the Ellsberg paradox, most economists will probably agree that the full array of Savage's axioms does not hold exactly in economic decision-making. However, some economists may argue that the degree of Knightian uncertainty is, if any, very small and that the foundations of rational expectations still hold in macroeconomics approximately. If this is the case, an extreme assumption on uncertain aversion such as the maxi-mini rule under complete ignorance might not provide a realistic counter-example for models of rational expectations.

The purpose of this section is to provide an example that even small Knightian uncertainty can cause a significant range of nominal price rigidity and may have a serious effect on real output. For analytical tractability, we consider the case of two states of nature discussed in section 6. In this case, the firm has smaller uncertainty aversion as μ is closer to 1/2. The nominal price level is rigid for some range when the condition (18) holds. Since the condition (18) is never satisfied when $\mu = 1/2$, the price rigidity never arises in the absence of

Knightian uncertainty. However, as the menu cost models suggested, the loss from the nominal price rigidity is a second order. Thus, even when μ is close to 1/2, the price rigidity can arise for some significant range and may cause a first-order loss of output, particularly when α_2 is large.

Table 1 reports the upper and lower values of $P_0/(zW)$ in (18) and their ratios for alternative values of α_1 and μ. They show that the ratio between the upper and lower values of $P_0/(zW)$ is large for various combinations of α_1 and μ. In particular, they show that when α_2 is close to one, the ratio can be sufficiently large even if μ is close to 0.5. For example, focus on the case where α_1 = 0.65 and α_2 = 0.99 in the table. In this case, the ratio between the upper and lower values of $P_0/(zW)$ is about 1.03 when μ = 0.49 and about 1.062 when μ = 0.48. Since zW is exogenously given, this implies that the upper value of P_0 is about 3.0% greater than the lower value of P_0 when μ = 0.49 and about 6.2% greater than the lower value of P_0 when μ = 0.48. Even when α_1 = 0.65 and α_2 = 0.95, the relative ratios indicate that the upper value of P_0 is about 2.6% greater than the lower value of P_0 when μ = 0.49 and about 5.2% greater than the lower value of P_0 when μ = 0.48. Hence, recalling that the firm has no uncertainty aversion when μ = 0.5, the results imply that the range of price rigidity can be sufficiently large even if aversion to Knightian uncertainty is very small.

When α_2 is close to α_1, the range of price rigidity becomes negligible when μ = 0.49. However, even when α_1 = 0.65 and α_2 = 0.75, the relative ratios show that the ratio between the upper and lower values of P_0 is about 3.3% when μ = 0.45 and about 6.8% when μ = 0.4. These results indicate that even if α_2 is not close to one, multiple equilibrium prices can exist for a significant range when the firm has some small aversion to Knightian uncertainty.

Because of (3), the multiplicity of equilibrium price implies the multiplicity of equilibrium real output. Thus, when the range of multiple equilibrium prices is large, the equilibrium real output can show large deviations from its natural level. For example, consider the case of two states of nature discussed above. In this special case, the equilibrium output q_0 is always equal to one as long as $P_i = P_0$. This implies that a productivity shock $1/z$, which has a positive impact on natural output level, has no impact on the equilibrium output as long as the condition (18) holds. In particular, as Table 1 suggests, the upper and lower values in (18) can be large even if μ is close to 1/2. Thus, even if aversion to Knightian uncertainty is very small, a series of positive productivity shocks may have no positive impact on real output when the equilibrium price level is close to the upper bound of the range.

9. Discussions

Except introducing multiple priors and ruling out exogenous menu costs, our model of Keynesian follows a standard New Keynesian model. One can interpret our model as differing from standard models of monopolistic competition only by replacing Savage's sure-thing principle by the Gilboa-Schmeidler set of axioms. Our model thus has several similarities with menu cost models. If "menu costs" are interpreted more broadly than the physical costs of changing price tags, one may classify our model as one of menu cost models. However, since there exists no exogenous transaction cost, our model can derive several new implications that menu cost models may not provide.

First, our model can explain why both small and large price changes occur for most commodities and transaction types. In menu cost models, small irregular price changes are rarely happen either with state-dependent adjustment or with time-dependent adjustment. In contrast, both frequent small price changes and infrequent large price changes can arise in our model depending on whether the equilibrium price is set within or outside the range of (16). When the price level is set outside the range of (16), the equilibrium price is adjusted to exogenous shocks instantaneously. However, when the price level is set within the range of (16) and the range is wide, the following equilibrium price tends to be constant until a series of exogenous shocks violate the condition (16). Our model thus can explain why frequent small price changes can occur after a long period of nominal price rigidity.

Secondly, in explaining the nominal price rigidity, our model does not need to impose an implicit assumption that firms have some price-adjustment costs but no quantity-adjustment cost. In menu cost models, this asymmetric assumption is crucial because real output becomes rigid when costs of quantity adjustment are large. However, in a frictionless economy, price determination is equivalent to quantity determination. It is, thus, far from evident why price adjustment is more costly than quantity adjustment. In contrast, without imposing any assumptions on adjustment costs, our model can derive rigid nominal price and flexible real output as a natural consequence of the price setting behavior of monopolistic firms.

Finally, our model may explain why nominal price rigidity can cause serious losses of real output under some pessimistic circumstances. Exogenous menu costs are very small and have few variations over time. The derived first-order losses of output are thus always moderate and are not sufficient to explain serious losses of output under depressions. In contrast, Knightian uncertainty aversion may show substantial variations because it is a consequence of psychological multiple-priors. Some pessimistic circumstances may therefore make the uncertainty aversion very large. If this is the case, our model can have a wide range of nominal price rigidity that may explain depressions.

In explaining the causes of depressions, Keynes (1921, Chapter 12 in 1936) emphasized the importance of "animal spirits" when individuals cannot estimate probabilities reliably and so cannot make a good calculation of expected values. To the extent that the degree of Knightian uncertainty becomes large when the "animal spirits" become pessimistic, this implies that pessimistic "animal spirits" can cause serious losses of output not only through causing serious decline of aggregate demand but also through causing persistent nominal price rigidity, particularly when money supply declines substantially.

10. Concluding Remarks

In this paper, we explored a source of nominal price rigidity and non-neutrality of money in a model of monopolistic competition under Knightian uncertainty. The decision-making theory we used in the analysis was that of expected utility under a nonadditive probability measure, that is, the Choquet expected utility model of preference. We applied this decision theory to a model of monopolistic competition without fixed cost of changing prices. A main result in the paper was that even if aversion to Knightian uncertainty is very small, nominal price becomes rigid in a model of monopolistic competition. The model therefore had a Keynesian feature that nominal disturbances, particularly anticipated changes of money supply, have real effects on aggregate output fluctuations.

A key assumption in the model was that the price-setting firm knows the quantity demanded at the status quo price but has imperfect information otherwise. To the extent that the firm maximizes its expected profit based on Savage's axioms, the assumption does not lead to nominal price rigidity unless imposing restrictive assumptions.[10] However, when the firm has aversion to unknown "uncertainty", its equilibrium price tends to be constant even if the status quo price does not maximize the profit on average. Thus, not only the assumption on

[10] Weinrich (1997) proposed that if the variance of the firm's subjective distribution over quantities demanded as a function of price displays a kink at the status quo, the assumption leads to nominal price rigidity when the firm is risk-averse. However, the variance of the firm's subjective distribution over quantities does not display a kink at the status quo for standard distributions.

the demand uncertainty but also the distinction between quantifiable “risks” and unknown “uncertainty” is crucial in having endogenous nominal price rigidity.

Since this is the first step to introduce the notion of Knightian uncertainty in a model of monopolistic competition, the analysis was based on specific utility and production functions in a static framework. Although this simplified the analysis, it is worthwhile to see how our results will change when we use more general utility and production functions in a dynamic framework. Our conjecture is that our essential results still hold using general utility and production functions. However, the results may be altered in a dynamic framework, particularly when the firms experience learning on unknown parameters.

Appendix: Proof of lemma in section 4.

As in the text, let S be a finite set of states of nature, and let Γ denote the set of all subsets of S. Suppose that θ is a convex probability capacity. Then, θ' is called a conjugate of θ if and only if $\theta'(A) = 1-\theta(A^c)$ for all $A \in \Gamma$, where A^c is the set of elements not in A.

Let $M(S, \Gamma)$ be the set of bounded additive probability measures. Define a core of θ as:

(A1) $\text{core}(\theta) = \{P | P \in M(S, \Gamma), \quad (\forall A \in \Gamma)\ \theta(A) \leq \mathrm{P}(A) \leq \theta'(A)\ \}$.

Define $\wp(\theta, u(\cdot, X)) \subseteq M(S)$ by

(A4) $\wp(\theta, u(\cdot, X)) \equiv \arg\min\{\int u(\alpha, X)P(d\alpha) \mid P \in \text{core}(\theta)\}$.

Since θ is a convex probability capacity, the core is non-empty. In addition, let $u(\alpha, X)$: S x $\Re \rightarrow \Re$ be a function such that $u(\cdot, X)$ is Borel-measurable for all $X \in \Re$ and that $u(\alpha, \cdot)$ is a differentiable concave function for all $\alpha \in S$. Then, under the definition of Choquet integral in section 4, it holds that

(A2) $E_Q\, u(\alpha, X) \equiv \int u(\alpha, X)\theta(d\alpha) = \min\{\int u(\alpha, X)P(d\alpha) \mid P \in \text{core}(\theta)\}$,

(A3) $-E_Q\,[-u(\alpha, X)] \equiv -\int [-u(\alpha, X)]\theta(d\alpha) = \int u(\alpha, X)\theta'(d\alpha)$

$$= \max\{\int u(\alpha, X)P(d\alpha) \mid P \in \text{core}(\theta)\},$$

(see Schmeidler (1986)). Proposition in Aubin (1979, p.116, Proposition 6) can thus derive the following proposition as its special case.

Proposition: Suppose that for all $\alpha \in$ S, $u(\alpha, X)$ is independent of α when $X = X_0$. Then, it holds that

$$\partial E_Q u(\alpha, X)/\partial X\Big|_{X=X_0-0} = \max\{\int[\partial u(\alpha, X)/\partial X]P(d\alpha) \mid P \in \wp(\theta, u(\cdot, X))\},$$

$$\partial E_Q u(\alpha, X)/\partial X\Big|_{X=X_0+0} = \min\{\int[\partial u(\alpha, X)/\partial X]P(d\alpha) \mid P \in \wp(\theta, u(\cdot, X))\}.$$

where integrals in parentheses are those for the bounded additive probability measure.

Now, suppose that for all $\alpha \in S$, $u(\alpha, X)$ is independent of α when $X = X_0$. Then, since $\wp(\theta, u(\cdot, X_0))\}=\text{core}(\theta)$, it holds that

$$\partial E_Q u(\alpha, X)/\partial X\big|_{X=X_0-0} = \max\{\int[\partial u(\alpha, X)/\partial X]P(d\alpha) \mid P \in \text{core}(\theta)\}$$

$$= -E_Q(-\partial u(\alpha, X)/\partial X\big|_{X=X_0}).$$

Similarly,

$$\partial E_Q u(\alpha, X)/\partial X\big|_{X=X_0+0} = \min\{\int[\partial u(\alpha, X)/\partial X]P(d\alpha) \mid P \in \text{core}(\theta)\}.$$

$$= E_Q(\partial u(\alpha, X)/\partial X\big|_{X=X_0}).$$

The above proposition thus proves the lemma in section 4.

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Table 1 The Range of the Price Rigidity

(a) $\mu = 0.49$

α_1	0.65	0.65	0.65	0.65	0.65	0.65
α_2	0.99	0.95	0.90	0.85	0.80	0.75
(1) Upper value	4.7175	4.4401	4.1200	3.8261	3.5553	3.3050
(2) Lower value	4.5770	4.3288	4.0390	3.7694	3.5182	3.2834
(1)/(2)	1.0307	1.0257	1.0201	1.0150	1.0106	1.0066

(b) $\mu = 0.48$

α_1	0.65	0.65	0.65	0.65	0.65	0.65
α_2	0.99	0.95	0.90	0.85	0.80	0.75
(1) Upper value	4.7920	4.4986	4.1622	3.8553	3.5743	3.3159
(2) Lower value	4.5106	4.2759	4.0000	3.7419	3.5000	3.2727
(1)/(2)	1.0624	1.0521	1.0405	1.0303	1.0212	1.0132

(c) $\mu = 0.45$

α_1	0.65	0.65	0.65	0.65	0.65	0.65
α_2	0.99	0.95	0.90	0.85	0.80	0.75
(1) Upper value	5.0345	4.6866	4.2958	3.9467	3.6329	3.3494
(2) Lower value	4.3256	4.1266	3.8889	3.6627	3.4471	3.2414
(1)/(2)	1.1639	1.1357	1.1046	1.0775	1.0539	1.0333

(d) $\mu = 0.4$

α_1	0.65	0.65	0.65	0.65	0.65	0.65
α_2	0.99	0.95	0.90	0.85	0.80	0.75
(1) Upper value	5.5158	5.0492	4.5455	4.1127	3.7368	3.4074
(2) Lower value	4.0570	3.9059	3.7209	3.5402	3.3636	3.1910
(1)/(2)	1.3596	1.2927	1.2216	1.1617	1.1110	1.0678

Notes 1) Upper values and lower values are those of $P_0/(zW)$ in the condition (18).
2) "(1)/(2)" is the ratio between the upper value and the lower value in the condition (18).