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Defense Expenditures and Allied Cooperation

Toshihiro Ihori

University of Tokyo

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Department of Economics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
(phone) 03-5841-5502, (fax) 03-5841-5521
E-mail:ihori@e.u-tokyo.ac.jp

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Spending on national defense is a good example of an international public good where one country's supply of national defense may be a substitute for another country's supply. When two or more nations or blocs with conflicting goals engage in a competitive increase in their national defense, an arms race will occur. In this paper I investigate the spillover effects of non-cooperative and cooperative spending on national defense of allied countries of the two conflicting blocs using static and leader-follower game models.

In their classic paper Olson and Zeckhauser (1966) apply the theory of the private provision of public goods to countries and conclude that all allied countries lose when they determine the levels of spending on national defense non-cooperatively. If allied countries can cooperate, it would benefit all of them.[1] On the contrary, Bruce (1990) considers a three-country model with two allies and an adversary and shows that all countries may be worse off when the allies cooperate on defense spending than when they do not. This is because defense spending by the adversary rises in response to a cooperative increase in defense spending by the alliance, so that cooperation among allies in setting defense spending is not necessarily welfare-improving. Even if allied countries can cooperate, cooperation will not attain a better outcome than non-cooperation. This is an interesting result. "The whole notion of suboptimality of defense provision must be reconsidered when adversaries' reactions are included". (Sandler and Hartley 1995, p.42) Bruce's (1990) analysis is, however, restrictive in that one bloc has two countries and another has a single country.

In the real world allied blocs usually have multiple countries. When the number of countries within the same bloc is large, gains from cooperation will also be large. Thus, if each bloc has a large number of allied countries, we might expect that cooperative behavior attains a better outcome than non-cooperative behavior. By developing a simple multi-country model of an arms race between two blocs, this paper investigates to what extent such a conjecture is plausible. It is shown that the countries in one bloc may gain by cooperating if the countries in the other bloc cooperate, while they may lose by cooperating if the adversaries do not cooperate. Furthermore, I show that in a leader-follower game the leader bloc will be better off when cooperating, while the follower bloc will be worse off when cooperating. These results suggest that there are cases where allied cooperation may still be beneficial even if the adversarial response of the opposing bloc is explicitly included.

The organization of this paper is as follows. The next section develops a simple analytical framework, while the third section investigates non-cooperative and cooperative solutions in a static game. The fourth section considers a leader-follower game. Finally the last section concludes the paper.

ANALYTICAL FRAMEWORK

Assume that there are n+m countries and two opposing blocs α and β in the world, consisting of n and m countries, respectively. Country i's utility function is specified as a Cobb-Douglas type:

$$U^i = U^i(c_i, G_i) = c_i(A + G_i), \tag{1}$$

where U^i is welfare of country i, c_i is private consumption of country i, and G_i is the benefit of an international public good (or national security) for country i. An initial endowment of national security A>0 is incorporated into equation (1) so that the total level of national security is positive; $A+G_i>0$. The population in each country is assumed fixed and normalized at unity so that we abstract from the public good nature of national defense within the country.

G_i is given by

$$G_i = g_i + \sum_{j \neq i} \varepsilon_{ij} g_j \,, \tag{2}$$

where g_i is the international public good (or national defense) provided by country i and ε_{ij} is the degree of externalities of the public good provided by country j to country i.[2] If $\varepsilon_{ij}=1$ for all i, j, then the public good is a standard pure public good where each country's defense is perfectly substitutable. If $\varepsilon_{ij}=-1$ between enemies, there exists an 'arms race' or 'red Queen' relationship where an equal increase in national defense by a country and its enemy leaves the national security of both unchanged. In this paper we assume that $\varepsilon_{ij}=1$ for $i,j \in \alpha$ or $i,j \in \beta$ and $\varepsilon_{ij}=-1$ for $i \in \alpha, j \in \beta$ or $i \in \beta, j \in \alpha$.[3] All countries within the same bloc are "perfect allies" and they treat the countries in the opposing bloc as a "perfect" enemy. Thus, from equation (2), we have

$$G_i = -G_j \,, \qquad \text{for } i \in \alpha, j \in \beta \text{ or } i \in \beta, j \in \alpha \,.$$

Country i's budget constraint is given by

$$c_i + g_i = Y_i \,, \tag{3}$$

where Y_i is exogenously given national income of country i. Adding $\sum_{j \neq i} \varepsilon_{ij} g_j$ to both sides of (3) and using (2), we have

$$c_i + G_i = Y_i + \sum_{j \neq i} \varepsilon_{ij} g_j \,. \tag{3'}$$

Production technologies are linear and identical across countries, and units are chosen such that the constant marginal rate of transformation between c_i and g_i is unity for all countries.

NASH EQUILIBRIUM

For simplicity we assume that all countries in the same bloc are identical and behave in the same way; namely, they either cooperate or do not cooperate with one another. We do not investigate the case of partial cooperation where some allied countries cooperate, while the rest do not. By investigating the potential gain of cooperation, we could explore the cost of free-riding situation where one country does not cooperate and the rest of the allied countries cooperate.

NON-COOPERATIVE SOLUTION

First, we investigate the case where each country determines its own national defense taking the national defense expenditure of every other country as given. In other words, in this section we will assume alliances do not undertake any cooperative decision making with respect to spending on allied national defense and demonstrate Nash behavior.[4]

The Nash reaction function of country i of bloc α follows from country i's maximizing its utility (1), subject to its budget constraint

$$c_{\alpha i} + A + G_\alpha = Y_\alpha + (n-1)g_{\alpha j} - mg_\beta + A, \tag{4}$$

while taking national defense of the other countries $g_{\alpha j}, g_\beta$ as given. Here $g_{\alpha j}$ denotes national defense by country j ($\neq$ i) of bloc α and g_β denotes national defense by any identical country of bloc β.

From the first-order condition with respect to G_α, we have

$$A + g_{\alpha i} + (n-1)g_{\alpha j} - mg_\beta = \frac{1}{2}(Y_\alpha + (n-1)g_{\alpha j} - mg_\beta + A). \tag{5}$$

Since all countries of bloc α are identical, we have $g_{\alpha i} = g_{\alpha j}$ at any Nash solution. Hence, substituting $g_{\alpha i} = g_{\alpha j} = g_\alpha$ into the above equation, we obtain the reduced-form reaction function of each country belonging to bloc α,

$$g_\alpha = \frac{1}{n+1}Y_\alpha + \frac{m}{n+1}g_\beta - \frac{1}{1+n}A. \tag{6}$$

g_α is an increasing function of both its own national income and defense spending by the enemy bloc. Similarly, the reaction function of country i of bloc β is

$$g_\beta = \frac{1}{m+1}Y_\beta + \frac{n}{m+1}g_\alpha - \frac{1}{1+m}A. \tag{7}$$

Henceforth, we call country α (or β) the representative country of bloc α (or β). In Figure 1 curve X represents country α's reaction curve, while curve Y represents country β's reaction curve, both of which are upward sloping. An increase in national defense in country α stimulates national defense in country β, and vice versa, so that national defense is a strategic complement, reflecting an arms race between rival blocs. Point N at the intersection of both curves represents the non-cooperative Nash equilibrium.

From equations (6) and (7) the Nash equilibrium levels of national defense for both countries at point N are respectively given as:

$$g_\alpha^N = \frac{2m+1}{1+m+n}(Y-A), \tag{8-1}$$

$$g_\beta^N = \frac{2n+1}{1+m+n}(Y-A). \tag{8-2}$$

For simplicity we assume henceforth that $Y_\alpha = Y_\beta = Y$. It follows from (8-1) that g_α^N increases with m; when the number of countries of the rival bloc increases, spending on national defense per country also increases. This is because an increase in the number of enemy countries raises the threat to countries of the other bloc, producing a negative income effect as security decreases. However, g_α^N decreases with n; when the number of allied countries increases, spending on national defense of those countries falls, due to the positive income effect from greater spillovers. National security of each country is increasing with the number of allied countries, which is consistent with McGuire (1974).

From equations (8-1) and (8-2), we can solve for $c_\alpha, A+G_\alpha$, $c_\beta, A+G_\beta$. Substituting these values into (1), we have

$$U^\alpha = \frac{[(n-m)Y+(1+2m)A]^2}{(1+m+n)^2},\ U^\beta = \frac{[(m-n)Y+(1+2n)A]^2}{(1+m+n)^2}. \tag{9}$$

Equation (9) means that welfare of each country increases with the number of allied countries, while it decreases with the number of enemy countries, which is intuitively plausible.

ALLIED COOPERATION

Bloc α Cooperates And Bloc β Cooperates

We next consider the case where allied countries may cooperate within each bloc although there is no cooperation (or negotiation) between the two blocs. First of all, let us investigate the case where all countries of bloc α cooperate together and all countries of bloc β cooperate together as well. Consider the joint optimization problem of country α. Adding equation (3) up over all n identical countries and allowing $g_{\alpha i} = g_{\alpha j}$, bloc α's consolidated budget constraint may be written as

$$nc_\alpha + G_\alpha + A = nY_\alpha - mg_\beta + A. \tag{10}$$

Thus country α jointly maximizes its utility (1) subject to the above consolidated budget constraint, taking g_β as given. From the first-order condition with respect to G_α, we then have

$$ng_\alpha - mg_\beta + A = \frac{1}{2}(nY_\alpha + A - mg_\beta),$$

so that the reaction function of country α under cooperation is

$$g_\alpha = \frac{1}{2}Y_\alpha + \frac{m}{2n}g_\beta - \frac{1}{2n}A. \tag{11}$$

Note that if n=1, equation (11) reduces to (6). When n>1, the slope of the reaction function, dg_α / dg_β, is given as m/2n, which is less than the slope of the reaction function in the non-cooperative case, m/(n+1). Namely, when the adversarial bloc β raises defense spending, countries of bloc α would react by spending more in the non-cooperative case than in the cooperative case. Due to the arms race, an increase in g_β induces bloc α to raise g_α. When bloc α cooperates, each member recognizes the positive spillover from the increase in defense spending by the other allied countries. However, when it does not cooperate, each member does not recognize the positive spillover from the allied countries' spending, so that it raises g_α more in response to the increase in g_β. Similarly, the reaction function of country β is

$$g_\beta = \frac{1}{2}Y_\beta + \frac{n}{2m}g_\alpha - \frac{1}{2m}A. \tag{12}$$

In Figure 1 curve S represents the reaction curve of country α when all countries of bloc α cooperate, and curve T represents the reaction curve of country β when all countries of bloc β cooperate. At the intersection of curve S and curve T, denoted by point C, a Nash equilibrium is reached that corresponds to the case where both bloc α and bloc β cooperate. The equilibrium levels of g_α, g_β at point C are respectively given as;

$$g_\alpha^C = \frac{(2n+m)Y - 3A}{3n}, \tag{13-1}$$

$$g_\beta^C = \frac{(n+2m)Y - 3A}{3m}. \tag{13-2}$$

Then, we have

$$U^\alpha = \frac{[(n-m)Y + 3A]^2}{9n},\ U^\beta = \frac{[(m-n)Y + 3A]^2}{9m}. \tag{14}$$

The total marginal rate of substitution of $A + G_\alpha$ with respect to c_α, $n[\partial U^\alpha / \partial(A + G_\alpha)/(\partial U^\alpha / \partial c_\alpha)] = nc_\alpha /(A + G_\alpha)$, equals 1, or the marginal cost of providing the public good, which is nothing but the Samuelson rule. Although the Samuelson rule holds for each bloc, there is no cooperation between the two blocs, so that the cooperative solution attains a second best equilibrium rather than the first best. The theory of second best cautions that utility is not necessarily higher at the cooperative solution than at the non-cooperative solution.

Bloc α Cooperates, While Bloc β Does Not Cooperate

In the case where countries of bloc α cooperate together, while countries of bloc β do not cooperate, country α's reaction curve is given as equation (11), while country β's reaction curve is given as equation (7). Then, the quasi-cooperative equilibrium levels of g_α, g_β at point P in Figure 1 are respectively given as;

$$g_\alpha^P = \frac{(m+n+mn)Y-(2m+1)A}{(m+2)n}, \tag{15-1}$$

$$g_\beta^P = \frac{(n+2)Y-3A}{m+2}. \tag{15-2}$$

We now have

$$U^\alpha = \frac{[(n-m)Y+(2m+1)A]^2}{(m+2)^2 n},\ U^\beta = \frac{[(m-n)Y+3A]^2}{(m+2)^2}, \tag{16}$$

where the Samuelson rule holds for bloc α only.

Bloc α Does Not Cooperate, While Bloc β Cooperates

Finally the case where countries of bloc α do not cooperate, while countries of bloc β cooperate together is a counterpart of the last subsection.

COMPARISON OF FOUR EQUILIBRIA

We are ready to compare four Nash equilibria; (1) both countries α and β do not cooperate at point N, (2) country α cooperates, while country β does not cooperate at point P, (3) country α does not cooperate, while country β cooperates at point Q, and (4) both countries α and β cooperate at point C, respectively.

In order to internalize the positive spillover effect between allies within the same bloc, it would be desirable for countries of the same bloc, say, α, to have a treaty for determining national defense cooperatively. By doing so, national defense of the bloc is stimulated, which benefits all countries of the bloc. We call this the OZ effect for Olson and Zeckhauser. However, countries of the rival bloc β would react by raising their spending on national defense, which would hurt countries of bloc α. We call this the Bruce effect.[5] If the negative spillover effect due to the reaction of the opposing bloc outweighs the positive spillover effect due to cooperation between allies, such cooperation hurts countries of bloc α. This possibility was first pointed by Bruce (1990) in the case of n=2 and m=1.

Table 1 depicts country α's welfare at four Nash equilibria points. Country α is likely better off when it does not cooperate within the bloc if country β does not cooperate. Namely, if n=m, we always have

$$(1+m+n)^2 < (m+2)^2 n.$$

However, if n $\geq$ 10 and m = 2, we have

$$(1+m+n)^2 > (m+2)^2 n,$$

which means that country α gains by cooperating when the number of allied countries is very large.

Table 1 also shows that country α loses by cooperating when country β cooperates if n is less than 4,

$(2+n)^2 < 9n$ if n<4.

However, for n ≥ 5, $(2+n)^2 > 9n$; the countries in one bloc gain by cooperating if the countries in the other bloc cooperate.

Several remarks are in order. First, when the number of countries within the same bloc increases, gains from cooperation would also increase. Thus, if each bloc has a large number of allied countries, we would expect that cooperative behavior improves welfare. We could say that the OZ effect may well dominate the Bruce effect in a world of endogenous threat by adversarial countries when the number of allied countries is relatively large.

Second, it is less likely the case that countries will gain from cooperation if their adversaries do not cooperate. This is because the model predicts that when one bloc raises defense spending by cooperating, countries of the adversarial bloc will react by spending more in the non-cooperative case than in the cooperative case. The arms-race reaction by the enemy bloc to an increase in the rival's defense spending is larger in the non-cooperative case than in the cooperative case since each country in the enemy bloc does not recognize the positive spillovers from allied members' increase in defense spending in the non-cooperative case. Thus, in such a case the Bruce effect may dominate the OZ effect even if the number of allied countries is large.

Third, while the alliance may gain from cooperation when the number of countries is large, this is also the case where the bargaining costs of reaching a cooperative solution are high. Hence in a small alliance, cooperation is likely but welfare reducing, while in a large alliance cooperation is unlikely, but welfare improving.

Finally, during the Cold War both NATO and the Warsaw Pact had many allies. We could say that the Warsaw Pact cooperated due to the strong leadership by USSR. If so, NATO might have large benefits from cooperation.

LEADER-FOLLOWER GAME

We consider the Stackelberg leader-follower game where one bloc, for whatever reason, can make a credible first move. This asymmetry may arise because the bloc is dominant in some sense or has a less flexible environment so that the level of defense spending it chooses is credibly maintained. Without loss of generality, we assume that bloc β acts as a Stackelberg leader.

BLOC α COOPERATES, WHILE BLOC β DOES NOT COOPERATE

We first consider the case where country β is a non-cooperative leader. A country of bloc β acts as a Stackelberg leader against country α but still behaves a Nash competitor with respect to the allies[6]. When country α cooperates, its reaction function is given by equation (11). Then, country i of bloc β non-cooperatively maximizes

$$U^{\beta i} = c_{\beta i}(A + g_{\beta i} + (m-1)g_{\beta j} - \frac{n}{2}Y_\alpha - \frac{1}{2}g_{\beta i} - \frac{m-1}{2}g_{\beta j} + \frac{1}{2}A),$$

subject to equation (3). From the first-order condition with respect to $g_{\beta i}$, we have

$$c_{\beta i} = 2(\frac{3A}{2} + mg_{\beta i} - \frac{n}{2}Y_{\alpha} - \frac{m}{2}g_{\beta i}),$$

which substituted into equation (3) gives

$$g_{\beta} = \frac{1}{1+m}Y_{\beta} - \frac{3}{1+m}A + \frac{n}{1+m}Y_{\alpha}, \qquad (17\text{-}1)$$

the first-stage response function of country β. Substituting equation (17-1) into (11), we also have country α's reduced form reaction function at the second stage;

$$g_{\alpha} = \frac{2m+1}{2(1+m)}Y_{\alpha} + \frac{m}{2n(1+m)}Y_{\beta} - \frac{1+4m}{2n(1+m)}A. \qquad (17\text{-}2)$$

When $Y_{\alpha} = Y_{\beta} = Y$, as the outcome in the leader-follower game, we have

$$U^{\alpha} = \frac{[(n-m)Y + (1+4m)A]^2}{4n^2(m+1)^2}, \; U^{\beta} = \frac{[(m-n)Y + 3A]^2}{2(1+m)^2}. \qquad (18)$$

The marginal rate of substitution of $A + G_{\beta}$ with respect to c_{β} now equals 2 in the leader-follower game, not 1 as in the static game. Since the leader country β now recognizes the negative reaction of the follower country α, the effective marginal cost of providing the public good rises for country β from 1 to 2.

BLOC α COOPERATES AND BLOC β COOPERATES

In the case where countries within bloc β cooperate in the first stage of the game, country β jointly maximizes

$$U^{\beta} = c_{\beta}(A + mg_{\beta} - \frac{n}{2}Y_{\alpha} - \frac{m}{2}g_{\beta} + \frac{A}{2}),$$

subject to equation (3). Considering the first-order condition with respect to

g_β, the reaction function of country β is

$$g_\beta = \frac{n}{2m}Y_\alpha + \frac{1}{2}Y_\beta - \frac{3}{2m}A. \tag{19-1}$$

Substituting equation (19-1) into (11), we have

$$g_\alpha = \frac{3}{4}Y_\alpha + \frac{m}{4n}Y_\beta - \frac{5}{4n}A, \tag{19-2}$$

at the second stage of the game. When $Y_\alpha = Y_\beta = Y$, the outcome in the leader-follower game is

$$U^\alpha = \frac{[(n-m)Y + 5A]^2}{16n},\ U^\beta = \frac{[(m-n)Y + 3A]^2}{8m}. \tag{20}$$

BLOC α DOES NOT COOPERATE AND BLOC β DOES NOT COOPERATE

When the follower bloc does not cooperate at the second stage of the game, the reaction function of country α is given as equation (6). Thus country i of bloc β non-cooperatively maximizes

$$U^{\beta i} = c_{\beta i}(A + g_{\beta i} + (m-1)g_{\beta j} - \frac{n}{n+1}Y_\alpha + \frac{1}{1+n}A - \frac{n}{n+1}g_{\beta i} + \frac{(m-1)n}{n+1}g_{\beta i}),$$

subject to equation (3). Considering the first-order condition with respect to $g_{\beta i}$, the reaction function of country β at the first stage of the game is

$$g_\beta = \frac{n}{1+m}Y_\alpha + \frac{1}{1+m}Y_\beta - \frac{2n+1}{1+m}A. \tag{21-1}$$

Substituting equation (21-1) into (6), we have

$$g_\alpha = \frac{mn+m+1}{(1+n)(1+m)}Y_\alpha + \frac{m}{(1+n)(m+1)}Y_\beta - \frac{(2n+1)m+1+m}{(1+n)(1+m)}A, \tag{21-2}$$

at the second stage of the game.

When $Y_\alpha = Y_\beta = Y$, the outcome in the leader-follower game consists of

$$U^{\alpha} = \frac{[(n-m)Y + (2mn+2m+1)A]^2}{[(1+n)(1+m)]^2}, \; U^{\beta} = \frac{[(m-n)Y + (2n+1)A]^2}{(1+n)(1+m)^2}. \quad (22)$$

BLOC α DOES NOT COOPERATE, WHILE BLOC β COOPERATES

In this case country β jointly maximizes

$$U^{\beta} = c_{\beta}(A + mg_{\beta} - \frac{n}{n+1}Y_{\alpha} + \frac{n}{n+1}A - \frac{nm}{n+1}g_{\beta}),$$

subject to equation (3). Considering the first-order condition with respect to g_{β}, the reaction function of country β is

$$g_{\beta} = \frac{n}{2m}Y_{\alpha} + \frac{1}{2}Y_{\beta} - \frac{2n+1}{2m}A. \quad (23\text{-}1)$$

Substituting equation (23-1) into (6), we have

$$g_{\alpha} = \frac{2+n}{2(n+1)}Y_{\alpha} + \frac{m}{2(n+1)}Y_{\beta} - \frac{2n+3}{2(n+1)}A, \quad (23\text{-}2)$$

at the second stage of the game.

When $Y_{\alpha} = Y_{\beta} = Y$, the outcome in the leader-follower game consists of

$$U^{\alpha} = \frac{[(n-m)Y + (2n+3)A]^2}{[2(1+n)]^2}, \; U^{\beta} = \frac{[(m-n)Y + (2n+1)A]^2}{4(1+n)m}. \quad (24)$$

COMPARISON OF FOUR EQUILIBRIA

Table 2 summarizes outcomes of the leader bloc β and the follower bloc α for n=m and A=1 in the leader-follower game[7]. The cooperative behavior in the leader bloc always provides a gain:

$$\frac{(2n+1)^2}{(1+n)^3} < \frac{(2n+1)^2}{4(1+n)n}, \quad \frac{9}{2(1+n)^2} < \frac{9}{8n}.$$

The intuition is as follows. When the leader bloc cooperates, it enjoys the positive spillover effect from increased spending on national defense by allied countries, while it suffers from the negative income effect from increased defense spending by enemy countries. The leader country may set its defense spending less than in the static game since it anticipates the reaction of the enemy bloc. By doing so, the leader bloc can choose the spending level in such a way that the positive spillover effect would outweigh the negative income effect; the OZ effect dominates the Bruce effect.

However, non-cooperative behavior in the follower bloc also provides a gain since

$$\frac{(2n^2+2n+1)^2}{(1+n)^4} > \frac{(4n+1)^2}{(1+n)^2 4n^2}, \quad \frac{(2n+3)^2}{4(1+n)^2} > \frac{25}{16n}.$$

The intuition is as follows. Since the leader bloc recognizes the response of the follower bloc, its national defense is larger when the follower bloc cooperates than when the follower bloc does not cooperate. Thus, the follower bloc receives greater negative spillovers when it cooperates than when it does not cooperate. The Bruce effect dominates the OZ effect.

We could say that the Warsaw Pact was the leader and NATO was the follower during the Cold War. If so, the Warsaw Pact might gain by cooperating and NATO might also gain by non-cooperating.

CONCLUSION

This paper has investigated the implications of non-cooperative and cooperative spending on defense expenditures of allied countries of the two rival blocs using static and leader-follower game models of arms races. It is well known that in the three-country world with two allies and an adversary all countries may be better off when the allies do not cooperate than they do.

By incorporating multi-countries into two opposing blocs respectively, we have shown that if the number of allied countries is large, the cooperative behavior may well attain a better outcome although the negative spillover from the rival bloc is high; the countries in one bloc gain by cooperating. Furthermore, in a leader-follower game cooperative behavior of the leader bloc will gain. The OZ effect may well be valid in several cases in the world of endogenous threat by adversarial countries. We have also shown that countries will likely lose from cooperation if their adversaries do not cooperate. In a leader-follower game the follower bloc will likely lose by cooperating. Thus, the Bruce effect is also important when adversaries' reactions are included.

It has been assumed that all countries in the same bloc behave in the same way. The analysis could be generalized to allow for partial cooperation among allied countries. The cost of cooperating has not been included in the model. And the cost of organizing the coalition is probably increasing in the number of coalition members. It would be useful to model the cost of cooperating explicitly. It will be also useful to investigate the impact of allied cooperation on arms races in a dynamic setting.

Notes:

1. See also Sandler (1977) and Kemp (1984) among others. They highlight the importance of allied cooperation in setting defense spending.

2. Although the weighted-sum technology is common in the literature, other formulations such as weakest-link and best-shot could be useful. See McGuire (1990) and Sandler (1998), among others.

3. Ihori (1992) considers the general case of adversarial relations with respect to ε.

4. As discussed in detail by Bergstrom, Blume and Varian (1986), Andreoni (1988), and Bruce (1990), a non-negativity constraint on providing public goods may well be binding as a solution if the number of enemy countries becomes large. In order to present the results in the simplest form possible, we only consider the case where all countries spend a positive amount on defense.

5. The assumption of $\varepsilon_{ij} = -1 (i \neq j)$ is the strongest case for the Bruce effect. Furthermore, the utility function (1) implies that the marginal propensity to consume the public good is 0.5, which is very high. This also raises the magnitude of the Bruce effect.

6. In this formulation it is assumed that the leader can credibly commit itself to its defense spending against the follower bloc and anticipate its response. The leader still takes as given defense spending of the allied countries. In this sense, the Stackelberg process is partial. Hayashi (2000) uses the similar concept.

7. The qualitative results are almost the same even if n $\neq$ m. Namely, we still obtain the result that the leader bloc gains by cooperating. However, the follower bloc might not lose by cooperating although such a case is unlikely to occur.

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Table 1: Welfare of country α in the static game

		country β	
		N	C
country α	N	$\frac{[(n-m)Y+(1+2m)A]^2}{(1+m+n)^2}$	$\frac{[(n-m)Y+3A]^2}{(n+2)^2}$
	C	$\frac{[(n-m)Y+(2m+1)A]^2}{(m+2)^2 n}$	$\frac{[(n-m)Y+3A]^2}{9n}$

N means non-cooperation and C means cooperation.

Table 2: Outcomes in the leader-follower game

		country β	
		N	C
country α	N	$\frac{(2n^2+2n+1)^2}{(1+n)^4}, \frac{(2n+1)^2}{(1+n)^3}$	$\frac{(2n+3)^2}{4(1+n)^2}, \frac{(2n+1)^2}{4(1+n)n}$
	C	$\frac{(4n+1)^2}{(1+n)^2 4n^2}, \frac{9}{2(1+n)^2}$	$\frac{25}{16n}, \frac{9}{8n}$

N means non-cooperation and C means cooperation.

Country β is a leader and country α is a follower.

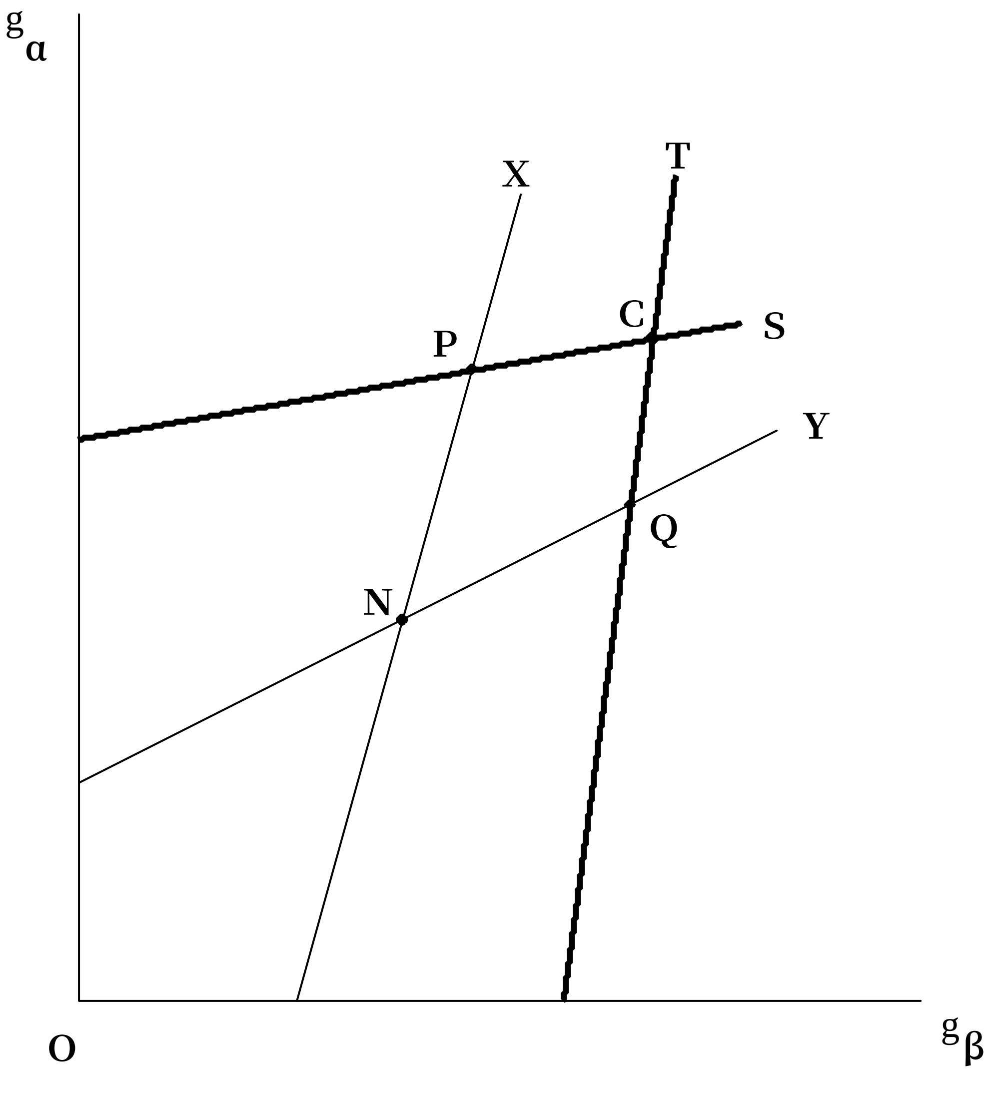

Figure 1 Four Nash Equilibria