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Abstract

We study the effects of a decrease in trade costs on the spatial distribution of industry in a multi-regional economy, when a rise in the regional population of workers generates higher urban costs. We show that high and low trade costs imply that all regions involve a positive share of the industrial sector. When urban costs are linear, there exists a stable equilibrium for almost all values of trade costs. Furthermore, as trade costs fall, there is a path of stable equilibria such that the industry is, first, agglomerated into a decreasing number of regions and, then, dispersed among a growing number of regions. The second phase arises because of the increasing urban costs associated with the process of agglomeration.

Keywords: multi-regional system, economic geography, agglomeration, transport costs, urban costs.

J.E.L. Classification: F12, L13, R13.

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1 Introduction

Our primary purpose is to study the impact of the secular decline in trade costs (Bairoch, 1985), broadly defined to include all impediments to the exchange of goods, on the spatial distribution of economic activities when the number of regions is arbitrary. Indeed, models of economic geography have so far focussed on a two-region setting (Krugman, 1991; Fujita, Krugman and Venables, 1999). This makes the dynamic analysis very simple since moving away from one region automatically implies that migrants (workers and firms) go to the other region. Furthermore, it is not clear what the main result obtained in economic geography, namely the existence of a core-periphery structure, becomes when there are more than two regions. Indeed, a multi-regional economy is able to sustain a much richer hierarchy. To the best of our knowledge, this is the first time that an analytical treatment of a multi-regional economy with mobile factors is addressed.

Our secondary purpose is to allow for urban costs to be paid by workers when residing in a particular region. In this perspective, the core-periphery model has been criticized because it does not account for the growing urban costs associated with the concentration of firms and workers within the same region (Helpman, 1998; Tabuchi, 1998; Papageorgiou and Pines, 1999). By ignoring the costs imposed by urban life, this model would remain in the tradition of international trade theory, and would thus fail to provide a fair description of the working of a spatial economy. Introducing urban costs is both reasonable and meaningful. It is reasonable because an increasing concentration of workers and firms within a region generates rising congestion costs. It is meaningful because, in the absence of such costs, when trade costs decrease the economy might move from full dispersion to full agglomeration without passing through intermediate stages, a result that strikes us as being very implausible.

In this paper, we extend the two-region model proposed by Ottaviano, Tabuchi and Thisse (2001) to study the impact of falling trade costs on the equilibrium distributions of firms and workers in the case of n regions, while permitting each region to have specific urban costs (e.g., commuting and housing), which vary with the number of workers. When the number of regions exceeds two, determining the equilibrium prices, wages, and (indirect) utilities in each region becomes a hard task. Indeed, these expressions typically depend on the whole distribution of the manufacturing sector across regions, while they also vary with the region under consideration. In order to be able to work with a tractable model, we make the simplifying assumption that regions are pairwise equidistant so that trade costs are the same regardless of the origin and destination regions. Such an assumption may be justified by the fact that distance-related transportation costs have become low enough while distance-unrelated costs such as tariffs, insurance, loading and unloading are still relatively high. Likewise, communication costs are not very sensitive to distance, but often involve high fixed costs (think of portable telephones).

Regarding urban costs, our modeling strategy is as follows. Although we acknowledge the fact that both trade and commuting costs have been decreasing since the beginning of the Industrial Revolution, we assume that interregional transport costs decrease while urban commuting costs are constant for a given population size. This assumption is made to capture the idea that, in modern economies, trade costs of manufactured goods keep decreasing at a fast pace, while the decrease in commuting costs tends to slow down (and maybe to rise) due to growing congestion and to higher opportunity time cost for urban residents.

Our concept of equilibrium is standard, while we borrow a dynamics that has been used in migration analysis (Ginsburgh, Papageorgiou and Thisse, 1985; Tabuchi, 1986; Zeng, 2000). More precisely, in our model, *the incentives to migrate away or toward a particular region are given by the sum of utility differentials between this region and the others.* It is well known that proving the existence of a stable equilibrium when there are more than two regions may be a problematic issue. For example, a limit cycle may arise. More generally, characterizing the eigenvalues of a nonnumerical system is often a formidable task. However, our model displays some nice features that allow us to apply recent stability theorems without having to compute eigenvalues (Tabuchi and Zeng, 2000). We will see that, under fairly weak conditions, *a stable equilibrium always exists.* To the best of our knowledge, such a result has not been proven for the original core-periphery model developed by Krugman (1993).

Previewing our main results, we will see that workers will move from small to large urban regions when the desirability of the differentiated product rises or when the size of the agricultural population falls (Proposition 1). Under some regularity conditions, we then show that the number of workers residing in a region with low urban costs is always larger than that in a region with high urban costs (Proposition 2). In Section 5, we study how the size of urban regions changes when trade costs fall. More precisely, we show that *large urban regions grow in the early stages of economic integration but decline in the late stages* (Theorem 1). Unfortunately, we have not been able to characterize the evolution of the urban regions when trade costs take "intermediate" values. Finally, in Section 6, we consider the more difficult case of a stable equilibrium in which some regions have no industrial sector. To this end, we restrict ourselves to linear urban costs. It is then shown that *the number of urban regions keeps decreasing when trade costs decrease from high values to intermediate values.* In this case, the core of the economy is made of a shrinking number of regions. However, *when trade costs keep decreasing, this process is reversed and the number of urban regions rise* (Theorem 2). In other words, once trade costs are sufficiently low, the market solves the congestion problem induced by the agglomeration of industry in a small number of regions by redistributing firms and workers among a larger number of regions. It should be clear that the implications of such results are important for the formation of integrating economies, such as the European Union or NAFTA.

Although we do not deal with differential regional growth, it seems fair to say that our paper contributes to the debate regarding the spatial implications of economic development. In the development literature, a high degree of urban concentration is expected to arise during the early phases of growth. As development proceeds, deconcentration would occur because the economy can afford to spread infrastructure, while the initial urban giants become high cost and congested places that are less attractive locations for producers and workers (Vining and Kontuly, 1978; Alonso, 1980). Since it is reasonable to interpret the value of internal trade costs as an index of economic development, we may conclude that our results suggest the existence of such a $\cap$-shaped relationship between economic development and the spatial distribution of activities. Interestingly, this relationship accords with the observations made in some developed economies, according to which industry would relocate outside the main urban regions (Champion, 1994; Geyer and Kontuly, 1996).

The remainder of the paper is organized as follows. The model is presented in Section 2, while existence and stability of an equilibrium are dealt with in Section 3. Some preliminary results are shown in Section 4. Sections 5 and 6 contain our main results discussed above, while Section 7 concludes.

2 The model

The space-economy is made of $n \geq 2$ regions ($i = 1, \cdots, n$). Each region has one city that has a given center but a variable size. As in urban economics, the city center stands for a central business district (CBD) in which all firms locate once they have chosen to set up in the corresponding region (Fujita and Thisse, 1996). The CBDs are given by n points of the location space.

There are two factors, called A and L. Factor A is evenly distributed across regions (A/n) and is spatially immobile. The assumption of a uniform distribution of A is made in order to focus on the impact of differential urban costs on the distribution of activities. Factor L is mobile between any two regions. Let $\lambda_i \in [0, 1]$ denotes its share in region i

and let

$$\Lambda \equiv \left\{ \lambda = (\lambda_1, \cdots, \lambda_n); \quad \sum_{i=1}^{n} \lambda_i = 1 \quad \text{and} \quad \lambda_i \geq 0 \right\}$$

For expositional purposes, we refer to the first sector as "agriculture" and to the second sector as "manufacturing". Accordingly, we call "farmers" the immobile factor A and "workers" the mobile factor L.[1] For this reason, we will refer to region accommodating workers ($\lambda_i > 0$) as *urban regions*, while regions with no workers ($\lambda_i = 0$) are called *rural regions*.

There are three goods in the economy. The first good is homogeneous. Consumers have a positive initial endowment of this good, which is also produced in the agricultural sector using factor A as the only input under constant returns to scale and perfect competition. Technology in agriculture requires one unit of A in order to produce one unit of output. We assume that this good can be traded freely between regions so that its price is the same across regions. Hence this good is chosen as the numéraire. As a result, farmers' income is equal to one in all regions.[2]

The second good is a horizontally differentiated product; it is supplied by using L as the only input under increasing returns to scale and monopolistic competition. Technology in manufacturing requires ϕ units of L in order to produce any amount of a variety, i.e. the marginal cost of production of a variety is set equal to zero. Each firm in the manufacturing sector has a negligible impact on the market outcome in the sense that it can ignore its influence on, and hence reactions from, other firms. To this end, we assume that there is a continuum of potential firms. There are no scope economies so that, due to increasing returns to scale, there is a one-to-one relationship between firms and varieties. Clearly, regardless of the regional distribution of firms and the value of trade costs, the total number of firms in the whole economy is given by $N = L/\phi$, which is assumed to be larger than 1. Although this might seem restrictive at first sight, this property allows us to focus on the spatial redistribution of industry per se.

Because each firm sells a differentiated variety, it faces a downward sloping demand. Since there is a continuum of firms, each one is negligible and the interaction between any two firms is zero. However, as will be seen below, aggregate market conditions of some

[1] We want to stress the fact, however, that the role of factor A is to capture the idea that some inputs (such as land or some services) are nontradable while some others have a very low spatial mobility (such as low-skilled workers). For example, the first sector could be reinterpreted as the traditional one and the second sector as the modern one.

[2] Recall that the choice of the numéraire is a difficult issue in general equilibrium model with imperfect competition.

kind affects any single firm. This provides a setting in which individual firms are not competitive (in the classic economic sense of having infinite demand elasticity) but, at the same time, have no strategic interactions with one another. Finally, interregional trade flows go from one CBD to another. As discussed in the introduction, the corresponding trade costs are assumed to be identical between any two regions:

$$\tau_{ij} = \begin{cases} \tau > 0 & \text{for } i \neq j \\ \\ 0 & \text{for } i = j \end{cases}$$

Thus, each variety can be traded at a positive cost of τ units of the numéraire for each unit carried from one region to another, regardless of the variety, τ accounting for all the impediments to trade. The underlying geography is simple: the n regions are located along a circumference, while shipping a good from one region to another involves going through the center of the circumference.

Housing is the third good in our economy. When they live in a certain region, workers are urban residents who use housing and commute to the regional CBD where they work. To keep things simple, all the urban costs borne by a worker who chooses to reside in region i (land rents, commuting and congestion costs, pollution) are subsumed in a cost function $\theta_i(\lambda_i)$, which varies with the size of the corresponding population of workers. This function is assumed to satisfy the following properties:

$$\theta_i(0) = 0 \qquad \theta_i(1) < \infty \qquad \theta_i'(y) \geq 0 \qquad i = 1, \cdots, n \qquad \text{and} \qquad y \in [0, 1]$$

Unlike trade costs that are the same between any pair of regions, urban costs are region-specific, reflecting the fact that living conditions may vastly differ across urban regions for the same population size (because of natural amenities, better transport facilities or local public services).

Preferences over the first two goods are identical across individuals and described by a quasi-linear utility with a quadratic subutility, which is supposed to be symmetric in all varieties:

$$U(q_0; q(x), x \in [0, N]) = \alpha \int_0^N q(x)dx - \frac{\beta - \gamma}{2} \int_0^N [q(x)]^2 dx - \frac{\gamma}{2} \left[\int_0^N q(x)dx \right]^2 + q_0 \qquad (1)$$

where $q(x)$ is the quantity of variety $x \in [0, N]$ and q_0 the quantity of the numéraire. The parameters in (1) are such that $\alpha > 0$ and $\beta > \gamma > 0$. In this expression, α expresses the intensity of preferences for the differentiated product, whereas $\beta > \gamma$ means that consumers are biased toward a dispersed consumption of varieties (*varietas delectat*).

If the consumption of the homogeneous good is positive, maximizing (1) under the budget constraint

$$\int_0^N p(x)q(x)dx + q_0 = w_i + \overline{q}_0 - \theta_i(\lambda_i) \tag{2}$$

(where w_i denotes the wage prevailing in region i and $\overline{q}_0$ is the initial endowment of the numéraire) yields the following first-order conditions:

$$\alpha - (\beta - \gamma)q(x) - \gamma \int_0^N q(y)dy = p(x) \qquad x \in [0, N]$$

or

$$q(x) = a - (b + cN)p(x) + c\int_0^N p(y)dy \qquad x \in [0, N] \tag{3}$$

where

$$a \equiv \frac{\alpha}{\beta + (N-1)\gamma} \qquad b \equiv \frac{1}{\beta + (N-1)\gamma} \qquad c \equiv \frac{\gamma}{(\beta - \gamma)[\beta + (N-1)\gamma]}$$

Substituting (2) and (3) into (1), we obtain the indirect utility of a worker residing in this region:

$$\begin{aligned} V_i \;=\; & \frac{a^2 N}{2b} - a\int_0^N p(x)dx + \frac{b+cN}{2}\int_0^N [p(x)]^2 dx - \frac{c}{2}\left[\int_0^N p(x)dx\right]^2 + \overline{q}_0 \\ & + w_i - \theta_i(\lambda_i) \end{aligned} \tag{4}$$

In accord with empirical evidence (Head and Mayer, 2000; McCallum, 1995), we assume that markets are regionally segmented so that each firm chooses a delivered price which is specific to the region in which its variety is sold. Let $p_{ij}(x)$ be the price of variety x produced in region i and sold in region j, and $q_{ij}(x)$ the demand in region j for variety x produced in region i. To ease the burden of notation, we drop x hereafter. Consequently, operating profits of a firm established in region i can be written as

$$\Pi_i(\lambda) = \sum_{j=1}^{n} (p_{ij} - \delta_{ij}\tau) q_{ij} \left(\frac{A}{n} + \lambda_j L\right)$$

where $\delta_{ij} = 1$ when $i \neq j$ and 0 otherwise. We assume throughout this paper that trade costs are such that it is always profitable for any firm to export from one region to another.

As to equilibrium wages, they are determined as follows. First, by maximizing firms' profits with respect to prices, we obtain[3]

$$\begin{aligned}
p_{ii} &= \frac{2a + c\tau(1-\lambda_i)N}{2(2b+cN)} \\
p_{ji} &= p_{ii} + \frac{\tau}{2} \qquad \text{for } i \neq j \\
q_{ii} &= a - (b+cN)p_{ii} + cN\sum_{k=1}^{n}\lambda_k p_{ki} = (b+cN)p_{ii} \\
q_{ji} &= a - (b+cN)p_{ji} + cN\sum_{k=1}^{n}\lambda_k p_{ki} = (b+cN)(p_{ji}-\tau) \qquad \text{for } i \neq j
\end{aligned}$$

Second, due to free entry and exit, profits net of fixed costs are zero in equilibrium. As in Krugman (1991), the equilibrium wages are determined by a bidding process between firms for workers, which ends when no firm can earn a strictly positive profit at the equilibrium market prices. In other words, all operating profits are absorbed by the wage bills. Hence, the wage prevailing in region i is determined as follows:

$$\begin{aligned}
w_i(\lambda) &= \frac{\Pi_i}{\phi} = \frac{1}{\phi}\sum_{j=1}^{n}(p_{ij} - \delta_{ij}\tau)q_{ij}\left(\frac{A}{n} + \lambda_j L\right) \\
&= \frac{(b+cN)N}{L}\sum_{j=1}^{n}\left(p_{jj} - \frac{\delta_{ij}\tau}{2}\right)^2\left(\frac{A}{n} + \lambda_j L\right) \\
&= \frac{(b+cN)N}{L}\left[\sum_{j\neq i}^{n}\left(p_{jj} - \frac{\tau}{2}\right)^2\left(\frac{A}{n} + \lambda_j L\right) + p_{ii}^2\left(\frac{A}{n} + \lambda_i L\right)\right] \\
&= \frac{(b+cN)N}{L}\left[\sum_{j=1}^{n}\left(p_{jj} - \frac{\tau}{2}\right)^2\left(\frac{A}{n} + \lambda_j L\right) + \left(p_{ii}\tau - \frac{\tau^2}{4}\right)\left(\frac{A}{n} + \lambda_i L\right)\right]
\end{aligned}$$

[3] It is reasonable to assume that each firm's demand is decreasing in the total number of varieties because consumers spread their purchases over more varieties. Furthermore, it is also reasonable to assume that a consumer's demand for the differentiated product increases with N because more varieties makes this good more attractive compared to the numéraire. Computing the partial derivatives of the above demand function, we immediately see that $\partial q_{ii}/\partial N < 0$ and $\partial(q_{ii}N)/\partial N > 0$.

Accordingly, the indirect utility of a worker living in region i can be computed as follows:

$$\begin{aligned} V_i(\lambda) &= \frac{a^2N}{2b} - a\sum_{j=1}^{n}\lambda_j N p_{ji} + \frac{b+cN}{2}\sum_{k=1}^{n}\lambda_j N p_{ji}^2 - \frac{c}{2}\left(\sum_{j=1}^{n}\lambda_j N p_{ji}\right)^2 \\ &\quad + \overline{q}_0 + w_i - \theta_i(\lambda_i) \\ &= \frac{a^2N}{2b} - aN\left[p_{ii} + \frac{\tau(1-\lambda_i)}{2}\right] + \frac{(b+cN)N}{2}\left[p_{ii}^2 + (1-\lambda_i)\tau\left(p_{ii}+\frac{\tau}{4}\right)\right] \\ &\quad - \frac{cN^2}{2}\left[p_{ii} + \frac{\tau(1-\lambda_i)}{2}\right]^2 + \frac{(b+cN)N}{L}\left[\sum_{j=1}^{n}\left(p_{jj}-\frac{\tau}{2}\right)^2\left(\frac{A}{n}+\lambda_j L\right)\right. \\ &\quad \left. + \tau\left(p_{ii}-\frac{\tau}{4}\right)\left(\frac{A}{n}+\lambda_i L\right)\right] + \overline{q}_0 - \theta_i(\lambda_i) \end{aligned} \tag{5}$$

As expected, the indirect utility $V_i(\lambda)$ depends on the whole distribution λ.

3 Existence and stability of a spatial equilibrium

We now move to the definition and the stability of a spatial equilibrium. The distribution $\lambda^* \in \Lambda$ is a *spatial equilibrium* when no worker may get a higher utility level by moving to another region. Formally, a distribution λ^* is an equilibrium if V^* exists such that

$$\begin{array}{lll} V_i(\lambda^*) = V^* & \text{if} & \lambda_i^* > 0 \\ V_i(\lambda^*) \leq V^* & \text{if} & \lambda_i^* = 0 \end{array} \tag{6}$$

In words, this means that, in equilibrium, workers' utility in urban regions is (weakly) higher than in rural regions, while the utility level is constant across urban regions. Since $V_i(\lambda)$ is continuous in $\lambda \in \Lambda$ as shown by (5), Proposition 1 of Ginsburgh *et al.* (1985) implies that *a spatial equilibrium exists.*

In order to study the stability of a spatial equilibrium, we assume that local labor markets adjust instantaneously when some workers move from one region to the other. More precisely, wages are adjusted in each region for each firm located therein to earn zero profits. Hence, during the adjustment process, the utility level of a worker residing in region i is given by $V_i(\lambda)$.

The above spatial equilibrium conditions turn out to be equivalent to the following zero migration conditions:

$$d\lambda_{ji}(t) = 0 \qquad \text{for all } j, i = 1, \ldots, n \tag{7}$$

where $d\lambda_{ji}(t)$ is the (net) migration from region j to region i during the infinitesimal time interval dt at time t. Following a now well-established tradition in migration modeling, we focus on an adjustment process in which *workers spread themselves among several regions, being attracted (repulsed) by regions providing high (low) utility levels.* In particular, we assume that migration $d\lambda_{ji}$ is proportional to the utility difference if population in region j is positive. Then, the dynamical system of equations is such as

$$\frac{d\lambda_i}{dt} \equiv \sum_{j=1}^{n} \frac{d\lambda_{ji}}{dt} \qquad \text{for } i = 1, \ldots, n \tag{8}$$

where the speed of adjustment has been normalized to one, and where

$$\frac{d\lambda_{ji}}{dt} \equiv \begin{cases} V_i(\lambda) - V_j(\lambda) & \text{if } \lambda_i > 0, \lambda_j > 0 \\ \min\{0, V_i(\lambda) - V_j(\lambda)\} & \text{if } \lambda_i > 0, \lambda_j = 0 \\ \max\{0, V_i(\lambda) - V_j(\lambda)\} & \text{if } \lambda_i = 0, \lambda_j > 0 \\ 0 & \text{if } \lambda_i = 0, \lambda_j = 0 \end{cases}$$

It is readily verified that $\sum_{i=1}^{n} d\lambda_i/dt = 0$ since the total population of workers remains constant during the adjustment process. This dynamics can be justified by the assumption that migration decisions are made on the basis of pairwise comparisons between regions in that the net migration from region j to region i is proportional to their utility differential $V_i - V_j$ if population in region j is positive. As a consequence, the sum of the net migration flows of region i is such that

$$\frac{d\lambda_i}{dt} = \sum_{j=1}^{n} [V_i(\lambda) - V_j(\lambda)] = n\left[V_i(\lambda) - \frac{1}{n}\sum_{j=1}^{n} V_j(\lambda)\right] \tag{9}$$

if population in region j is positive. Expression (9) also means that regions with a utility level higher (lower) than the average level across regions have a growing (declining) population of workers (and firms).[4]

[4] Observe that (9) bears some resemblance with Weibull's (1995) replicator dynamics used recently by Fujita *et al.* (1999): $d\lambda_i/dt = [V_i(\lambda) - \sum_{j=1}^{n} \lambda_j V_j(\lambda)]\lambda_i$. The two dynamics yield identical stationary states since they both solve (6). Furthermore, the stability conditions of equilibrium in both dynamics turn out to be the same as (12) and (13) as shown by Tabuchi and Zeng (2000). There are differences, however. Workers out-migrate (in-migrate) from region i if its utility V_i is lower (higher) than the interregional weighted average utility in the replicator dynamics whereas workers out-migrate (in-migrate) if V_i is lower than the interregional unweighted average utility in ours. By using the replicator, one makes the regions with high utility even more attractive, thus affecting the pace of adjustment.. Yet, (9) is simpler to handle because it does not involve any crossed term $\lambda_i \lambda_j$ ($i, j = 1, \ldots, n$ and $i \neq j$) and leads to analytical results.

In order to study the stability of a spatial equilibrium, we must evaluate the sum of the pairwise utility differentials used in (9). To this end, we set

$$S_i(\lambda_i) \equiv (C_1\tau - C_2\tau^2)\lambda_i - C_3\tau^2\lambda_i^2 - \theta_i(\lambda_i)$$

where

$$\begin{aligned} C_1 &\equiv \frac{aN(b+cN)(3b+2cN)}{(2b+cN)^2} \\ C_2 &\equiv \frac{N(b+cN)}{8(2b+cN)^2}\left[4(2b+cN)\frac{cNA}{nL} + 12b^2 + 4bcN - 3c^2N^2\right] \\ C_3 &\equiv \frac{cN^2(b+cN)(8b+5cN)}{8(2b+cN)^2} \end{aligned}$$

It is readily verified that $C_1 > 0$, $C_3 > 0$, $C_2 + C_3 > 0$. However, C_2 may be negative when c is very large, namely when varieties are very close substitutes. Throughout the rest of paper, we will assume that the product is sufficiently differentiated for C_2 to be positive. This entails very little loss of generality. Clearly, $S_i(0) = 0$.

Unlike $V_i(\lambda)$ that depends on the whole distribution λ, the function $S_i(\lambda_i)$ depends only upon the size of region i. In addition, the following lemma will allow us to use $S_i(\lambda_i)$ instead of $V_i(\lambda)$ in the stability analysis of equilibria. The proof is given in Appendix A.

Lemma 1 *For $i = 1, \cdots, n$, we have:*

$$\sum_{j=1}^{n}[V_i(\lambda) - V_j(\lambda)] = \sum_{j=1}^{n}[S_i(\lambda_i) - S_j(\lambda_j)] \tag{10}$$

Hence, the RHS of (10) is additively separable with respect to the λ_i's, i.e., there are no crossed terms $\lambda_i\lambda_j$ with $i \neq j$. This lemma implies that

$$V_i(\lambda) - \frac{1}{n}\sum_{j=1}^{n} V_j(\lambda) = S_i(\lambda_i) - \frac{1}{n}\sum_{j=1}^{n} S_j(\lambda_j) \qquad i = 1, ..., n$$

For a given distribution λ, this means region i yields a welfare level higher (lower) than the average welfare if and only if $S_i(\lambda_i)$ is larger (smaller) than the average value of the $S_j(\lambda_j)$'s. Hence, the migration equation (9) becomes

$$\frac{d\lambda_i}{dt} = n\left[S_i(\lambda_i) - \frac{1}{n}\sum_{j=1}^{n} S_j(\lambda_j)\right]$$

thus making the stability analysis much simpler. From now on, we refer to $S_i(\lambda_i)$ as the "pseudo-surplus" of region i. This function may be used to study the properties of an equilibrium. If λ^* is a spatial equilibrium with $m \leq n$ urban regions i_j $(j = 1, \ldots, m)$ then it must be that

$$\begin{array}{rl} S_{i_1}(\lambda^*_{i_1}) = \cdots = S_{i_{m-1}}(\lambda^*_{i_{m-1}}) = S_{i_m}(\lambda^*_{i_m}) & \text{if} \quad \lambda^*_{i_j} > 0 \\ S_{i_j}(\lambda^*_{i_j}) = 0 & \text{if} \quad \lambda^*_{i_j} = 0 \end{array} \tag{11}$$

and conversely. So, (6) and (11) are equivalent. Although $S_i(0) = 0$, observe that there may exist an equilibrium at which all the pseudo-surpluses are negative and equal. However, if the equilibrium involves at least one rural region, the pseudo-surpluses of all urban regions are nonnegative and equal.

Consider an equilibrium with m urban regions such that

$$S'_{i_1}(\lambda^*_{i_1}) \leq \cdots \leq S'_{i_{m-1}}(\lambda^*_{i_{m-1}}) \leq S'_{i_m}(\lambda^*_{i_m})$$

When $m < n$, Tabuchi and Zeng (2000) show that λ^* is (locally) *stable* if the following two conditions hold:

$$S'_{i_{m-1}}(\lambda^*_{i_{m-1}}) < 0 \qquad \text{and} \qquad \sum_{j=1}^{m-1} \frac{S'_{i_m}(\lambda^*_{i_m})}{S'_{i_j}(\lambda^*_{i_j})} > -1 \tag{12}$$

$$S_{i_j}(\lambda^*_{i_j}) > 0 \qquad j = 1, \cdots, m \tag{13}$$

When the manufacturing sector is concentrated into a single region ($m = 1$), these two conditions boil down to $S_1(1) > 0$. Furthermore, when $m = n$, the sole condition (12) ensures stability. Finally, the equilibrium is *unstable* when the second inequality in (12) is reversed.[5]

4 On the size of regions

It turns out to be possible to figure out how the size of urban regions is affected by an increase in the desirability of the industrial good or by a decrease in the number of farmers, two trends that have characterized the evolution of developed economies since the beginning of the Industrial Revolution. Let m be the number of regions with manufacturing workers. We may then predict the directions of migration between large and small regions as follows.

Proposition 1 *When the desirability of the differentiated good* (α) *rises or when the agricultural population* (A) *falls, workers migrate from regions whose industrial share is smaller than the average* ($1/m$) *to large regions whose industrial share is larger than the average.*

[5]When the second inequality in (12) becomes an equality, the equilibrium may be stable or unstable.

Proof. Assume first that α increases up to $\hat{\alpha}$. From the definition of S_i, it follows that C_2 and C_3 are unchanged while a increases up to $\hat{a} = a\hat{\alpha}/\alpha$ so that C_1 increases up to $\hat{C}_1 \equiv C_1\hat{\alpha}/\alpha$. Set

$$\hat{S}_i(\lambda_i) \equiv (\hat{C}_1\tau - C_2\tau^2)\lambda_i - C_3\tau^2\lambda_i^2 - \theta_i(\lambda_i) \qquad i = 1, \ldots, m$$

Since λ^* is an equilibrium, we have

$$\begin{aligned}
\sum_{j=1}^{m}[\hat{S}_i(\lambda_i^*) - \hat{S}_j(\lambda_j^*)] &= \sum_{j=1}^{m}[\hat{S}_i(\lambda_i^*) - \hat{S}_j(\lambda_j^*) - S_i(\lambda_i^*) + S_j(\lambda_j^*)] \\
&= \sum_{j=1}^{m}[(\hat{C}_1 - C_1)\tau\lambda_i^* - (\hat{C}_1 - C_1)\tau\lambda_j^*] \\
&= (\hat{C}_1 - C_1)\tau\sum_{j=1}^{m}(\lambda_i^* - \lambda_j^*) = (\hat{C}_1 - C_1)\tau(m\lambda_i^* - 1)
\end{aligned}$$

Therefore, when α increases, $d\lambda_i/dt$ has the same sign as $m\lambda_i^* - 1$, thus implying that large regions become larger while small regions become smaller.

Similarly, when A decreases, C_2 decreases while C_1 and C_3 remain unchanged. Hence the conclusion follows. $\square$

Hence, *a stronger preference for the differentiated product as well as a smaller population of farmers fosters a higher level of geographical concentration of the manufacturing sector*. Unfortunately, the effects of other parameters' change are ambiguous.

Let us re-index for the moment the regions as follows:

$$\lambda_i^* > 0 \text{ for } i = 1, \ldots, m \qquad \text{and} \qquad \lambda_j^* = 0 \text{ for } j = m+1, \ldots, n$$

Since λ^* is a stable equilibrium, we know from the stability condition (12) that at most one expression S_i' is nonnegative, all the others being negative.

Definition 1 *The spatial equilibrium $\boldsymbol{\lambda}^*$ is said to be* regular *if $S_i'(\lambda_i^*) < 0$ for $i = 1, \ldots, m$; otherwise, it is called* irregular.

Clearly, any regular equilibrium is stable. The next result states some sufficient conditions allowing us to rank regions in terms of the size of their manufacturing sector in the case of a regular spatial equilibrium.

Proposition 2 *Consider a regular equilibrium $\boldsymbol{\lambda}^* = (\lambda_1^*, \ldots, \lambda_i^*, \ldots, \lambda_n^*)$. If the urban costs are convex and if $\theta_i(y) \leq \theta_j(y)$ for some $i, j \in \{1, \ldots, n\}$, then we have $\lambda_i^* \geq \lambda_j^*$. Furthermore, $\lambda_i^* = \lambda_j^* > 0$ implies that $\theta_i(\lambda_i^*) = \theta_j(\lambda_j^*)$.*

Proof: (i) The statement is obvious for $i = m+1, \ldots, n$ since $\lambda_i^* = 0$. Let $i \in \{1, \ldots, m\}$ and assume that $\theta_i(\lambda_i^*) \leq \theta_j(\lambda_j^*)$ while $\lambda_j^* > \lambda_i^*$ holds for some $j \in \{1, \ldots, n\}$. Then, it must be that $\lambda_j^* > 0$ so that $j \in \{1, \ldots, m\}$. Since the equilibrium is regular, we have

$$C_1\tau - C_2\tau^2 < 2C_3\tau^2\lambda_i^* + \theta_i'(\lambda_i^*) \quad i = 1, ..., m \tag{14}$$

Furthermore, since λ^* is an equilibrium while both λ_i^* and λ_j^* are strictly positive, we have $S_i(\lambda_i^*) = S_j(\lambda_j^*)$ so that

$$C_1\tau - C_2\tau^2 = C_3\tau^2(\lambda_i^* + \lambda_j^*) + \frac{\theta_j(\lambda_j^*) - \theta_i(\lambda_i^*)}{\lambda_j^* - \lambda_i^*} \tag{15}$$

Combining (14) and (15), we get

$$C_3\tau^2(\lambda_j^* - \lambda_i^*) + \frac{\theta_j(\lambda_j^*) - \theta_i(\lambda_i^*)}{\lambda_j^* - \lambda_i^*} < \theta_i'(\lambda_i^*)$$

Then, the mean value theorem implies that $\xi \in [\lambda_i^*, \lambda_j^*]$ exists such that

$$C_3\tau^2(\lambda_j^* - \lambda_i^*) + \frac{\theta_j(\lambda_j^*) - \theta_i(\lambda_j^*)}{\lambda_j^* - \lambda_i^*} + \theta_i'(\xi) < \theta_i'(\lambda_i^*) \tag{16}$$

Because $\theta_i(\cdot)$ is convex, we have $\theta_i'(\xi) \geq \theta_i'(\lambda_i^*)$. Furthermore, since $\theta_j(\lambda_j^*) - \theta_i(\lambda_j^*) \geq 0$, $C_3 > 0$ and $\lambda_j^* - \lambda_i^* > 0$, we obtain

$$C_3\tau^2(\lambda_j^* - \lambda_i^*) + \frac{\theta_j(\lambda_j^*) - \theta_i(\lambda_j^*)}{\lambda_j^* - \lambda_i^*} + \theta_i'(\xi) > \theta_i'(\lambda_i^*)$$

thus contradicting (16). Accordingly, we have $\lambda_i^* \geq \lambda_j^*$.

(ii) Using $S_i(\lambda_i^*) = S_j(\lambda_j^*)$ and the definition of $S_i(\lambda_i)$, it is readily verified that $\lambda_i^* = \lambda_j^* > 0$ implies that $\theta_i(\lambda_i^*) = \theta_j(\lambda_j^*)$. $\square$

The assumption of convex urban costs has been shown to hold under fairly general conditions in urban economics (Fujita, 1989, p.145). The regular equilibrium condition $S_i'(\lambda_i^*) < 0$ ($i = 1, \ldots, m$) is more demanding, but it constitutes a simple sufficient condition for stability. Under these two assumptions, the proposition above says that, regardless of the value of τ, *regions with low (high) urban costs always have a large (small)* share of the *industrial sector*. In other words, regions with poorer urban infrastructure attract less firms from the industrial sector, although workers' welfare is the same in all urban regions ($i = 1, ..., m$). By contrast, *farmers enjoy a lower welfare level in regions having poor urban infrastructure than those living in regions having good infrastructure* because the former regions accommodate less firms and, therefore, produce fewer varieties than the latter.

5 The effect of decreasing trade costs

In this section, we focus on the case in which all regions are urban and study how their size is affected by decreasing trade costs. To this end, it is convenient to renumber the regions as follows:

$$S_1'(\lambda_1^*) \leq \cdots \leq S_{n-1}'(\lambda_{n-1}^*) \leq S_n'(\lambda_n^*)$$

Since λ^* is a stable equilibrium, we know from the stability condition (12) that, while the sign of S_n' may be positive, zero or negative, all the other S_i' $(i = 1, \ldots, n-1)$ must be negative.

When trade costs are given by τ, we denote the corresponding interior equilibrium by

$$\boldsymbol{\lambda}^*(\tau) = (\lambda_1^*(\tau), \ldots, \lambda_n^*(\tau))$$

with $\lambda_i^*(\tau) > 0$. Assume that (12) holds so that $\boldsymbol{\lambda}^*(\tau)$ is stable for each τ. Since $\sum_{k=1}^n \lambda_k^*(\tau) = 1$, it must be that

$$\frac{d\lambda_n^*(\tau)}{d\tau} = -\sum_{k=1}^{n-1} \frac{d\lambda_i^*(\tau)}{d\tau} \tag{17}$$

Since $S_i(\lambda_i)$ is also a function of τ, we may denote it as $S_i(\lambda_i, \tau)$. For convenience, we also set

$$S_i' \equiv \left.\frac{\partial S_i(\lambda_i, \tau)}{\partial \lambda_i}\right|_{\lambda_i = \lambda_i^*}$$

$$z_i \equiv \sum_{j=1}^n \frac{\partial(S_i - S_j)}{\partial \tau} = (C_1 - 2C_2\tau)(n\lambda_i^* - 1) - 2C_3\tau\left[n(\lambda_i^*)^2 - \sum_{j=1}^n (\lambda_j^*)^2\right] \tag{18}$$

In equilibrium, it must be that $\sum_{j=1}^n [S_i(\lambda_i^*(\tau), \tau) - S_j(\lambda_j^*(\tau), \tau)] = 0$. Differentiating this equation yields the following system of linear equations whose unknowns are $d\lambda_i^*(\tau)/d\tau$:

$$-(n-1)S_i'\frac{d\lambda_i^*(\tau)}{d\tau} + \sum_{\substack{j=1 \\ j \neq i}}^n S_j'\frac{d\lambda_j^*(\tau)}{d\tau} = z_i \qquad i = 1, \ldots, n \tag{19}$$

Let

$$D = \begin{pmatrix} -(n-1)S_1' - S_n' & S_2' - S_n' & \cdots & S_{n-1}' - S_n' \\ S_1' - S_n' & -(n-1)S_2' - S_n' & \cdots & S_{n-1}' - S_n' \\ \vdots & \vdots & \ddots & \vdots \\ S_1' - S_n' & S_2' - S_n' & \cdots & -(n-1)S_{n-1}' - S_n' \end{pmatrix}$$

and D_i be the matrix obtained from D by replacing the i-th column with

$$\begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_{n-1} \end{pmatrix} = (C_1 - 2C_2\tau) \begin{pmatrix} n\lambda_1^* - 1 \\ n\lambda_2^* - 1 \\ \vdots \\ n\lambda_{n-1}^* - 1 \end{pmatrix} - 2C_3\tau \begin{pmatrix} n(\lambda_1^*)^2 - \sum_{j=1}^n \left(\lambda_j^*\right)^2 \\ n(\lambda_2^*)^2 - \sum_{j=1}^n \left(\lambda_j^*\right)^2 \\ \vdots \\ n(\lambda_{n-1}^*)^2 - \sum_{j=1}^n \left(\lambda_j^*\right)^2 \end{pmatrix}$$

Using (17), it is readily verified that the solution to the system (19) is given by

$$\frac{d\lambda_i^*(\tau)}{d\tau} = \frac{|D_i|}{|D|} \qquad i = 1, \ldots, n-1 \tag{20}$$

where $|D_i|$ (respectively $|D|$) is the determinant of the matrix D_i (respectively D).

Next, we may establish the following result which will be useful in studying the evolution of the industry distribution.

Lemma 2 *If* (12) *holds at equilibrium* $\boldsymbol{\lambda}^*$*, then*

$$\frac{d\lambda_i^*(\tau)}{d\tau} = \frac{1}{n^2|D|} \prod_{\substack{k=1 \\ k \neq i}}^{n} (-nS_k') \left[\sum_{k=1}^{n} \frac{z_i - z_k}{(-S_k')} \right] \qquad i = 1, \ldots, n \tag{21}$$

The proof is given in Appendix C.

We can now sketch the idea underlying the results that will be proven below. The last term of (21) may be rewritten as follows:[6]

$$\frac{z_i - z_k}{-S_k'} = \frac{n\frac{\partial(S_i - S_k)}{\partial\tau}}{-S_k'} = n\frac{\frac{\partial(S_i - S_k)}{\partial\tau}}{\frac{\partial(S_i - S_k)}{\partial\lambda_k}} \tag{22}$$

In order to understand the meaning of this expression, we fix λ_i and consider the impact on λ_k of a change in τ, while keeping the equilibrium condition $S_i = S_k$. Then, we have

$$-\frac{d\lambda_k(\tau)}{d\tau} = -\frac{\frac{\partial(S_i - S_k)}{\partial\tau}}{\frac{\partial S_k}{\partial\lambda_k}} = \frac{\frac{\partial(S_i - S_k)}{\partial\tau}}{\frac{\partial(S_i - S_k)}{\partial\lambda_k}}$$

so that the third expression in (22) measures the marginal impact of τ on λ_k (up to n). Therefore, if (22) is negative, region k experiences net in-migration from region i at a regular equilibrium with $S_i' < 0$ for all $i = 1, \cdots, n$.

[6]Recall that S_i (resp. S_k) depends on τ and λ_i (resp. τ and λ_k).

Since the population of workers is fixed, when some regions become larger due to the change in trade costs, some others must become smaller. Among the regions that experience a decreasing population, let h be the region with the largest increase (or smallest decrease) in the equilibrium utility level. On the other hand, among the regions whose population rises, let l be the region with the smallest increase (or largest decrease) in the equilibrium utility level. Finally, let region e be a region with unchanging population size. Formally, we have:

$$\begin{cases} z_h = \min_i \left\{ \frac{\partial S_i}{\partial \tau}; \frac{d\lambda_i^*(\tau)}{d\tau} > 0 \right\} \\ \frac{d\lambda_e^*(\tau)}{d\tau} = 0 \\ z_l = \max_i \left\{ \frac{\partial S_i}{\partial \tau}; \frac{d\lambda_i^*(\tau)}{d\tau} < 0 \right\} \end{cases} \tag{23}$$

Note that regions h, e, l may not exist simultaneously, but at least one of them does. By construction, a decrease in τ induces migration from region h to region l, implying that utility is higher in region l. That is, $\partial(S_l - S_h)/\partial\tau < 0$, a result that is consistent with the fact that $z_h > z_l$.

Consider now a regular equilibrium. Then, in (21), the sign of a change in $\lambda_i^*(\tau)$ is determined by the sign of $\sum_{k=1}^{n}(z_i - z_k)/(-S_k')$. From (18), it follows that each $z_i - z_k$ in (21) is given by

$$n(\lambda_i^* - \lambda_k^*)\{C_1 - 2[C_2 + C_3(\lambda_i^* + \lambda_k^*)]\tau\}$$

Hence, *when trade costs are high* ($\tau > C_1/2[C_2 + C_3(\lambda_i^* + \lambda_k^*)]$), *a decrease in* τ *makes larger regions* ($\lambda_i^* > \lambda_k^*$) *larger, while smaller regions* ($\lambda_i^* < \lambda_k^*$) *become smaller.* By contrast, *when trade costs are low* ($\tau < C_1/2[C_2 + C_3(\lambda_i^* + \lambda_k^*)]$), *the opposite holds.*

This argument is developed in a more systematic way in what follows. For analytical simplicity, in the remainder of this section we consider asymmetric equilibria in which there exist regions i, j such that $\lambda_i^* \neq \lambda_j^*$. Furthermore, we assume

$$\tau \neq \frac{C_1}{2C_2 + 2C_3(\lambda_i^* + \lambda_j^*)} \quad \text{for all} \quad i, j = 1, \ldots, n \text{ and } i \neq j \tag{24}$$

Therefore, there exist regions i and j such that $z_i \neq z_j$. Condition (24) excludes only a finite number of values of τ out of a continuum. This does not induce any significant loss of generality.

Three types of interior equilibria may emerge according to the sign of S_n'. In Figure 1a, a regular equilibrium in which $S_n' < 0$ is represented. The cases of irregular equilibria with $S'_n = 0$ and $S'_n > 0$ are depicted in Figures 1b and 1c, respectively. In what follows, each case is discussed in order.

Figure 1: Regular and irregular equilibria with urban regions

5.1 Regular equilibrium ($S'_n < 0$)

In the case of a regular equilibrium, Lemma 2 allows us to rewrite (23) as follows:

$$\begin{cases} z_h = \min\left\{ z_i; \sum_{k=1}^{n} \frac{z_i - z_k}{-S'_k} > 0 \right\} \\ \sum_{k=1}^{n} \frac{z_e - z_k}{-S'_k} = 0 \\ z_l = \max\left\{ z_i; \sum_{k=1}^{n} \frac{z_i - z_k}{-S'_k} < 0 \right\} \end{cases} \tag{25}$$

By definition, $z_h > z_e > z_l$. Region e may not exist. However, both regions h and l do exist in any regular equilibrium as shown by Lemma D in Appendix. In what follows, we use the following definition of "large" and "small" regions.

Definition 2 *Region i is said to be large if $\lambda_i^* \geq \max\{\lambda_l^*, \lambda_e^*, \lambda_h^*\}$ and small if $\lambda_i^* \leq \min\{\lambda_l^*, \lambda_e^*, \lambda_h^*\}$ for $i \neq l, e, h$. When $\max\{\lambda_l^*, \lambda_e^*, \lambda_h^*\} > \lambda_i^* > \min\{\lambda_l^*, \lambda_e^*, \lambda_h^*\}$, the region is called medium.*

Since we consider asymmetric equilibria, both region h and region l exist by Lemma E in Appendix and, hence, large and small regions exist. Let

$$\tau_{ih} \equiv \frac{C_1}{2[C_2 + C_3(\lambda_i^* + \min\{\lambda_l^*, \lambda_e^*, \lambda_h^*\})]} \tag{26}$$

$$\tau_{il} \equiv \frac{C_1}{2[C_2 + C_3(\lambda_i^* + \max\{\lambda_l^*, \lambda_e^*, \lambda_h^*\})]} \tag{27}$$

for $i = 1, \cdots, n$, where λ_e^* is deleted when region e does not exist. Clearly, we have $\tau_{il} \leq \tau_{ih}$; the larger λ_i^*, the smaller τ_{ih} and τ_{il}. Although $\tau_{il} \leq \tau_{jl}$ and $\tau_{ih} \leq \tau_{jh}$ when $\lambda_i^* > \lambda_j^* > 0$, it is not possible to rank τ_{jl} and τ_{ih}.

We have:

Lemma 3 *Consider any asymmetric regular equilibrium. If $i \neq e$, we have:*

(i) when $\tau > \tau_{ih}$, large regions become larger while small regions become smaller;

(ii) when $\tau < \tau_{il}$, large regions become smaller while small regions become larger.

Proof: From Lemmas D and E as well as from Definition 2, exactly one of the following two cases applies to a large region or a small region. First, $z_i \geq z_h$ is equivalent to

$$(\lambda_i^* - \lambda_h^*)[C_1 - 2C_2\tau - 2C_3\tau(\lambda_i^* + \lambda_h^*)] \geq 0 \tag{28}$$

Second, $z_i \leq z_l$ is equivalent to

$$(\lambda_i^* - \lambda_l^*)[C_1 - 2C_2\tau - 2C_3\tau(\lambda_i^* + \lambda_l^*)] \leq 0 \tag{29}$$

Consider now the situation in which $\tau > \tau_{ih}$. We have

$$C_1 - 2C_2\tau - 2C_3\tau(\lambda_i^* + \lambda_l^*) \leq C_1 - 2[C_2 + C_3\tau(\lambda_i^* + \min\{\lambda_l^*, \lambda_e^*, \lambda_h^*\})] < 0$$

In a large region i with $\lambda_i^* \geq \lambda_l^*$, (29) holds, and hence $z_i \leq z_l$. This implies that region i becomes larger by Lemma D. Similarly, in a small region i with $\lambda_i^* < \lambda_l^*$, (28) holds, and hence $z_i \geq z_h$, implying that region i becomes smaller.

When $\tau < \tau_{il}$, we can similarly show that a large region becomes smaller while a small region becomes larger. $\square$

Assume that τ decreases from some large threshold $\overline{\tau}$. First, let i be any large region. Since τ_{ih} is inversely related to λ_i^*, *the larger the regional size of* i, *the larger the interval* $[\tau_{ih}, \overline{\tau}]$ *for which the size of region* i *necessarily expands.* By contrast, since τ_{il} is inversely related to λ_i^*, *the larger the regional size, the smaller the interval* $[0, \tau_{il}]$ *of trade costs for which the regional size must shrink.* When $\tau \in (\tau_{il}, \tau_{ih})$, we do not know how the size of region i evolves. For example, as shown by the analysis of the two-region case, the whole manufacturing sector may agglomerate into a single region (Ottaviano *et al.*, 2001). The reason for the existence of the domain (τ_{il}, τ_{ih}) lies in the fact that some regions may become rural when τ falls in this interval. This explains why the previous analysis cannot cover the whole domain of τ-values.

Second, consider any small region i. As τ decreases from $\overline{\tau}$, the size of region i must decrease up to $\tau = \tau_{ih}$. Below τ_{il}, this region recoups some firms/workers and keeps growing as τ falls. Again, for the same reason as in the case of large regions, we do not know how a small region changes when $\tau \in (\tau_{il}, \tau_{ih})$.

Last, for a medium region, the evolution of its size as trade costs fall is undetermined.

5.2 Irregular equilibrium ($S'_n = 0$)

In this case, we rewrite (21) as follows.

$$
\begin{cases}
\dfrac{d\lambda_i^*(\tau)}{d\tau} = \dfrac{1}{n^2|D|}\displaystyle\prod_{k=1}^{n-1}(-nS'_k)\dfrac{z_i - z_n}{-S'_i} & \text{for } i = 1, \dots, n-1 \\
\dfrac{d\lambda_n^*(\tau)}{d\tau} = \dfrac{1}{n^2|D|}\displaystyle\prod_{k=1}^{n-1}(-nS'_k)\sum_{k=1}^{n-1}\dfrac{z_n - z_k}{-S'_k}
\end{cases}
\tag{30}
$$

The expression for region n in (30) is the same as that in (21), whereas the expression for region $i \neq n$ is different from that in (21). In this case, we cannot use (22) anymore since $S'_n = 0$. However, we can proceed as follows:

$$
\frac{z_i - z_n}{-S'_i} = -n\frac{\frac{\partial(S_n - S_i)}{\partial\tau}}{\frac{\partial(S_n - S_i)}{\partial\lambda_i}}, \qquad i = 1, \dots, n-1
$$

which slightly differs from (22).

The first equation of (30) implies that region n plays a role similar to that of regions l, e, h. That is, the changes in regions $i = 1, \cdots, n-1$ are determined only by the sign of $z_i - z_n$. As a result, (26) and (27) reduce to

$$
\tau_{ih} = \tau_{il} = \frac{C_1}{2[C_2 + C_3(\lambda_i^* + \lambda_n^*)]} \equiv \tau_{in}
$$

For the same reason as before, i is a large region if $\lambda_i^* > \lambda_n^*$ and a small region if $\lambda_i^* < \lambda_n^*$. On the other hand, whether region n is large or small is determined by regions l, e, h redefined as follows:

$$
\begin{cases}
z_h = \min\left\{z_i; i < n,\ \displaystyle\sum_{k=1}^{n-1}\dfrac{z_i - z_k}{-S'_k} > 0\right\} \\
e < n,\ \displaystyle\sum_{k=1}^{n-1}\dfrac{z_e - z_k}{-S'_k} = 0 \\
z_l = \max\left\{z_i; i < n,\ \displaystyle\sum_{k=1}^{n-1}\dfrac{z_i - z_k}{-S'_k} < 0\right\}
\end{cases}
\tag{31}
$$

Comparing with the regular equilibrium case, the new definitions exclude region n. With these new definitions of h, e and l, τ_{nh} and τ_{nl} are the same as in (26) and (27) respectively. Hence, Definition 2 is valid for region n.

If $z_n \geq z_k$ ($k = 1, \dots, n-1$) while at least one inequality is strict, then $d\lambda_n^*(\tau)/d\tau > 0$ and, hence, region n becomes smaller. If $z_n \leq z_k$ ($k = 1, \dots, n-1$), and at least one inequality is strict, then $d\lambda_n^*(\tau)/d\tau < 0$ and, hence, region n becomes larger. Otherwise, there exist two regions i^*, $j^* \in \{1, \ldots, n-1\}$ such that

$$\min\{z_k; k = 1, \ldots, n\} = z_{j^*} < z_n < z_{i^*} = \max\{z_k; k = 1, \ldots, n\}$$

Then,

$$\sum_{k=1}^{n-1} \frac{z_{i^*} - z_k}{-S_k'} > 0 \qquad \text{and} \qquad \sum_{k=1}^{n-1} \frac{z_{j^*} - z_k}{-S_k'} < 0$$

hold, implying that regions h and l exist. In this case, we have the following result.

Lemma 4 *The statements of Lemma 3 hold for all regions $i = 1, \ldots, n$ at any irregular equilibrium such that $S_n' = 0$ provided that*

$$\begin{aligned} \tau_{ih} &= \tau_{il} = \tau_{in} \quad \text{for } i = 1, \ldots, n-1 \\ \tau_{nh} &= \frac{C_1}{2C_2 + 2C_3(\lambda_n^* + \min\{\lambda_l^*, \lambda_e^*, \lambda_h^*\})} \\ \tau_{nl} &= \frac{C_1}{2C_2 + 2C_3(\lambda_n^* + \max\{\lambda_l^*, \lambda_e^*, \lambda_h^*\})} \end{aligned}$$

where regions h, e and l are defined by (31).

5.3 Irregular equilibrium ($S_n' > 0$)

We redefine regions h, e and l as follows:

$$\begin{cases} z_h = \min\left\{ z_i; i < n,\ \displaystyle\sum_{k=1}^{n} \frac{z_i - z_k}{-S_k'} < 0 \right\} \\ e < n,\ \displaystyle\sum_{k=1}^{n} \frac{z_e - z_k}{-S_k'} = 0 \\ z_l = \max\left\{ z_i; i < n,\ \displaystyle\sum_{k=1}^{n} \frac{z_i - z_k}{-S_k'} > 0 \right\} \end{cases}$$

By comparison with the other two cases, the inequalities of the summations are reverse. This is because the sum in (21) is multiplied by a negative term $(-nS_n')$. However, since z_h is still given by (23), regions h and l can be re-interpreted as in the regular equilibrium case.

From $S_n' > 0$, the stability condition (12) now becomes $\sum_{k=1}^{n} 1/(-S_k') < 0$. Therefore, if $z_i \geq z_h$ for region $i \neq n$, we have

$$\sum_{k=1}^{n} \frac{z_i - z_k}{-S_k'} = (z_i - z_h) \sum_{k=1}^{n} \frac{1}{-S_k'} + \sum_{k=1}^{n} \frac{z_h - z_k}{-S_k'} < 0$$

This implies that $d\lambda_i^*(\tau)/d\tau > 0$ and, hence, region i becomes smaller. Similarly, we can show that region $i \neq n$ with $z_i \leq z_l$ becomes larger. For region n, the conclusion is opposite because of the negative sign of $S_n' > 0$. That is, region n becomes larger when $z_n \geq z_h$, and region n becomes smaller when $z_n \leq z_l$.

It should be noticed that region h or region l exists at an asymmetric equilibrium, but the regions may not exist simultaneously. Nevertheless, we obtain the same results as in Lemma 3, except for region n.

Lemma 5 *At any irregular equilibrium with $S_n' > 0$,*

(a) Lemma 3 holds for regions $i = 1, \cdots, n-1$

(b) The opposite results of Lemma 3 hold for region n.

Proof: (a1) If h exists but l does not exist, then $z_i \geq z_h$ holds for all $i \neq n$, or

$$(\lambda_i^* - \lambda_h^*)[C_1 - 2C_2\tau - 2C_3\tau(\lambda_i^* + \lambda_h^*)] \geq 0$$

This is equivalent to

$$\tau > \frac{C_1}{2C_2 + 2C_3(\lambda_i^* + \lambda_h^*)} \quad \text{and} \quad \lambda_i^* \leq \lambda_h^* \tag{32}$$

$$\tau < \frac{C_1}{2C_2 + 2C_3(\lambda_i^* + \lambda_h^*)} \quad \text{and} \quad \lambda_i^* \geq \lambda_h^* \tag{33}$$

Condition (32) means that a small region with $\lambda_i^* \leq \lambda_h^*$ becomes smaller, which corresponds to the latter part of Lemma 3 (i).

On the other hand, (33) implies that a large region with $\lambda_i^* \geq \lambda_h^*$ becomes smaller, which is the former part of Lemma 3 (ii).

(a2) If l exists but h does not exist, we can similarly show that the former part of Lemma 3 (i) and the latter part of Lemma 3 (ii).

(a3) If both regions h and l exist and if $\tau > \tau_{nh}$, then $C_1 - 2C_2 - 2C_3\tau(\lambda_n^* + \lambda_l^*) < 0$. If region i is large, $(\lambda_i^* - \lambda_l^*)[C_1 - 2C_2 - 2C_3\tau(\lambda_n^* + \lambda_l^*)] < 0$ holds, and hence, $z_i < z_l$. That is, region $i \neq n$ becomes larger. The other part can be shown in a similar way.

(b) The opposite results for region n can be shown similarly. $\square$

5.4 Dispersion/agglomeration/re-dispersion

Putting together Lemmas 3, 4 and 5 while using the inequalities $\tau_{ih} \leq C_1/2C_2$ as well as $\tau_{il} \geq C_1/2(C_2 + C_3)$, we obtain the following result.

Theorem 1 *Assume that trade costs fall and disregard the finite number of values of τ given by (24). If $S'_n \leq 0$, then large regions become larger and small regions become smaller as long as $\tau > C_1/2C_2$, while large regions become smaller and small regions become larger once $\tau < C_1/2(C_2 + C_3)$. Furthermore, when $S'_n > 0$, the direction of migration is reverse for at most one region.*

This theorem has several interesting implications. First, when trade costs are high ($\tau > C_1/2C_2$), their decrease triggers an agglomeration process in which each large region attracts workers and firms from the small regions which shrink. By contrast, when trade costs are small ($\tau < C_1/2(C_2 + C_3)$), the large regions lose workers and firms while the small regions grow. Hence, *agglomeration takes place in the early stages of economic integration, while re-dispersion should occur in the late stages of the economic integration process.*

In order to show the importance of fixed costs for this process, it is worth noting that the interval $[C_1/2(C_2 + C_3), C_1/2C_2]$ collapses at a single value zero when $\phi = 0$. Since

$$C_1/2(C_2 + C_3) \leq \tau_{il} \leq \tau_{ih} \leq C_1/2C_2$$

each region becomes an autarky producing the whole range of varieties. In this case, when the urban cost functions are the same across regions, the market outcome implies an even distribution of activities. When ϕ starts rising from zero, symmetry is broken, implying that ups and downs arise in the regional distribution.

Second, it is not clear how region sizes change with intermediate trade costs ($C_1/2(C_2 + C_3) \leq \tau \leq C_1/2C_2$). In other words, it seems hard to predict the evolution of a multi-regional system once trade costs are neither high nor small.

6 On the number of urban regions

So far, all regions were urban ($\lambda_i^* > 0$ for all i). However, it is important to figure out *how the number of urban regions is affected by a fall in trade costs.* This means that we must deal with equilibrium in which some regions have no manufacturing sector. In order to achieve this goal, we impose additional restrictions on the urban cost functions. More precisely, we assume that each urban cost is linear and regions are re-indexed according to the values of unit costs:

$$\theta_i(y) = \theta_i \cdot y \qquad \text{and} \qquad \theta_i \leq \theta_{i+1} \qquad i = 1, ..., n-1$$

This assumption is justified when the urban space is linear, each worker consumes a fixed lot size for housing, and the commuting cost is proportional to distance. In this case, $S_i(\lambda_i)$ can be rewritten as follows:

$$S_i(\lambda_i) = C_3\tau^2[\lambda_i^o(\tau) - \lambda_i]\lambda_i$$

where

$$\lambda_i^o(\tau) \equiv \frac{C_1\tau - C_2\tau^2 - \theta_i}{C_3\tau^2}$$

denotes the size of the manufacturing sector in region i for which $S_i(\lambda_i) = 0$; we have $\lambda_i^o(\tau) \leq \lambda_j^o(\tau)$ when $i < j$. Clearly, $S_i(\lambda_i)$ is a concave parabola passing through the origin. Since $\lambda_i^o(\tau)$ may be negative, we set

$$\lambda_i^\sharp \equiv \max\{0, \lambda_i^o(\tau)\} \qquad i = 1, ..., n$$

and we also define

$$\mathcal{L}_m \equiv \sum_{j=1}^{m} \lambda_j^\sharp \qquad m = 1, ..., n$$

Since $\theta_i \leq \theta_{i+1}$ and since $S_i(\lambda_i)$ is a concave parabola, we have:

$$\begin{array}{ll} \lambda_i^\sharp \geq \lambda_{i+1}^\sharp & \text{for } \; i = 1, \cdots, n-1 \\ S_i(y) \geq S_{i+1}(y) & \text{for all } \; y \geq 0 \; \text{ and } \; i = 1, \cdots, n-1 \\ S_i'(y) \leq S_{i+1}'(y) & \text{for all } \; y \geq 0 \; \text{ and } \; i = 1, \cdots, n-1 \end{array}$$

As expected, for the same industrial size, pseudo-surpluses are higher in the regions endowed with efficient transport infrastructure. Hence, in equilibrium, there is a negative relationship between commuting costs and the size of urban regions.

In Figure 2, we depict the case of an equilibrium in which regions 1 and 3 are urban while regions 2 and 4 are rural because the regional surplus of region 4 is lower than the equilibrium surplus $S_i(\lambda_i^*)$ in the other three regions or because the initial endowment of region 3 is zero. This figure is sufficient to show that several such stable equilibria may exist. For example, regions 1, 2 and 3 could also be active in equilibrium if the initial endowment of 2 were positive.

Figure 2: Equilibrium with urban and rural regions

In the next lemma, we identify sufficient conditions for a stable equilibrium to exist.

Lemma 6 *If*

$$\lambda_i^o(\tau) \neq 0 \qquad \textit{and} \qquad \mathcal{L}_j \neq 1 \qquad j = 1, \ldots, n \tag{34}$$

the system (8) has at least one stable equilibrium.

Proof: For any $\lambda_1 \geq \lambda_1^\sharp$, it must be that $S_1(\lambda_1) \leq 0$ and $\lambda_1 \geq \lambda_1^o(\tau)$. Thus, for $i = 2, ..., n$ we may define $\lambda_i(\lambda_1|S_i = S_1)$ as the larger solution to

$$S_i(\lambda_i(\lambda_1)) = S_1(\lambda_1)$$

so that

$$\lambda_i(\lambda_1|S_i = S_1) = \frac{\lambda_i^o(\tau) + \sqrt{[\lambda_i^o(\tau)]^2 + 4[\lambda_1 - \lambda_1^o(\tau)]\lambda_1}}{2}$$

Clearly, $\lambda_i(\lambda_1) \leq \lambda_i^\sharp$ for $i = 2, \ldots, n$. Furthermore, $S_i(y)$ is strictly decreasing over $(\lambda_i^\sharp, \infty)$.

Three cases may arise.

(i) If $\mathcal{L}_n < 1$, then there exists a unique $\lambda_1^* > \lambda_1^\sharp$ such that $\lambda_1^* + \sum_{i=2}^n \lambda_i(\lambda_1^*|S_i = S_1) = 1$. Hence,

$$\boldsymbol{\lambda} \equiv (\lambda_1^*, \lambda_2(\lambda_1^*|S_2 = S_1), \ldots, \lambda_n(\lambda_1^*|S_n = S_1))$$

is a stable equilibrium.

(ii) If $\mathcal{L}_1 > 1$, then $\boldsymbol{\lambda} \equiv (1, 0, \cdots, 0)$ is a stable equilibrium.

(iii) If the two conditions above are not met, there exists a region $m \in \{2, \ldots, n\}$ such that

$$\mathcal{L}_{m-1} < 1 < \mathcal{L}_m \tag{35}$$

Since $\mathcal{L}_{m-1} < \mathcal{L}_m$, $\lambda_m^o(\tau) > 0$ and, hence, $\lambda_i^o(\tau) > 0$ for all $i = 1, \ldots, m$. Under (35), $S_i(\lambda_i) = S_m(\lambda_m)$ has a single solution that belongs to the interval $[\lambda_i^\sharp/2, \lambda_i^\sharp]$. This solution is given by:

$$\lambda_i(\lambda_m|S_i = S_m) = \frac{\lambda_i^o(\tau) + \sqrt{[\lambda_i^o(\tau)]^2 + 4[\lambda_m - \lambda_m^o(\tau)]\lambda_m}}{2} \tag{36}$$

for $i = 1, \cdots, m-1$. Let

$$f_m(\lambda_m, \tau) \equiv \lambda_m + \sum_{i=1}^{m-1} \lambda_i(\lambda_m|S_i = S_m)$$

Since $f_m(0,\tau) < 1$ and $f_m(\lambda_m^\sharp, \tau) > 1$ from (35), we can always find a value λ_m^* in $(0, \lambda_m^\sharp)$ such that $f_m(\lambda_m^*, \tau) = 1$, thus implying that

$$\boldsymbol{\lambda} \equiv (\lambda_1(\lambda_m^*|S_1 = S_m), \dots, \lambda_{m-1}(\lambda_m^*|S_{m-1} = S_m), \lambda_m^*, 0, \dots, 0)$$

is an equilibrium.

By direct calculation, we obtain

$$\begin{aligned}
\frac{\partial f_m(\lambda_m,\tau)}{\partial \lambda_m} &= 1 + \sum_{i=1}^{m-1} \frac{2\lambda_m - \lambda_m^o(\tau)}{\sqrt{(\lambda_i^o(\tau))^2 + 4(\lambda_m - \lambda_m^o(\tau))\lambda_m}} \\
&= 1 + \sum_{i=1}^{m-1} \frac{S_m'(\lambda_m)}{S_i'(\lambda_i(\lambda_m|S_i = S_m))} \qquad \text{and} \\
\frac{\partial^2 f_m(\lambda_m,\tau)}{\partial \lambda_m^2} &= \sum_{i=1}^{m-1} \frac{2[(\lambda_i^o(\tau))^2 - (\lambda_m^o(\tau))^2]}{[(\lambda_i^o(\tau))^2 + 4(\lambda_m - \lambda_m^o(\tau))\lambda_m]^{3/2}} \geq 0 \qquad (37)
\end{aligned}$$

where the inequality follows from the definition of $\lambda_i^o(\tau)$ and $\theta_i \leq \theta_m$ for $i = 1, \cdots, m-1$. Since

$$\frac{\partial f_m^2(\lambda_m,\tau)}{\partial \lambda_m^2} \geq 0, \quad f_m(\lambda_m^*,\tau) = 1 > f(0,\tau) \;\; \text{and} \;\; \lambda_m^* > 0$$

we have $\partial f_m(\lambda_m^*,\tau)/\partial\lambda_m > 0$. Consequently,

$$1 + \sum_{i=1}^{m-1} \frac{S_m'(\lambda_m^*)}{S_i'(\lambda_i(\lambda_m^*|S_i = S_m))} > 0$$

Therefore, if

$$\lambda_{m-1}(\lambda_m^*|S_{m-1} = S_m) \in (\lambda_{m-1}^\sharp/2, \lambda_{m-1}^\sharp] \quad \text{and} \quad S_{m-1}'(\lambda_{m-1}(\lambda_m^*|S_{m-1} = S_m)) < 0$$

then $\boldsymbol{\lambda}$ is a stable equilibrium. Indeed, otherwise, (36) would imply $\lambda_{m-1}(\lambda_m^*|S_{m-1} = S_m) \in [\lambda_{m-1}^\sharp/2, \lambda_{m-1}^\sharp]$ so that we would have $\lambda_{m-1}(\lambda_m^*|S_{m-1} = S_m) = \lambda_{m-1}^\sharp/2$. In this case, the inequality $S_i(y) \geq S_{i+1}(y)$ would entail $S_m(y) = S_{m-1}(y)$ and, hence, $\lambda_m^* = \lambda_m^\sharp/2$. Accordingly, we would have

$$1 > \mathcal{L}_{m-1} = \mathcal{L}_{m-2} + \frac{\lambda_{m-1}^\sharp}{2} + \frac{\lambda_m^\sharp}{2} \geq f_m(\frac{\lambda_m^\sharp}{2}, \tau) = f_m(\lambda_m^*, \tau) = 1$$

a contradiction. $\square$

This lemma implies that *a stable equilibrium always exists* (except for a finite number of τ-values) but it does not say anything about the uniqueness of such an equilibrium.

However, when $\mathcal{L}_n < 1$, there is a single stable equilibrium. This is because $S_i(\lambda)$ is negative and decreasing on $(\lambda_i^\sharp, \infty)$.

In the sequel, we consider the evolution of the manufacturing sector distribution when trade costs decrease from a sufficiently large value down to zero. In doing so, we assume that (34) holds, thus excluding only a finite number of τ-values.

Let

$$m_o \equiv \min\{m; m < n \quad \text{and} \quad \exists\tau \text{ such that } \mathcal{L}_m(\tau) > 1\}$$

If m_o does not exist, then the equilibrium configuration involves dispersion for any trade cost value. So for the problem to be meaningful, we assume from now on that m_o exists.

For $m = 1, \ldots, n-1$, solve $\lambda_m^o = 0$. If $C_1^2 > 4C_2\theta_m$, then there exist two real roots:

$$\tau_m^+ \equiv \frac{C_1 + \sqrt{C_1^2 - 4C_2\theta_m}}{2C_2} \qquad \tau_m^- \equiv \frac{C_1 - \sqrt{C_1^2 - 4C_2\theta_m}}{2C_2}$$

Set

$$m^+ \equiv \begin{cases} \min\{m; \mathcal{L}_m(\tau_m^+) > 1\} & \text{if there is } m \text{ satisfying } \mathcal{L}_m(\tau_m^+) > 1 \\ n & \text{if no } m \text{ satisfies } \mathcal{L}_m(\tau_m^+) > 1 \end{cases}$$

$$m^- \equiv \begin{cases} \min\{m; \mathcal{L}_m(\tau_m^-) > 1\} & \text{if there is } m \text{ satisfying } \mathcal{L}_m(\tau_m^-) > 1 \\ n & \text{if no } m \text{ satisfies } \mathcal{L}_m(\tau_m^-) > 1 \end{cases}$$

Since $\mathcal{L}_i(\tau_i^-) = \mathcal{L}_{i-1}(\tau_i^-) < 1$ for $i = 1, \ldots, m_o$ (we set $\mathcal{L}_0(\tau) \equiv 0$), it always holds that $m_o < m^-$. Furthermore, since

$$\lambda_i^o(\tau_m^-) = \frac{\theta_m - \theta_i}{C_3(\tau_m^-)^2} \geq \frac{\theta_m - \theta_i}{C_3(\tau_m^+)^2} = \lambda_i^o(\tau_m^+) \qquad \text{for} \quad i = 1, \ldots, m$$

we have $\mathcal{L}_m(\tau_m^-) \geq \mathcal{L}_m(\tau_m^+)$, so it always holds that $m^- \leq m^+$. Hence we have

$$m_o < m^- \leq m^+$$

Since we assume that m_o exists, we know that $\mathcal{L}_n(\tau) > 1$ holds for some τ. Therefore, each of the two equations $\mathcal{L}_{m^+}(\tau) = 1$ and $\mathcal{L}_{m^-}(\tau) = 1$ always has two real roots. Define the larger root of $\mathcal{L}_{m^+} = 1$ as τ^+ and the smaller root of $\mathcal{L}_{m^-} = 1$ as τ^-. Then, since $\mathcal{L}_{m^+}(\tau_{m^+}^+) > 1$ and $\mathcal{L}_{m^+}(\tau^+) = 1$ it must be that $\tau^+ > \tau_{m^+}^+$; similarly, we have $\tau^- < \tau_{m^-}^+$.

For any m, set

$$\tau_m \equiv \frac{2\sum_{i=1}^m \theta_i}{mC_1}$$

For any given value of m, computing $\partial\mathcal{L}_m/\partial\tau$ shows that $\mathcal{L}_m$ is non-decreasing in τ for $\tau < \tau_m$ and non-increasing in τ for $\tau > \tau_m$. Therefore, $\mathcal{L}_m(\tau_m) = \max_\tau \mathcal{L}_m(\tau)$.

Although there may exist several stable equilibria, we can choose a typical one which displays the feature of *agglomeration cascades*.

Theorem 2 *Assume that urban costs are linear. As trade costs decrease, there is a path of stable equilibria such that the number of urban regions varies as follows.*

(i) For large trade costs ($\tau > \tau^+$), there is a single stable equilibrium and each region accommodates a positive share of the manufacturing sector. The number of urban regions suddenly decreases from n to $m^+ - 1$ when τ reaches τ^+.

(ii) For $\tau \in (\tau_{m_o}, \tau^+)$, the number of urban regions decreases or remains constant. The number of urban regions is never smaller than m_o.

(iii) For $\tau \in (\tau^-, \tau_{m_o})$, the number of urban regions increases or remains constant.

(iv) At $\tau = \tau^-$ the number of urban regions suddenly increases from $m^- - 1$ to n. For low trade costs ($\tau < \tau^-$), there is a single stable equilibrium and each region accommodates a positive share of the manufacturing sector.

Proof: By definition, we have

$$\mathcal{L}_m(\tau) = \sum_{i=1}^{m} \lambda_i^\sharp(\tau) = \sum_{i=1}^{m} \frac{C_1\tau - C_2\tau^2 - \theta_i}{C_3\tau^2} \quad \text{for} \quad \tau \in [\tau_m^-, \tau_m^+]$$

For all m, $\mathcal{L}_m$ is non-decreasing on $[0, \tau_m]$ and non-increasing on $[\tau_m, \infty)$. Therefore, it is quasi-concave and reaches its maximum at τ_m. From $\mathcal{L}_{m_o}(\tau_{m_o}) > 1$ and $m_o \leq m^+$, it follows that $1 < \mathcal{L}_{m_o}(\tau_{m_o}) \leq \mathcal{L}_{m^+}(\tau_{m_o})$. Since $\mathcal{L}_{m^+}(\tau)$ is quasi-concave and $\mathcal{L}_{m^+}(\tau^+) = 1$, we see that $\tau_{m_o} < \tau^+$. Similarly, $\tau_{m_o} > \tau^-$.

(i). For $\tau > \tau^+$, we have

$$0 \leq \lambda_n^\sharp(\tau) \leq \ldots \leq \lambda_{m^+}^\sharp(\tau) \leq \lambda_{m^+}^\sharp(\tau^+) \leq \lambda_{m^+}^\sharp(\tau_{m^+}^+) = 0$$

so $\lambda_i^\sharp(\tau) = 0$ for $i = m^+, \ldots, n$. Therefore,

$$\mathcal{L}_i(\tau) \begin{cases} \leq \mathcal{L}_{m^+}(\tau) & \text{if } i \leq m^a \\ = \mathcal{L}_{m^+}(\tau) & \text{if } i \geq m^a \end{cases}$$

Since $\mathcal{L}_{m^+}(\tau) < \mathcal{L}_{m^+}(\tau^+) = 1$, we have $\mathcal{L}_i(\tau) < 1$ for $i = 1, \ldots, n$ in either case. Consequently, the unique stable equilibrium is such that each region has firms and workers. For τ very close to τ^+, the population in regions $m^+, \cdots, n$ becomes very small. Eventually, when $\tau = \tau^+$, the population in each region $i = m^+, \cdots, n$ becomes simultaneously zero, thus implying that the number of urban regions drops down to $m^+ - 1$. This equilibrium is regular and, hence, stable.

The same argument applies, mutatis mutandis, to case (iv) in which $\tau \leq \tau^-$.

Before proceeding, observe that there may exist several equilibria when $\tau \in (\tau^-, \tau^+)$. Among them, we choose the following one. For any given $\tau \in (\tau^-, \tau^+)$, there exists a unique value $m(\tau) \geq m_o$ such that $\mathcal{L}_{m(\tau)-1}(\tau) < 1 < \mathcal{L}_{m(\tau)}(\tau)$. Using the proof of Lemma 6, there is a stable equilibrium given by

$$\boldsymbol{\lambda}^\sharp = (\lambda_1(\lambda^*_{m(\tau)}|S_1 = S_{m(\tau)}), \ldots, \lambda_{m(\tau)-1}(\lambda^*_{m(\tau)}|S_{m(\tau)-1} = S_{m(\tau)}), \lambda^*_{m(\tau)}, 0, \ldots, 0)$$

where $f_{m(\tau)}(\lambda^*_{m(\tau)}, \tau) = 1$. Note that in this equilibrium, only $m(\tau)$ regions are urban.

(ii). Let us first show that the following inequality holds for any $\tau \in (\tau_{m_o}, \tau^+)$:

$$\mathcal{L}_{m(\tau)}(x) > 1 \qquad \text{for all} \;\; x \in [\tau_{m_o}, \tau] \tag{38}$$

If $\tau \geq \tau_{m(\tau)}$, then

$$\mathcal{L}_{m(\tau)}(x) \geq \mathcal{L}_{m(\tau)}(\tau) > 1 \qquad \text{for all} \quad x \in [\tau_{m(\tau)}, \tau]$$

$$\mathcal{L}_{m(\tau)}(x) \geq \mathcal{L}_{m(\tau)}(\tau_{m_o}) \geq \mathcal{L}_{m_o}(\tau_{m_o}) > 1 \qquad \text{for all} \quad x \in [\tau_{m_o}, \tau_{m(\tau)}] \tag{39}$$

If $\tau < \tau_{m(\tau)}$, then (39) implies that (38) holds.

The inequality (38) implies that the number of urban regions cannot exceed $m(\tau)$ when trade costs take the value $x \in [\tau_{m_o}, \tau]$. Since this holds true for any $\tau \in (\tau_{m_o}, \tau^+)$, the number of urban regions does not increase when τ decreases from τ^+ to τ_{m_o}.

Likewise, the following inequality tells us that the number of urban regions does not decrease when $\tau < \tau^o$:

$$\mathcal{L}_{m(\tau)-1}(x) < 1 \quad \text{for all} \quad x \in (0, \tau)$$

This inequality holds since $\tau < \tau_{m_o} \leq \tau_{m(\tau)-1}$. This covers Case (iii).

In addition, for any $m < m_o$, we have $\mathcal{L}_m(\tau) < 1$ for all τ. Then, at any equilibrium with $m < n$ urban regions, it must be that $S_i(\lambda_i^*) < 0$ for any urban region. This implies that this equilibrium is unstable. As a result, m_o is the smallest number of urban regions at any stable equilibrium. $\square$

Consequently, for sufficiently large or sufficiently small trade costs, each region has a share of the manufacturing sector. In this case, the market outcome satisfies the main assumption of the foregoing section, explaining why results are consistent. However, for intermediate values of these costs, the number of urban regions typically varies. Under linear urban costs, it first decreases and then increases. Hence, *urban concentration first arises while re-dispersion comes afterwards.* In addition, the minimum number of urban regions may exceed 1, implying that the highest degree of agglomeration within the economy may involve several regions. This shows how *the presence of urban costs may prevent the full agglomeration into a single core region.*

On the other hand, if the urban costs in region 1 are sufficiently small such that $\mathcal{L}_1(\tau_1) > 1$, or equivalently, $\theta_1 \leq C_1^2/4(C_2 + C_3)$, then industry fully agglomerates into a single region for intermediate values of the trade costs. Consequently, when trade costs decrease while urban costs do not, *the economy would move from dispersion to the emergence of an urban giant and, then, would display gradual deconcentration.*

Of course, some parts of the equilibrium path described in Theorem 2 may not arise. This is so when trade costs are so high (Case (i)) for no interregional trade to occur. Even the part corresponding to Case (ii) may not show. This happens when there are no farmers ($A = 0$). Because of the existence of urban costs, the decrease in trade costs induces a gradual dispersion of the industrial sector over a growing number of regions, as in Helpman (1998).

Although our path of stable equilibria seems to involve unit changes in the number of cities, we cannot exclude the simultaneous disappearance or emergence of several urban regions. For example, starting from $S_m > 0$, the equilibrium becomes irregular ($S_m' > 0$) before the smallest urban region m becomes rural. In this case, the stability condition (12) is violated so that the size of region m may jump down to zero. From this moment on, we do not know which path the economy will follow when there are multiple equilibria.

7 Concluding remarks

This paper suggests that the secular fall in transport and communication costs should lead to a possibly strong concentration of mobile activities, which will eventually be followed by a re-dispersion of these activities. In other words, the general pattern of activities as trade costs fall would be more or less $\cap$-shaped. However, much work remains to be done in order to understand how regions evolve when trade costs take intermediate values, while it is also important to figure out how the medium regions react to decreasing trade costs.

Our model has dismissed the fact that commuting costs have decreased together with trade costs. So, it would be interesting to study the impact of their relative change on the spatial structure of industry. Finally, it should be kept in mind that our model considers a given and fixed set of economic activities. In particular, the number of firms is the same regardless of the value of trade costs. In this respect, the observed decline of the industrial sector within big cities does not necessarily imply the economic and social decline of these areas. The continuous decrease in communication and transport costs gives rise to new economic activities that are typically information-oriented, and which, therefore, tend to grow in large metropolises. Thus, one task for future research is to investigate this question in a setting allowing firms and workers to locate "out in the burds".

APPENDIX

A. Proof of Lemma 1

$$\begin{aligned}
\sum_{j=1}^{n}[V_i(\boldsymbol{\lambda})-V_j(\boldsymbol{\lambda})] &= -aN\left[np_{ii}-\sum_{j=1}^{n}p_{jj}-\frac{\tau}{2}\left(n\lambda_i-\sum_{j=1}^{n}\lambda_j\right)\right] \\
&+\frac{(b+cN)N}{2}\left[np_{ii}^2-\sum_{j=1}^{n}p_{jj}^2+\tau n(1-\lambda_i)\left(p_{ii}+\frac{\tau}{4}\right)-\tau\sum_{j=1}^{n}(1-\lambda_j)\left(p_{jj}+\frac{\tau}{4}\right)\right] \\
&-\frac{cN^2}{2}\left\{\sum_{j=1}^{n}\left[\left(p_{ii}+\frac{\tau(1-\lambda_i)}{2}\right)^2-\left(p_{jj}+\frac{\tau(1-\lambda_j)}{2}\right)^2\right]\right\} \\
&+(b+cN)N\tau\left[n\left(p_{ii}-\frac{\tau}{4}\right)\lambda_i-\sum_{j=1}^{n}\left(p_{jj}-\frac{\tau}{4}\right)\lambda_j\right]+\frac{(b+cN)NA\tau}{nL}\left(np_{ii}-\sum_{j=1}^{n}p_{jj}\right) \\
&-n\theta_i(\lambda_i)+\sum_{j=1}^{n}\theta_j(\lambda_j).
\end{aligned}$$

However,

$$np_{ii}-\sum_{j=1}^{n}p_{jj}=\frac{cN\tau}{2(2b+cN)}(1-n\lambda_i),$$

$$np_{ii}^2-\sum_{j=1}^{n}p_{jj}^2=\frac{(2a+Nc\tau)Nc\tau}{2(2b+cN)^2}(1-n\lambda_i)+\frac{c^2N^2\tau^2}{4(2b+c)^2}\left(n\lambda_i^2-\sum_{j=1}^{n}\lambda_j^2\right),$$

$$\begin{aligned}
&\tau n(1-\lambda_i)\left(p_{ii}+\frac{\tau}{4}\right)-\tau\sum_{j=1}^{n}(1-\lambda_j)\left(p_{jj}+\frac{\tau}{4}\right) \\
&\quad=\tau\left[(np_{ii}-\sum_{j=1}^{n}p_{jj})-\left(n\lambda_i p_{ii}-\sum_{j=1}^{n}\lambda_j p_{jj}\right)-\frac{\tau}{4}(n\lambda_i-1)\right] \\
&\quad=\tau\left(\frac{\tau}{4}+\frac{a+cN\tau}{2b+cN}\right)(1-n\lambda_i)+\frac{cN\tau^2(n\lambda_i^2-\sum_{j=1}^{n}\lambda_j^2)}{2(2b+cN)},
\end{aligned}$$

$$\sum_{j=1}^{n}\left\{\left[p_{ii}+\frac{\tau(1-\lambda_i)}{2}\right]^2-\left[p_{jj}+\frac{\tau(1-\lambda_j)}{2}\right]^2\right\}$$

$$
\begin{aligned}
&= \sum_{j=1}^{n} \left[p_{ii} + p_{jj} + \tau - \frac{\tau}{2}(\lambda_i + \lambda_j)\right] \left(p_{ii} - p_{jj} + \frac{\lambda_j - \lambda_i}{2}\tau\right) \\
&= \frac{b + cN}{2b + cN}\tau \left[\frac{2(a + b\tau + N\tau c)}{2b + cN}(1 - n\lambda_i) + \frac{b + cN}{2b + cN}\tau \left(n\lambda_i^2 - \sum_{j=1}^{n} \lambda_j^2\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
&n\left(p_{ii} - \frac{\tau}{4}\right)\lambda_i - \sum_{j=1}^{n}\left(p_{jj} - \frac{\tau}{4}\right)\lambda_j \\
&= -\frac{(2a + cN\tau)(1 - n\lambda_i) - cN\tau(n\lambda_i^2 - \sum_{j=1}^{n}\lambda_j^2)}{2(2b + cN)} + \frac{\tau}{4}(1 - n\lambda_i) \\
&= \left[\frac{\tau}{4} - \frac{2a + cN\tau}{2(2b + cN)}\right](1 - n\lambda_i) - \frac{cN\tau}{2(2b + cN)}\left(n\lambda_i^2 - \sum_{j=1}^{n}\lambda_j^2\right).
\end{aligned}
$$

Then,

$$
\begin{aligned}
\sum_{j=1}^{n}[V_i(\boldsymbol{\lambda}) - V_j(\boldsymbol{\lambda})] &= -\frac{aN\tau(b + cN)}{2b + cN}(1 - n\lambda_i) \\
&\quad + \frac{(b + cN)N}{2}\left\{\left[\frac{Nc\tau(2a + Nc\tau)}{2(2b + cN)^2} + \tau\left(\frac{\tau}{4} + \frac{a + cN\tau}{2b + cN}\right)\right](1 - n\lambda_i)\right. \\
&\qquad \left. + \left[\frac{cN\tau^2}{2(2b + cN)} + \frac{c^2N^2\tau^2}{4(2b + c)^2}\right]\left(n\lambda_i^2 - \sum_{j=1}^{n}\lambda_j^2\right)\right\} \\
&\quad - \frac{cN^2\tau(b + cN)}{2(2b + cN)}\left[\frac{2(a + b\tau + N\tau c)}{2b + cN}(1 - n\lambda_i) + \frac{b + cN}{2b + cN}\tau(n\lambda_i^2 - \sum_{j=1}^{n}\lambda_j^2)\right] \\
&\quad + (b + cN)N\tau\left\{\left[\frac{\tau}{4} - \frac{2a + cN\tau}{2(2b + cN)}\right](1 - n\lambda_i) - \frac{cN\tau}{2(2b + cN)}(n\lambda_i^2 - \sum_{j=1}^{n}\lambda_j^2)\right\} \\
&\quad + \frac{(b + cN)NA\tau}{nL}\frac{cN\tau}{2(2b + cN)}(1 - n\lambda_i) - n\theta_i(\lambda_i) + \sum_{j=1}^{n}\theta_j(\lambda_j) \\
&= (1 - n\lambda_i)\left\{-\frac{aN\tau(b + cN)}{2b + cN} + \frac{(b + cN)N^2c\tau(2a + Nc\tau)}{4(2b + cN)^2} + \frac{(b + cN)N\tau}{2}\left(\frac{\tau}{4} + \frac{a + cN\tau}{2b + cN}\right)\right. \\
&\quad \left. - \frac{cN^2\tau(b + cN)(a + b\tau + N\tau c)}{(2b + cN)^2} + (b + cN)N\tau\left[\frac{\tau}{4} - \frac{2a + cN\tau}{2(2b + cN)}\right] + \frac{c(b + cN)N^2A\tau^2}{2nL(2b + cN)}\right\} \\
&\quad + (n\lambda_i^2 - \sum_{j=1}^{n}\lambda_j^2)\left\{\frac{(b + cN)N}{2}\left[\frac{cN\tau^2}{2(2b + cN)} + \frac{c^2N^2\tau^2}{4(2b + cN)^2}\right]\right. \\
&\qquad \left. - \frac{cN^2\tau^2(b + cN)^2}{2(2b + cN)^2} - \frac{(b + cN)N^2\tau^2c}{2(2b + cN)}\right\} - n\theta_i(\lambda_i) + \sum_{j=1}^{n}\theta_j(\lambda_j) \\
&= n(-C_1\tau + C_2\tau^2)\left(\frac{1}{n} - \lambda_i\right) + C_3\tau^2\left(\sum_{j=1}^{n}\lambda_j^2 - n\lambda_i^2\right) - n\theta_i(\lambda_i) + \sum_{j=1}^{n}\theta_j(\lambda_j)
\end{aligned}
$$

which is identical to (10).

B. Lemma B

Let D_{ij} be the submatrix of D obtained by deleting the i-th row and j-th column. We then have:

Lemma B. *If* (12) *holds at the equilibrium* $\boldsymbol{\lambda}^*$, *then*

$$|D| > 0$$

$$|D_{ii}| = \frac{1}{n}\left(1 + \sum_{\substack{k=1 \\ k \neq i}}^{n} \frac{S'_n}{S'_k}\right) \prod_{\substack{k=1 \\ k \neq i}}^{n-1} (-nS'_k) \qquad i = 1, \ldots, n-1 \tag{40}$$

$$|D_{ij}| = (-1)^{i-j+1}(S'_i - S'_n) \prod_{\substack{k=1 \\ k \neq i,\ k \neq j}}^{n-1} (-nS'_k) \quad i, j = 1, \ldots, n-1,\ i \neq j \tag{41}$$

Proof.(i) Using some basic properties of determinants, we obtain

$$\begin{aligned}
|D| &= \begin{vmatrix} -(n-1)S'_1 - S'_n & S'_2 - S'_n & S'_3 - S'_n & \cdots & S'_{n-1} - S'_n \\ nS'_1 & -nS'_2 & 0 & \cdots & 0 \\ nS'_1 & 0 & -nS'_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ nS'_1 & 0 & 0 & \cdots & nS'_{n-1} \end{vmatrix} \\
&= \left[-(n-1)S'_1 - S'_n + \sum_{k=2}^{n-1} \frac{S'_1(S'_k - S'_n)}{S'_k}\right] \prod_{k=2}^{n-1}(-nS'_k) \\
&= \frac{1}{n}\left(1 + \sum_{k=1}^{n-1} \frac{S'_n}{S'_k}\right) \prod_{k=1}^{n-1}(-nS'_k) > 0,
\end{aligned}$$

where the inequality follows from the stability condition (12).

(ii) We first consider the case of $i > 1$. By definition of D_{ii} and some properties of determinants, we have

$$|D_{ii}| = \begin{vmatrix} -(n-1)S'_1 - S'_n & \cdots & S'_{i-1} - S'_n & S'_{i+1} - S'_n & \cdots & S'_{n-1} - S'_n \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ S'_1 - S'_n & \cdots & -(n-1)S'_{i-1} - S'_n & S'_{i+1} - S'_n & \cdots & S'_{n-1} - S'_n \\ S'_1 - S'_n & \cdots & S'_{i-1} - S'_n & -(n-1)S'_{i+1} - S'_n & \cdots & S'_{n-1} - S'_n \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ S'_1 - S'_n & \cdots & S'_{i-1} - S'_n & S'_{i+1} - S'_n & \cdots & -(n-1)S'_{n-1} - S'_n \end{vmatrix}$$

$$
= \begin{vmatrix}
-(n-1)S_1' - S_n' & S_2' - S_n' & \cdots & S_{i-1}' - S_n' & S_{i+1}' - S_n' & \cdots & S_{n-1}' - S_n' \\
nS_1' & -nS_2' & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
nS_1' & 0 & \cdots & -nS_{i-1}' & 0 & \cdots & 0 \\
nS_1' & 0 & \cdots & 0 & -nS_{i+1}' & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
nS_1' & 0 & \cdots & 0 & 0 & \cdots & -nS_{n-1}'
\end{vmatrix}
$$

$$
= \left[-(n-1)S_1' - S_n' + \sum_{\substack{k=2 \\ k \neq i}}^{n-1} \frac{S_1'(S_k' - S_n')}{S_k'} \right] \prod_{\substack{k=2 \\ k \neq i}}^{n-1} (-nS_k')
$$

$$
= \frac{1}{n} \left(1 + \sum_{\substack{k=1 \\ k \neq i}}^{n} \frac{S_n'}{S_k'} \right) \prod_{\substack{k=1 \\ k \neq i}}^{n-1} (-nS_k').
$$

Next, for $i = 1$, we have

$$
\begin{aligned}
|D_{11}| &= \begin{vmatrix}
-(n-1)S_2' - S_n' & S_3' - S_n' & S_4' - S_n' & \cdots & S_{n-1}' - S_n' \\
nS_2' & -nS_3' & 0 & \cdots & 0 \\
nS_2' & 0 & -nS_4' & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
nS_2' & 0 & 0 & \cdots & -nS_{n-1}'
\end{vmatrix} \\
&= \left[-(n-1)S_2' - S_n' + \sum_{k=3}^{n-1} \frac{S_k' - S_n'}{S_k'} S_2' \right] \prod_{k=3}^{n-1} (-nS_k') \\
&= \frac{1}{n} \left(1 + \sum_{k=2}^{n} \frac{S_n'}{S_k'} \right) \prod_{k=2}^{n-1} (-nS_k')
\end{aligned}
$$

(iii) We consider only the case where $j < i$. By straightforward calculation, we know

$$
|D_{ij}|
$$

$$
= \begin{vmatrix}
-(n-1)S_1' - S_n' & S_2' - S_n' & \cdots & S_{j-1}' - S_n' & S_{j+1}' - S_n' & \cdots & S_{i-1}' - S_n' & S_i' - S_n' & S_{i+1}' - S_n' & \cdots & S_{n-1}' - S_n' \\
nS_1' & -nS_2' & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
nS_1' & 0 & \cdots & -nS_{j-1}' & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
nS_1' & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
nS_1' & 0 & \cdots & 0 & -nS_{j+1}' & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
nS_1' & 0 & \cdots & 0 & 0 & \cdots & -nS_{i-1}' & 0 & 0 & \cdots & 0 \\
nS_1' & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & -nS_{i+1}' & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
nS_1' & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -nS_{n-1}'
\end{vmatrix}
$$

$$= \quad (-1)^{i-j+1}(S'_i - S'_n) \prod_{\substack{k=1 \\ k \neq i,\ k \neq j}}^{n-1} (-nS'_k)$$

C. Proof of Lemma 2

For $i = 1, \ldots, n-1$, it follows from (20) that

$$\frac{d\lambda_i^*(\tau)}{d\tau} = \frac{(-1)^i}{|D|} \sum_{j=1}^{n-1} (-1)^j z_j D_{ji}$$

Using (40) and (41) in Appendix B, this expression becomes

$$\frac{(-1)^i}{|D|} \sum_{\substack{j=1 \\ j \neq i}}^{n-1} z_j (-1)^{-i+1}(S'_j - S'_n) \prod_{\substack{k=1 \\ k \neq i \\ k \neq j}}^{n-1} (-nS'_k) + \frac{z_i}{n|D|}\left(1 + \sum_{\substack{k=1 \\ k \neq i}}^{n} \frac{S'_n}{S'_k}\right) \prod_{\substack{k=1 \\ k \neq i}}^{n-1} (-nS'_k)$$

which is equal to

$$\frac{1}{n|D|} \prod_{\substack{k=1 \\ k \neq i}}^{n-1} (-nS'_k) \left[z_i + \sum_{j=1}^{n-1} z_j + \sum_{j=1}^{n-1} \frac{S'_n}{S'_j}(z_i - z_j) \right]$$

$$= \frac{1}{n^2|D|} \prod_{\substack{k=1 \\ k \neq i}}^{n} (-nS'_k) \sum_{k=1}^{n} \frac{z_i - z_k}{-S'_k}$$

For region n,

$$\begin{aligned}
\frac{d\lambda_n^*(\tau)}{d\tau} &= -\sum_{i=1}^{n-1}\frac{d\lambda_i^*(\tau)}{d\tau} \\
&= -\frac{1}{n^2|D|}\prod_{k=1}^{n}(-nS_k')\left(\sum_{i=1}^{n-1}\sum_{k=1}^{n}\frac{z_i}{-S_k'}-\sum_{i=1}^{n-1}\frac{1}{-nS_i'}\sum_{k=1}^{n}\frac{z_k}{-S_k'}\right) \\
&= -\frac{1}{n^2|D|}\prod_{k=1}^{n}(-nS_k')\Bigg(\sum_{i=1}^{n}\frac{z_i}{-nS_i'}\sum_{k=1}^{n}\frac{1}{-S_k'}+\frac{z_n}{nS_n'}\sum_{k=1}^{n}\frac{1}{-S_k'} \\
&\qquad -\sum_{i=1}^{n}\frac{1}{-nS_i'}\sum_{k=1}^{n}\frac{z_k}{-S_k'}-\frac{1}{nS_n'}\sum_{k=1}^{n}\frac{z_k}{-S_k'}\Bigg) \\
&= \frac{1}{n^2|D|}\prod_{\substack{k=1\\k\neq n}}^{n}(-nS_k')\sum_{k=1}^{n}\frac{z_n-z_k}{-S_k'}
\end{aligned}$$

D. Lemma D

Lemma D. *If λ^* is an asymmetric regular equilibrium, then regions h and l always exist. Furthermore, when trade costs decrease, one and only one of the following three relationships holds for each region i:*

$$\text{Case 1} \;:\; z_i \geq z_h \Rightarrow \text{region i shrinks} \tag{42}$$

$$\text{Case 2} \;:\; z_i = z_e \Rightarrow \text{the size of region i does not change} \tag{43}$$

$$\text{Case 3} \;:\; z_i \leq z_l \Rightarrow \text{region i expands} \tag{44}$$

Proof: Let $z_{i^*} = \max\{z_i; i = 1, \ldots, n\}$ and $z_{j^*} = \min\{z_i; i = 1, \ldots, n\}$. Since λ^* is asymmetric, $z_{i^*} > z_{j^*}$ must hold and, hence,

$$\begin{aligned}
\sum_{k=1}^{n}\frac{z_{i^*}-z_k}{-S_k'} &\geq \frac{z_{i^*}-z_{j^*}}{-S_{j^*}'} > 0 \\
\sum_{k=1}^{n}\frac{z_{j^*}-z_k}{-S_k'} &\leq \frac{z_{j^*}-z_{i^*}}{-S_{i^*}'} < 0
\end{aligned}$$

Consequently, regions h and l always exist.

In any regular equilibrium, $-S'_k > 0$ holds for all $k = 1, \ldots, n$. In Case 1, since $z_i \geq z_h$, we have:

$$\sum_{k=1}^{n} \frac{z_i - z_k}{-S'_k} = (z_i - z_h) \sum_{k=1}^{n} \frac{1}{-S'_k} + \sum_{k=1}^{n} \frac{z_h - z_k}{-S'_k} \geq \sum_{k=1}^{n} \frac{z_h - z_k}{-S'_k} > 0$$

Therefore, using (21) yields $d\lambda_i^*(\tau)/d\tau > 0$, implying that i becomes smaller. Cases 2 and 3 can be dealt with in a similar way.

For any region i, if neither (42) nor (43) holds, then $\sum_{k=1}^{n}(z_i - z_k)/(-S'_k) < 0$ holds by (25). In this case, $z_i \leq z_l$ must be satisfied, which is precisely (44). Finally, more than one of (42)-(44) cannot hold simultaneously. □

E. Lemma E

Lemma E. *The following holds for any region k:*
(i) $z_k = z_h$ if and only if $\lambda_k^* = \lambda_h^*$;
(ii) $z_k = z_m$ if and only if $\lambda_k^* = \lambda_m^*$;
(iii) $z_k = z_l$ if and only if $\lambda_k^* = \lambda_l^*$.

Proof: (i) From (18), we have

$$0 = z_k - z_h = n(\lambda_k^* - \lambda_h^*)[C_1 - 2C_2\tau - 2C_3\tau(\lambda_k^* + \lambda_h^*)]$$

Then, (i) follows from (24). It can be shown that (ii) and (iii) hold by a similar argument. □

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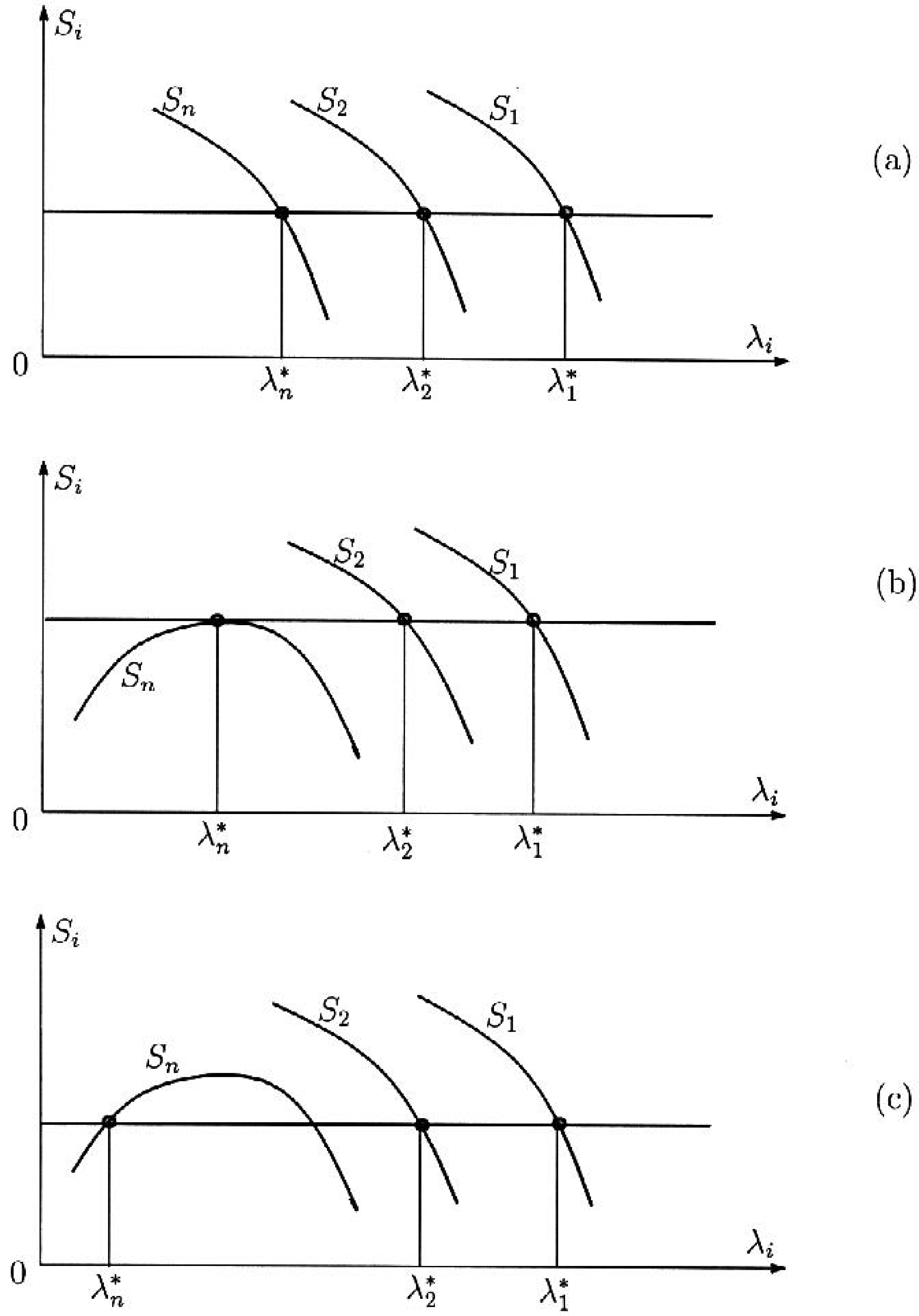

Figure 1: Regular and irregular equilibria with urban regions

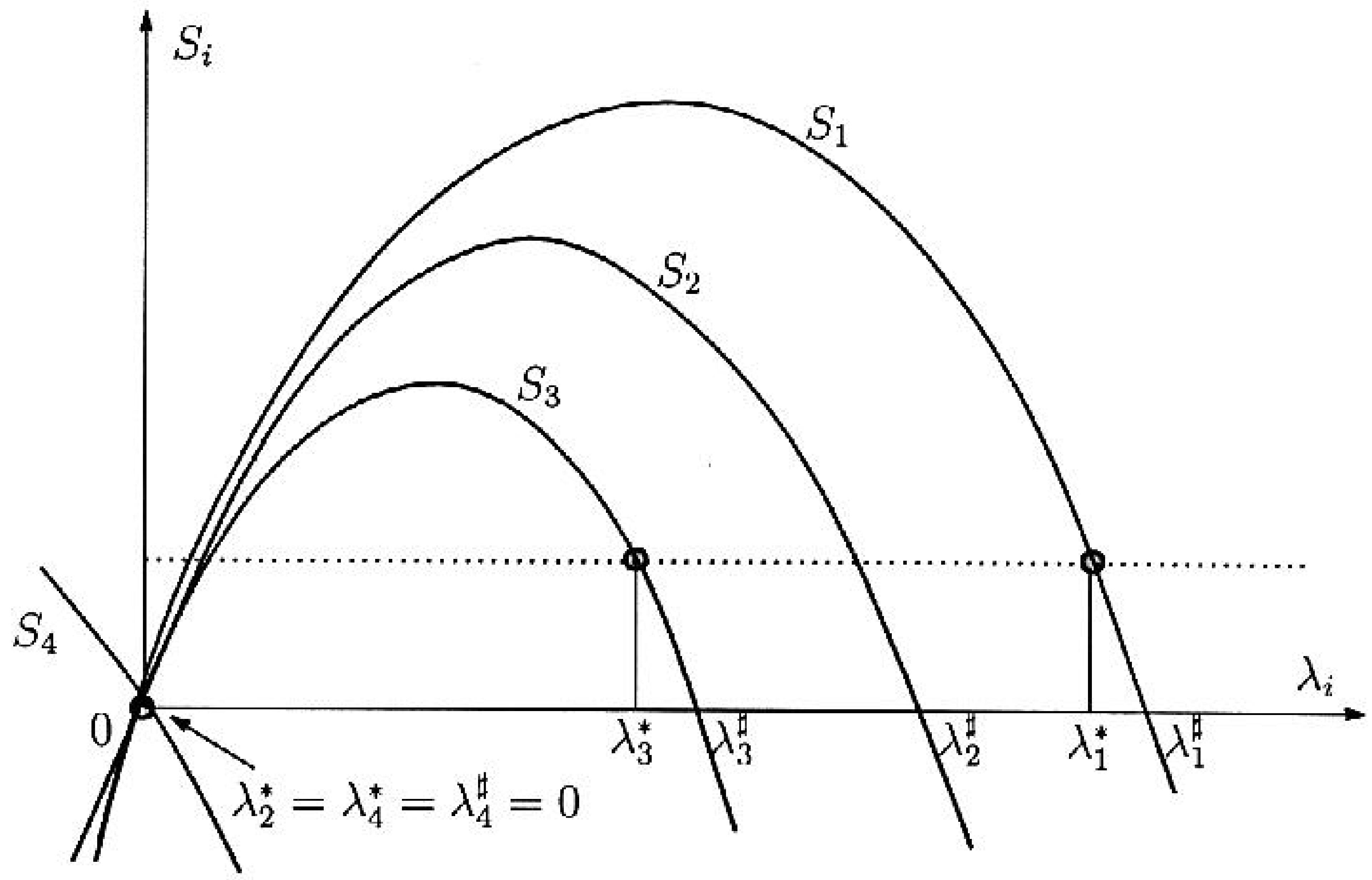

Figure 2: Equilibrium with urban and rural regions