The saving rate in Japan: Why it has fallen and why it will remain low

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September 12, 2006

Abstract

This paper quantifies the role of alternative shocks in accounting for the recent declines in the Japanese saving rate and provides some projections about its future course. We consider three distinct sources of variation in the saving rate: changes in fertility rates, changes in survival rates, and changes in technology. The empirical relevance of these factors is explored using a computable dynamic OLG model. Our model successfully explains historical variation in the saving rate and other aggregate variables including the after-tax real interest rate, hours per worker and output. Model projections indicate that the Japanese saving rate will be much lower in future years and will not recover to levels of 15 percent that were seen as recently as 1990.

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†Joines acknowledges financial support from the Japan Foundation and the Social Science Research Council under an Abe Fellowship.

1 Introduction

Between 1961 and 1990 the net national saving rate in Japan averaged over 16 percent. It exceeded 10 percent in all years except 1983 and as recently as 1990 was 15 percent. For purposes of comparison, the United States saving rate in 1990 was 9 percent lower or about 6 percent.[1] Since 1990, however, Japan's saving rate has experienced a sharp decline. By 2000 it had fallen to 5.7 percent. Associated with this decline in the Japanese saving rate has been a concurrent decline in the after-tax real return on capital, or *after-tax real interest rate*, from 6 percent in 1990 to 4 percent in 2000, and low economic growth.[2]

Is this sharp decline in Japan's national saving rate a temporary aberration from its historical average of 16 percent or is the national saving rate likely to remain low in future years? In this paper we find that the Japanese saving rate will be low in future years. We project that the average value of Japan's saving will be less than 5 percent for the remainder of the century.

We support this claim by developing a general equilibrium model of the national saving rate. Our model maintains the life-cycle hypothesis of Modigliani and Brumberg (1954). This choice is motivated by recent findings of Hayashi (1995) and Horioka, et al. (2000). Hayashi (1995) estimates Engel curves for Japanese households and finds that they are inconsistent with the hypothesis that bequest motives are important. Horioka, et al. (2000) argue, more generally, that survey evidence of Japanese households is much more consistent with the life-cycle hypothesis than the alternatives of altruistic or dynastic households. Under the life-cycle hypothesis household saving varies with age. With the further assumption of overlapping generations, demographic changes such as the aging of a baby boom generation can have important implications for saving rates. Japan is a particularly interesting case to analyze because it will experience unprecedented changes in demographics during the next several decades. The level of the population is projected to decline from 127.7 million to 100.6 million between 2006 and

[1]The net national saving rate is defined as net national saving divided by net national product. Our data source for Japan is Hayashi and Prescott (2002) and for the United States it is the Department of Commerce, Bureau of Economic Analysis.

[2]Our measure of the after-tax real interest rate is from Hayashi and Prescott (2000).

2050 and the fraction of the population over the age of 65 is projected to rise from 0.21 to 0.36 over the same interval of time.

In the model, households are formed when individuals reach age 21 and become economically active. Households have one adult and a varying number of children who consume a fixed fraction of the adult's consumption. The number of children varies with the age of the adult and over time. Households may survive until a maximum age of 100 and are assumed to interact in perfectly competitive markets in a closed economy.[3]

We consider three distinct sources of variation in saving rates and real interest rates: changes in fertility rates, changes in survival rates, and changes in the growth rate of total factor productivity (TFP).[4] The interaction of fertility rates and survival rates jointly determines the age distribution of the population at any point in time. By varying fertility rates and survival rates, we capture the effects of the Japanese baby boom, the ensuing permanent decline in fertility and the permanent increase in longevity on the age distribution and thus on the aggregate saving rate and other macroeconomic variables. In a model calibrated to Spanish data, Rios-Rull (2002) has found that permanent shocks to demographics have large effects on saving and interest rates.

Changes in the growth rate of productivity can also have large effects on the national saving rate. Hayashi and Prescott (2002), for instance, have found that the the productivity slowdown in the 1990s produces big declines in private investment in a representative agent real business cycle model. Chen, İmrohoroğlu, and İmrohoroğlu (2005, 2006a, 2006b) find that changes in TFP growth alone can explain much of the variation in the Japanese saving

[3]Japan is one of the largest economies in the world both in terms of aggregate and per capita GDP. Japan also has the smallest trade-to-GDP ratios for both goods and services in the OECD. For instance, in 2001 the trade-to-GDP ratio for goods was 9.3% in the United States and 8.4% in Japan and the ratio of services to GDP was 2.4% and 2.3% respectively. For these reasons we think it reasonable to assume that real interest rates are determined in the domestic market in Japan.

[4]In explaining the historical behavior of Japanese saving and interest rates, we also permit time variation in the depreciation rate and various indicators of fiscal policy, including government purchases, tax rates, the public debt, and the size of the public pension system.

rate over the last four decades of the twentieth century.[5]

We assess our theory by calibrating the model to Japanese data and conducting a perfect foresight dynamic simulation analysis starting from 1961. This solution technique requires that the entire trajectory of demographic variables and TFP be specified. Our baseline specification uses historical Japanese data for the demographic variables and TFP for the period up to 2000. For future years we use the Japanese government's intermediate population projections and assume that annual TFP growth recovers to 2 percent between 2000 and 2010.

Our model is reasonably successful in reproducing the observed year-to-year pattern of Japanese saving rates in the 1970s, 1980s and 1990s. The Japanese national saving rate was 24 percent in 1970, 11 percent in 1980, 15 percent in 1990 and 6 percent in 2000. Our model yields a saving rate of 20 percent in 1970, 8 percent in 1980, 14 percent in 1990 and 7 percent in 2000. The model also reproduces movements in the after-tax return on capital, output growth and the secular decline in Japanese hours worked between 1961 and 1990.

Projections from the baseline model indicate that the net national saving rate will not exceed 3.3 percent through the end of the century.[6] The aging of Japan's baby-boom generation and lower birth rates play an important role in these projections. If instead the demographic variables are held fixed

[5]Changes in unemployment risk can also affect saving and interest rates. Unemployment rates in Japan rose from 2.2 percent in 1990 to 5.5 percent in 2003. Moreover, between 1990 and 2000 the median duration spell of unemployment rose from 3.5 months to 5.5 months and the replacement rate fell from 0.84 to 0.68. If this risk is largely uninsurable then households will respond to it by increasing their demand for savings. The general equilibrium effects described in Aiyagari (1994) then imply that the real interest rate will also fall. Braun et al. (2005) simulated steady-state versions of our model incorporating unemployment risk and found that the measured increase in unemployment risk during the 1990s had a much smaller impact on saving and interest rates than either TFP or fertility rates. TFP and fertility rates had about equal sized effects on the saving rate.

[6]These projections are long-run trend values of the saving and interest rates. They are based on the assumption that fertility and mortality rates, the TFP growth rate, and fiscal policy variables evolve smoothly over time. As with any projection high frequency shocks to any of these variables would produce additional fluctuations in saving and interest rates. In addition, shocks to variables not present in our model, e.g., monetary policy could also induce high frequency variation in these variables. However, over intervals of e.g. ten years the affects of these high-frequency shocks should average out.

at their values from the 1980s, the saving rate rises to nearly 8 percent by 2045.

We assess the robustness of the model projections by varying the conditioning assumptions for the demographic variables and TFP. In all cases, the saving rate remains at or below 5 percent through the year 2093.

Our work is related to research by Hayashi, Ito, and Slemrod (1988), who investigate the role of imperfections in the Japanese housing market in accounting for the Japanese saving rate in an overlapping generations endowment economy. They find that the combination of rapid economic growth, demographics, and housing market imperfections explains the level of Japanese saving rates in 1980. Their projections, which condition on an unchanged real interest rate, show declines in the saving rate of about 10 percent between 2000 and 2030.

Our work is also closely related to but distinct from the work of Chen, İmrohoroğlu, and İmrohoroğlu (2005, 2006b). They also consider an overlapping generations model but assume that labor supply is exogenous and that the family scale is fixed.[7] We have an endogenous labor supply decision and allow family scale to vary over time in a way that is consistent with the population of Japanese under 21 years of age. Both of these generalizations have implications for household saving decisions. Modeling variations in family scale also turns out to play an important role in reproducing the secular decline in Japanese hours worked. They find that convergence from a low initial capital stock in conjunction with changes in TFP growth explain most of the variation in the Japanese saving rate in historical data prior to 2000.

Our objective is to assess the roles of TFP and demographics in future years. In this regard we find that variation in TFP also plays the most important role in our model's projections prior to 2020. However, over longer horizons demographic factors are much more important and account for more than half of the decline in the national saving rate from its 1990 level. While the elderly share of the population is increasing in many countries, aging is both relatively recent and quite pronounced in Japan. As recently as 1990,

[7]Chen, İmrohoroğlu, and İmrohoroğlu (2006a) consider an infinite horizon representative agent model with a labor supply decision.

12 percent of the population was aged 65 and above in Japan, the lowest percentage among the G6 large, developed economies. By 2005, this figure had risen to 20 percent, the highest among the G6, and by 2050 it is projected to triple to 36 percent. Thus, the effects of aging on saving rates are likely to be large in Japan in coming decades compared either to the effects seen in other countries or to those observed historically in Japan itself.

The remainder of the paper is divided into six sections. In section 2 we describe the model economy, while section 3 reports its calibration. Section 4 evaluates the model's ability to explain the observed behavior of saving and interest rates since 1961 and section 5 reports our projections. Section 6 contains our conclusions.

2 Model

2.1 Demographic Structure

This economy evolves in discrete time. We will index time by t where $t \in \{..., -2, -1, 0, +1, +2, ...\}$. Households can live at most J periods and J cohorts of households are alive in any period t. They experience mortality risk in each period of their lifetime.

Let $N_{j,t}$ denote the number of households of age j in period t. Then the dynamics of population are governed by the first-order Markov process:

$$\mathbf{N}_{t+1} = \begin{bmatrix} (1+n_{1,t}) & 0 & 0 & \dots & 0 \\ \psi_{1,t} & 0 & 0 & \dots & 0 \\ 0 & \psi_{2,t} & 0 & \dots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \psi_{J-1,t} & 0 \end{bmatrix} \mathbf{N}_t \equiv \mathbf{\Gamma}_t \mathbf{N}_t, \tag{1}$$

where $\mathbf{N}_t$ is a $J \times 1$ vector that describes the population of each cohort in period t, $\psi_{j,t}$ is the conditional probability that a household of age j in period t survives to period $t+1$ and $\psi_{J,t}$ is implicitly assumed to be zero. The growth rate of the number of age-1 households between periods t and $t+1$ is $n_{1,t}$, which we will henceforth refer to as the net fertility rate.[8] The

[8]Note that this usage differs from other common definitions of the fertility rate and

aggregate population in period t, denoted by N_t, is given by

$$N_t = \sum_{j=1}^{J} N_{j,t}. \tag{2}$$

The population growth rate is then given by $n_t = N_{t+1}/N_t$. The unconditional probability of surviving from birth in period $t-j+1$ to age $j > 1$ in period t is:

$$\pi_{j,t} = \psi_{j-1,t-1}\pi_{j-1,t-1} \tag{3}$$

where $\pi_{1,t} = 1$ for all t.

2.2 Firm's Problem

Firms combine capital and labor using a Cobb-Douglas constant returns to scale production function

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}, \tag{4}$$

where Y_t is the output which can be used either for consumption or investment, K_t is the capital stock, H_t is effective aggregate labor input and A_t is total factor productivity.[9] Total factor productivity grows at the rate $\gamma_t = A_{t+1}^{1/(1-\alpha)}/A_t^{1/(1-\alpha)}$. We will assume that the the market for goods and the markets for the two factor inputs are competitive. Then labor and capital inputs are chosen according to

$$r_t = \alpha A_t K_t^{\alpha-1} H_t^{1-\alpha} \tag{5}$$

$$w_t = (1-\alpha) A_t K_t^{\alpha} H_t^{-\alpha}, \tag{6}$$

that the net fertility rate, as we have defined it, can be negative, indicating a decline in the size of the youngest cohort from one period to the next. We compute quantities analogous to $n_{1,t}$ from Japanese data and use these values to parameterize our model. We use our definition of the fertility rate to describe both the model quantities and their empirical counterparts.

[9]As described below, labor efficiency is assumed to vary with age, so that changes in the age distribution of the population alter the average efficiency of the labor force. This effect is measured by H_t, while changes in efficiency due to technical progress are captured by A_t.

where r_t is the rental rate on capital and w_t is the wage rate per effective unit of labor. The aggregate capital stock is assumed to follow a geometric law of motion

$$K_{t+1} = (1 - \delta_t)K_t + I_t \tag{7}$$

where, I_t, denotes aggregate investment and δ_t is the depreciation rate which is assumed to vary over time.

2.3 Household's Problem

All households have one adult and a varying number of children the number of which varies with the age of the adult and also over time.[10] The utility function for a household born (and thus of age 1) in period s is given by

$$U_s = \sum_{j=1}^{J} \beta^{j-1} \pi_{j,t} u(c_{j,t}, \ell_{j,t}; \eta_{j,t}), \tag{8}$$

where β is the preference discount rate, $c_{j,t}$ is total household consumption for a household of age j in period $t = s + j - 1$ and $\eta_{j,t}$ is the scale of a family of age j in period t.

Households are born with zero assets but may borrow against their future income. Labor supply of a household of age j in period t is $1 - \ell_{j,t}$. Labor income is determined by an efficiency-weighted wage rate $w_t \varepsilon_j$ per unit of labor supplied, where w_t denotes the market wage rate per unit of effective labor in period t and ε_j denotes the time-invariant efficiency of an age-j worker. The efficiency index ε_j is assumed to drop to zero for all $j \geq J_r$, where J_r is the retirement age. The budget constraint for a household of age j in period t is:

$$c_{j,t} + a_{j,t} \leq R_t a_{j-1,t-1} + w_t \varepsilon_j (1 - \ell_{j,t}) + b_{j,t} + \xi_t - \theta_{j,t} \tag{9}$$

where $a_{j,t}$ denotes assets held at the end of period t (with $a_{0,t} = 0$ for all t), $\theta_{j,t}$,are taxes imposed by the government, $b_{j,t}$ denotes public pension (social

[10]We thank a referee for suggesting that we model time-variation in the family scale.

security) benefits, and ξ_t is a uniform, lump-sum government transfer to all individuals alive in period t, and $R_t = 1 + r_t - \delta_t$. Here, δ_t denotes the depreciation rate of capital in period t. The pension benefit $b_{j,t}$ is assumed to be zero before age J_r and a lump-sum payment thereafter.

Taxes imposed by the government are given by

$$\theta_{j,t} = \tau_t^a (R_t - 1) a_{j-1,t-1} + \tau_t^\ell w_t \varepsilon_j (1 - \ell_{j,t}) \tag{10}$$

where τ^a and τ^ℓ are the tax rates on income from labor and capital, respectively.

2.4 Household's Decision Rules

We summarize the individual situation of an age-j household in period t with the state variable $x_{j,t}$. The individual state consists solely of asset holdings $a_{j-1,t-1}$: $x_{j,t} = \{a_{j-1,t-1}\}$. The aggregate state of the economy, denoted X_t, is composed of total factor productivity, A_t, the depreciation rate, δ_t, the family scale, $\eta_t = \{\eta_{1,t}, \eta_{2,t}, ...\eta_{J,t}\}$, government policy, Ψ_t, the period t age-asset profile $\mathbf{x}_t = \{x_{1,t}, x_{2,t}, ..., x_{J,t}\}$, and the population distribution, $\mathbf{N}_t$ or $X_t \equiv \{A_t, \delta_t, \eta_t, \Psi_t, \mathbf{x}_t, \mathbf{N}_t\}$.[11] Households are assumed to know the entire path of X_t except $\mathbf{x}_t$ when they solve their problems. With these various definitions and assumptions in hand, we can now state Bellman's equation for a typical age-j household in period $t = s + j - 1$:

$$\begin{aligned} & V_j(x_{j,t}; X_t) \\ = & \max \left\{ u(c_{j,t}, \ell_{j,t}; \eta_{j,t}) + \beta \psi_{j+1} V_{j+1}(x_{j+1,t+1}; X_{t+1}) \right\} \end{aligned} \tag{11}$$

subject to

$$c_{j,t} + a_{j,t} \leq R(X_t) a_{j-1,t-1} + w(X_t) \varepsilon_j (1 - \ell_{j,t}) + b_{j,t} + \xi_t - \theta_{j,t} \tag{12}$$

$$c_{j,t} \geq 0, \quad 0 \leq \ell_{j,t} \leq 1 \tag{13}$$

$$K_{t+1} = K(X_t) \tag{14}$$

$$H_t = H(X_t) \tag{15}$$

[11] The elements of Ψ_t are defined in Section 2.5 below.

and given $\{A_t, \delta_t, \eta_t, \Psi_t, \mathbf{N}_t\}_{t=s}^{\infty}$ and the laws of motion for the aggregate capital stock and labor input where s is the household's birth year. Since a household dies at the end of period J, $V_{J+1,t} = 0$ for all t. A solution to the household's problem consists of a sequence of value functions: $\{V_j(x_{j,t}; X_t)\}_{j=1}^{J}$ for all t, and policy functions: $\{a_{j,t}(x_{j,t}; X_t), c_{j,t}(x_{j,t}; X_t), \ell_{j,t}(x_{j,t}; X_t)\}_{j=1}^{J}$ for all t.

2.5 Government

The government raises revenue by taxing income from labor and capital at the flat rates τ^{ℓ}, and τ^{a}, respectively. It receives additional revenue by imposing a 100-percent tax on all accidental bequests. Total accidental bequests in period t are:

$$Z_t = \sum_{j=2}^{J+1} (1 - \psi_{j-1,t-1}) R(X_t) a_{j-1,t-1}(x_{j-1,t-1}; X_{j-1,t-1}) N_{j-1,t-1} \tag{16}$$

and total government tax revenue is

$$T_t = \sum_{j=1}^{J} \theta_{j,t}(x_{j,t}; X_{j,t}) N_{j,t} + Z_t \tag{17}$$

Note that $\theta_{j,t}$ depends on $\{x_{j,t}; X_{j,t}\}$ since it is a function of $\ell_{j,t}$ by (10).

Total government expenditure is the sum of government purchases, public pension benefits, interest on the public debt, and lump-sum transfers. Government purchases are set exogenously to G_t. Aggregate pension benefits are given by

$$B_t = \sum_{j=J_r}^{J} b_{j,t} N_{j,t} \tag{18}$$

We assume that the household's pension benefit $b_{j,t}$ is proportional to its average wage before retirement and is constant after retirement. The households pension benefit $b_{j,t}$ is given by

$$b_{j,t} = \begin{cases} 0 & \text{for } j = 1, 2, ..., j_r - 1 \\ b_{j_r, t+j_r-j} & \text{for } j = j_r, j_r + 1, ..., J \end{cases} \tag{19}$$

where j_r is the retirement age. And the constant amount of payment drawn by the new retiree at time $t + j_r - j \leq t$, $b_{j_r,t+j_r-j}$, in (19) is given by

$$b_{j_r,t+j_r-j} = \lambda_{t+j_r-j} \frac{1}{j_r - 1} \sum_{i=1}^{j_r-1} w_{t+i-j} \epsilon_j (1 - l_{j,t+i-j}) \tag{20}$$

where λ is the replacement ratio of the pension benefit. The public debt is set exogenously and evolves according to

$$D_{t+1} = R(X_t) D_t + G_t + B_t + \Xi_t - T_t. \tag{21}$$

Aggregate lump-sum transfers, Ξ_t, are set so as to satisfy this equation, and the per capita transfer, ξ_t, is determined from the equation

$$\Xi_t = \sum_{j=1}^{J} \xi_t N_{j,t} \tag{22}$$

Given the above definitions the government policy in period t, is given by $\Psi_t \equiv \{\{\theta_{j,t}\}_{j=1}^{J}, \tau_t^l, \tau_t^a, G_t, D_{t+1}, \lambda_t\}$. Observe that given this definition of the government policy variables and an initial level of government debt, the transfer Ξ_t can be derived from the period government budget constraint (21).

2.6 Recursive Competitive Equilibrium

Having completed the description of the economy we can now define a recursive competitive equilibrium.

Definition 1: Recursive Competitive Equilibrium

Given $\{A_t, \delta_t, \Psi_t, \mathbf{N}_t\}_{t=0}^{\infty}$, a recursive competitive equilibrium is a set of household value functions $\{V_j(x_{j,t}; X_t)\}_{j=1}^{J}$ for all t, and associated policy functions: $\{a_{j,t}(x_{j,t}; X_t), c_{j,t}(x_{j,t}; X_t), \ell_{j,t}(x_{j,t}; X_t)\}_{j=1}^{J}$ for all t, factor prices $\{w(X_t), r(X_t)\}_{t=0}^{\infty}$ and aggregate policy functions for capital $K_{t+1} = K(X_t)$ and labor input $H_t = H(X_t)$ such that:

- Given the functions of factor prices $\{w(X_t), R(X_t)\}$ and the aggregate policy functions for labor and capital the household policy functions $\{a_{j,t}(x_{j,t}; X_t), c_{j,t}(x_{j,t}; X_t), \ell_{j,t}(x_{j,t}; X_t)\}$ solve the household's dynamic program (11)-(15).

- The factor prices are competitively determined so that (5) and (6) hold, and $R_t = R(X_t) \equiv 1 + r_t - \delta_t$ and $w_t = w(X_t)$.

- The commodity market clears:

$$Y_t = C_t + I_t + G_t$$

  where $C_t = \sum_j c_{j,t}(x_{j,t}; X_t)N_{j,t}$ is aggregate consumption and $I_t = K_{t+1} - (1 - \delta_t)K_t$ is aggregate investment, and G_t is government purchases.

- The laws of motion for aggregate capital and the effective labor input are given by:

$$K(X_t) = \sum_j a_{j,t}(x_{j,t}; X_t)N_{j,t}$$

$$H(X_t) = \sum_j^{J_r-1} \varepsilon_j(1 - \ell_{j,t}(x_{j,t}; X_t))N_{j,t}.$$

- The government budget constraint is satisfied in each period:

$$D_{t+1} + T_t = R(X_t)D_t + G_t + B_t + \Xi_t$$

In our simulations we assume that the economy eventually approaches a stationary recursive competitive equilibrium. Before we can define a stationary recursive competitive equilibrium we need to define some of the building blocks.

Definition 2: Stationary population distribution

Suppose that the fertility rate and the conditional survival probabilities are constant over time: $n_{1,t} = n_1$ for all t and $\psi_{j,t} = \psi_j$ for all t and j.

Then a stationary population distribution, $\mathbf{N}_t^*$, satisfies $\mathbf{N}_{t+1}^* = \mathbf{\Gamma}^* \mathbf{N}_t^*$ and $\mathbf{N}_{t+1}^* = (1+n_1) \cdot \mathbf{N}_t^*$ where

$$\mathbf{\Gamma}^* = \begin{bmatrix} (1+n_1) & 0 & 0 & \dots & 0 \\ \psi_1 & 0 & 0 & \dots & 0 \\ 0 & \psi_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \psi_{J-1} & 0 \end{bmatrix}$$

A stationary population distribution has two desirable properties. First, cohort shares in the total population are constant over time: $N_{j,t+1}^*/N_{t+1}^* = N_{j,t}^*/N_t^*$ for all t. Second, the aggregate population growth rate is time-invariant: $n_t = N_{t+1}^*/N_t^* = n_1$ for all t. This allows us to convert the growth economy into a stationary economy using the following transformations:

$$\tilde{c}_{j,t} = \frac{c_{j,t}}{A_t^{1/(1-\alpha)}}, \quad \tilde{a}_{j,t} = \frac{a_{j,t}}{A_t^{1/(1-\alpha)}}$$

Other per-capita variables in the household budget constraint are transformed in same way. Aggregate variables in period t are transformed by dividing by $A_t^{1/(1-\alpha)} N_t$ except for aggregate labor input, which is transformed by dividing by N_t.

Definition 3: Stationary recursive competitive equilibrium

Suppose the population distribution is stationary and the growth rate of total factor productivity is constant over time: $\gamma_t = \gamma^*$ for all t. Then a stationary recursive competitive equilibrium is a recursive competitive equilibrium that satisfies:

$$\tilde{c}_{j,t} = \tilde{c}_j^*, \quad \tilde{a}_{j,t} = \tilde{a}_j^*, \quad \tilde{\ell}_{j,t} = \ell_j^*$$

for all t and j, i.e., the factor prices are constant over time: $\{r_t, \tilde{w}_t\} = \{r^*, \tilde{w}^*\}$ for all t where $\tilde{w}^* = w_t^*/A_t^{1/(1-\alpha)}$.

This completes the description of the model.

3 Calibration

The model is calibrated to Japanese data. The values of the parameters and sources of the exogenous variables are reported in Table 1. We assume that each household has one adult member. New households are formed when individuals reach the age of 21 and households die no later than the end of the 100th year of life, i.e., $J = 80$.

Table 1: Model calibration and data sources for exogenous variables

Preferences		
Subjective discount factor	β	0.977
Share of leisure	ϕ	0.361
Technology		
Capital share	α	0.363
Depreciation rate	δ_t	Hayashi and Prescott(2002)
Labor efficiency profile	ϵ_j	Braun, et al.(2005)
Tax, expenditure and annuity		
Capital income tax rate	τ_t^a	Hayashi and Prescott(2002)
Wage income tax rate	τ_t^w	Hayashi and Prescott(2002)
Social security replacement rate	λ_t	Oshio and Yashiro(1997)
Government purchases	G_t/Y_t	Hayashi and Prescott(2002)
Demographics		
Population growth rate	n_t	IPSS
Survival probabilities	$\psi_{j,t}$	IPSS
Family scale	$\eta_{j,t}$	See data appendex
Initial conditions		
Initial capital stock	k_0	Hayashi and Prescott(2002)
Initial asset holdings by age	$a_{j,0}$	Hayashi, Ando and Ferris (1988)

We assume that the period utility function is logarithmic

$$u(c_{j,t}, \ell_{j,t}; \eta_{j,t}) = \phi[\eta_{j,t} \log(c_{j,t}/\eta_{j,t})] + (1-\phi)\log(1-\ell_{j,t}). \tag{23}$$

The calibration of the other structural parameters is done in the following way. We set the capital share parameter, α, to reproduce the average capital share of output in Japanese data over the period 1984-2000. The preference discount factor is chosen to reproduce the average capital-output ratio observed in Japanese data over the period 1984-2000 in a steady state. The preference parameter for leisure, ϕ, is chosen so that steady-state hours per worker equals average weekly hours per worker in Japanese data over the period 1984-2000.[12]

Dynamic simulations require values for the initial state of the economy in 1961 and for the entire future time path of the exogenous elements of the state vector. Hayashi, Ando and Ferris (1988) report asset holdings by generation using data from 1983-1984. We use their data to determine the asset shares of each cohort in 1961 and then re-scale to reproduce the value of the aggregate Japanese capital stock in 1961.

The aggregate state vector X_t consists of total factor productivity, the depreciation rate, the family scale, the age distribution of the population, the asset holding of each cohort and the government policy variables. Total factor productivity is calculated by the standard growth accounting method using a calibrated capital share θ and data on the capital stock and labor input reported in Hayashi and Prescott (2002) for the period 1961 through 2000. In our baseline model, we assume that TFP recovers linearly to a growth rate of 2 percent per annum between 2000 and 2010, and then grows thereafter at a constant rate of 2 percent per year. We also report results below that examine the robustness of our conclusions to this assumption. The depreciation rate varies over time and is measured using data provided by Hayashi and Prescott (2002) up through 2001. After 2001 the depreciation rate is assumed to remain constant at its 2001 value of 0.076. Household's labor efficiencies vary with age but the efficiency profile is assumed to be constant over time. The labor efficiency profile, ε_j, is constructed from Japanese data on employment, wages, and weekly hours following the methodology described in Hansen

[12]Even though we have data extending back to 1960, the sample period used in calibrating the parameters is restricted to 1984-2000. The reason for this is that sample averages of e.g. the capital output ratio are likely to be closer to their long-run averages when data from the 1960s and 1970s are omitted. Under the maintained null hypothesis of our model, data during this period are dominated by convergence to the steadystate from a low initial capital stock.

(1993).[13]

The net fertility rate, $n_{1,t}$, is calibrated to data on the growth rate of 21-year-olds for the period 1961-2000, and the series is extended to 2050 using projections of the National Institute of Population and Social Security Research (IPSS). After 2050 we assume that the growth rate of 21-year olds recovers over a 15 year period to zero and is then constant at zero thereafter. Conditional survival probabilities, $\psi_{j,t}$, are based on life tables produced by IPSS through 2050. After 2050 the survival probabilities are held fixed at their 2050 levels.[14] These assumptions about fertility and survival rates in conjunction with an initial age-population distribution are used to produce an age distribution of the population at each date using equations (1)-(3).

Figure 1 shows the implications of our baseline demographic assumptions for the time path of fractions of different age groups in total population. The figure also displays the actual cohort shares and the official IPSS open-economy projections. These are quite close to the model predicted series which abstract from immigration and emigration flows. Our demographic assumptions imply that the Japanese population will fall by about 50 percent over the next 100 years.

We allow family scale to vary over time. Our calibration requires several simplifying assumptions about how families evolve over time. A key assumption is that the number of children born to a household of age j in period t is given by $m_{j,t} = f_t m_j$, where m_j is a time-invariant indicator of the relative number of births occuring in each year of the parent's life cycle and f_t is a time-varying shock to aggregate fertility. The time series of $f_(t)$ together with the $m_(j)$ determine the number of children in a household of a given age at each date. We calibrate $f_(t)$ and $m_(j)$ from cross-sectional data on the number of children in households of different ages in 2000 and the time series of 21-year-olds, $N_(j, t)$.

Government purchases, the labor income tax rate, and the capital income tax rate are taken from data provided by Hayashi and Prescott (2002) for the 1961-2001 period and after that the tax rates are held fixed at their 2001 levels. The capital income tax rate is measured as the tax on capital income

[13]See the data appendix in Braun, et al. (2005) for more details.

[14]More details on the construction of these variables is found in the Appendix.

divided by capital income, and the wage income tax rate is measured as the sum of direct tax on households and the social security tax payments divided by wage income.

Our baseline specification assumes that the amount of government debt is fixed at zero. One might wonder whether this assumption is innocuous. Households are selfish in our model and the timing of debt and lump-sum transfers may have real effects on economic activity depending on the nature of the intervention. In Section 5.3 we extend the baseline model to allow for time-variation in government debt. This extension has only a negligible effect on our baseline results. In this sense, Ricardian equivalence is an excellent benchmark for our economy.

All variants of the model assume public pension benefits to be equal to 17 percent of average earnings in working periods up through 1976 and 40 percent thereafter following Oshio and Yashiro (1998). Chen, İmrohoroğlu, and İmrohoroğlu (2005) make this same assumption in their overlapping generations model.

4 Assessing the Model's Performance using Historical Data

In this section, we use our model to simulate the Japanese saving rate from 1961 to 2000. Our ultimate objective is to use our model to make projections about the future course of the saving rate. However, before doing that we first demonstrate that we have a good model by documenting its in-sample performance.

The Japanese net national saving rate and after-tax real interest rate have exhibited substantial variation during the decades following 1960. The saving rate peaks in excess of 25 percent in the late 1960s, then fluctuates between 10 and 15 percent from the early 1970s until 1990, and finally falls to about 5 percent during the 1990s. The after-tax real return on capital varies between 12 and 21 percent between 1961 and 1973. From the mid 1970s to 1990 it ranges between 5 and 6 percent and then falls below 4 percent in the 1990s.

To what extent are the large historical variations in Japanese saving rates a puzzle for economic theory? Christiano(1989) investigates whether the reconstruction hypothesis can account for these movements. He posits a low capital stock in a neoclassical growth model and finds that the large observed swings in the Japanese saving rate are a puzzle for standard economic theory. Chen, İmrohoroğlu, and İmrohoroğlu (2005) revisit this same question and find that a model similar to the one used here, but with constant birth and death rates over time and exogenous labor, can account for much of the variation in the Japanese saving rate in historical Japanese data. The major reason for their success is that they allow TFP growth to vary over time.

More recently, Chen, İmrohoroğlu, and İmrohoroğlu (2006b) incorporate time-varying birth and death rates into their model, as in the analysis reported here. The model continues to perform well in accounting for historical saving behavior. However, allowing for demographic variation results in little increase in explanatory power as compared to a specification with only time-varying TFP growth. This conclusion contrasts with our findings in Braun, Ikeda and Joines (2005). We compare steady states and conduct a dynamic analysis calibrated to Japanese data from 1990 to 2000 and find that demographics and TFP growth are roughly equally important in accounting for the observed declines in saving and interest rates in the 1990s.

Our model differs from Chen, İmrohoroğlu, and İmrohoroğlu (2005, 2006b) in several respects. Our households have an endogenous labor supply decision.[15] Allowing for a labor supply decision provides another way for households to smooth consumption and thus can affect households saving decisions. Secondly, we allow the size of families to vary over time in a way that is consistent with the number of under-21-year-olds in the Japanese economy in any given year. Time variation in family scale also affects consumption-saving decisions. With these extensions our model does a reasonably good job of accounting for observed variation in Japanese saving. The model also reproduces some of the principal movements in the after-tax real interest

[15] Chen, İmrohoroğlu, and İmrohoroğlu (2006a) and Braun, Okada and Sudou (2006) apply infinite horizon representative agent models with flexible labor supply to Japanese data. Chen, İmrohoroğlu, and İmrohoroğlu (2006a) find that their model successfully reproduces movements in the Japanese saving rate but don't report simulation results for labor input.

rate, output, and hours per worker.

Figure 2 displays our baseline results for the period 1961-2001. Figure 2 has four panels that show the behavior of the net national saving rate, the after-tax real interest rate, hours per worker and the growth rate of GNP.[16] The data are all taken from Hayashi and Prescott (2002).

The model tracks the observed saving rate reasonably well. It reproduces the 1961 value of the saving rate in Japanese data. The empirical saving rate reaches its maximum value of 27 percent in 1970. The simulated series reaches its maximum of 25 percent in the same year. From 1970 to 1991 the model understates the level of the Japanese saving rate with a maximum gap of 5.5 percent between the two saving rates in 1983. But the model performs well again in the 1990s. The observed series declines from 14.9 percent in 1990 to 5.7 percent in 2000, while the simulated series declines from 13.7 percent to 6.9 percent.

Our data set, which is based on the 1968 system of national accounts (SNA), stops on 2000.[17] We compare the model's predictions with more recent saving data by constructing a measure of national saving using the new 1993 SNA data. This is not directly comparable to our primary data set. Depreciation is based on book value and as noted by Hayashi (1997) using a book value based measure of depreciation induces a large upward bias in the national saving rate. In 1990 this measure of the national saving rate is in excess of 20 percent as compared to 15 percent for the Hayashi-Prescott (2002) measure. However, we can still use the 93 SNA data to draw two qualitative conclusions about the performance of the model. First, the 1993 SNA based measure of the national saving rate also declines steadily during the 1990s. Second, the 1993 SNA based measure of the national saving rate continues to decline between 2000 and 2004. Both of these facts are consistent with the pattern of saving from the model.

The model also does reasonably well in reproducing the after-tax real

[16] The saving national saving rate is defined as the ratio of Net National Product minus private consumption minus government consumption to NNP. The after-tax real interest rate is the after-tax real return on capital.

[17] 1968 SNA data are not reported by the Japanese government after 2001. Our data also use a replacement cost measure of depreciation constructed by Hayashi and Prescott (2002). which is only available through 2000.

interest rate. The gap between the model and data is largest between 1966 and 1976. The model reproduces the general year-to-year movements in the data during this period but understates the high real return to capital. The model does much better from 1976 to 2000. During that period the gap between the model and the data is always less than 60 basis points. The model predicts a decline of 130 basis points during the 1990s, which is 80 basis points smaller than the observed decline of 210 basis points.

Interestingly, the model also reproduces the secular decline in Japanese average hours per worker between 1961 and 1990.[18] Empirical hours per worker decrease from 50.3 hours to 43.5 hours during that period, and the simulated series decreases from 49.6 hours to 41.4 hours. The match is particularly good prior to 1976. Braun, Okada and Sudou (2006) find that a one-sector representative agent model has considerable trouble matching the movements of hours in Japanese data. Their model fails to reproduce the trend in Japanese labor input and simulated labor input is only weakly correlated with Japanese labor input. Modeling variations in family scale helps match the trend in the data. Over the 1961-2000 sample period family scale has fallen substantially, and this acts to increase household's demand for leisure relative to consumption goods.

One puzzling feature of these results is that model hours per worker decline from 43.4 hours per week in 1979 to 39.9 hours per week in 1983, whereas Japanese hours per worker remain above 43 hours per week through 1989. We have explored the source of this discrepancy and found that the reason model hours fall is a rising tax rate on labor income. Between 1961 and 1978, the labor income tax increases at an annualized rate of 0.47 percentage points per year. Then in the next 3 years it jumps by 4.5 percentage points and then rises by another percentage point in the next 2 years. After that the growth rate of the labor income tax rate slows to 0.28 percent per annum on average. When we simulate the model with a constant labor income tax rate the model no longer predicts a decline in hours between 1979 and 1983.

Finally, the model predictions for per capita output growth are also quite good. The model reproduces both the amplitude and timing of movements

[18] The model expresses hours worked as a share. When converting this share to a measure of weekly hours we assume a weekly time endowment of 112 hours (16 hours per day).

in the growth rate of Japanese output.

5 Projections

5.1 Baseline Projections

The success of our model in reproducing much of the year-to-year pattern of saving rates as well as the long-term decline in interest rates suggests that we have a good theory of the Japanese national saving rate. We now use this same theory to project the future course of the national saving rate. Figure 3 displays baseline projections and two other sets of projections that are designed to isolate the role of demographics and TFP. Recall that our baseline conditioning assumptions rely on projections from IPSS for the net fertility rates and mortality rates through 2050. The annual growth rate of TFP is assumed to recover gradually to two percent between 2000 and 2010. These assumptions are discussed in more detail in the calibration section above.

The single most important fact about Japanese saving in the post-World War II period has been its magnitude. As recently as 1990 the Japanese saving rate was 15 percent, or about three times as large as the U.S. saving rate. Our baseline results indicate that in future years the trend level of the Japanese saving rate will never exceed 5 percent. Saving rates fall to a low of −0.2 percent in 2009 and eventually rise to a new steady-state value of 5.1 percent by the year 2140. This pattern is not monotonic, however. The saving rate increases to 3.0 percent in 2025 as a result of the echo of the baby boom. It then falls again to 1.7 percent in 2045 before increasing gradually to the new steady state.

One way to identify the distinct roles of demographics and TFP for the aggregate saving rate is to run counterfactual simulations. Figure 3 reports results from two such simulations. The *1980s no change* simulation, holds the net fertility rate from 1990 on fixed at 1 percent, which is close to the average growth rate of the population aged 21 and above during the 1980s. In addition, the mortality rates are held fixed at their 1990 levels. TFP growth from 1990 on is set to 3.1 percent, which is the average value of TFP growth in

Japanese data during the 1980s. This set of assumptions is meant to illustrate what might have happened if the demographic and TFP growth patterns of the 1980s had persisted forever. The second counterfactual simulation, *1980s population*, differs from the first in assuming that TFP growth follows our baseline conditioning assumptions and only the fertility rate and the mortality rates are held at levels representative of the 1980s.

Consider the *1980s no change* simulation. The most striking thing about this simulation is that the variation in the saving rate during and after the 1990s is very small. Observe next that even though the population growth and mortality rates are fixed at their 1980s levels, the saving rate does decline until 2014 to a low of 8.1 percent. This is due to the aging of the baby-boom generation. The new long-run steady-state value is 9.2 percent. Next compare the *1980s no change* simulation with the *1980s population* simulation, which shows a large drop in the saving rate in the early part of the twenty-first century. From this we can see that low TFP growth between 1990 and 2010 plays the dominant role in the evolution of the baseline saving rate through about 2020. By 2020, though, demographics account for one half of the gap between the baseline and *1980s no change* simulation. The contribution of demographics to the gap then rises to 70 percent in 2031 and remains between 70 and 80 percent until 2107. In the final steady state, demographics account for 60 percent of the total gap between the baseline simulation and the *1980s no change* simulation.

Taken together these results suggest that demographic variation will exert considerable influence on the Japanese saving rate in the twenty-first century.

Figure 3 also reports projections for the after-tax real interest rate. There are some striking differences among the three projections. The baseline results presented in the lower panel of Figure 3 suggest that after-tax real interest rates have bottomed out and will gradually recover to levels experienced by Japan between the mid-1970s and the mid-1990s. After reaching a minimum value of 4.0 percent in 2006, the after-tax real interest rate rises to 5 percent by 2025 and to 5.1 percent in 2055 before settling at its final steady-state value of 5.2 percent. Comparing the two counterfactual simulations, we see that TFP plays a more significant role than demographics in after-tax real interest rate projections. The *1980s no change* simulation is particularly interesting. This specification has the after-tax interest rate rising during the 1990s. We will return to discuss this final point in more detail in Section 5.2.

The rich demographic structure of our model provides us with a way to understand what changes in the microeconomic structure of this economy are driving variations in the aggregate saving rate. The net national saving rate is defined by the net increase in the aggregate capital stock divided by the net national product

$$s_t = \frac{K_{t+1} - K_t}{Y_t - \delta_t K_t}. \tag{24}$$

The net national saving rate in turn can be decomposed into a weighted sum of age-specific household saving rates.

$$\begin{aligned}
s_t =& \frac{\sum_{j=1}^{J} a_{j,t} N_{j,t}}{Y_t - \delta_t K_t} - \frac{\sum_{j=1}^{J} a_{j,t-1} N_{j,t-1}}{Y_t - \delta_t K_t} \\
=& \sum_{j=1}^{J} \frac{N_{j,t}}{Y_t - \delta_t K_t} [a_{j,t} - a_{j-1,t-1}] + \sum_{j=1}^{J} \frac{a_{j-1,t-1}}{Y_t - \delta_t K_t} [N_{j,t} - N_{j-1,t-1}] \\
=& \sum_{j=1}^{J} \frac{y_{j,t} N_{j,t}}{Y_t - \delta_t K_t} \frac{a_{j,t} - a_{j-1,t-1}}{y_{j,t}} + \sum_{j=1}^{J} \frac{a_{j-1,t-1} N_{j-1,t-1}}{Y_t - \delta_t K_t} [\psi_{j-1,t-1} - 1] \\
=& \sum_{j=1}^{J} \frac{y_{j,t} N_{j,t}}{Y_t - \delta_t K_t} \frac{a_{j,t} - a_{j-1,t-1} - q_t}{y_{j,t}} \\
\equiv& \sum_{j=1}^{J} \chi_{j,t} s_{j,t}
\end{aligned} \tag{25}$$

where

$$\chi_{j,t} \equiv \frac{y_{j,t} N_{j,t}}{Y_t - \delta_t K_t}, \quad q_t = \sum_{j=1}^{J} \frac{a_{j-1,t-1} N_{j-1,t-1}}{N_t} (1 - \psi_{j-1,t-1})$$

and where $s_{j,t}$ is the individual saving rate, $a_{j,t}$ is the asset holding of an individual of age j at the end of time t, $N_{j,t}$ is the population of age j at time t, and $\psi_{j,t}$ is the age-j survival probability at time t.[19] The weight $\chi_{j,t}$, is simply the share of net national income accruing to households of age j.

[19]Note that $s_{j,t}$, the individual saving rate, corresponds to a situation where the gov-

Let $\mu_{j,t} \equiv N_{j,t}/N_t$ denote cohort j's share in total population in period t. Then using equation (25) we can express the change in the net national saving rate from t to $t-k$ as the sum of three components

$$s_t - s_{t-k} = \sum_{j=1}^{J} \chi_{j,t-k}(s_{j,t} - s_{j,t-k}) + \sum_{j=1}^{J} s_{j,t} z_{j,t-k}(\mu_{j,t} - \mu_{j,t-k}) + \sum_{j=1}^{J} s_{j,t}\mu_{j,t}(z_{j,t} - z_{j,t-k}) \qquad (26)$$

where $z_{j,t} = \frac{y_{j,t}}{(Y_t - \delta_t K_t)/N_t}$ is the per capita income of individuals of age j relative to overall per capita income in the economy. We will refer to the first, second, and third terms in equation (26) as respectively the saving rate component, the cohort size component and the relative income component. The saving rate component is a weighted average of changes in individual saving rates. It summarizes the endogenous response of household saving rates to variations in wages and interest rates.[20] We want to emphasize that prices and thus the saving rate component respond to a variety of shocks, including technology, demographics, and fiscal policy. The cohort size component is a weighted average of changes in the relative size of each cohort and the relative income component is a weighted average of changes in the income of an age-j household relative to overall per capita income.

Figure 4 reports two plots of this decomposition of the national saving rate using data from the baseline simulation. The upper panel shows decade changes of the national saving rate for the period 1961-2000. The lower panel shows differences over successively longer horizons starting from a base year of 1990. Consider first the upper panel. According to the model the saving rate component has been the primary source of historical decade level

ernment gathers accidental bequests and redistributes them in a lump-sum way equally among all surviving individuals. An individual's saving during period t is defined as assets held at the end of the period, $a_{j,t}$, less initial assets. Initial assets are the sum of assets held by the individual at the end of the previous period, $a_{j-1,t-1}$, and q_t, the individual's share of the assets held at the end of period $t-1$ by individuals who die before the beginning of period t.

[20]It should be kept in mind that the age-specific saving rates depend on the entire profile of wages and interest rates that a household experiences in its lifetime.

variations in the national saving rate. It is the largest component in all but one decade. In the first decade (1961-1970), changes in the relative income component are largest. The cohort size component is small in historical data. Chen, İmrohoroğlu, and İmrohoroğlu (2006a, 2006b) find that modeling demographics is not important for understanding the evolution of the saving rate over a similar sample period. The upper panel of Figure 4 suggests that their finding may stem from the fact that cohort size movements were relatively small during this period.[21] Are cohort effects *always* small and in particular smaller than saving rate effects? Results reported in the lower panel of Figure 4 suggest that the answer is no. The size of the cohort effect steadily increases as the forecast horizon is expanded. Through 2030 the saving rate component is the largest source of variations in the national saving rate. But after that the cohort size component is always larger. By 2100 the cohort size component is 2.5 times as large as the saving rate component.

Decomposing the saving rate into these three components offers some insight into one role of demographics but does not tell the whole story. This is because the saving rate and relative income components reflect movements in both demographics and TFP, as well as in other relevant exogenous shocks including fiscal policy. The cohort size component, on the other hand, is affected only by demographic change and thus measures only the direct effects of such change on saving rates. The information in Figure 4 reinforces the conclusion from Figure 3 that TFP shocks are the primary determinant of variations in the saving rate in historical Japanese data. But both figures also imply that demographic change will be the dominant factor in explaining a long-run decline in trend saving rates from the levels seen in the late 1980s and early 1990s.

5.2 Projections using Alternative Conditioning Assumptions

How sensitive are the model's projections to our conditioning assumptions about total factor productivity and demographics? In order to answer this

[21] Although not reported here due to space constraints, the saving rate component can be further decomposed by age. Doing so reveals that saving rates change in the same direction for almost all age groups during a given decade.

question we report four other simulations in Figure 5. Two of these variants maintain our baseline assumptions for TFP growth but use either the high or low IPSS population projections rather than the intermediate projections which we use in our baseline model. The intermediate population projection implies that the Japanese population in 2050 will be 105.2 million and that 36 percent of the population will be of age 65 or above. The high population projection is 108.2 million with 33 percent of the population aged 65 and above, and the low projection yields an estimate of 92 million with 39 percent of the total aged 65 and above. The third and fourth variants retain the baseline population projections but make alternative assumptions about the TFP growth rate. The low TFP simulation assumes TFP growth does not recover and instead remains at 0.33 percent per year, its average value for the 1990s. This assumption of permanently low total factor productivity growth is maintained by Hayashi and Prescott (2002). The high TFP simulation assumes that TFP growth recovers to 3.1 percent per annum.

Consider first the results for alternative demographic assumptions. These assumptions have no discernible effect on the saving rate either in the very long run, by which point they all yield the same age structure of the population, or up until the local peak associated with the echo of the baby boom around 2025. Over intermediate forecast horizons, however, demographics exert a noticeable influence on saving. The saving rate under the high population assumption is uniformly above the baseline projection, and the decline after the local peak in 2029 is muted. The corresponding decline under the low population assumption is quite pronounced, however, with the saving rate falling to zero between 2060 and 2068. The effect of these alternative demographic assumptions on saving rates is nevertheless much smaller than the decline in saving compared with 1990. This is because the alternative population projections all result in age distributions of the population that are similar to each other, while the 1990 age distribution is quite different. As noted above, the elderly (aged 65 and above) are projected to constitute between 33 and 39 percent of the population in 2050, compared to 12 percent in 1990.

Varying the demographic assumptions have smaller effects on interest rates. The low (high) population assumption results in interest rates that are below (above) those predicted by the baseline model during much of the transition to the new steady-state. The differences from the baseline projection are largest during the years 2035-2086, when they range between five and twenty basis points.

The results would look very different if the low growth rate of TFP of the 1990s is assumed to be permanent while the demographic variables are set to their baseline values. Consider first the net saving rate. It remains negative into the next century and eventually approaches a new long-run value below one percent, as compared to 5.1 percent in the baseline specification. However, the saving rate with low TFP growth is above the baseline case for the years 2001-2013. This is because an anticipated recovery of TFP depresses saving in the short term.

We see similar patterns in the real interest rate. There are two distinctions between the low TFP simulation and the baseline. The real interest rate does not increase after 2007 as it does under the baseline parameterization of a recovery of total factor productivity growth. Instead, the real interest rate stays in the neighborhood of 3.5 percent between 2001 and 2052. In addition, the new steady-state interest rate is only 3.9 percent, versus 5.2 percent in the baseline case.

Finally, consider the high TFP simulation which uses actual data on TFP and demographics during the 1990s but posits a stronger recovery of TFP growth to 3.1 percent per annum between 2001 and 2010. Recall from Section 5.2 that TFP grew at an average rate of 3.1 percent during the 1980s. There are three most noteworthy features about the saving rate in this simulation. First, the model predicts a permanent decline in the saving rate from 13.6 percent in 1990 to 7 percent in the final steady-state. Second, the saving rate remains at or below 5 percent through the year 2093. Third, the model fit with the data is good during the 1990s. We saw above that the *1980s no change* simulation, fails to account for the saving rate in the early 1990s. By comparing these two simulations which have identical assumptions for the long-run growth rate of TFP we see that expectations about what happens to TFP growth during the 1990s really matters.

The role of expectations is even more pronounced for the real interest rate. The real interest rate increases during the 1990s in the *1980s no change* simulation reported in Figure 3 . However, in the high TFP simulation which feeds through actual TFP growth and demographic changes, the real interest rate falls throughout the 1990s as in the data. These differences reveal that the model's success in accounting for the experience of Japan during the 1990s depends crucially on the assumption that households correctly perceived that TFP growth would be low during this period.

From this analysis we see that both changing demographics and lower productivity growth contribute to reproducing the observed decline in the interest rate from 6 percent in 1990 to 3.9 percent by the year 2000. These results also indicate that observed and projected changes in fertility rates produce very persistent responses in the saving rate, but much smaller responses in the after-tax real interest rate. Sustained but temporary shocks to total factor productivity growth have large contemporaneous effects but do not produce much propagation over time in the model. Finally, these simulations add further support to our contention that the average value of saving rates in future years will be low relative to levels experienced in Japan before 1990. The saving rate remains low through the end of the century even under the most optimistic assumptions about TFP growth and demographics.

5.3 Government Debt

Here we consider the robustness of our conclusions to our maintained assumption that government debt is zero. In an infinite horizon model this assumption is innocuous when lump-sum transfers are present and free to adjust. However, in an overlapping generations model the timing of government borrowing and lump-sum transfers may benefit particular generations. To explore this issue we conducted a simulation in which we used data on government borrowing. Following Broda and Weinstein (2004) we construct the net government debt using data from the Bank of Japan Flow of Funds website for 1979-2004 and from government sources for 1961-1978. The debt-output ratio is held fixed at its 2004 level in future years. Net government debt constructed in this way varies from about 2 percent of GDP in the 1960s to 72.7 percent in 2004. Lump-sum transfers are adjusted each period to insure that the government budget constraint is satisfied. Even though the net government debt in 2004 is more than 10 times as large as that in 1990, the results from generalizing the baseline model in this way are imperceptibly different from the baseline specification. The maximum difference between the baseline saving rate and the specification with government debt is 0.35 percent and this occurs in 1962. Ricardian equivalence is a very good approximation in our model.

6 Conclusion

In this paper we have shown that the measured declines in saving rates and real interest rates in Japan during the 1990s are consistent with the predictions of theory. Both low total factor productivity growth and the life cycle hypothesis play important roles in accounting for these facts. A variety of theories have been put forth to explain Japan's development miracle in the post WWII period (Eaton and Kortum (1999), Parente and Prescott (1994)), Japan's high saving rate between the 1961 and 1990 (Hayashi(1997), Horioka (1990), Slemrod et al (1988)), Japan's economic stagnation in the 1990s (Caballero, Kayshup and Hoshi (2006), Ito and Mishikin (2005) , and the secular decline in hours per worker between 1961 and 2000 (Parente and Prescott (1994)). An attractive selling point of our specification is that it provides a single coherent explanation for all of these phenomena.

Our theory also has sharp implications for the future evolution of saving rates. According to our projections, the average value of Japanese saving rates will remain at or below 5 percent for the remainder of the 21st century. Moreover, this finding is reasonably robust to alternative assumptions about demographics and future TFP growth. The population distribution, which is a key determinant of saving, changes only gradually over time in a highly predictable way. Thus, even when we posit a robust recovery in total factor productivity growth, saving rates remain low by historical standards.

Appendix

A1. Data set

Demographics and survival probabilities

We can construct the model's complete demographic dynamics from an initial age distribution of the population, a series of age-1 population, and a series of survival probabilities. We measure the initial population by age using Japanese data for 1961. A series of age-1 population is constructed using the historical (1961-2000) and projected (2001-2050) age-1 population.[22] We calculate a series of survival probabilities in three steps. First, given the initial population by age and by sex and a series of survival probabilities by age and by sex we construct a series of population by age and by sex.[23] Second, summing over sexes, we get a closed-economy series of population by age for the period 1961-2050. Third, we use the series of population by age to construct a series of survival probabilities by age. The survival probability at age j and time t is calculated as $\psi_{j,t} = N_{j+1,t+1}/N_{j,t}$, where $N_{j,t}$ is the population of age j at time t. Assuming that survival probabilities remain constant after 2050 and that the age-1 population growth rate recovers to zero in 15 years and remains constant thereafter, we recursive construct time-series of population by age using equation (1).

Labor efficiency profile

The labor efficiency profile, ε_j, is constructed from Japanese data on employment, wages, and weekly hours from 1990 to 2000 following the methodology described in Hansen (1993). The data source is the *Basic Survey in Wage Structure* by the Ministry of Health, Labor and Welfare. The constructed labor efficiency profiles are 0.646 (age 20-24), 0.834 (age 25-29), 0.999 (age 30-34), 1.107 (age 35-39), 1.165 (age 40-44), 1.218 (age 45-49), 1.233 (age 50-54), 1.127 (age 55-59), 0.820 (age 60-64), 0.727 (over age 65). We interpolate these values to get labor efficiency by age. For more detail on the methodology constructing those values, see the data appendix in Braun, et al. (2005).

Capital and wage income tax rates

[22]The data are available in the National Institute of Population and Social Security (IPSS) home-page. The IPSS projection has three different levels of population: low, medium and high. The differences among the three projections come entirely from differences in assumptions about fertility. The three projections use common survival probabilities. We take the medium projection as our baseline.

[23]The data on survival probabilities are available only every five years, and we interpolate between those years. These data are also available in the IPSS home-page.

The capital income tax rate is measured by revenue from the tax on capital income divided by capital income, and the wage income tax rate is measured by the sum of direct tax payments by households and social security tax payments divided by wage income. We use data provided by Hayashi and Prescott (2002) to get capital income and wage income as well as capital income tax revenue. We take data on direct taxes on households and the social security tax from the 2000 Annual Report on National Accounts.

Government debt

we calculate net government debt for 1979-2004 following Broda and Weinstein (2004). The net government debt is sum of the net debts of the Japanese government, the postal savings system, and government financial institutions. The data are available only from 1979 and their source is the Bank of Japan Flow of Funds website. We calculate net government debt for 1961-1978 using data on the gross government debt and assuming that the ratio of net debt to gross debt is the same as the average value for 1979-2001. The data source for the gross government debt is Financial Bureau, Ministry of Finance.

Family scale

The baseline model allows family scale to vary over time in a way that makes family scale consistent with Japanese data on the under 21 year old population which are children under the assumption of our model. This section describes how we calibrate the family scale variables ($\eta_{j,t}$).

A secular decline in the net fertility rate, $n_{1,t}$, implies a corresponding decline in the number of children per household and thus in the family scale, $\eta_{j,t}$, for ages when children are present in the home. We do not have data to allow measurement of $\eta_{j,t}$ on a frequent basis. Instead, we adopt simplifying assumptions that allow us to estimate $\eta_{j,t}$ from information on family scale in 2001 and observations on the time series of the number of twenty-one-year-olds in the population, $N_{1,t}$.

Suppose that the number of children born to a household of age j in period t is given by $m_{j,t} = f_t m_j$, where m_j is a time-invariant indicator of the relative number of births occuring in each year of the parent's life cycle and f_t is a time-varying shock to aggregate fertility. In our model each household contains one adult, so that the empirical analogue of $m_{j,t}$ is births per adult of age j in period t. Assume that no births occur before the parent reaches real-time age 21 (model age 1). Assume further that the mortality rate is zero before real-time age 21. Finally, assume that children remain in the household until they reach real-time age 21, at which time they form their own households.

Given the above assumptions, the number of individuals of real-time age 21 (or model age 1) in period t is

$$N_{1,t} = f_{t-20} \sum_{i=1}^{J} m_i N_{i,t-20}, \tag{27}$$

where the right-hand side is simply the total number of births twenty periods ago. We have time-series data on $N_{j,t}$, the population of age j at each date.

Let $M_{j,t}$ denote the total number of children in a household of age j in period t. The number of children in a household of model age 1 is thus $M_{1,t} = m_{1,t} = f_t m_1$ and the number of children in a household of model age 2 is $M_{2,t} = m_{1,t-1} + m_{2,t} = f_{t-1} m_1 + f_t m_2$. More generally, the number of children in a household of age j is

$$M_{j,t} = \sum_{i=1}^{j} f_{t-(j-i)} m_i$$

for $j \leq 20$ and

$$M_{j,t} = \sum_{i=j-19}^{j} f_{t-(j-i)} m_i$$

for $j > 20$. Note that because f_t and m_j enter multiplicatively in all relevant expressions, some normalization assumption is needed to pin down one value of either f_t or m_j. The specific normalization is unimportant for the results, and we assume $f_{2001} = 1.0$.

Given values of f_t and m_j, we can calculate $M_{j,t}$ for all j and t. We have data for 2001 that allow us to estimate the number of children per adult for age intervals of parents that generally span five years. From these, we construct by interpolation an empirical measure of $M_{j,2001}$ for each age j. We try to choose values for f_t and m_j so that, given our simplifying assumptions, the model values of $M_{j,2001}$ and $N_{1,t}$ closely match their empirical analogues. Because we have data on age-specific mortality rates over time, matching $N_{1,t}$ implies that we match the entire time series of population by age, $N_{j,t}$. Note from equation (27) that there exists one observation on f_t for each time-series observation of the population, $N_{j,t}$. Suppose that $m_j = 0$ for $j > \widehat{j}$. Our assumption that each child remains at home for exactly 20 years implies that there are $\widehat{j} + 19$ nonzero model values of $M_{j,2001}$ corresponding to the $\widehat{j}$ nonzero values of m_j, i.e., the system is overdetermined. Therefore, we are unable to match all the values of $M_{j,2001}$ and $N_{1,t}$ exactly. Note from equation (27) that, given vales for m_j, we can pick a sequence of f_t so that the ratio of our model $N_{1,t}$ to the empirical value is constant across t, thus exactly reproducing the observed values of $n_{1,t}$, the growth rate of the youngest cohort. We could, of course, choose values of f_t so that this ratio is unity and we match $N_{1,t}$ exactly but do not match $M_{j,2001}$. Achieving a closer fit to $M_{j,2001}$ generally requires a less exact match to the level of the population series. We consider two calibrations. In one we closely match the $N_{1,t}$. In the other we closely match the $M_{j,2001}$. Presumably, any other calibration would lie between these two extremes.

We do not employ any analytically derived metric to judge the closeness of the match to $M_{j.2001}$, but instead make judgments based on the visual appearance of the measured object and its model counterpart. In our baseline simulations, we employ values of f_t and m_j that result in a model population series that is three percent higher than the observed data. As a robustness check, we use alternate values that match $M_{j,2001}$ about as closely as seems possible, resulting in a model population series that is 17 percent lower than the data. Both sets of assumptions result in matrices $M_{j,t}$ that are hump-shaped in the j dimension, reaching a peak at about model age 23 in each year. The baseline $M_{j,t}$ declines from this peak somewhat more slowly than the alternative. The most striking feature, however, is that the peak value of $M_{j,t}$ over the life cycle varies substantially over time. The peak value of children per adult in 1960 is 1.35 for the baseline calibration and 1.27 for the alternative. By 2000, the peak has fallen to 0.54 for the baseline calibration and 0.55 for the alternative.

Children receive a weight of one-half in calculating family scale, so that the family scale of a household of age j in period t is $1 + M_{j,t}/2$. We have simulated our baseline model using the alternative calibration for family scale and find no qualitative differences and only very slight quantitative differences compared to the baseline model. For instance, between 1961 and 2001 the maximum difference in the saving rate occurs in 1962 and is 0.37 percent.

A2. Simulation methodology

We use first-order conditions of the household problem (11)-(15) to compute an equilibrium. Given factor prices and the condition that the initial and final asset holding is zero, the household problem is a fixed point problem to solve for an initial consumption to satisfy the first order conditions and the budget constraint from age 1 to J. We can get factor prices if we know $\tilde{k}/h$ where $h = H/N$ is the labor input divided by total population. The superscript $\tilde{}$ indicates a variable measured in per-capita efficiency units.

Stationary equilibrium

1. Derive the stationary distribution of the population in the steady state.

2. Let $(k/h)^o$ and ξ^o be the guesses of $\tilde{k}/h$ and $\tilde{\xi}$ in the steady state. Compute factor prices $\{r, \tilde{w}\}$ and the output $\tilde{y}$ using $(k/h)^o$.

3. Let c^o be the guess of $\tilde{c}_1$. Calculate $\{\tilde{c}_j, \tilde{a}_j, l_j\}$ forward using the first order conditions and the budget constraint. Reset c^o so that $\tilde{a}_J = 0$. Then recalculate $\{\tilde{c}_j, \tilde{a}_j, l_j\}$ by setting $\tilde{c}_1 = c^0$. Set $(k/h)^o = (k/h)^n$ if $|(k/h)^o - (k/h)^n| < tol$ where $(k/h)^n$ is the new value given $(k/h)^o$ and tol is the convergence tolerance. Otherwise repeat this process until $|(k/h)^o - (k/h)^n| < tol$.

4. Given the $(k/h)^o$ computed in stage 3, re-do a simulation as stage 3 to get ξ^o such that $|\xi^o - \xi^n| < tol$, and calculate new $(k/h)^n$ in this loop.

5. If $|(k/h)^o - (k/h)^n| < tol$, stop.[24]. Otherwise set $\xi^o = \xi^n$ and go back to the stage 3.

Transitional Dynamics

1. Calculate the final steady state.

2. Let $\{(k/h)_t^o\}$ and $\{\xi_t^o\}$ be the guesses of $\{\tilde{k}_t/h_t\}$ and $\{\tilde{\xi}_t\}$ in a transition. The guess of the final period must be same as the corresponding variables of the final steady state. Compute factor prices $\{r_t, \tilde{w}_t\}$ and the output $\{\tilde{y}_t\}$ using $\{(k/h)_t^o\}$.

3. For households of age $j = 1$ and for $t = 1, 2, ..., T$ compute the series of consumption, asset and leisure $\{\tilde{c}_{j,t}, \tilde{a}_{j,t}, l_{j,t}\}$ forward. For households of age $j > 1$ at time 1 compute the series $\{\tilde{c}_{j,t}, \tilde{a}_{j,t}, l_{j,t}\}$ forward given the initial distribution of asset.

4. Compute the new series $\{(k/h)_t^n, \tilde{\xi}_t^n\}$. If the series converge we get an equilibrium. Otherwise, set new $\{(k/h)_t^o, \tilde{\xi}_t^o\}$ as the convex combination of the old $\{(k/h)_t^o, \tilde{\xi}_t^o\}$ and $\{(k/h)_t^n, \tilde{\xi}_t^n\}$ and go back to stage 2.[25]

[24] In this simulation, the factor markets clear, the household first-order conditions including the budget constraint hold, and the government budget constraint holds. Then the goods market clears automatically. We calculate excess demand in the goods market as a consistency check.

[25] If it takes too many iterations we may switch the iteration method to the Broyden method after n iterations. For example n is set to 100. See Judd (1998) for details on the Broyden method.

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Figure 1
Demographics: Model and IPSS Data

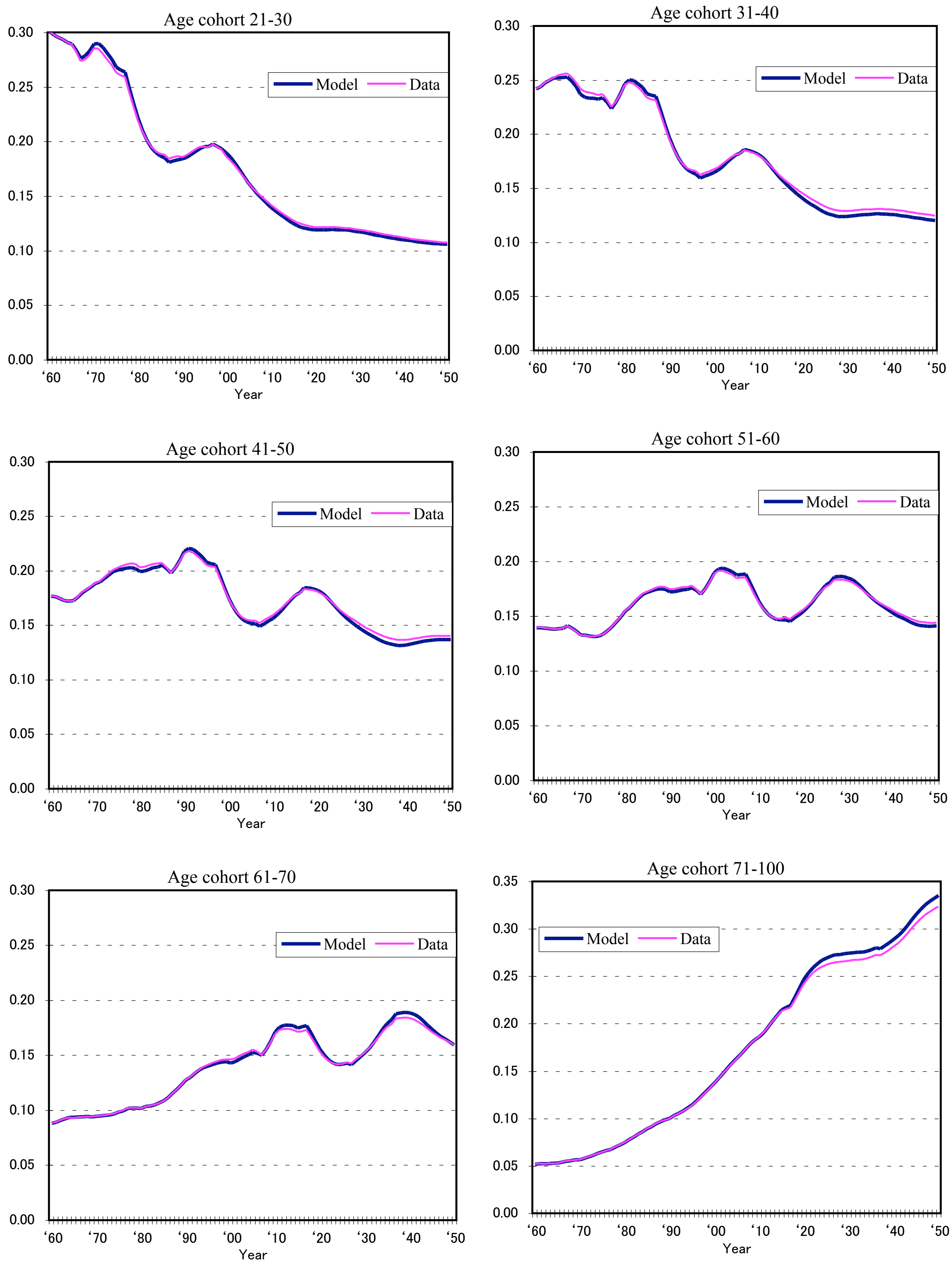


Figure 2
In-sample Performance of the Model
1961-2000

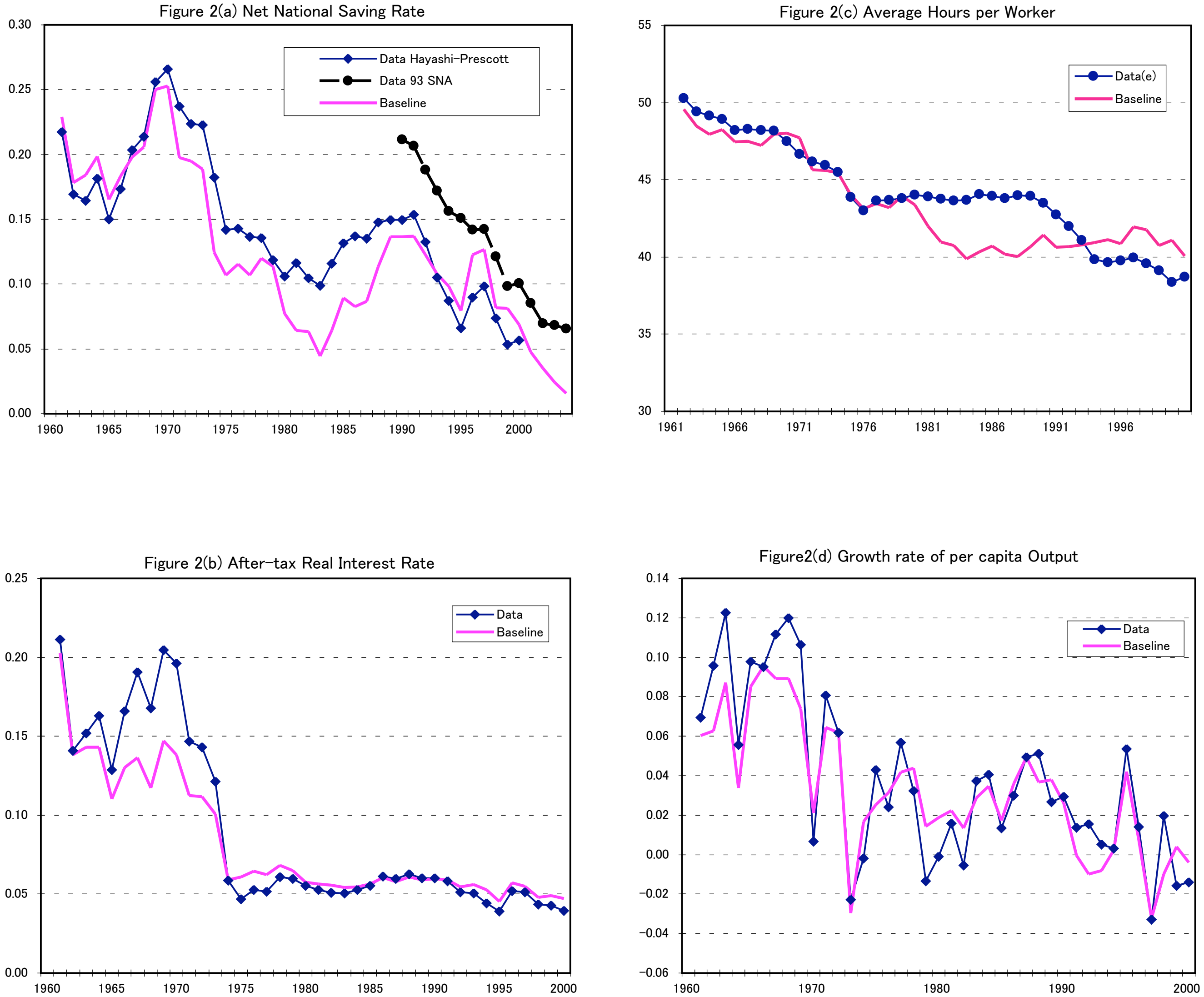
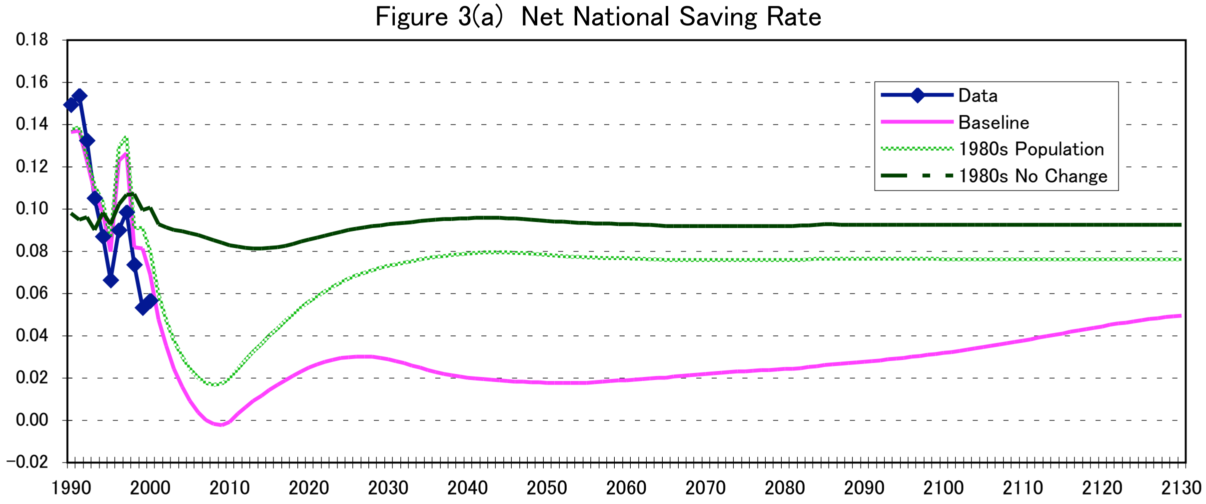


Figure 3
Model Projections



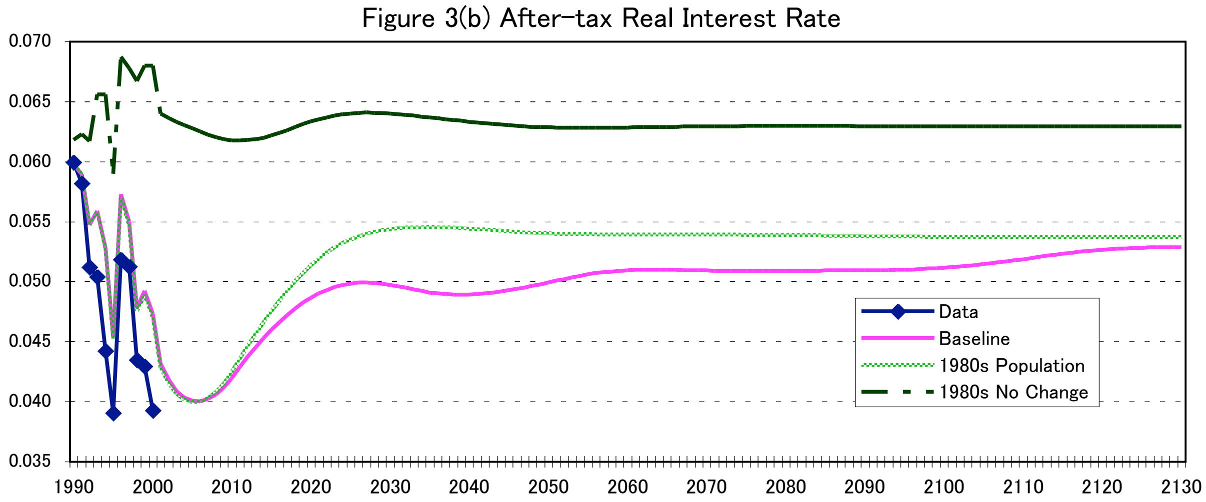
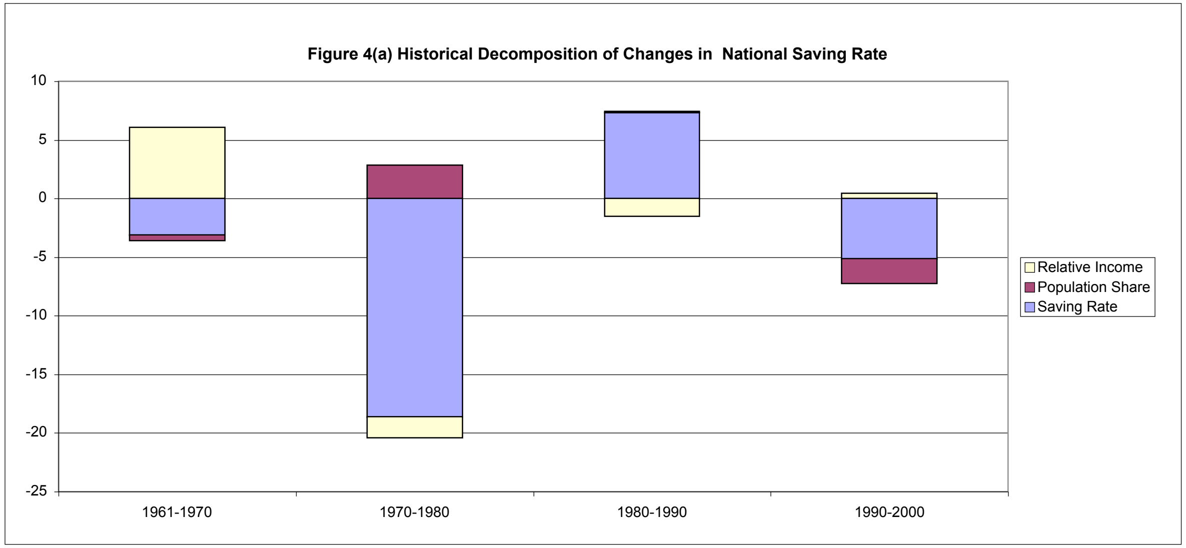


Figure 4
Decomposition of Changes Japan National Saving rate into three components



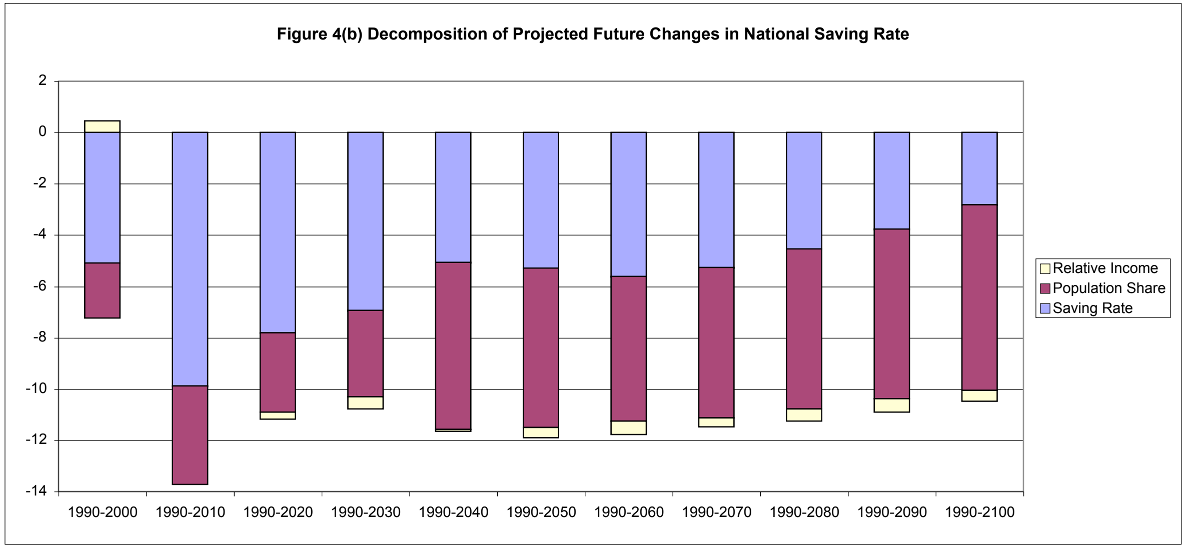


Figure 5
Projections: Baseline and Alternative Scenarios