From Shirtsleeves to Shirtsleeves in a Long Lifetime

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Abstract

This paper is a quantitatively-oriented theoretical study into the interaction between housing prices, aggregate production, and household behaviour over a lifetime. We develop an overlapping generations model of a production economy in which land and capital are combined into residential and commercial structures. We find that, in the economy where land is more important for structures, the housing price is more sensitive to changes in technology; and, in the steady state, households face a higher house price-rental ratio and buy houses later in life. On the other hand, relaxing collateral constraints has a limited impact on housing prices and aggregate production in the transition to a new steady state, even though it encourages households to buy houses earlier in life, resulting in first order welfare gains.

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1 Introduction

Over the last few decades, we observe considerable fluctuations of real estate value and aggregate economic activities in some economies. In Japan, both the real capital gains on land during the decade of 1980s and the losses during 1990s are in the order of multiple years worth of GDP for the respective periods. Recent increases in housing prices in many developed countries (except in Japan and Germany) raise concerns. To what extent are these housing booms consistent with fundamental conditions? Is the Japanese 1990s-style recession a possible outcome? In this paper, we develop a theoretical framework to investigate how the housing price and aggregate production react to changes in technology and financial conditions. At the same time, we would like to use the same framework to explore the life-cycle of home ownership and consumption. By doing so, we can examine which groups of households are most affected by changes in macroeconomic fundamentals. The life-cycle framework also helps us in explaining the difference in home-ownership rates across countries and time.

One unique aspect of housing (and real estates more generally) arises from the fact that land (or location) is an important input for building residential and commercial structures. Because the supply of land is limited, the supply of structures does not grow as fast as final output with steady growth of technology and population, causing an upward trend in the real rental price and the purchase price of real estate. Because per capita land supply decreases with population growth, land scarcity becomes severer with population growth.

Another unique aspect of real estate is incomplete contract enforcement. Often, landlords are afraid that the tenant may modify (or depreciate) the property against their interests. Even if the modifications are beneficial, disputes may arise over splitting the costs. In order to mitigate these problems, landlords restrict tenant freedom and the tenant enjoys lower utility from renting the house compared to owning the same house. If there were no other frictions, then the household would buy the house straight away. The household, however, may face a financing constraint, because the creditor is afraid that the borrower may default. The creditor demands the borrower to put his house as collateral for a loan and asks him to provide a downpayment from his own net worth.

In this paper, we take the importance of land for structures, the tightness of collateral constraints and the loss of utility from rented housing as exogenous parameters, and examine how these parameters affect household life-cycle choices, prices and aggregate quantities. For this purpose, we develop a simple overlapping generations model of a production economy in which land and capital are combined into residential and commercial structures. We are also interested in the way households cope with idiosyncratic and uninsurable shocks to their labour productivity and wage income.

The interaction between the collateral constraint and the loss of utility from renting a house turns out to generate a typical pattern of consumption and housing over the life cycle. When the household is born without inheritance, it cannot afford a sufficiently high downpayment for buying a house. Thus the household rents and consumes modestly to save for a downpayment. When some net worth has been accumulated, the household buys a house subject to the collateral constraint, which is smaller than a house that would be bought without the collateral constraint. As net worth rises, households upgrade along the housing ladder with the collateral constraint continuing to be binding. At some stage, it becomes better to start repaying the debt rather than maximizing the size of the house, — the collateral constraint is no longer binding. When the time comes for retirement, possibly with idiosyncratic risk attached, the household moves to a smaller house, anticipating a lower income in the future.

Because people tend to save substantially in order to cope with the downpayment requirement and uninsurable shocks to their wage income, there is relatively abundant capital stock in our economy. Thus the rate of return tends to be low relative to the time preference and economic growth rate in equilibrium. Then, after retirement, people tend not to save enough to keep up with economic growth, slowly shrinking their assets relative to the average wage of the working population, if they live long enough. Thus comes our title: "From Shirtsleeves to Shirtsleeves in a Long Lifetime".

In equilibrium, the more important land is for structures, the higher is the expected growth rate of the rental price and the higher is the house price-rental ratio. (The price-rental ratio is an increasing function of the importance of land, also because the effective depreciation rate of structures decreases as land becomes more important for structure since land does not depreciate). This is true for a country like Japan or a metropolitan area. In such an economy, the household needs a larger downpayment relative to wage income in order to buy a house, and tends to buy a house later in life, resulting in lower home ownership rates. In an economy where land is more important for structures, we find the housing price is more sensitive to changes in the level and the expected growth rate of labour productivity.

We also examine the effect of an improvement of the financial system. When households can borrow a larger fraction of the cost associated with purchasing a house under a better financial system, the households tend to buy houses earlier in the life-cycle, and take longer in moving up the housing ladder. The more relaxed collateral constraint leads to higher home-ownership rates with the first order welfare gains because a smaller fraction of households suffer from the utility loss of renting. We find, however, that relaxing the collateral constraint has limited effects on prices and aggregate resource allocation during the transition to a new steady state, because most of the adjustment is achieved through the conversion of houses from being rented to being owned.

Theoretically our work broadly follows two strands of the literature. One is the literature on consumption and saving of a household facing an idiosyncratic and uninsurable earnings shock and a borrowing constraint, which includes Bewley (1977), Deaton (1991), Carroll (1997), Attanasio et. al. (1999) and Gourinchas and Parker (2002). Huggett (1993), Aiyagari (1994), den Haan (1994), and Krusell and Smith (1997, 1998)) have examined the general equilibrium implications of such models. The second strand is the literature on the investment behavior of firms under liquidity constraints. In particular, Kiyotaki and Moore (1997) is closely related since they study the dynamic interaction between asset prices, collateral value, credit limits and aggregate economic activity for an economy in which entrepreneurs face collateral constraints. When many households borrow substantially against their collateral assets (houses), consume, and move up and down along the housing ladder, these households are more like small businesses rather than simple consumers.

Our attention to housing collateral is in line with substantial micro-level evidence in the UK (Campbell and Cocco (2004)) and the US (Hurst and Stafford (2004)) which suggests that dwellings are an important source of collateral for households. Given the importance of home equity in the household portfolio and the empirical connection between housing prices, home equity and aggregate consumption, there has been substantial research on building models that capture these relationships, either with a representative agent (Aoki et. al. (2004), Davis and Heathcote (2005) and Iacoviello (2005)), or with heterogeneous agents (Ortalo-Magne and Rady (2006), Fernandez-Villaverde and Krueger (2001, 2006), Chambers, Garriga and Schlagenhauf (2004) and Rios-Rull and Sanchez (2005)). Distinguishing features of our analysis include an investigation of the interaction between household life-cycle choices and the aggregate economy, and an explicit account of the role of land as a limiting factor in a general equilibrium production economy.

Our paper is organized as follows. Section 2 sets out the model and section 3 presents long-run observations of US aggregate economy and home-ownership rates. Section 4 investigates the individual and aggregate predictions of the model using calibration, and Section 5 concludes.

2 The Model

2.1 Framework

We consider an economy with homogeneous product and labour, and homogeneous reproducible capital stock and non-reproducible land. Capital and land are combined for structures. The structures are fully furnished or equipped, and can be used as houses or productive structures (such as offices and factories) interchangeably. There is a continuum of heterogeneous households of population size $\overline{N}_t$ in period t, a representative firm and a representative foreigner.

The representative firm has a constant returns to scale production technology to produce output (Y_t) from labour (N_t) and productive structures (Z_{Yt}) as:

$$Y_t = F(A_t N_t, Z_{Yt}) = (A_t N_t)^{1-\eta} Z_{Yt}^{\eta}, \quad 0 < \eta < 1, \tag{1}$$

where A_t is aggregate labour productivity which grows at a constant rate, $A_{t+1}/A_t = G_A$. Structures (Z_t) are produced according to a constant returns to scale production function using aggregate capital (K_t) and land (L), and become either a productive structure or a house:

$$\begin{aligned} Z_t &= L^{1-\gamma} K_t^{\gamma}, \quad 0 < \gamma < 1, \\ &= Z_{Yt} + \int_0^{\overline{N_t}} h_t(i) di, \end{aligned} \tag{2}$$

where $h_t(i)$ is housing used by household i in period t. Without loss of generality, we normalize the aggregate supply of land L to be unity. The capital stock depreciates at a constant rate $1 - \lambda \in (0, 1)$ every period, but can be accumulated through investment of goods (I_t) as:

$$K_t = \lambda K_{t-1} + I_t \tag{3}$$

Structures built this period can be used immediately.

Households are heterogeneous in labour productivity, and can have either low productivity, medium productivity, high productivity, or be retired. Every period, there is a flow of new households born with low productivity without any inheritance of the asset. Each low productivity household may switch to medium productivity in the next period with a constant probability δ^l. Each medium productivity household has a constant probability δ^m to become a high productivity one in the next period. Once a household has switched to high productivity it remains at this high productivity until retirement. All the households with low, medium and high productivity are called *workers*, and the flow of new born workers is $G_N - \omega$ fraction of the workforce in the previous period, where $G_N > \omega > \delta^i$ for $i = l, m$. All the workers have a constant probability $1 - \omega \in (0, 1)$ of retiring next period. Once retired, each household has constant probability $1 - \sigma \in (0, 1)$ of dying before the next period. (In other words, a worker continues to work with probability ω, and a retiree survives with probability σ in the next period). The productivity level of the individual household is private information (so that the low productivity household can pretend to be retired, for example). All the transitions are i.i.d. across a continuum of households and over time, and thus there is no aggregate uncertainty on the distribution of individual labour productivity. Let N_t^l, N_t^m and N_t^h be populations of low, medium and high productive workers, and let N_t^r

be population retired households in period t. Then, we have:

$$\begin{aligned}
N_t^l &= (G_N - \omega)\left(N_{t-1}^l + N_{t-1}^m + N_{t-1}^h\right) + (\omega - \delta^l)N_{t-1}^l \\
N_t^m &= \delta^l N_{t-1}^l + (\omega - \delta^m)N_{t-1}^m \\
N_t^h &= \delta^m N_{t-1}^m + \omega N_{t-1}^h \\
N_t^r &= (1-\omega)(N_{t-1}^l + N_{t-1}^m + N_{t-1}^h) + \sigma N_{t-1}^r
\end{aligned}$$

Solving these for the steady state growth of populations, which we are considering in the following, we get:

$$\begin{aligned}
N_t^l &= \frac{G_N - \omega}{G_N - \omega + \delta^l}(N_t^l + N_t^m + N_t^h) \\
N_t^m &= \frac{\delta^l}{G_N - \omega + \delta^m} N_t^l \\
N_t^h &= \frac{\delta^m}{G_N - \omega} N_t^m \\
N_t^r &= \frac{1-\omega}{G_N - \sigma}(N_t^l + N_t^m + N_t^h) \\
\overline{N}_t &= N_t^l + N_t^m + N_t^h + N_t^r = G_N \overline{N}_{t-1}
\end{aligned}$$

The fraction of low productivity workers is larger, the larger is the entry of new low productivity workers, and the more difficult it is for them to transit to the medium productivity state.[1]

Each household derives utility from the consumption of output (c_t) and housing services (h_t) of rented or owned housing, and suffers disutility from supplying labour (n_t). (We suppress the index of household i when we describe a typical household). We assume the household enjoys smaller utility from a rented house than the owned house of the same size by a factor $\psi \in (0,1)$. This disadvantage of rented housing reflects the tenant's limited freedom to adjust the way the house is modified according to her tastes. Household preferences are

[1] We choose to formulate the life-cycle of household in this stylized way, following Diaz-Gimenez, Prescott, Fitzgerald and Alvarez (1992) and Gertler (1999), because we are mainly interested in the interaction between the life-cycles of households and the aggregate economy. The three levels of wage income give us enough flexibility to mimic a typical life-cycle of wage income for our aggregate analysis.

given by the expected discounted utility as:

$$E_0\left(\sum_{t=0}^{\infty}\beta^t u(c_t - v(n_t,\varepsilon_t),\left[1-\psi I(rent_t)\right]h_t)\right),\ 0<\beta<1, \tag{4}$$

where $I(rent_t)$ is an indicator function which takes the value of unity when the household rents the house in period t and zero when she owns it.[2] Disutility of labour $v(n_t,\varepsilon_t)$ is a convex and increasing function of labour supply in terms of efficiency n_t, and is subject to idiosyncratic shocks to its labour productivity ε_t. The value of ε_t is either high (ε^h), medium (ε^m), low (ε^l), or 0, depending on whether the household has high, medium or low productivity, or is retired, and follows the stationary Markov process described above. $E_0(X_t)$ is the expected value of X_t conditional on survival at date t and conditional on information at date 0. For most of our computation, we choose a particular utility function with inelastic labour supply as:

$$u\left(c_t,h_t\right)=\frac{\left(\left(\frac{c_t-v_t}{\alpha}\right)^{\alpha}\left(\frac{\left[1-\psi I(rent_t)h_t\right]}{1-\alpha}\right)^{1-\alpha}\right)^{1-\rho}}{1-\rho}, \tag{5}$$

and $v_t=0$ if $n_t\leq\varepsilon_t$, (and v_t becomes arbitrarily large if $n_t>\varepsilon_t$). We normalize the labour productivity of the average worker to unity as:

$$N_t^l\varepsilon^l+N_t^m\varepsilon^m+N_t^h\varepsilon^h=N_t^l+N_t^m+N_t^h. \tag{6}$$

The parameter ρ is the coefficient of relative risk aversion (as well as the inverse of the elasticity of intertemporal substitution) and α reflects the share of consumption of goods (rather than housing services) in total expenditure.

We focus on the environment in which there are problems in enforcing contracts and thus there are restrictions on trades in markets. There is no insurance market against the idiosyncratic shock to labour productivity of each household. The only assets that households own and trade are the ownership shares of structures, and the annuity contract upon this share. Because we analyze the economy under the assumption of perfect foresight

[2] We assume that, in order to enjoy full utility of the house, the household must own the entire house used. If the household rents a fraction of house used, then she will not enjoy full utility even for the owned fraction of the house.

about the aggregate states, this restriction on tradeable assets is not important (because all the tradeable assets would earn the same rate of return), except for the case of an unanticipated aggregate shock. Because the production function of structures is constant returns to scale, the shareholder of a structure effectively owns capital and land together and receives the returns as an owner of land and the capital stock. There is no separate market for ownership of capital and land. Each household can live in their own house, which is partly financed by outside equity held by outside creditors. But the outside creditors only provide credit up to a fraction $1-\theta \in [0,1)$ of the house. Thus, in order to own the house and enjoy full utility of an owner-occupied house of size h_t, the household must own share s_t at least as large as:

$$s_t \geqslant \theta h_t. \tag{7}$$

We can think of this constraint as a collateral constraint for a residential mortgage — even though in our economy the mortgage is financed by equity rather than debt[3] —, and we take θ as an exogenous parameter of the collateral constraint. Also, because the tenant household does not have a collateral asset, we assume the tenant cannot borrow (or issue shares):

$$s_t \geq 0. \tag{8}$$

Although we do not attempt to derive these restrictions on market transactions explicitly as the outcome of an optimal contract, the restrictions are broadly consistent with our environment in which agents can default on contracts, misrepresent their labour productivity, and can trade assets anonymously (if they wish).[4] The rented house yields lower utility than the owned house, because landlords try to mitigate the disputes over the house renovations by

[3] Caplin, Chan, Freeman and Tracy (1997) provide arguments that equity is superior to debt for housing finance. Recently, we have observed the rapid development of a market for shares of real estate trust funds in many developed countries. Later, we are going to examine the effects of an unanticipated shock on the economy in which only debt is used for financing in order to compare the effects on the economy with only equity.

[4] Cole and Kocherlakota (2001) show that, if agents can misrepresent their idiosyncratic income and can save privately, the optimal contract is a simple debt contract with a credit limit. For more explicit analysis of optimal contracts with tangible assets as collateral, see Lustig (2004) and Lustig and Van Nieuwerburgh (2005).

limiting tenant freedom. There is no separate market for ownership of land and capital upon it, because people prefer to own land and capital together in order to avoid the complications. The outside creditor asks the home owners to own some fraction of the housing equity to prevent default.

Let w_t be the real wage rate, r_t be the rental price of structures, and q_t be the price of a share of structures of unit size at the beginning of this period (before used in this period). The shareholder of the unit size structure of this period receives rental income r_t this period and gross dividend d_{t+1} in the next period and no payoffs afterwards. (In order to maintain the ownership of the structure, the agent has to buy another share in the next period). The flow-of-funds constraint of the worker is given by:

$$c_t + r_t h_t + q_t s_t = (1-\tau) w_t \varepsilon_t + r_t s_t + d_t s_{t-1}, \tag{9}$$

where τ is a constant tax rate on wage income[5]. The left hand side (LHS) of this equation is consumption, the rental cost of housing, and purchases of shares of structures. The right hand side (RHS) is gross revenue, which is the sum of after tax wage income, the rental income from shares purchased this period, and the gross dividend for the share purchased in the previous period.

For the retiree who only survives until the next period with probability σ, there is a competitive annuity market in which the owner of a unit annuity will receive the gross dividend d_{t+1}/σ if and only if the owner survives, and receive nothing if dead.[6] The retiree also receives the benefit b_t per person from the government, which is financed by the tax

[5] The firm pays uniform payroll tax before paying wages to the workers. The firm observes each worker's labour contribution to its production, but it does not observe whether the worker works elsewhere as well.

[6] When the retiree who owned the house dies, then the house becomes the creditor's – similar to the reverse mortgage.

revenue on wage income of the workers as[7]

$$b_t N_t^r = \tau w_t (N_t^l + N_t^m + N_t^h). \tag{10}$$

Because the productivity of each household is private information and a low productivity worker can pretend to be retired, the viable retirement benefit does not exceed after-tax wage income of the low productivity worker[8], or:

$$b_t / w_t = \tau \frac{G_N - \sigma}{1 - \omega} \leq (1 - \tau) \varepsilon^l.$$

The flow-of-funds constraint for the retiree is

$$c_t + r_t h_t + q_t s_t = b_t + r_t s_t + (d_t / \sigma) s_{t-1}. \tag{11}$$

Each household takes the share from the previous period (s_{t-1}) and the joint process of prices, dividends and idiosyncratic labour productivity shocks $\{w_t, r_t, q_t, d_t, \varepsilon_t\}$ as given, and chooses the plan of consumption of goods and housing, and the shareholding $\{c_t, h_t, s_t\}$ to maximize the expected discounted utility subject to the constraints of flow-of-funds and collateral.

The representative firm takes the wage rate, the rental price of structure and the rate of returns on the share

$$R_t = \frac{d_{t+1}}{q_t - r_t} \tag{12}$$

as given. The firm owns land and capital from the last period, and chooses production plan $\{N_t, Z_{Yt}, Y_t, I_t, K_t\}$ to maximize the value of the firm, i.e., the present value of net cash flow

[7] More generally, if the governement consumes C_t^G and purchases the equity S_t^G, then the flow-of-funds constraint of the government is given by

$$C_t^G + b_t N_t^r + q_t S_t^G = \tau_t w_t N_t + r_t S_t^G + d_t S_{t-1}^G$$

[8] Although the government does not observe the productivity of each household, it observes whether the household works or not, at least with some probability by random monitoring. We assume that the penalty of getting caught for cheating is sufficiently high (say, a prohibition to receive any benefit in the future), so that no worker receives the benefit while working.

from production:

$$V_t = Y_t - w_t N_t + r_t(Z_t - Z_{Yt}) - I_t + \sum_{s=t+1}^{\infty} \frac{1}{R_t R_{t+1} \cdot \cdot R_{s-1}} \left[Y_s - w_s N_s + r_s(Z_s - Z_{Ys}) - I_s \right]$$

subject to the constraints of technology (1), (2) and (3). The net cash flow consists of output net of wage costs as well as net rental income net of investment cost. The gross dividend is defined as the value of the firm per unit of the shares of the structure from the last period:

$$d_t = V_t / Z_{t-1}$$

The representative foreigner purchases goods C_t^* and shares on home structures S_t^* in net (thus both C_t^* and S_t^* can be negative), subject to the international flow-of-funds constraint against home agents as:

$$C_t^* + q_t S_t^* = r_t S_t^* + d_t S_{t-1}^*. \tag{13}$$

The LHS is gross expenditure of foreigners on home goods and shares, which means gross inflow of funds to home agents. The RHS is the gross receipts of foreigners. Although the foreigner maximizes their objective subject to their technological constraint and the flow-of-funds constraint, here we posit the reduced form demand function for home shares of the representative foreigner as an increasing function of the gap between the rate of return on home shares and the rate of return on foreign asset, R_t^*, as:

$$S_t^* = S^*(R_t,\ R_t^*) = \overline{S}^* + \xi(R_t - R_t^*),$$

where $\xi > 0$ is the sensitivity of demand with respect to the gap in the rates of returns, and $\overline{S}^*$ is the parameter which summarizes the other determinants of their demand[9]. One special case is a small open economy in which $\xi \to \infty$, and another special case is a closed economy in which $\overline{S}^* = \xi = 0$.

Given the above choice of many households and a representative firm and a foreigner, the competitive equilibrium of our economy is characterized by the prices $\{w_t, r_t, q_t\}$ which clear the markets for labour, output, the ownership share and the rental of structures as:

$$N_t = \int_0^{\bar{N}_t} n_{it} di = \varepsilon^l N_t^l + \varepsilon^m N_t^m + \varepsilon^h N_t^h = N_t^l + N_t^m + N_t^h, \tag{14}$$

[9] The rates of returns on home and foreign assets can differ under perfect foresight because of the transaction costs.

$$Y_t = \int_0^{\bar{N}_t} c_{it} di + I_t + C_t^*, \tag{15}$$

$$Z_t = \int_0^{\bar{N}_t} s_{it} di + S_t^*. \tag{16}$$

and (2) [10]. Because of Walras' Law, one of these four market clearing conditions is not independent.

2.2 Behavior of Representative Firm

The representative firm chooses a plan of production to maximize the value of the firm. The first order conditions for the maximization are:

$$w_t = (1-\eta) Y_t / N_t \tag{17}$$

$$r_t = \eta Y_t / Z_{Yt} = \eta \left(\frac{N_t'}{f_t} \right)^{1-\eta}, \text{ where } N_t' \equiv A_t N_t \text{ and } f_t \equiv Z_{Yt}/Z_t \tag{18}$$

$$1 - \frac{\lambda}{R_t} = r_t \gamma K_t^{\gamma - 1} = \gamma \eta \left(\frac{N_t'}{f_t} \right)^{1-\eta} K_t^{\gamma\eta - 1} \tag{19}$$

The first two equations are the familiar equality of price and marginal products of factors of production. The value of N_t' is the labour in efficiency unit, and f_t is a fraction of structures used for production. The last equation says that the opportunity cost of holding capital for one period – the cost of capital – should be equal to the marginal value product of capital. Thus we have

$$K_t = \left[\frac{\gamma\eta}{1 - \frac{\lambda}{R_t}} \left(\frac{N_t'}{f_t} \right)^{1-\eta} \right]^{1/(1-\gamma\eta)} \tag{20}$$

$$Y_t = f_t \left[\left(\frac{\gamma\eta}{1 - \frac{\lambda}{R_t}} \right)^{\gamma\eta} \left(\frac{N_t'}{f_t} \right)^{1-\eta} \right]^{1/(1-\gamma\eta)} \tag{21}$$

Because the production function of output is constant returns to scale, there is no profit

[10] The name of individual household i is such that a fraction of new-born housholds named after the names of the deceiced households and the remaining fraction of new-borns are given new names for $i \in \left(\overline{N}_{t-1}, \overline{N}_t\right]$. In this way, the name of households are always destributed unifromely in $\left[0, \overline{N}_t\right]$ at date t.

associated with regular production. The resulting value of the firm is:

$$\begin{aligned} d_t Z_{t-1} &= V_t = r_t Z_t - (K_t - \lambda K_{t-1}) + \frac{1}{R_t}\left[r_{t+1}Z_{t+1} - (K_{t+1} - \lambda K_t)\right] + ... \\ &= \lambda K_{t-1} + \eta(1-\gamma)\left(\frac{Y_t}{f_t} + \frac{1}{R_t}\frac{Y_{t+1}}{f_{t+1}} + \frac{1}{R_t R_{t+1}}\frac{Y_{t+2}}{f_{t+2}} + ...\right) \end{aligned} \tag{22}$$

The second term of the RHS is the value of land, which is proportional to the present value of augmented output $Y_t' = Y_t/f_t$. Thus, the value of the representative firm is equal to the sum of capital stock inherited from the last period and the value of the land, and the shareholders own capital and land indirectly through the equity.

2.3 Household Behavior

The household chooses one among three modes of housing - becoming a tenant, a credit constrained home-owner, and an unconstrained home-owner. The flow-of-funds constraint of the worker and retiree can be rewritten as

$$\begin{aligned} c_t + r_t h_t + (q_t - r_t)s_t &= (1-\tau)w_t\varepsilon_t + d_t s_{t-1} \equiv x_t, \\ c_t + r_t h_t + (q_t - r_t)s_t &= b_t + (d_t/\sigma)s_{t-1} \equiv x_t, \end{aligned}$$

where x_t is the liquid wealth of the household. Liquid wealth is the wealth of the household, excluding illiquid human capital (the expected discounted value of future wage and pension income). We call liquid wealth "net worth" hereafter.

2.3.1 The tenant

The tenant chooses consumption of goods and housing services to maximize the utility, which leads to:

$$c_t : r_t h_t = \alpha : 1 - \alpha$$

Using the flow-of-funds constraint we can express housing and consumption as functions of current expenditure:

$$c_t = \alpha[x_t - (q_t - r_t)s_t]$$

and

$$h_t = \frac{(1-\alpha)\left[x_t - (q_t - r_t)s_t\right]}{r_t}$$

Substituting these into the utility function we get the following indirect utility function:

$$u^T(s_t, x_t; r_t, q_t) = \left(\frac{r_t}{1-\psi}\right)^{(\alpha-1)(1-\rho)} \frac{[x_t - (q_t - r_t)s_t]^{1-\rho}}{1-\rho}$$

Due to the limited freedom from living in a rented house, the tenant effectively faces a higher rental price than the home owner for the same utility, i.e., $[r_t/(1-\psi)]$ rather than r_t.

2.3.2 The constrained home-owner

The constrained home owner faces a binding collateral constraint as:

$$s_t = \theta h_t$$

Thus he consumes $h_t = s_t/\theta$ amount of housing service, and consumes the remaining on goods as:

$$c_t = x_t - \left(q_t - r_t + \frac{r_t}{\theta}\right) s_t$$

The indirect period utility of the constrained home owner is now:

$$u^C(s_t, x_t; r_t, q_t) = \left\{ \left[\frac{x_t - \left(q_t - r_t + \frac{r_t}{\theta}\right) s_t}{\alpha}\right]^{\alpha} \left[\frac{s_t/\theta}{1-\alpha}\right]^{1-\alpha} \right\}^{1-\rho} / (1-\rho)$$

2.3.3 The unconstrained home-owner

The unconstrained home-owner does not face a binding collateral constraint. His intra-temporal choice is identical to the tenant's but he does not suffer from the limited freedom associated with renting a house.

$$u^U(s_t, x_t; r_t, q_t) = r_t^{(\alpha-1)(1-\rho)} \frac{[x_t - (q_t - r_t)s_t]^{1-\rho}}{1-\rho}$$

2.3.4 Value functions

Let $\overline{A}_t$ be the vector of variables characterizing the aggregate state of the economy at the beginning of period t

$$\overline{A}_t = (A_t,\ N_t^l,\ N_t^m, N_t^h,\ N_t^r,\ K_{t-1},\ S_{t-1}^*,\ (s_{t-1}(i))_{i\in[0,\bar{N}]})'.$$

The prices and dividend (w_t, r_t, q_t, d_t) would be a function of this aggregate state in equilibrium. We can express the value functions of the retiree, high, medium and the low productivity worker by $V^r(x_t, \overline{A}_t)$, $V^h(x_t, \overline{A}_t)$, $V^m(x_t, \overline{A}_t)$, and $V^l(x_t, \overline{A}_t)$ as functions of the individual net worth and the aggregate state.

First consider the choice of the retiree. The retiree chooses the mode of housing and an annuity contract on shares, s_t, subject to the flow-of-funds constraint. Then, the retiree's value function satisfies the Bellman equation:

$$\begin{aligned} V^r(x_t, \overline{A}_t) &= Max(\max_{s_t} \left\{ u^T(s_t, x_t; r_t, q_t) + \beta\sigma V^r(b_{t+1} + (d_{t+1}/\sigma)s_t, \overline{A}_{t+1}) \right\}, \\ & \max_{s_t} \left\{ u^C(s_t, x_t; r_t, q_t) + \beta\sigma V^r(b_{t+1} + (d_{t+1}/\sigma)s_t, \overline{A}_{t+1}) \right\}, \\ & \max_{s_t} \left\{ u^U(s_t, x_t; r_t, q_t) + \beta\sigma V^r(b_{t+1} + (d_{t+1}/\sigma)s_t, \overline{A}_{t+1}) \right\}) \end{aligned}$$

Now consider the choice of the worker. The worker chooses whether to own or rent a house, and whether to consume or save to buy the shares. Let us denote $\epsilon^i = \varepsilon^i(1-\tau)$ after tax labour productivity of the worker of type i (high or low). Then the value function of the worker of high productivity satisfies the Bellman equation:

$$V^h(x_t, \overline{A}_t) = Max($$

$$\begin{aligned} &\max_{s_t} \left\{ \begin{array}{c} u^T(s_t, x_t; r_t, q_t) + \beta[\omega V^h(\epsilon^h w_{t+1} + d_{t+1}s_t, \overline{A}_{t+1}) \\ +(1-\omega)V^r(b_{t+1} + d_{t+1}s_t, \overline{A}_{t+1})] \end{array} \right\}, \\ &\max_{s_t} \left\{ \begin{array}{c} u^C(s_t, x_t; r_t, q_t) + \beta[\omega V^h(\epsilon^h w_{t+1} + d_{t+1}s_t, \overline{A}_{t+1}) \\ +(1-\omega)V^r(b_{t+1} + d_{t+1}s_t, \overline{A}_{t+1})] \end{array} \right\}, \\ &\max_{s_t} \left\{ \begin{array}{c} u^U(s_t, x_t; r_t, q_t) + \beta[\omega V^h(\epsilon^h w_{t+1} + d_{t+1}s_t, \overline{A}_{t+1}) \\ +(1-\omega)V^r(b_{t+1} + d_{t+1}s_t, \overline{A}_{t+1})] \end{array} \right\}). \end{aligned}$$

The high productivity worker may retire with probability $1-\omega$ next period, and continues to work with probability ω.

The value function of a medium productivity worker satisfies:

$$V^m(x_t, A_t) = Max($$

$$\max_{s_t}\left\{\begin{array}{c} u^T(s_t,x_t)+\beta[(\omega-\delta^m)V^m(\epsilon^m w_{t+1}+d_{t+1}s_t,A_{t+1}) \\ +\delta^m V^h(\epsilon^h w_{t+1}+d_{t+1}s_t,A_{t+1})+(1-\omega)V^r(b_{t+1}+d_{t+1}s_t,A_{t+1})] \end{array}\right\},$$
$$\max_{s_t}\left\{\begin{array}{c} u^C(s_t,x_t)+\beta[(\omega-\delta^m)V^m(\epsilon^m w_{t+1}+d_{t+1}s_t,A_{t+1}) \\ +\delta^m V^h(\epsilon^h w_{t+1}+d_{t+1}s_t,A_{t+1})+(1-\omega)V^r(b_{t+1}+d_{t+1}s_t,A_{t+1})] \end{array}\right\},$$
$$\max_{s_t}\left\{\begin{array}{c} u^U(s_t,x_t)+\beta[(\omega-\delta^m)V^m(\epsilon^m w_{t+1}+d_{t+1}s_t,A_{t+1}) \\ +\delta^m V^h(\epsilon^h w_{t+1}+d_{t+1}s_t,A_{t+1})+(1-\omega)V^r(b_{t+1}+d_{t+1}s_t,A_{t+1})] \end{array}\right\}).$$

Next period, the medium productivity worker switches to high productivity with probability δ^m, retires with probability $1-\omega$, and remains with medium productivity with probability $\omega-\delta^m$.

The value function of a low productivity worker is similar to the value function of a medium productive worker, except for the fact that superfix m is replaced by l and h is replaced by m. (Remember a low productive worker only may move up to a medium productive worker with probability δ^l).

Growth in the economy with land presents a unique problem for the solution of the individual agent problem because wages grow at different rates from the rental price and the purchase price of structures even in the steady state. This means that we need to transform the non-stationary per capita variables in the model into stationary per capita units. In Appendix B, we describe how to convert the value functions of the household into a stationary representation in the growing economy with scarce land.

2.4 Steady State Growth

Before calibrating, it is useful to examine the property of the steady state growth of our economy. Let $G_X = X_{t+1}/X_t$ be the steady state growth factor of variable X_t. In the following we simply call the growth factor as the "growth rate". In steady state, the growth rate of aggregate output variables should be equal:

$$\frac{Y_{t+1}}{Y_t}=\frac{I_{t+1}}{I_t}=\frac{K_{t+1}}{K_t}=G_Y. \tag{23}$$

The growth rate of structures need not be equal to the growth rate of output, but it should be equal to the growth rate of productive structures:

$$\frac{Z_{t+1}}{Z_t} = \frac{Z_{Yt+1}}{Z_{Yt}} = G_Z. \tag{24}$$

Then, from the production functions, these growth rates depend upon the growth rates of aggregate labour productivity and population as:

$$G_Y = (G_A G_N)^{1-\eta}\ G_Z^{\eta}, \text{ and } G_N = G_Y^{\gamma}.$$

Thus

$$G_Y = (G_A G_N)^{(1-\eta)/(1-\gamma\eta)}$$

$$G_Z = (G_A G_N)^{\gamma(1-\eta)/(1-\gamma\eta)}$$

Because the supply of land is fixed, to the extent that land is an important input for structures, the growth rates of output and structures are both smaller than the growth rate of labour in efficiency units. Moreover, because structures are more directly affected by the limitation of land than output, the growth rate of structures lags behind the growth rate of output.

In the steady state of the competitive economy, we learn that the real rental price and the purchase price of structures rise at the ratio of the growth rate of output and the growth rate of structures to keep the expenditure share of rental expenditure constant under our Cobb Douglas utility function:

$$G_r \equiv \frac{r_{t+1}}{r_t} = \frac{q_{t+1}}{q_t} = \frac{G_Y}{G_Z} = G_Y^{1-\gamma}.$$

Because land is scarce ($\gamma < 1$), the rate of increase of the rental price and the purchase price of structures is an increasing function of the growth rate of workers in efficiency units in steady state. The wage rate grows in the steady state with the same rate as the per capita output as

$$G_w = \frac{G_Y}{G_N} = \left[G_A^{1-\eta} G_N^{-\eta(1-\gamma)}\right]^{1/(1-\gamma\eta)}$$

Because the per capita supply of land decreases with population growth, the growth rate of the wage rate is a decreasing function of the population growth rate.

3 Observations

Here, we gather some observations, which give us some guidance for our calibrations.

3.1 Features of U.S. Economy

Table 1 summarizes the features of the US. economy, relevant for our aggregate economy

Table 1: Long run aggregate features of the U.S. economy

	1900	1939	1958	Average
Reproducible tangible assets/GDP	3.07	3.34	2.92	3.3
Land/GDP	1.61	0.96	0.66	-
Net foreign assets/GDP	-0.12	0.02	0.05	-
Fraction of productive structures	0.47	0.43	0.42	0.53

Notes to Table 1: National wealth is from Raymond Goldsmith, (1962). GDP is from GDP - Millennial Edition Series of Table Ca9-19 of Volume 3 of Carter et. al. (2006). The fraction of productive structures is defined as the ratio of nonfarm nonresidential structures plus producer durables to the sum of nonfarm residential and nonresidential structures and producer and consumer durables. Average refers to the average quarterly estimates between 1952:Q1 and 2005:Q5 for the US economy based on Flow of Funds data (see data appendix for details on the construction of these variables).

We observe that the share of land in total tangible assets (land plus reproducible tangible assets) falls from 34% in 1900 to 18% in 1958. In the United States, the elasticity of substitution between land and reproducible capital in production of fully equipped structures appears to exceed unity, because the share of land decreases as the ratio of prices of land and capital increases. (Roughly speaking, the scarcity of land is relatively easily overcome by using technology with higher capital-land ratio). Thus, our assumption of a Cobb-Douglas production function (equation (2)) is a rough approximation of the production of structures, which is valid only for not very long periods of time.[11] On the other hand, the fraction of

[11] For Japan, Kiyotaki and West (2006) provide evidence that the elasticity of substitution between land and capital is not significantly larger than unity for the period 1961-1995.

productive structures (our Zt_{Yt}/Z_t) shows only mild decline over the long period of time.

3.2 Evolution of home-ownership rates

There exists considerable variation in home ownership rates across countries and over time. *Table* 2 shows the home ownership rates (fraction of households that own houses) of selected developed countries between 1970 and 2003 taken from IMF World Economic Outlook. The table shows a general upward trend in home-ownership rates across countries since 1970.

Table 2: Home ownership rates in %	1970	1980	1990	2003
United States	64.2	65.6	64.0	68.3
Germany	-	41.0	39.0	43.6
Italy	-	59.0	68.0	80.0
United Kingdom	50.0	55.0	66.0	70.0
Japan	-	60.0	61.0	62.0

Notes to Table 2: See Table 2.1 in page 73 of World Economic Outlook (September 2004).

Focussing on the U.S., *Table* 3 shows the evolution of home ownership rates for white and black households for the 1900-1990 period derived from Collins and Margo (2001).

Table 3: U.S. Home-Ownership Rates (in %)						
	1900	1920	1940	1960	1980	1990
whites	48.5	47.1	42.1	64.0	68.6	66.5
blacks	24.1	24.6	20.5	35.8	43.8	40.9

We observe that there is a substantial gap between white and black households, reflecting the difference in their income and access to the credit market. The home ownership rates for both whites and blacks declined during the Great Depression, before increasing after WWII. During the 1980s, average home ownership rate declined, perhaps because of the high nominal and real interest rates[12].

[12] The high nominal interest rate often tightens the credit constraint, because lenders tend to restrict loans to households with a high ratio of mortgage payments to disposable income, and because the payment of traditional fixed interest mortgage in earlier stage increases with a higher nominal interest rate.

4 Calibrations

4.1 Parameters for Calibration

The parameters for the baseline calibration are as in *Table* 4.

Table 4: Parameters for Baseline Calibration
$\eta = 0.258$: share of productive structures in production of output
$\gamma = 0.9$: share of capital in the production of structures
$\lambda = 0.9 : 1 -$ depreciation rate
$\overline{S}^* =$: exogenous foreign demand for domestic shares
$\xi = 20$: elasticity of foreign demand with respect to return gap
$\beta = 0.96$: utility discount factor
$\alpha = 0.75$: share of consumption of goods
$\rho = 2$: coefficient of relative risk aversion
$\psi = 0.09$: fraction of utility loss from renting a house
$\theta = 0.3$: fraction of house that needs downpayment
$\delta^l = 0.08, \ \delta^m = 0.014$: probability of switching to a higher wage
$\varepsilon^l = 0.331, \ \varepsilon^m = 0.663$ and $\varepsilon^h = 2.650$: labour productivities
$\frac{b}{w} = 0.2$: ratio of retirement benefit to pre-tax wages of average worker
$\omega = 0.978$: probability of continuing working
$\sigma = 0.945$: surviving probability
$G_A = 1.02$: labour productivity growth
$G_N = 1.01$: population growth

We consider one period of our model to be roughly one year and think of the baseline economy as the U.S.. The share of productive structures in the production of final output (η) is a bit lower than the one used in other studies because the theoretical model includes explicitly housing tangible assets. Consistent with the Cooley and Prescott (1995) methodology of aligning the data to their theoretical counterparts, Appendix D outlines how the U.S. Flow of Funds and NIPA data for the period 1952:Q1 to 2005:Q5 are used to derive an estimate for η. A key parameter in our model is the importance of land in the production of structures $(1 - \gamma)$. When γ approaches 1, land plays a very limited role in the model (land is plentiful). A higher γ therefore captures a state like Nebraska instead of a city like New York or a country like the U.S. instead of a country like Japan. This parameter will be the key parameter we will be changing when performing comparative statics results across countries. Thinking of the U.S. economy as our baseline, we set $\gamma = 0.9$ since Haughwout and Inman (2001) calculate the share of land in capital income between 1987 and 2005 to be about 10.9% while Heathcote and Morris (2004) also use $\gamma = 0.9$.

The depreciation rate $(1 - \lambda)$ is set at 10 percent per annum, while the annual discount factor is set at 0.96 and the coefficient of relative risk aversion at 2, all standard parameter choices. The parameters determining the foreign demand for domestic shares $\overline{S}^{*}$is chosen to generate net foreign liability (around 20% of GDP), which implies current account surplus of around 0.9% of GDP in the steady state. The utility loss from renting a house is set to generate reasonable implications for aggregate home-ownership/tenant rates: a small value for ψ at around 0.09 worked well. The fraction of a house that needs a downpayment (θ) is set at 30% but we perform extensive comparative statics relative to this parameter since one of our goals is to better understand the role of collateral constraints on home-ownership rates, house prices and allocation. The probability (δ^l, δ^m) of switching earnings states is set so that population ratio of low, medium and high productive workers is approximately equal to 30%, 50%, and 20%. The probability of continuing to work (ω) is set so that the expected duration of working life is 45.5 years, while the conditional probability of surviving (σ) implies an expected retirement duration of 18.2 years. The replacement ratio (b) implies that the ratio of the government retirement benefit to the after-tax wage is equal to $b/\left[(1-\tau)\,\varepsilon^l\right] = 0.647$ for a low productive worker, and is equal to $b/\left[(1-\tau)\,\varepsilon^h\right] = 0.081$ for a high productive worker. Thus, the retirement benefit is roughly equal to the two-third of after-tax earnings of the low-wage worker, while it is about one-twelves of the after-tax wage of the highly productive worker, generating the intended redistribution of the pension system. Labour productivity (G_A) and population growth (G_N) are set to two and one percent, respectively.

4.2 General Features of Household Behavior

The household chooses present consumption, saving, and mode of housing, taking into account its net worth – the result of past saving – and its expectations of future income. *Figure* 1*A* illustrates the consumption of goods, housing services and the mode of housing of the worker with low productivity as a function of net worth. In order to explore the stable relationship between the household choice and the state variable, we detrend all the variables using their own theoretical trend as in *Appendix B*. When the worker does not have much net worth, $x < x_{1l}$, he does not have enough to pay for a downpayment of even a tiny house. He chooses to rent a modest house and consume a modest amount. Hoping to become more productive in the future, the low productivity worker hardly saves. *Figure* 1*B* shows the transition of the share-holdings for the low productivity worker. The locus $s' = s(s, q, yl)$ shows the share-holding at the end of present period as the function of the share-holding of the end of the last period for the low productivity worker. Everyone enters the labour market with low productivity and no inheritance $s_0 = 0$. As long as the worker continues to be with low productivity, he does not save, and continues to live in a rented house.[13]

Figure 2*A* shows the choice of a worker in the medium productivity state. When she does not have much net worth to pay for a downpayment to buy a house, $x < x_{1m}$, she chooses to rent a place, a similar behaviour with the low productivity worker. The main difference is that the medium productivity worker saves vigorously to accumulate the downpayment to buy a house in the future. In *Figure* 2*B*, the $s' = s(s, q, ym)$ locus (the transition of share-holdings of the high productive worker from this to the next period) lies above the 45-degree line for $s < sm^*$, so that the shareholding at the end of this period is larger than at the end of the last period. When the medium productivity worker accumulates modest net

[13]No saving by low productivity worker is not always true for an economy with different parameters. If the income gap between low productive and more productive workers is not so large and/or the transition probability from low to more productive states is not so high, then the low productive worker saves to accumulate net worth in order to buy a house. The low income worker also would save if the pension is very limited. If the low productivity households had substantial inheritance, then they would decumulate the share-holding until $s = 0.12$ at the intersection between $s(s, q, yl)$ locus and the 45-degree line.

worth, $x \in [x_{1m}, x_{2m}]$ in *Figure* 2*A*, she buys her own house subject to the binding collateral constraint. The size of the house at net worth $x = x_{1m}$ is smaller than the house rented at net worth slightly below x_{1m}, because she can only afford to pay downpayment on a smaller house. (Nonetheless, she is happier than before, because she derives more utility from the owned home than a rented place). Consumption is lower too, because she tries to mitigate the collateral constraint by saving vigorously. For $x \in [x_{1m}, x_{2m}]$, the size of an owned house is a sharply increasing function of net worth, because the worker maximizes the size of the house subject to the downpayment constraint. When the medium productive worker has substantial net worth $x > x_{2m}$, she becomes an unconstrained home owner, using her saving partly to repay the debt (or increase the housing equity ownership). In *Figure* 2*B*, the medium productivity worker continues to accumulate her shareholding along $s' = s(s, q, ym)$ until she reaches the level of shareholding at sm^*, the intersection of $s(s, q, ym)$ and the 45-degree line.

The behavior of the high productivity worker is similar to the medium productivity one, except that she accumulates more shares: $s' = s(s, q, yh)$ lies above $s' = s(s, q, ym)$ and her converging share-holding sh^* is larger than that of medium productive worker sm^*. Therefore, the shareholding of all the workers is distributed in $s \in [0, sh^*]$, with mass of workers at both $s = 0$, $s = sm^*$ and $s = sh^*$.

Figure 3*A* illustrates the consumption and housing choices of the retiree. Because pension income and the probability of death are the same for every retiree (by assumption), he consumes goods and housing services as a function of only net worth. *Figure* 3*B* illustrates the transition of share-holding of the retiree. Because in our economy, the productive workers have strong incentives to save for retirement and mitigate the collateral constraint, the equilibrium level of capital stock and structures tend to be fairly large. Then, for a large set of parameters, the rate of return on share-holding (in terms of utility) is not high relative to the time preference rate, taking into account the effect of growth. (Note that the real rate of return should be sufficiently higher than the time preference rate in a growing economy for the retiree to maintain their relative shareholding). Thus, the transition of share-holding of the retirees, the locus $s' = s(s, q, b)$, lies below the 45-degree line for $s > sr^*$. Thus the retiree slowly decreases his share-holding along the locus $s(s, q, b)$ until $s = sr^*$. The relative decumulation of shareholding of the retiree stops at $s = sr^*$, the threshold for him to become a constrained home owner, and his holding stays at sr^* afterwards.[14]

Putting together these arguments, we can draw a picture of a typical life-cycle. *Figure* 4 illustrates a typical life for a household. The horizontal axis is age, and the vertical axis measures housing consumption (h) and share-holding (s). Starting from no inheritance, he chooses to live in a rented house without saving during the young and low wage periods until the $19th$ year. When he becomes a medium productive wage worker at the $20th$ year, he starts saving vigorously. Quickly, he buys a house subject to the collateral constraint. Then he moves up fast the housing ladder to become a unconstrained home owner at the $23th$ year.[15] Afterwards, he starts increasing the fraction of his own equity of the house (similar to repaying the debt), instead of moving to the maximum size house within the collateral constraint. By the time of retirement, he has repaid all the mortgage and has accumulated shares more than the value of the his own house. (Remember that the aggregate share-holding of structures of all the households is the sum of all the houses and productive structures in equilibrium). When the worker hits the wall of retirement (with the arrival of

[14] In the Baseline economy, there is a small population of the retirees who had never been highly productive during the working period and thus retired without the net worth. Because the low productive workers give up hope to become a productive worker at the time of retirement and their pension is not much lower than the thier after-tax wage income in Baseline economy, they actually save to become a constrained home owner by accumulating shareholding along the locus of $s' = s(s, q, b)$ (which lies above 45-degree line for $s < sr^*$).

This behavior will disappear in an economy in which there are sufficient incentives for low wage workers to save (because of a small pension, for an example). In such an economy, the equilibrium real interest rate is low and the retiree's sharholding rule $s' = s(s, q, b)$ lies below 45-degree line for all $s > 0$. Then, the retiree becomes a constrained home owner and then becomes a tenant as he gets older. Eventually, the shareholding of the retiree will stop when he eats up all the shares at point $s = sr^* = 0$. After that, the retiree will rely entirely on the benefit to pay for rent and consumption. Then the retiree goes back to a real shirtsleeves again if he lives long enough.

[15] The worker moves to a bigger house every period in our model because there are no transaction costs. With a transaction cost, the worker moves infrequently, and changes housing consumption by discrete amounts, rather than continuously. The housing ladder would become a true ladder, instead of having a continual upward slope. He may even buy first a larger house than the house rented before, anticipating the transaction cost. But the basic features remain the same.

a retirement shock) at the $51th$ year, his permanent income drops, and he moves to a smaller house. He also sells all the shares to buy an annuity contract on the shares, because the annuity earns the gross rate of return which is $(1/\sigma) > 1$ times as much as the straightforward share-holding, (as long as he survives with probability σ). But his effective utility discount factor shrinks by a factor σ too. Thus as the rate of return on the annuity is not sufficiently high (relative to the effective time preference rate) to induce the retiree to save enough, he decumulates slowly the relative share-holding, downsizing his consumption of goods and housing services as he gets older relative to the working populations. When he dies, his assets drop to zero, according to the annuity contract (which pays zero if the contract holder dies).

4.3 Comparison of Steady States

4.3.1 Large Open Economy

We present our results of the baseline calibration in *Table* 5. In the baseline calibration the fraction of tenants in the population is about 25%, which is substantial but a bit lower than the number from Collins and Margo (2001). The fraction of constrained home owners is 8.3%. The fraction of houses lived in by tenants and constrained home owners are smaller than the fraction of their population, because they live in smaller houses than the unconstrained home owner on average. The average size of a tenant's house is about 34% of the average house size of unconstrained home owners, and the average house size of constrained home owners is about 22% that of unconstrained home owners. The tenants and the constrained home owners live in smaller houses than the unconstrained home owners, mainly because the former have lower permanent income. The constrained home owners, in addition, tend to choose smaller housing and larger saving in order to meet the collateral constraint. The distribution of share-holding is even more unequal among the groups of households in different modes of housing. The fraction of total shares held by tenants is negligible (0.08%), the fraction of total shares held by constrained home owners is 0.33% and the remainder is held by unconstrained home owners. This is consistent with the conventional wisdom that the distribution of wealth is much more skewed than the distribution of income. Perhaps a new insight would be that, when the distribution of wealth and income are difficult to observe, we can infer inequality by looking at the home ownership rates across different groups of people, as Collins and Margo (2001) do.

Turning to prices and aggregate variables, the gross rate of return on share-holding is 1.066 in terms of goods, and is equal to $1.066 \div G_r^{1-\alpha} = 1.065$ in terms of consumption basket. The latter is smaller than the inverse of the time preference rate, which, adjusted for growth effects, equals $(1/\beta)\left(G_w/G_r^{1-\alpha}\right)^{\rho} = 1.080$. This is not because people are impatient, but because people tend to save substantially during the working period in order to cope with the labour productivity and retirement shocks and to mitigate the collateral constraint. Many general equilibrium models with uninsurable idiosyncratic risk have such a feature, including Aiyagari (1994). Even though some aggregate variables are not the same as the numbers in *Table* 3, they are broadly consistent with the main features of the US economy. The ratio of average housing value to the average wage is 2.5 years, while the housing price to rental ratio is 8.7 years in the Baseline economy. The ratio of value of total structures to GDP is 3.0 years, while the share of housing in total structures is 45%.

Columns 2 and 3 of *Table* 5 report the results for a different level of financial development. Column 2 is the case of a more advanced financial system, where the collateral requirement is 0.1 instead of 0.3 (the baseline number). The main difference relative to the baseline economy is that now there are more constrained home owners instead of tenants. Intuitively, because borrowing becomes easier, relatively poor households buy a house with high leverage (outside equity ownership) instead of renting. Column 3, by comparison, is the case of no housing mortgage ($\theta = 1$) so that the household must buy the house from its own net worth. In this economy, the fraction of tenants is significantly larger. Financial development affects substantially the home-ownership rate. On the other hand, the degree of financial development has limited effects on prices and aggregate quantities in the steady state. This result arises because the shareholding of tenants and constrained households is a small fraction of aggregate wealth, and because the required adjustment is mostly achieved through conversion of houses between being rented and being owned. Nevertheless, the utility gain from higher home ownership associated with better housing finance is substantial, for a given parameter of utility loss from renting a house.

In column 4 we present an economy in which the growth rate of the population is two percent, instead of one per cent. A greater growth in the population rate implies a greater percentage of tenants and a lower home-ownership rate, mainly because there are a larger fraction of low productivity workers with a larger inflow of new low productive workers. The house price-rental ratio is also higher than in the baseline, anticipating that land becomes more scarce with a larger population in the future (despite a higher real interest rate).

In column 5, we consider an economy in which labour productivity growth is three percent instead of two. A higher productivity growth leads to a higher rate of return on the asset, with a low elasticity of intertemporal substitution, (even with a larger foreign ownership of shares) in the steady state. The housing price to rental ratio is slightly lower because of the higher real interest rate which offsets the effect of a larger expected growth rate of the rental price. The workers expect higher wages in the future and do not save much. This leads to a lower home-ownership rate (despite a slightly lower housing price-rental ratio).

In Column 6, we decrease the ratio of the retirement benefit to average pre-tax wage to 0.1 from 0.2 (in Baseline). This has big overall effects on both the distribution of the mode of housing and aggregate allocations because households save more in preparing for the retirement shock. As a result of the more vigorous saving among workers, the foreign ownership of shares declines, the rate of return of shares is much lower than the rate of time preference, and the home ownership rate increases in the new steady state.

In column 7 we consider a closed economy, by shutting down the demand coming from the representative foreigner by setting $S^* = 0 = \xi$. With a lower demand for shares the real rate of return increases relative to the baseline while the housing price is reduced. There is a greater number of renters in the economy but most other comparative statics remain similar to the baseline case.

In Columns 8 and 9, we consider an economy in which land is more important for structures than in the Baseline: $\gamma = 0.5$, instead of 0.9. (Remember $1 - \gamma$ is the share of land in structures). In column 8 we only adjust γ relative to the baseline while in column 9 we also adjust the parameters associated with the foreign demand for the domestic shares to maintain an approximately similar current account (0.9% of GDP). Because land is more scarce, the house price-rental ratio is substantially higher (13.9 years when the current account is left free to adjust (11.9 years in column 9) instead of 8.7 years in the baseline), reflecting the higher expected growth rate of the rental price. Focussing on column 9, the rate of return in terms of goods is substantially higher, even through the rate of return on the consumption basket is muted by the increase of rental price: $R/G_r^{1-\alpha} = 1.083$ instead of 1.086. The home-ownership rate is higher because more vigorous saving due to the higher real rate of return on asset overcomes the negative effects of the need for larger downpayment.

4.3.2 Small Open Economy

We can conduct the above comparative steady state for the special case of a small open economy, i.e., $\xi = 0$ instead of $\xi = 20$, by keeping constant the real interest rate. Even though the general features of the economy are similar to those in $Table$ 5, there are interesting differences arising across comparative statics. Given that this specification could be more appropriate for a city within a country (like London in the U.K. or New York in the U.S.), Table 6 reports a set of comparative statics exercises to understand the general features of this specification for $\gamma = 0.9$.

The baseline results (column 1) in table 6 are identical to their closed economy counterparts (column 1 of table 5) since the world real return is chosen to be the same. Changing the parameter controlling the ability to use collateral (θ) again predominantly affects home-ownership rates rather than prices, in a similar way as in the previous section. A substantial number of differences arise, however, in the response of endogenous variables to population and productivity growth. Faced with higher growth rates in these variables, and with the real return not adjusting, there is a pronounced positive effect on house prices, the house price to rental ratio and the value of housing to wages. This contrasts sharply with the results from the economy where the real return was allowed to adjust and suggests that the level of international capital market integration may be key in assessing the extent to which fundamentals affect house prices.

4.4 Transition of Small Open Economy against a Change in Fundamentals

We now examine how the small open economy reacts to a once-for-all change in the fundamental condition in technology and finance. In order to highlight the importance of land, we compare the reactions of Baseline economy ($\gamma = 0.9$) with the reaction of the economy in which land is more important ($\gamma = 0.5$) . This gives us a sense of how the housing market in an economy like Japan or the UK might respond to different shocks, relative to the U.S. baseline (which is a relatively land-abundant economy).

$Figure$ 5 shows how these two economies react to a once-for-all fall in the world real interest rate by 1%. Because the economy is growing, all the following figures show the percentage difference from the steady state growth path of the respective economies. In both economies, housing prices (figure 5A) and output (figure 5B) increase, and the adjustment of both housing prices and output is relatively fast. Net exports (figure 5C) initially experience a large deficit, before recording a surplus. But in the economy where land is more important, the increase in house prices is larger (about 11%), the swing of net exports is larger, the increase in output is smaller (up to 1%) and the adjustment is slower. In this economy, output does not increase in the initial period despite the large increase in the capital stock, because a large fraction of structures gets reallocated from production to housing. Reflecting on the larger swing of net exports, consumption swings more (from 10% initial increase to 4% eventual decrease) in the economy with a larger land share in structures. Perhaps the swing of next exports is unrealistically large in our model where there are no adjustment costs of capital but we ignore the adjustment cost to keep the analysis simple and clear.

$Figure$ 6 shows the reactions to a once-for-all increase in the *level* of labour productivity by 5%. In both economies, the housing price rises (figure 6A), and output (figure 6B) and consumption (figure 6D) rise almost immediately to new steady state levels. (Again the lack of any adjustment costs is responsible for such a fast adjustment). Net exports (figure 6C) move initially to deficit before moving to the surplus region. Again, we observe that the housing price is larger (2.6%), and the swing of net exports is more dramatic, in the economy with larger land share, while the increase in consumption is smaller and slower (both about

4% after 3 periods).

Figure 7 shows the responses to a once-for-all increase in the *growth rate* of labour productivity from 2% to 2.25%. In both economies the housing price increases substantially initially and continues to increase afterwards. In the economy with a more significant land component, the housing price rises more initially (2.9% instead of 1.9%), and real house price inflation rises by 0.11% at an annual rate thereafter (instead of 0.024%). The economy with a larger land share also experiences a delayed increase in output, a larger swing in net exports and in consumption.

Putting together the simulation results from these experiments, we can conclude that, if we were to explain the large increase in housing prices in many developed countries in the last decades, we have to look for increases in the expected growth rate of labour productivity and for decreases in the real interest rate. Suppose that the expected growth rate of labour productivity rises from 2% to 3%. Then in the Baseline economy, housing prices would increase initially by 7.6% and the real housing inflation rate would afterwards increase from 0.29% to 0.38% in terms of output (to 0.29% in terms of the consumption basket). In an economy in which land is more important ($\gamma = 0.5$), the housing price would initially increase by 11.6%, followed by the real housing price inflation of 1.7% in terms of output (1.3% in terms of the consumption basket).

Suppose that the world real rate of return on assets falls from 6.62% to 5.62%, in addition to the above 1% increase in the growth rate of labour productivity. Then, in the Baseline economy, the housing price would increase initially by approximately 15.6%, followed by an annual real housing price inflation of 0.4% in terms of output. In the economy where land is more important, the initial increase in the real housing price would be 22.6%, followed by the real housing price inflation of 1.7% annually (in terms of output). In 10 years, the cumulative increase in real housing price in terms of output would be about 20% in the Baseline economy and would be 45% in the economy where land is more important ($\gamma = 0.5$). Thus, if half the population lives and works in the area in which land is not important ($\gamma = 0.9$) and another half lives and works in the area in which land is important for structures, *and* the mobility of labour is restricted while capital freely moves between the areas, then the cumulative housing price increase would be roughly $(20+45)/2 = 32.5\%$.

This number is significantly lower than the increase in real housing prices in the U.S. and the U.K. in the last decades. Arguably, this is very crude calculation, ignoring how regional agglomeration takes place. Nonetheless, it gives us some guidance that a significant fraction of the increase in real housing prices may be explained by a combination of an increase in the growth rate of labour productivity, a decrease in the real interest rate, and the property that the largest fraction of economic activity is taking place in the area in which land has a larger share in the value of residential and commercial structures.

5 Conclusions

This paper develops a heterogeneous agent model to investigate the interaction between housing prices, rental rates, aggregate production, and household behaviour over a lifetime. A key innovation involves the explicit introduction of land as a fixed factor of production and analyzing the implications of this innovation for time series (aggregate) and cross sectional (distributional and life-cycle) outcomes. In particular, where the share of land value for commercial and residential structure is large, households face a higher house price-rental ratio and they buy houses later in life in the steady state. On the other hand, relaxing collateral constraints has a limited impact on housing prices and aggregate production, even though it encourages households to buy houses earlier in life. The perfect foresight comparisons illustrate that, where land is more important, aggregate shocks generate more volatile responses in the housing market and can generate a substantial increase in house prices in response to real macroeconomic shocks.

Appendix A: Solving the model

Solving the household's decision problem

We discretize the net worth (x_t^i) using 200 grid points, with denser grids closer to zero to take into account the higher curvature of the value function in this region. The grid range for the continuous state variable is verified ex-post by comparing it with the values obtained in the simulations. For points which do not lie on state space grid, we evaluate the value function using cubic spline interpolation along the net worth dimension. We simulate the idiosyncratic exogenous productivity shock from its three-point distribution. The realizations of these exogenous random variables are held constant when searching for the market clearing prices (q and r). We use the policy functions to simulate the behavior of 20000 agents and aggregate the individual housing and share ownership demands to determine the market clearing rental and housing price and the equilibrium household allocations.

Solving perfect foresight model

We guess a sequence of structure rental rates $\{r_t\}_{t=1}^{T}$ such that the rental rate has converged to the new steady state. Use (19) to calculate a sequence of capital stocks $\{K_t\}_{t=1}^{T}$ and then use (12) to compute the sequence of structure prices $\{q_t\}_{t=1}^{T}$. Given these guessed prices, we solve the household's problem backwards from period T when the economy is assumed to have converged to the new steady state. Households are assumed to know the realization of the entire path of structure prices and rental rates. The value function in period T is the value function for the new steady state. Then the value function in period T-1 is computed as follows:

$$V_{T-1}\left(x_{T-1}|r_{T-1}, q_{T-1}\right) = \max_{c_T, h_T}\left[u\left(c_{T-1}, h_{T-1}\right) + \beta V_T\left(x_T|r_T, q_T\right)\right]$$

We simulate the model forward, starting from the capital stock and the wealth distribution of the original steady state. In each period, we simulate a cross-section of 20000 agents and aggregate their individual housing choices, computing the excess demand for housing in each period. We increase the rental rate in periods with a positive excess demand and decrease the rental rate in periods with a negative excess demand. We repeat this until markets clear in all periods and successive paths of the rental rate are close to one another.

Appendix B: Stationary Representation of Value Func-

tions

The stationary representation of the worker's problem

Using the property of the steady state equilibrium of Section 2.4, we normalize the quantities and prices using the power function of labour in efficiency units $N_t' \equiv A_t N_t$ and population N_t. Both variables are exogenous state variables, and there can be a jump or a kink in the trend if labour productivity experiences a once-for-all change in its level or growth rate. Let us denote the normalized variable X_t as $\widetilde{X}_t$. Then we have:

$$\begin{aligned}
\widetilde{K}_t &= K_t / N_t'^{\frac{1-\eta}{1-\gamma\eta}}, \qquad \widetilde{S}_t^* = S_t^* / N_t'^{\gamma\frac{1-\eta}{1-\gamma\eta}} \\
(\widetilde{w}_t, \widetilde{x}_t) &= (w_t, x_t) / (N_t'^{\frac{1-\eta}{1-\gamma\eta}} / N_t) \\
(\widetilde{h}_t, \widetilde{s}_t) &= (h_t, s_t) / (N_t'^{\gamma\frac{1-\eta}{1-\gamma\eta}} / N_t) \\
\left(\widetilde{r}_t, \widetilde{q}_t, \widetilde{d}_t\right) &= (r_t, q_t, d_t) / N_t'^{(1-\gamma)\frac{1-\eta}{1-\gamma\eta}} \\
\widetilde{V}_t^i &= V_t^i / \left[\frac{N_t'^{\frac{1-\eta}{1-\gamma\eta}} / N_t}{N_t'^{(1-\alpha)(1-\gamma)\frac{1-\eta}{1-\gamma\eta}}} \right]^{1-\rho}, \text{ for } i = l, m, h, \text{ or } r
\end{aligned}$$

We also define the normalized discount factor as:

$$\widetilde{\beta} = \beta \left(\frac{G_w}{G_r^{1-\alpha}} \right)^{1-\rho}.$$

Let us assume population grows along the steady state path. Let $\widetilde{A}_t$ be deviation of labour productivity from the trend. Then the vector of normalized state variables adjusted by the productivity change are:

$$\overline{\widetilde{A}}_t = \left(\widetilde{A}_t, \widetilde{K}_{t-1}, \widetilde{S}_{t-1}^*, (\widetilde{s}_{t-1}(i))_{i \in [0, \overline{N}_t]} \right)'.$$

Using these normalized variables, we can define the normalized value function. For an example, the stationary representation of the retiree's problem is

$$\widetilde{V}^r(\widetilde{x}, \overline{\widetilde{A}}_t) = Max($$

$$\max_{\widetilde{s}}\left\{\begin{array}{c}(1-\psi)^{(1-\alpha)(1-\rho)}\frac{(\widetilde{x}-(\widetilde{q}_t-\widetilde{r}_t)\widetilde{s})^{1-\rho}}{1-\rho}\left(\frac{\widetilde{w}_t}{\widetilde{r}_t^{1-\alpha}}\right)^{1-\rho}\\ +\beta\sigma\left(\frac{G_w}{G_r^{1-\alpha}}\right)^{1-\rho}\widetilde{V}^r(\widetilde{b}+\left(\widetilde{d}/\sigma G_z\right)\widetilde{s},\overline{\widetilde{A}}_{t+1})\end{array}\right\},$$

$$\max_{\widetilde{s}}\left\{\begin{array}{c}\left\{\left[\frac{\widetilde{x}-\left(\widetilde{q}_t-\widetilde{r}_t+\frac{\widetilde{r}_t}{\theta}\right)\widetilde{s}}{\alpha}\right]^{\alpha}\left[\frac{\widetilde{s}\widetilde{r}/\theta}{1-\alpha}\right]^{1-\alpha}\right\}^{1-\rho}/(1-\rho)\\ +\beta\sigma\left(\frac{G_w}{G_r^{1-\alpha}}\right)^{1-\rho}\widetilde{V}^r(\widetilde{b}+\left(\widetilde{d}/\sigma G_z\right)\widetilde{s},\overline{\widetilde{A}}_{t+1})\end{array}\right\},$$

$$\max_{\widetilde{s}}\left\{\begin{array}{c}\frac{(\widetilde{x}-(\widetilde{q}_t-\widetilde{r}_t)\widetilde{s})^{1-\rho}}{1-\rho}\\ +\beta\sigma\left(\frac{G_w}{G_r^{1-\alpha}}\right)^{1-\rho}\widetilde{V}^r(\widetilde{b}+\left(\widetilde{d}/\sigma G_z\right)\widetilde{s},\overline{\widetilde{A}}_{t+1})\end{array}\right\})$$

where $G_z = (G_A G_N)^{\gamma\frac{1-\eta}{1-\gamma\eta}}/G_N$.

Appendix C: Representative Agent Model

We consider a special case of the economy in which there is no heterogeneity of labour productivity, retirement , or death, i.e., $\delta = 0$ and $\omega = 1$. Everyone lives forever with the same labour productivity (no idiosyncratic shock to labour productivity), and population is equal to the number of workers. Because there are no retirees, there is no pension and no tax to finance to pension. The technology can be written in per capita terms as:

$$y_t = A_{Nt}^{1-\eta} z_{Yt}^{\eta}$$

$$z_t = \left(\frac{L}{N_t}\right)^{1-\gamma} k_t^{\gamma}$$

$$i_t = k_t - \frac{\lambda}{G_N} k_{t-1}$$

Consider a closed economy with the representative agent who maximizes utility under certainty. Because nobody lends or borrows in equilibrium, the collateral constraint does not bind. The conditions for utility maximization are:

$$r_t = \frac{u_{h_t}}{u_{c_t}} = \frac{1-\alpha}{\alpha}\frac{c_t}{h_t} \tag{A1}$$

$$1 = \frac{d_{t+1}}{q_t - r_t}\beta\frac{u_{c_{t+1}}}{u_{c_t}},$$

where u_{c_t} is shorthand notation of $\partial u/\partial c_t$. From and profit maximization conditions (18), (19) and (20) , we have

$$r_t = \eta y_t / z_{Yt} \tag{A2}$$

$$\frac{d_{t+1}}{q_t - r_t} = R_t = \frac{\lambda}{1 - \gamma \frac{z_t}{k_t} r_t} \tag{A3}$$

Consider a planner who maximizes the social welfare function:

$$N_t V(k_{t-1}, A_t) = \sum_{j=t}^{\infty} \beta^{j-t} N_j u(c_j, h_j)$$

subject to the constraint of technology and resource allocation. Let $A_t = (A_{Nt}, N_t)'$ be the exogenous state variables. The value function of the planner would be:

$$\begin{aligned} V(k_{t-1}, A_t) &= Max \left\{ u(c_t, h_t) + \beta G_N V(k_t, A_{t+1}) \right\} \\ &= \underset{k_{Yt}, k_t}{Max} \left\{ \begin{array}{c} u\left(A_t z_{Yt}{}^{\eta} - k_t + \frac{\lambda}{G_N} k_{t-1}, \left(\frac{L}{N_t}\right)^{1-\gamma} k_t^{\gamma} - z_{Yt} \right) \\ +\beta G_N V\left(k_t, A_{t+1}\right) \end{array} \right\} \end{aligned}$$

The first order conditions are:

$$\eta \frac{y_t}{z_{Yt}} = \frac{u_{h_t}}{u_{c_t}} = \frac{1-\alpha}{\alpha} \frac{c_t}{h_t}, \tag{A4}$$

$$\begin{aligned} 1 &= \gamma \frac{z_t}{k_t} \frac{1-\alpha}{\alpha} \frac{c_t}{h_t} + \beta G_N \frac{\partial V(k_t, A_{t+1})}{\partial k_t} \\ &= \gamma \frac{z_t}{k_t} \frac{1-\alpha}{\alpha} \frac{c_t}{h_t} + \beta \frac{u_{c_{t+1}}}{u_{c_t}} \lambda. \end{aligned} \tag{A5}$$

From the utility maximization condition of households $(A1)$ and the profit maximization condition, we learn that the competitive equilibrium satisfies the first order conditions of the planner's problem $(A4)$ and $(A5)$.

In the steady state, per capita quantities satisfy:

$$G_y = G_Y / G_N, \quad \text{and } G_z = G_Z / G_N.$$

Then,

$$\frac{u_{c_{t+1}}}{u_{c_t}} = \left(G_y^{\alpha} \; G_z^{1-\alpha} \right)^{1-\rho} G_y^{-1} = G_u \; G_y^{-1}$$

where $G_u = \left(G_y^{\alpha} \; G_z^{1-\alpha} \right)^{1-\rho}$ is the growth rate of utility. Let ψ_K be the capital-output ratio and ψ_Y be share of productive structures in the steady state:

$$\begin{aligned} f_K &= \frac{k_t}{y_t}, \\ f &= \frac{z_{Yt}}{z_t}. \end{aligned}$$

Then we learn:

$$\frac{c_t}{y_t} = 1 - \frac{i_t}{k_t}\frac{k_t}{y_t} = 1 - \left(1 - \frac{\lambda}{G_K}\right) f_K.$$

From the two first order conditions $(A4), (A5)$, we learn:

$$\begin{aligned}
\frac{\eta\alpha}{1-\alpha} &= \frac{c_t}{y_t}\frac{z_{Yt}}{h_t} = \left[1 - \left(1 - \frac{\lambda}{G_K}\right) f_K\right]\frac{f}{1-f}, \\
1 &= \frac{\gamma\eta}{f_K f} + \beta\lambda\frac{G_u}{G_y}.
\end{aligned}$$

Solving these with respect to f_K and f, we get

$$\begin{aligned}
f &= \eta\frac{\frac{\alpha}{1-\alpha} + \gamma\frac{1-\frac{\lambda}{G_K}}{1-\beta\lambda G_u/G_y}}{\frac{\eta\alpha}{1-\alpha} + 1}, \\
f_K &= \gamma\frac{\frac{\eta\alpha}{1-\alpha} + 1}{\frac{\alpha}{1-\alpha}\left[1 - \beta\lambda G_u/G_y\right] + \gamma\left(1 - \frac{\lambda}{G_K}\right)}.
\end{aligned}$$

From $(A3)$ and $(A5)$, we learn

$$\frac{G_y}{\beta G_u} = R = \frac{G_K{}^{\gamma} q_t - (G_K - \lambda)\frac{k_t}{z_t}}{q_t - \frac{\eta}{\psi_K\psi_Y}\frac{k_t}{z_t}}.$$

Thus

$$Q \equiv \frac{q_t z_t}{k_t} = \frac{1}{\gamma}\frac{R - (1-\gamma)\lambda - \gamma G_K}{R - G_K{}^{\gamma}}.$$

We can also compute price-rental ratio as

$$\frac{q_t}{r_t} = \frac{q_t}{\eta y_t/z_{Yt}} = \frac{q_t z_t/k_t}{\eta}\frac{z_{Yt}}{z_t}\frac{k_t}{y_t} = \frac{1}{\eta} f_K f Q.$$

The ratio of housing value to wage is:

$$\frac{q_t h_t}{w_t} = \frac{q_t (1-f) z_t}{(1-\eta) y_t} = \frac{1}{1-\eta} f_K (1-f) Q.$$

Thus, in our Baseline calibration, we learn

$$\begin{aligned}
f &= \frac{z_{Yt}}{z_t} = 0.543 \\
f_K &= \frac{k_t}{y_t} = 2.78 \\
R &= 1.0636 \\
Q &= \frac{q_t z_t}{k_t} = 1.56 \\
\frac{q_t}{r_t} &= 9.12 \\
\frac{q_t h_t}{w_t} &= 2.64.
\end{aligned}$$

Appendix D: Data sources and definitions

We use quarterly data from the US Flow of Funds accounts and from the NIPA for the 1952 Q1 - 2005Q4 period. We follow Cooley and Prescott (1995) in aligning the model economy with the data. A crucial parameter we need to calibrate is the productive structures' share in production (η). Given that the model economy includes residential structures explicitly we need to adjust GDP and flow of funds data to derive a reasonable estimate for this parameter.

We define unambiguous capital income as the sum of rental income (r), corporate profits (π) and net interest (i) from the NIPA (table 1.12). We allocate the share of proprietors' income (Y_P, NIPA, Table 1.12) arising from productive structures using η, while a measure for the depreciation of capital (DEP) is given by the consumption of fixed capital (NIPA, table 1.14). Defining Y_{KP} as income from productive structures, Y_{KP} can be computed as the sum of unambiguous capital income plus $\eta*$Proprietors' Income plus DEP. $Y_{KP} = \eta Y$, where Y is GDP excluding explicit and implicit rents from housing. Solving for η, we have

$$\eta = \frac{r + \pi + i + DEP}{Y - Y_P}$$

which is a similar expression for the share of capital in output found in Cooley and Prescott (1995, p.19).

Averaging the quarterly data for the U.S. from 1952 to 2005, we obtain a value of η equal to 0.26. This is lower than the share of capital in output in the real business cycle literature (estimates there range between 0.3 and 0.4) because we have explicitly included housing services in the theoretical model and, consistent with the logic in Cooley and Prescott (1995), we are not including housing in this computation.

The presence of housing in both the firm and household side also allows us to decompose economy-wide tangible assets between the household and the firm. The exact definitions in the data and their counterparts in the theoretical model are given in the following table:

Economic concept	Flow of Funds concept
qK_y	Non-farm, non-financial tangible assets (Non-residential structures+Equipment+software+Inventories) Flow of funds, Tables B.102 and B.103 FL102010005.Q+FL112010005
qH	Household tangible assets (Residential structures+Equipment+software+Consumer durables) Flow of funds, Table B.100 FL152010005.Q

Using this definitions, we compute the average numbers between 1952:Q1 and 2005:Q4. The ratio of household tangible assets to firm tangible assets (H/K_y) is 0.913 (equivalently the ratio of household tangible assets to total capital is 0.47) and the ratio of total capital to GDP ($q\left(H+K_y\right)/Y$) is 3.3. If farm corporate and non-corporate tangible assets (FL132010005.Q in the Flow of Funds)[16] are added to the non-farm tangible assets, then the ratio of household tangible assets to total capital falls from 0.47 to 0.44 while the ratio of total capital to GDP rises from 3.3 to 3.6.

[16]Thanks to Michael Palumbo (Board of Governors) of kindly sending us this series in private correspondence.

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FIGURE 1A: Policy functions for a low productivity worker

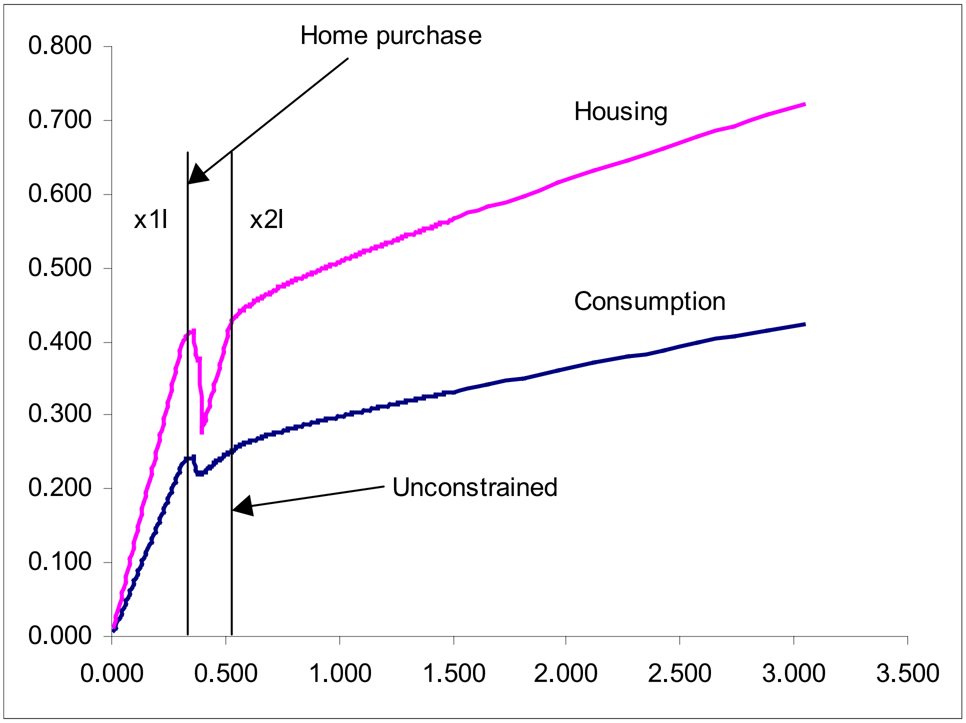


FIGURE 1B: Evolution of savings for a low productivity household

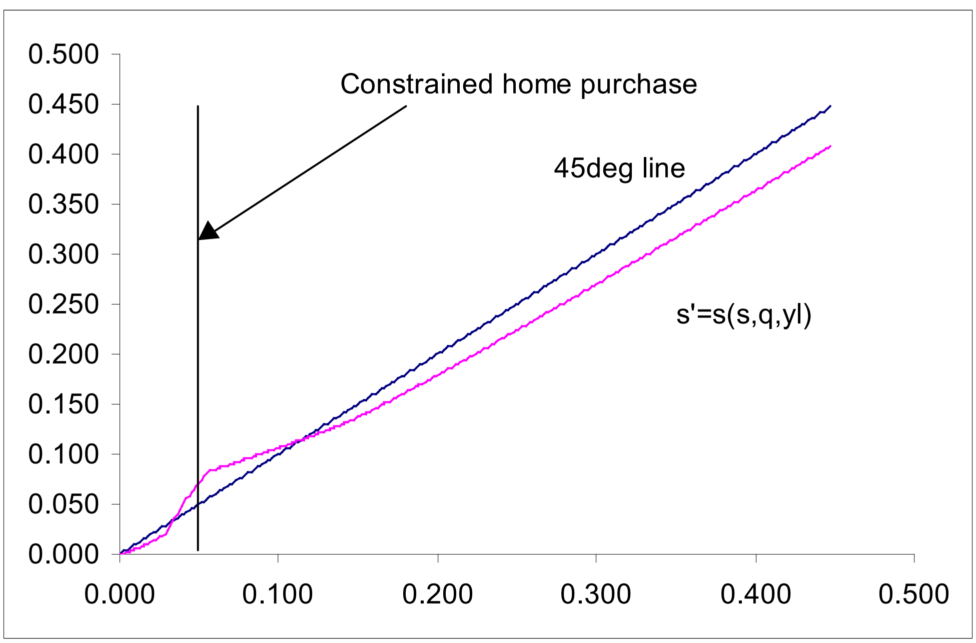


FIGURE 2A: Policy functions for a high productivity worker

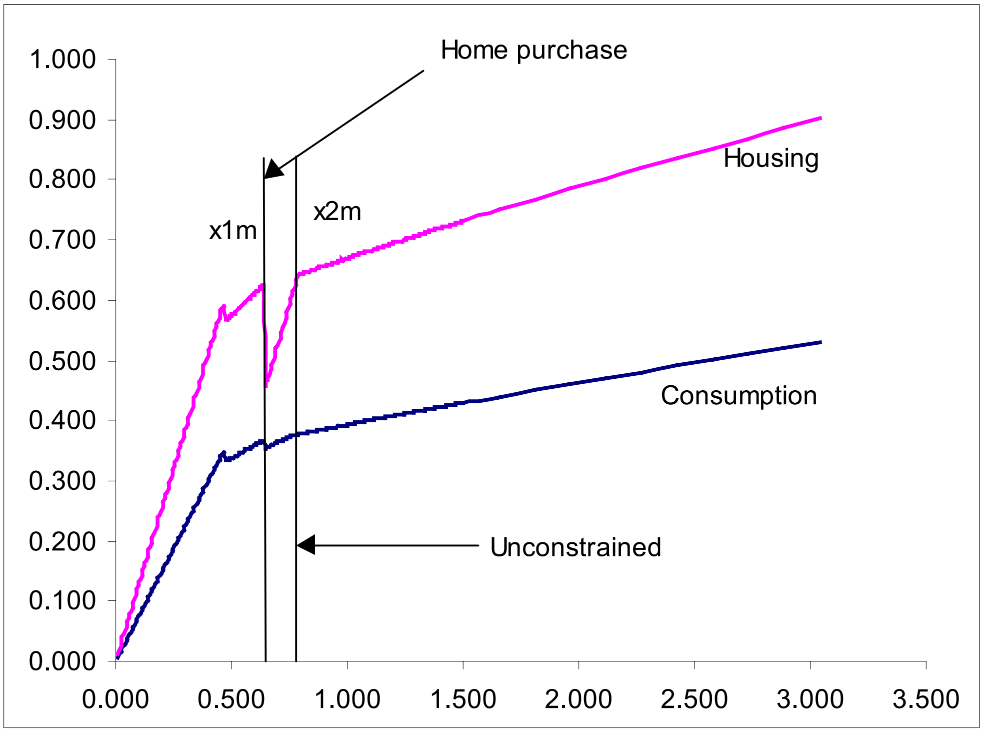


FIGURE 2B: Evolution of savings for a high productivity household

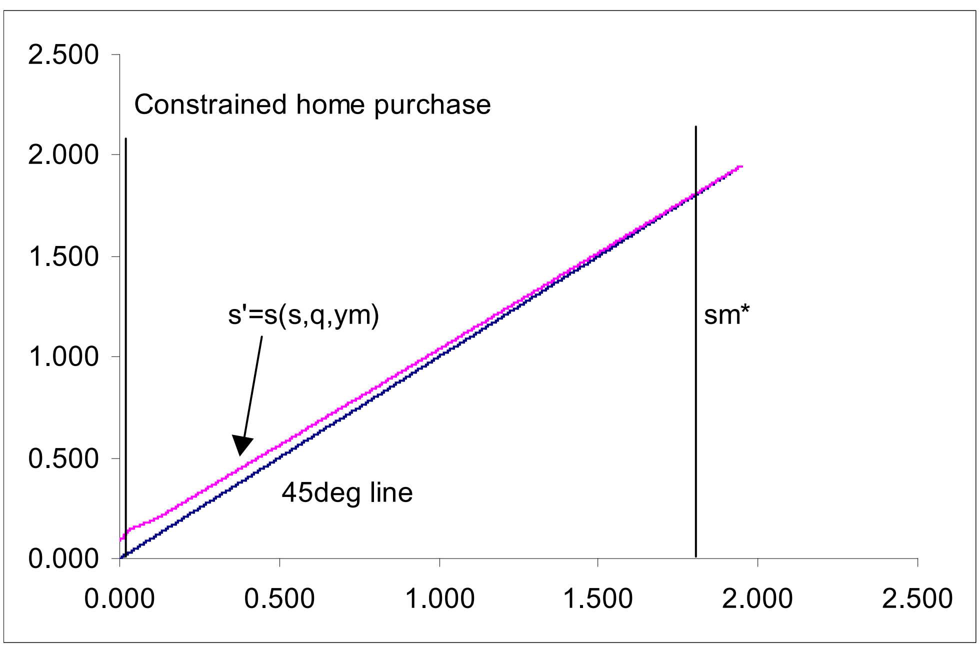


FIGURE 3A: Policy functions for the Retiree

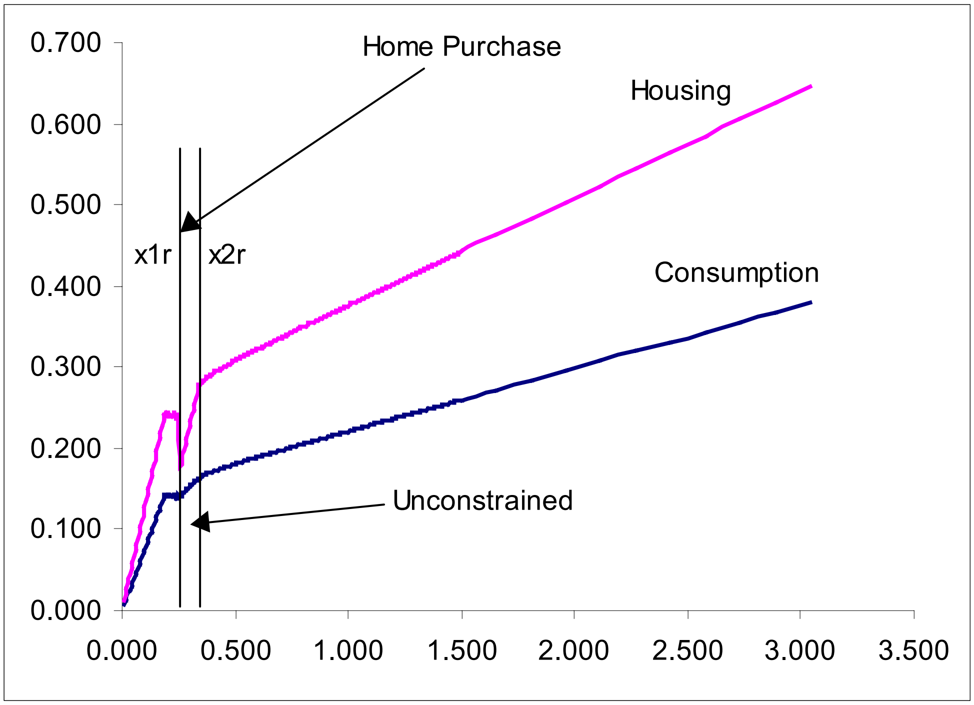


FIGURE 3B: Evolution of savings for the retiree

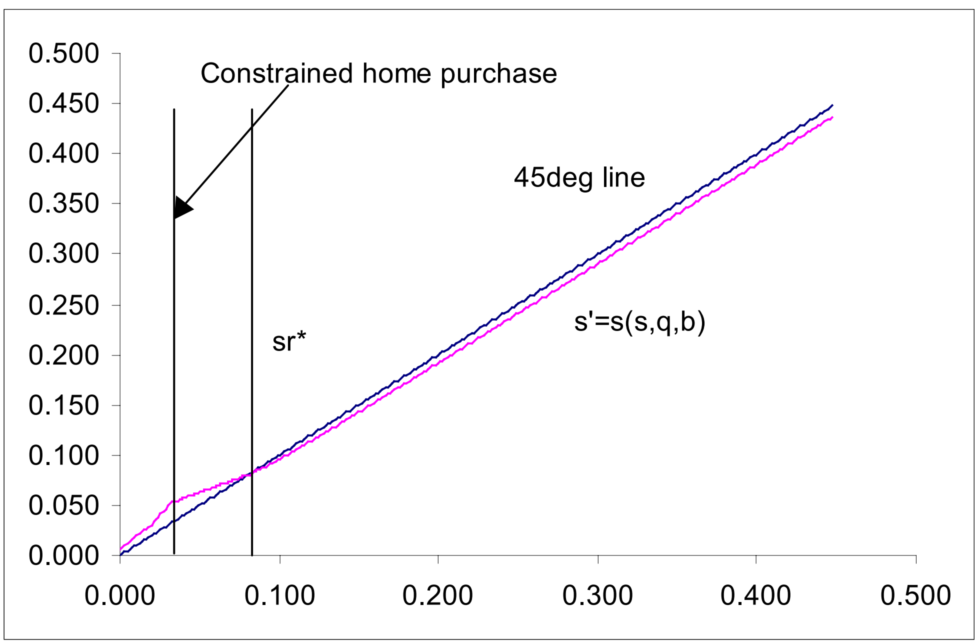


FIGURE 4: An example life time

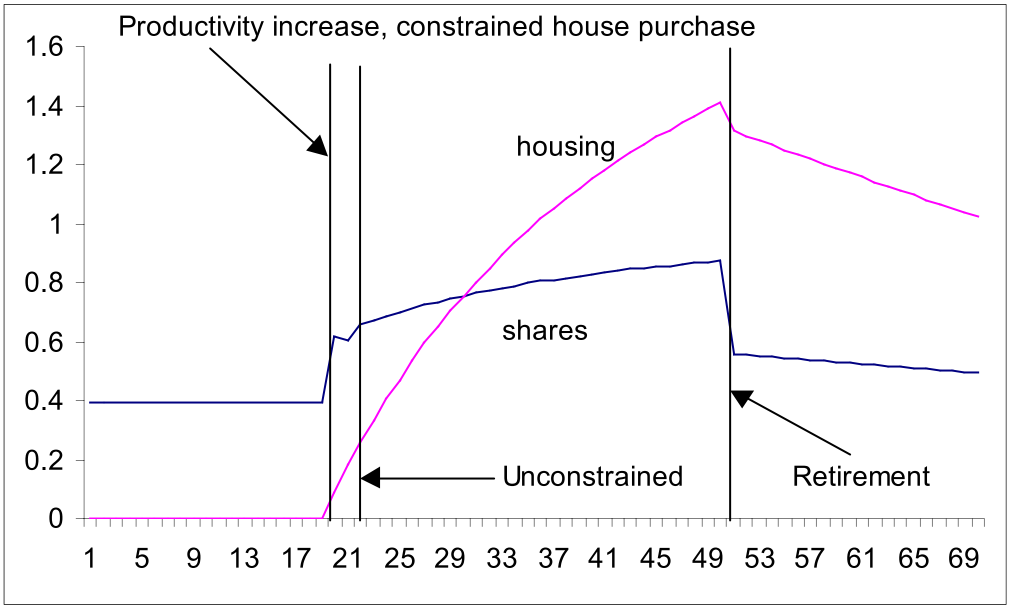


Table 5

	baseline	θ=0.1	θ=1.0	gn=1.02	ga=1.03	b=0.1	S*=0	γ=0.5	γ=0.5
Column	1	2	3	4	5	6	7	8	9
% of tenants	24.76	2.59	37.32	42.35	38.77	21.10	36.85	25.34	12.84
% of constrained households	8.27	25.49	11.83	11.76	14.95	15.12	10.99	5.54	11.09
% of unconstrained homeowners	66.97	71.92	50.85	45.88	46.28	63.78	52.17	69.12	76.06
% of shares owned by tenants	0.08	0.02	0.80	0.93	1.06	1.07	0.87	0.07	0.16
% of shares owned by constrained	0.33	0.36	3.15	3.73	4.10	2.70	2.93	0.28	0.47
% of housing used by tenants	8.61	0.76	13.87	17.26	15.44	6.74	13.53	7.87	3.47
% of housing used by constrained	2.42	8.05	6.99	8.30	9.12	5.98	6.40	1.90	3.09
Current account as % of GDP	0.90	0.89	0.86	1.93	2.17	0.14	0.00	8.12	0.93
Net foreign Assets as % of GDP	-19.49	-19.32	-18.77	-34.76	-42.69	-3.17	0.00	-137.70	-13.26
Value of total structures to GDP	2.98	2.98	2.99	3.00	2.83	3.33	2.90	5.18	4.49
Housing structures to total structures	0.45	0.45	0.45	0.45	0.45	0.45	0.46	0.48	0.51
Value of housing to wages	2.50	2.50	2.50	2.48	2.35	2.74	2.46	4.49	4.22
Housing price to rental rate	8.65	8.66	8.69	8.77	8.31	9.61	8.43	13.89	11.86
Real return	6.62	6.61	6.58	7.27	7.69	5.86	6.84	7.54	8.64
House price (N=An=1)	1.66	1.66	1.66	1.71	1.67	1.78	1.63	4.95	4.40
Output (N=An=1)	1.11	1.11	1.11	1.10	1.09	1.12	1.10	0.89	0.88

Notes to Table 5: Results from the closed economy with a given demand for domestic shares by a representative foreigner (world interest rate is 6%). In the baseline economy, the collateral constraint is denoted by θ and is equal to 0.3, gn denotes population growth and is equal to 1.01 (one percent per annum), ga=1.02 denotes a two percent annual productivity growth, and b=0.2 denotes a twenty percent gross replacement rate during retirement. S* controls the amount of foreigner demand so that S*=0 is the completely closed economy. The results from reducing γ from its baseline value of 0.9 to 0.5 are reported in column (8) labeled {γ=0.5, (8)}. Given the large change in current accounts from this comparative statics exercise, we re-calibrate the foreigner demand parameters to deliver a similar current account for γ=0.5 as for when γ=0.9. These results are reported in the last column and are labeled {γ=0.5, column (9)}.

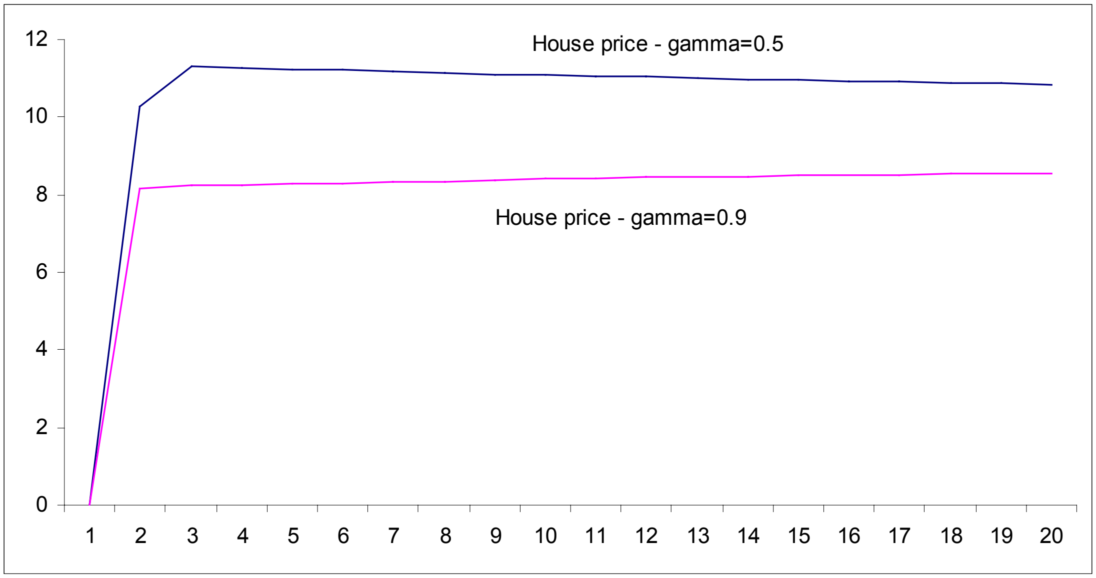
Table 6

	baseline	θ=0.1	θ=1.0	gn=1.02	ga=1.03	b=0.1	R*=5.62
Column	*1*	*2*	*3*	*4*	*5*	*6*	*7*
% of tenants	24.76	2.60	37.32	19.44	30.25	2.60	30.25
% of constrained households	8.27	25.49	10.83	16.67	7.04	5.82	5.56
% of unconstrained homeowners	66.97	71.92	51.85	63.89	62.71	91.60	64.19
% of shares owned by tenants	0.08	0.02	0.93	0.48	0.18	0.06	0.61
% of shares owned by constrained	0.33	0.41	3.17	1.52	1.44	0.13	0.75
% of housing used by tenants	8.61	0.76	14.01	7.89	10.80	0.68	10.61
% of housing used by constrained	2.42	8.05	6.27	5.65	2.74	1.31	2.39
Current account as % of GDP	0.90	0.96	0.85	4.59	5.38	-2.94	4.24
Net foreign Assets as % of GDP	-19.49	-20.60	-18.30	-98.37	-147.10	63.40	-116.10
Value of total structures to GDP	2.98	2.98	2.97	3.33	3.31	3.01	3.45
Housing structures to total structures	0.45	0.45	0.45	0.44	0.44	0.46	0.44
Value of housing to wages	2.50	2.60	2.60	2.76	2.77	2.71	2.86
Housing price to rental rate	8.65	8.65	8.65	9.69	9.68	8.65	9.99
Real return	6.62	6.62	6.62	6.62	6.62	6.62	5.62
House price (N=An=1)	1.66	1.66	1.66	1.84	1.86	1.66	1.83
Output (N=An=1)	1.11	1.11	1.11	1.11	1.11	1.11	1.13

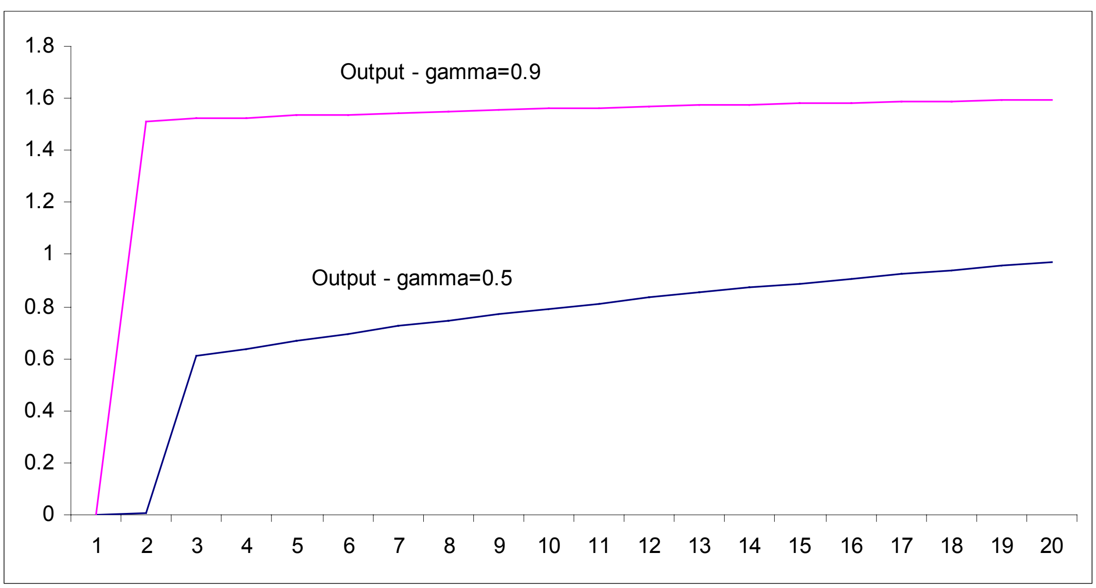
Notes to Table 6: Results from the small open economy with a given demand for domestic shares by a representative foreigner (world interest rate is 6.62% and γ=0.9). In the baseline economy, the collateral constraint is denoted by θ and is equal to 0.3, gn denotes population growth and is equal to 1.01 (one percent per annum), ga=1.02 denotes a two percent annual productivity growth, and b=0.2 denotes a twenty percent gross replacement rate during retirement. R* is the world real return.

Figure 5: The world real interest rate declines by 1pp

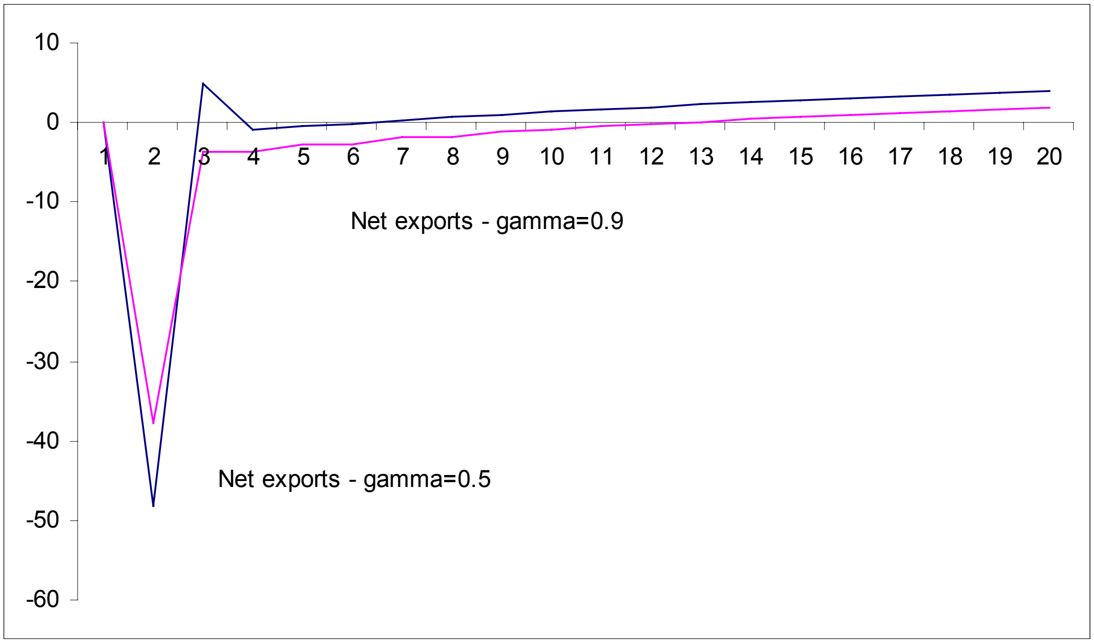
5A: Housing price (percent difference from baseline steady state)



5B: Output (per cent difference from baseline steady state)



5C: Net exports as a percentage of GDP



5D: Consumption (per cent difference from baseline steady state)

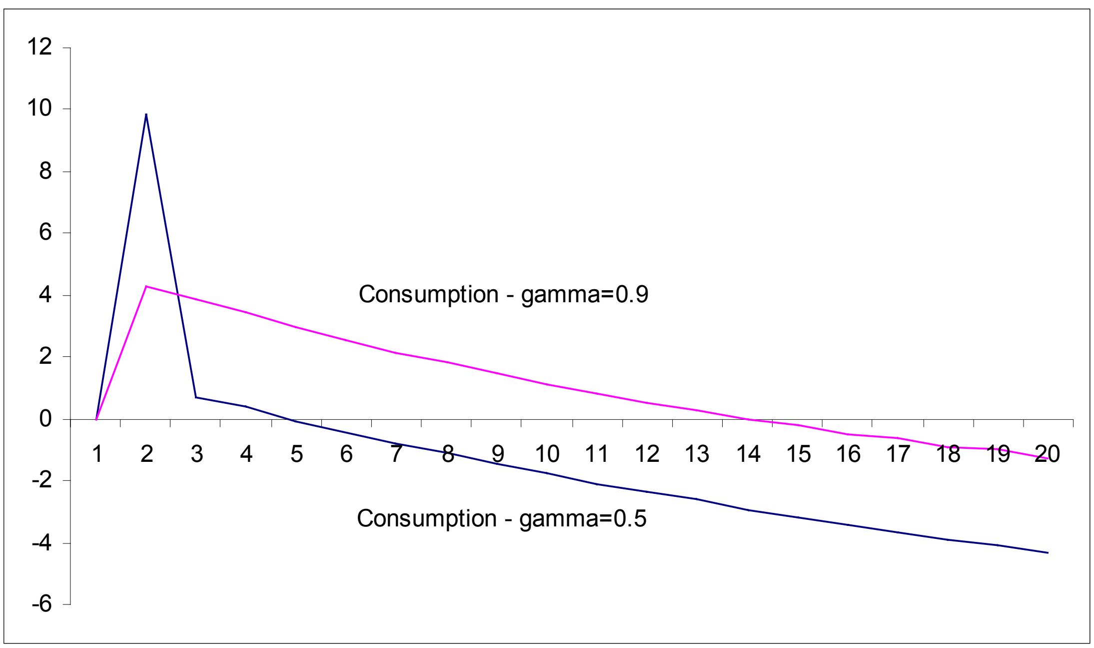
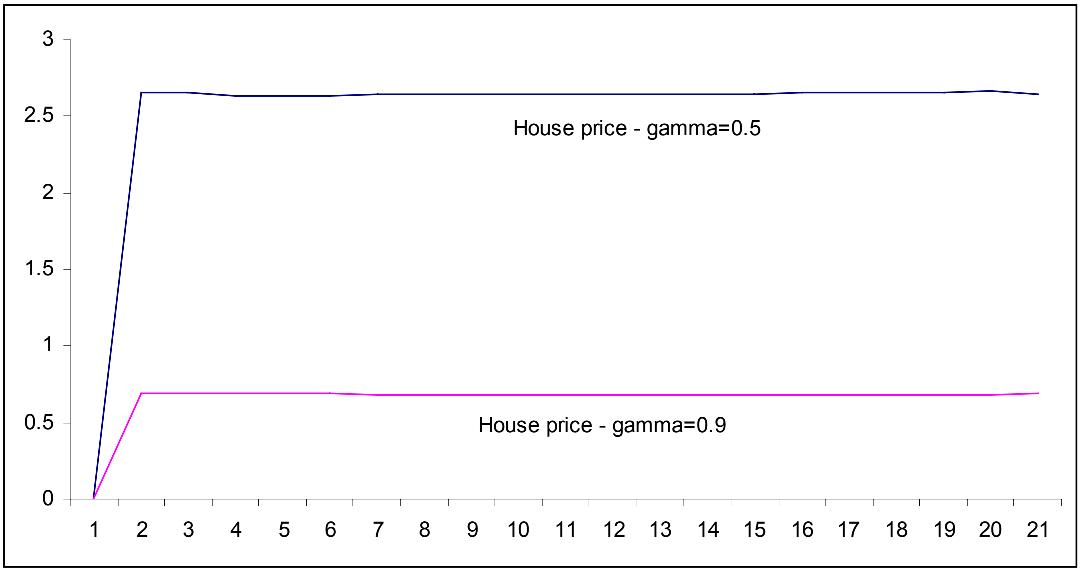
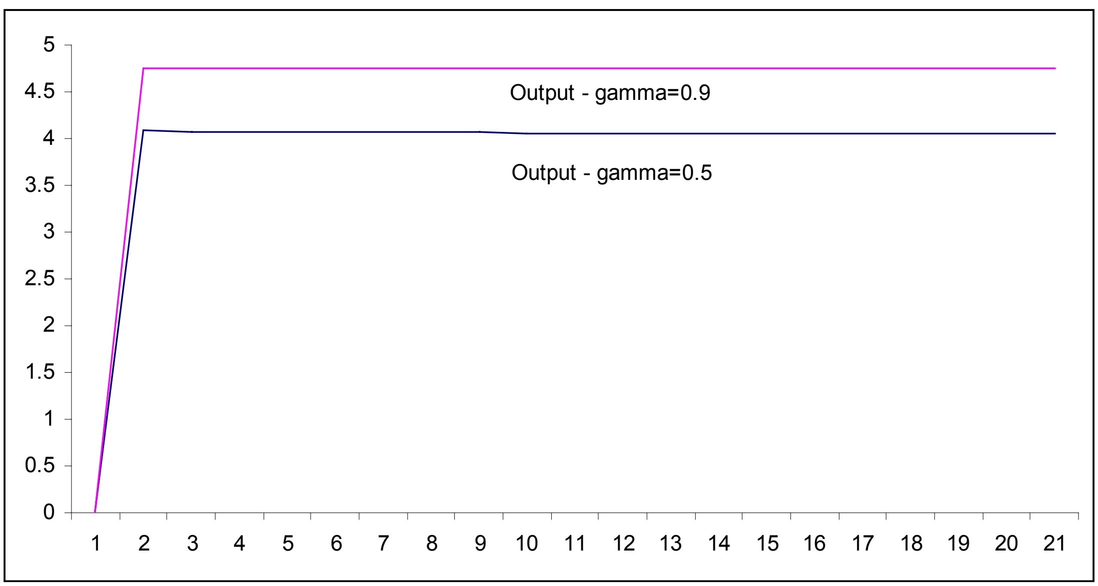


Figure 6: The level of labour productivity increases by 5%

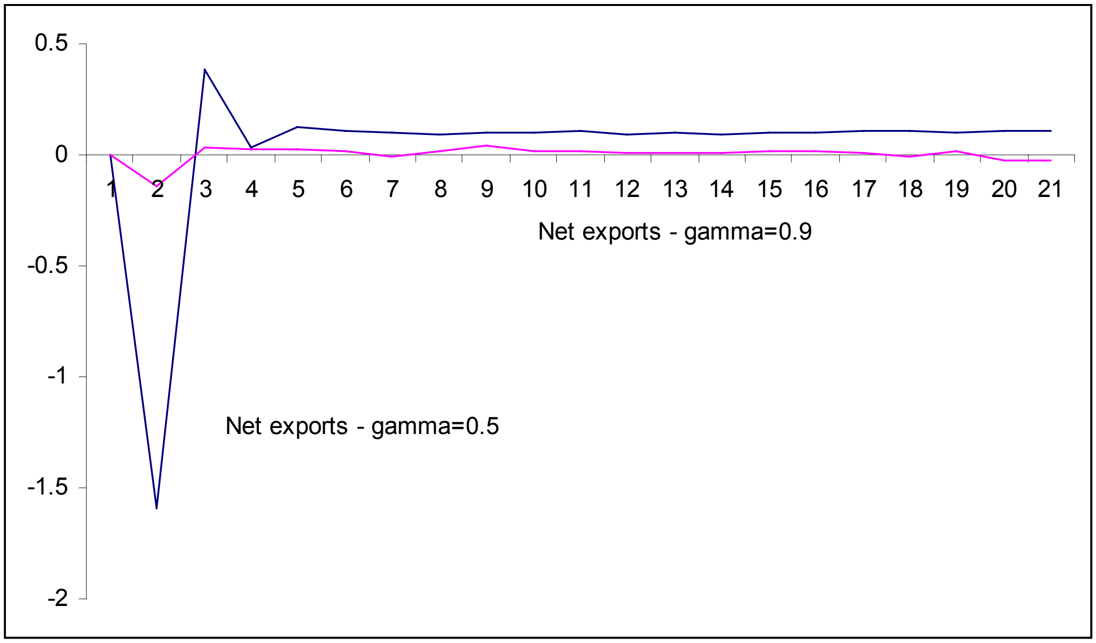
6A: Housing price (percent difference from baseline steady state)



6B: Output (per cent difference from baseline steady state)



6C: Net exports as a percentage of GDP



6D: Consumption (per cent difference from baseline steady state)

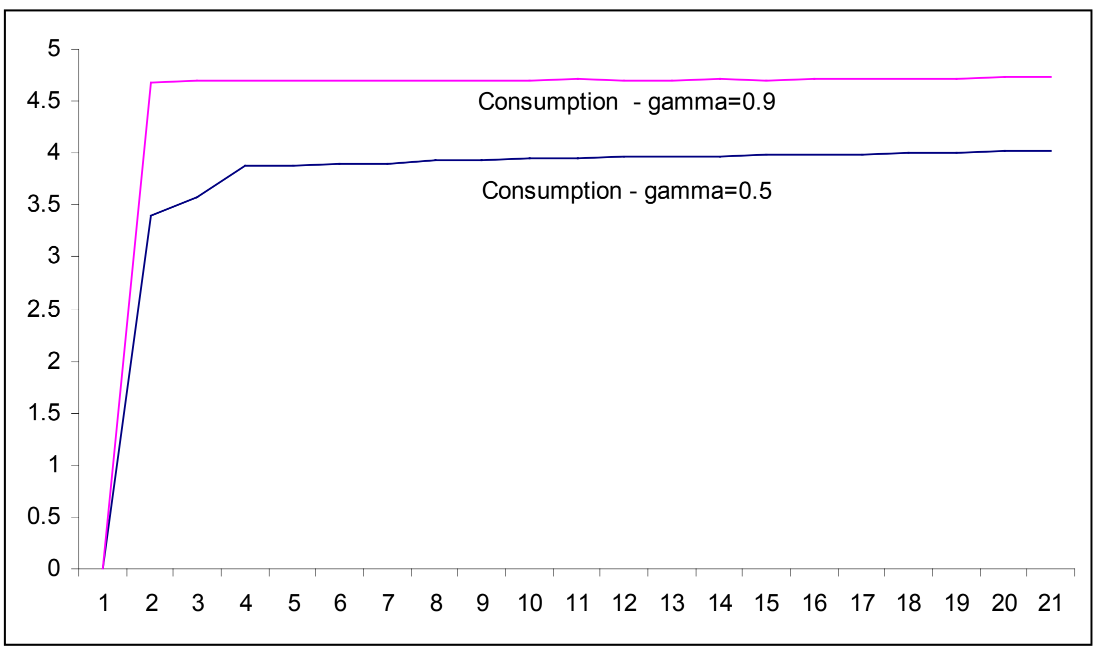
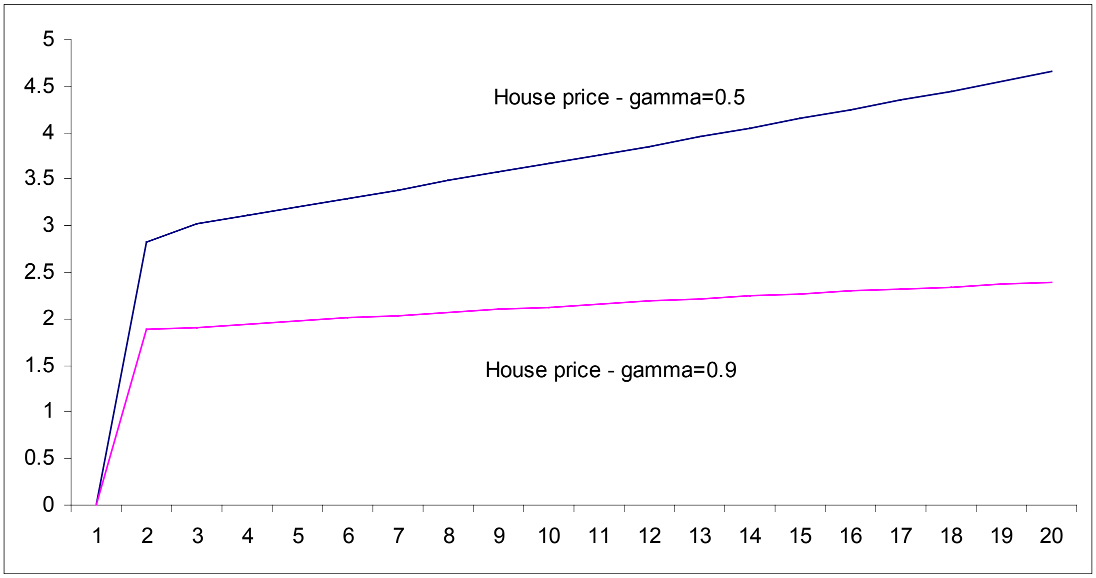
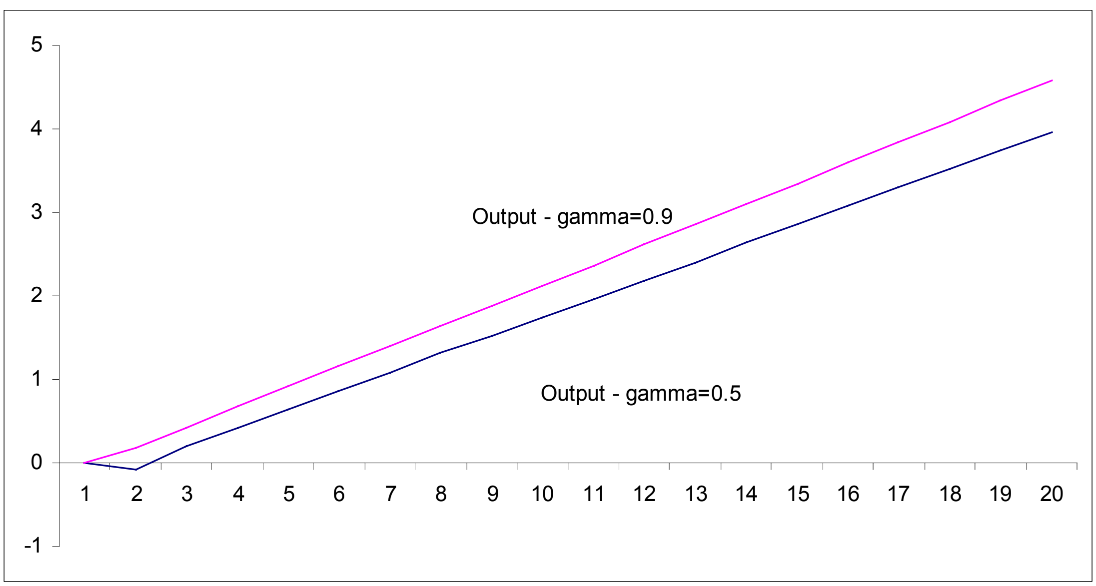


Figure 7: The growth rate of labour productivity increases by 0.25pp

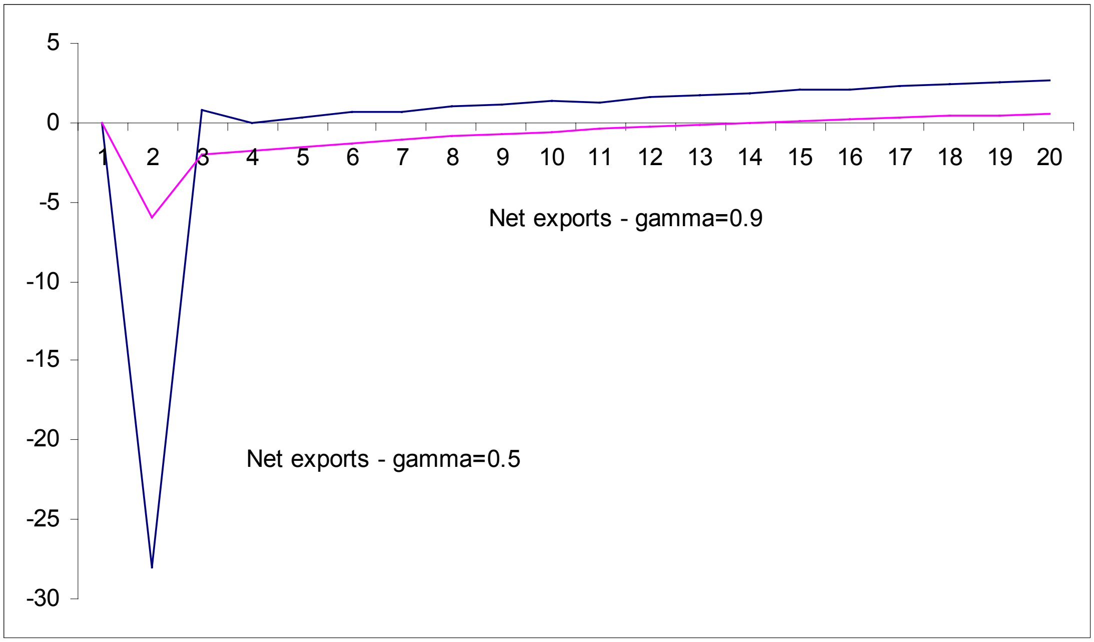
7A: Housing price (percent difference from baseline steady state)



7B: Output (per cent difference from baseline steady state)



7C: Net exports as a percentage of GDP



7D: Consumption (per cent difference from baseline steady state)